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SORET AND CHEMICAL REACTION EFFECTS ON THE RADIATIVE MHD FLOW FROM AN INFINITE VERTICAL POROUS PLATE

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ABSTRACT. In this present article, we analyzed the heat and mass transfer characteristics of the nonlinear unsteady radiative MHD flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate under the influence of Soret and chemical reaction effects. The effect of physical parameters are accounted for two distinct types of thermal boundary conditions namely prescribed uniform wall temperature thermal boundary condition and prescribed heat flux thermal boundary condition. Based on the flow nature, the dimensionless flow governing equations are resolved to harmonic and non harmonic parts. In particular skin friction coefficient, Nusselt number and Sherwood number are found to evolve into their steady state case in the large time limit. Parametric study of the solutions are conducted and discussed.

NOMENCLATURE

a^*	mean absorption coefficient	M	magnetic parameter
B_0	uniform magnetic field	N_r	radiation parameter
C	species concentration	Nu	Nusselt number
C_f	skin-friction coefficient	n	frequency of oscillation
C_∞	uniform concentration	Pr	Prandtl number
c_p	heat at constant pressure	Q	non dimensional heat source parameter

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K_1	permeability of porous medium	q_r	radiating flux vector
D_m	chemical molecular diffusivity	q_w	heat flux
Gm	Solutal Grashof number	Sc	Schmidt number
Gr	thermal Grashof number	Sh	Sherwood number
g	acceleration due to gravity	T	fluid temperature
j_w	mass flux	T_∞	uniform temperature
K	permeability parameter	t	dimensional time
K_2	dimensional chemical reaction	U	A scaled velocity
Kr	non-dimensional chemical reaction	u	fluid velocity in x-direction
k_T	thermal conductivity of the fluid	v	fluid velocity in y-direction

Greek symbols

β_c	coefficient of concentration expansion	σ	electrical conductivity
β_T	coefficient of thermal expansion	τ	non dimensional time
ν	kinematic coefficient of viscosity	τ_w	shear stress
ω	A scaled frequency	η	A scaled coordinate
ϕ	A scaled concentration	θ	A scaled temperature
ρ	fluid density	σ^*	Stefan-Boltzmann constant

1. INTRODUCTION

Several investigations on natural convection flow are performed using both analytical and numerical methods under different types of thermal boundary conditions which are continuous and well defined. Natural convection flows are frequently encountered in science and technological problems such as chemical catalytic reactors, materials processing, nuclear heat transfer control, electronic circuits, nuclear waste repositories, petroleum reservoirs, fiber and granular insulation, geothermal systems etc. Natural convection flows from bodies with different geometries are extensively investigated as it is evident from review articles and books published so far [1–4]. Therefore, the present investigation focuses on the problem of MHD natural convection, heat and mass transfer flow of a viscous, incompressible and electrically conducting fluid past a suddenly started infinite vertical plate taking into account the Soret effect, chemical reaction and two kinds of thermal boundary conditions.

A convective heat transfer flow from bodies with different geometries embedded in a porous medium is of significant importance due to its varied and wide applications in many areas of science and technology, namely, drying of porous solids, thermal insulation, enhanced recovery of oil and gases, cooling of nuclear reactors, underground energy transport etc. Keeping in view the importance of such fluid flow problems, a number of investigations on natural convection flow near a vertical plate embedded in a porous medium have been carried out. Ahmed and Sarmah [5, 6] presented thermal radiation and Soret effects on a transient MHD flow with mass transfer past an infinite vertical plate. Muthucumaraswamy and Ganesan [7] studied the radiation effects on flow past an impulsively started infinite vertical plate with variable

temperature. Chamkha [8] investigated on the transient MHD free convection from a porous medium supported by a surface. Venkateswarlu et al. [8–12] investigated analytically thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion. Postelnicu [13] studied the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Raptis and Kafousias [14] presented a steady free convection flow past an infinite vertical porous plate through a porous medium in the presence of a magnetic field. Aldoss et al. [15] presented a combined free and forced convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde [16] investigated a hydromagnetic mixed convection flow and mass transfer past a vertical porous plate with constant heat flux embedded in a porous medium.

In most of the chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications viz. polymer production, manufacturing of ceramics or glassware, food processing etc. Chamkha [17] investigated MHD flow over a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. Afify [18] studied the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Ibrahim et al. [19] analyzed the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Bakr [20] discussed the effects of chemical reaction on MHD free convection and mass transfer flow of a micro polar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Chamkha et al. [21] discussed the effects of Joule heating, chemical reaction and thermal radiation on unsteady hydromagnetic natural convection boundary layer flow with heat and mass transfer of a micro polar fluid from a semi-infinite heated vertical porous plate in the presence of a uniform transverse magnetic field. Bhattacharya and Layek [22] obtained similarity solution of MHD boundary layer flow with mass diffusion and chemical reaction over a porous flat plate with suction/blowing. Kishore et al. [23] studied the effects of radiation and chemical reaction on unsteady hydromagnetic natural convection flow of a viscous fluid past over an exponentially accelerated vertical plate. Venkateswarlu et al. [24–27] presented thermal radiation and chemical reaction effects on MHD flow through porous medium.

The objective of the present investigation is to study an unsteady natural convection transient flow of a viscous, incompressible and electrically conducting fluid with radiative heat transfer past an impulsively moving vertical plate embedded in a fluid saturated porous medium. We should in prior emphasize that our intention is not to reproduce the results of Turkyilmazoglu and Pop [28]. In fact, the model that we consider differs considerably from that of Turkyilmazoglu and Pop [28] in that we use a better approach in the formulation, introduce a permeability term and chemical reaction, use of two kinds of thermal boundary conditions, namely, prescribed wall temperature thermal boundary condition (PST) and prescribed heat flux thermal

boundary condition (PHF). Analytical closed form solutions are presented for the momentum, the energy and the concentration equations using some proper change of non dimensional variables and parameters.

2. MATHEMATICAL FORMULATION

Consider an unsteady radiative MHD heat and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a suddenly started infinite vertical porous plate in presence of a uniform transverse magnetic field of strength B_0. Let the x-axis be taken in vertically upward direction along the plate and y-axis is normal to the plate. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate K_2 between the diffusing species and the fluid. Initially i.e. at time $t \leq 0$, both the fluid and plate are at rest and at uniform temperature T_∞. Also species concentration within the fluid is maintained at uniform concentration C_∞. At time $t > 0$, plate starts with constant velocity u_o in x-direction. Geometry of the problem is presented in Figure. 1. Since plate is of infinite extent along x-direction and is electrically non conducting, all physical quantities depend on y and t only. Under the assumptions made by Turkyilmazoglu and Pop [28], the equations of conservation of mass, momentum, energy and concentration governing the natural convective nonlinear boundary layer flow over an infinite vertical plate can be expressed as:

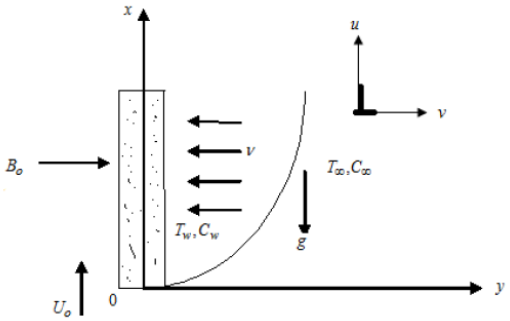

FIGURE 1. Geometry of the problem.

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T \left(T - T_\infty\right) + g\beta_C \left(C - C_\infty\right) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K_1} u \tag{2.2}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} - \frac{Q_o}{\rho c_p} \left(T - T_\infty\right) \tag{2.3}$$

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} - K_2 \left(C - C_\infty\right) \tag{2.4}$$

where $u-$ fluid velocity in x-direction, $v-$ fluid velocity along y-direction, $g-$ acceleration due to gravity, $\rho-$ fluid density, β_T- coefficient of thermal expansion, β_C- coefficient of concentration volume expansion, $t-$ time, K_1- permeability of porous medium, B_0- magnetic induction, $T-$ fluid temperature, $T_\infty-$ uniform temperature, $\alpha-$ thermal conductivity of the fluid, k_T- thermal diffusivity of the fluid, $C-$ species concentration in the fluid, $\sigma-$ fluid electrical conductivity, $C_\infty-$ uniform concentration, c_p- specific heat at constant pressure, D_m- chemical molecular diffusivity, T_m-mean fluid temperature, q_r- radiating flux vector, $\nu-$ kinematic viscosity of the fluid, Q_0- heat source term and K_2- chemical reaction respectively.

We should in prior warn the reader that our model is not the same as that by Turkyilmazoglu and Pop [28] in which the permeability of porous medium, and chemical reaction effects were not taken into account. Assuming that no slipping occurs between the plate and the fluid, the corresponding initial and boundary conditions of the system of partial differential equations for the fluid flow problem are given below

$$\left.\begin{array}{l} u=0,\ T=T_\infty,\ C=C_\infty \text{ when } t\leq 0\,,\ y\geq 0 \\ u=u_o,\ \ C=C_w+\varepsilon\left(C_w-C_\infty\right)e^{nt},\ \ T=T_w\ +\varepsilon\left(T_w-T_\infty\right)e^{nt}\ \text{ for PST}, \\ \frac{\partial T}{\partial y}=-\frac{q_w}{\alpha}\left(1+\varepsilon e^{nt}\right)\ \text{ for PHF when } t>0\,, y=0 \\ u\to 0,\ T\to T_\infty,\ C\to C_\infty\ \ as\ \ y\to\infty \end{array}\right\} \tag{2.5}$$

where T_w-fluid temperature at the surface of the plate, C_w- species concentration at the surface of the plate, u_o- characteristic velocity, q_w- heat flux at the surface of the plate, $n-$frequency of oscillation respectively and $\varepsilon \ll 1$ is a very small positive constant.

Following Magyari and Pantokratoras [29], by using the Rosseland approximation, the radiative flux vector q_r can be written as:

$$q_r=-\frac{4\sigma^*}{3a*}\frac{\partial T^4\left(y,t\right)}{\partial y} \tag{2.6}$$

where, σ^* and a^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. We assume that the difference between fluid temperature T and free stream temperature T_∞ within the flow is sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in Taylor series T^4 about the free stream temperature T_∞ and neglecting the second and higher order terms, we have

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{2.7}$$

Using equations (2.6) and (2.7) in equation (2.3) we obtain

$$\frac{\partial T}{\partial t}=\alpha\left(1+\frac{16\sigma^*T_\infty^3}{3k_Ta^*}\right)\frac{\partial^2T}{\partial y^2}-\frac{Q_0}{\rho c_p}\left(T-T_\infty\right) \tag{2.8}$$

In order to represent equations (2.2) , (2.4) and (2.8) along with initial and boundary conditions (2.5) in non dimensional form, we introduce the following non-dimensional variables.

$$\eta = \frac{u_o}{\nu}y,\ U = \frac{u}{u_o},\ \omega = \frac{n\nu}{u_o^2}\ ,\ \tau = \frac{u_o^2}{\nu}t,\ \theta = \frac{T-T_\infty}{T_r},\ \phi = \frac{C-C_\infty}{C_w-C_\infty} \tag{2.9}$$

It may be noticed that characteristic velocity u_o may be defined according to the non-dimensional process mentioned above as (Turkyilmazoglu and Pop [28])

$$u_o = [g\beta_T \nu T_r]^{\frac{1}{3}} \tag{2.10}$$

where the reference temperature $T_r = T_w - T_\infty$ for PST case and $T_r = \frac{q_w \nu}{\alpha u_o}$ for PHF case.

By substituting equations (2.9) and (2.10) into equations (2.2), (2.4) and (2.8), we obtain the following non-dimensional partial differential equations

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial \eta^2} + \theta + N\phi - \left[M + \frac{1}{K}\right]U \tag{2.11}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \eta^2} - Q\theta \tag{2.12}$$

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial \eta^2} + S_r\frac{\partial^2 \theta}{\partial \eta^2} - Kr\phi \tag{2.13}$$

Here $Gr = \frac{g\beta_T \nu T_r}{u_o^3}$ is the thermal buoyancy force, $Gm = \frac{g\beta_C \nu (C_w - C_\infty)}{u_o^3}$ is the concentration buoyancy force, $N = \frac{Gm}{Gr}$ is the ratio of concentration buoyancy force and the thermal buoyancy force, $M = \frac{\sigma B_0^2 \nu}{\rho u_o^2}$ magnetic parameter, $K = \frac{K_1 u_o^2}{\nu^2}$ is the permeability parameter, $Nr = \frac{16\sigma^* T_\infty^3}{3k_T a^*}$ is the radiation parameter, $P_r = \frac{\nu}{\alpha(1+Nr)}$ is the Prandtl number, $Q = \frac{Q_o \nu}{\rho c_p u_o^2}$ is the heat source parameter, $S_c = \frac{\nu}{D_m}$ is the Schmidt number , $Sr = \frac{D_m k_T T_r}{T_m \nu (C_w - C_\infty)}$ is the Soret number, $Kr = \frac{K_2 \nu}{u_o^2}$ is the chemical reaction parameter respectively.

Initial and boundary conditions (2.5) in non-dimensional form, are presented below

$$\left.\begin{aligned} &U = 0,\ \theta = 0,\ \phi = 0 \ \text{ when } \ \tau \le 0\,,\ \eta \ge 0 \\ &U = 1,\ \phi = 1 + \varepsilon e^{\omega\tau},\ \theta = 1\ + \varepsilon e^{\omega\tau} \text{ for PST}, \\ &\frac{\partial \theta}{\partial \eta} = -\left(1 + \varepsilon e^{\omega\tau}\right) \text{ for PHF when } \tau > 0\,, \eta = 0 \\ &U \to 0, \theta \to 0, \phi \to 0 \ \ as\ \ \eta \to \infty \end{aligned}\right\} \tag{2.14}$$

It is now important to calculate physical quantities of primary interest, which are the local wall shear stress or skin friction coefficient, the local surface heat flux and the local surface mass flux. Given the velocity, temperature and concentration fields in the boundary layer, the shear stress τ_w, the heat flux q_w and mass flux j_w are obtained by

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0} \tag{2.15}$$

$$q_w = -\alpha \left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{2.16}$$

$$j_w = -D_m \left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{2.17}$$

In non-dimensional form the skin-friction coefficient Cf, heat transfer coefficient Nu and mass transfer coefficient Sh are defined as

$$Cf = \frac{\tau_w}{\rho \nu^2} \tag{2.18}$$

$$Nu = \frac{\nu}{u_o} \frac{q_w}{\alpha T_r} \tag{2.19}$$

$$Sh = \frac{\nu}{u_o} \frac{j_w}{D_m \left(C_w - C_\infty\right)} \tag{2.20}$$

Using non-dimensional variables in equation (2.9) and equations (2.15) to (2.17) into equations (2.18) to (2.20), we obtain the physical parameters

$$Cf = \left[\frac{\partial U}{\partial \eta}\right]_{\eta=0} \tag{2.21}$$

$$Nu = -\left[\frac{\partial \theta}{\partial \eta}\right]_{\eta=0} \tag{2.22}$$

$$Sh = -\left[\frac{\partial \phi}{\partial \eta}\right]_{\eta=0} \tag{2.23}$$

3. SOLUTION OF THE PROBLEM

In order to reduce the system of partial differential equations (2.11) to (2.13) into a system of ordinary differential equations in non-dimensional form, we represent the trial solutions for the velocity, temperature and concentration of the fluid as (see, Siva Kumar et al. [30] and Venkateswarlu et al. [31, 32])

$$U(\eta, \tau) = U_0(\eta) + \varepsilon e^{\omega \tau} U_1(\eta) + o\left(\varepsilon^2\right) \tag{3.1}$$

$$\theta(\eta, \tau) = \theta_0(\eta) + \varepsilon e^{\omega \tau} \theta_1(\eta) + o\left(\varepsilon^2\right) \tag{3.2}$$

$$\phi(\eta, \tau) = \phi_0(\eta) + \varepsilon e^{\omega \tau} \phi_1(\eta) + o\left(\varepsilon^2\right) \tag{3.3}$$

Substituting equations (3.1) to (3.3) into equations (2.11) to (2.13), then equating the harmonic and non–harmonic terms and neglecting the higher order terms of $o\left(\varepsilon^2\right)$, we obtain

$$U_0'' - \left[M + \frac{1}{K}\right] U_0 = -\left[\theta_0 + N\phi_0\right] \tag{3.4}$$

$$U_1'' - \left[M + \frac{1}{K} + \omega\right] U_1 = -\left[\theta_1 + N\phi_1\right] \tag{3.5}$$

$$\theta_0'' - P_r Q \theta_0 = 0 \tag{3.6}$$

$$\theta_1'' - P_r(Q+\omega)\theta_1 = 0 \tag{3.7}$$

$$\phi_0'' - S_c K_r \phi_0 = -S_c S_r \theta_o'' \tag{3.8}$$

$$\phi_1'' - S_c(K_r+\omega)\phi_1 = -S_c S_r \theta_1'' \tag{3.9}$$

where the prime denotes the ordinary differentiation with respect to η.

Initial and boundary conditions, presented by equation (2.14), can be written as

$$\left.\begin{array}{l} U_o = 1, U_1 = 0,\ \phi_o = 1, \phi_1 = 1, \\ \theta_o = 1, \theta_1 = 1, \text{ for PST case, } \frac{d\theta_o}{d\eta} = -1, \frac{d\theta_1}{d\eta} = -1 \text{ for PHF case } \quad at\ \ \eta = 0 \\ U_o \to 0, U_1 \to 0, \theta_o \to 0, \theta_1 \to 0, \phi_o \to 0, \ \ \phi_1 \to 0\ as\ \ \eta \to \infty \end{array}\right\} \tag{3.10}$$

Solving (3.4) to (3.9) together with (3.10), we get

$$U_o(\eta,\tau) = A_{12}e^{-A_1\eta} + A_{13}e^{-A_3\eta} + A_{14}e^{-A_6\eta} \tag{3.11}$$

$$U_1(\eta,\tau) = A_{15}e^{-A_2\eta} + A_{16}e^{-A_5\eta} + A_{17}e^{-A_7\eta} \tag{3.12}$$

$$\theta_o(\eta,\tau) = e^{-A_1\eta} \tag{3.13}$$

$$\theta_1(\eta,\tau) = e^{-A_2\eta} \tag{3.14}$$

$$\phi_o(\eta,\tau) = A_8 e^{-A_1\eta} + A_9 e^{-A_3\eta} \tag{3.15}$$

$$\phi_1(\eta,\tau) = A_{10}e^{-A_2\eta} + A_{11}e^{-A_5\eta} \tag{3.16}$$

By substituting equations (3.11) to (3.16) into equations (3.1) to (3.3), we obtained solutions for the fluid velocity, temperature and concentration in the case of PST and are presented in the following form

$$U(\eta,\tau) = A_{12}e^{-A_1\eta} + A_{13}e^{-A_3\eta} + A_{14}e^{-A_6\eta} + \varepsilon e^{\omega\tau}\left[A_{15}e^{-A_2\eta} + A_{16}e^{-A_5\eta} + A_{17}e^{-A_7\eta}\right] \tag{3.17}$$

$$\theta(\eta,\tau) = e^{-A_1\eta} + \varepsilon e^{\omega\tau}\left[e^{-A_2\eta}\right] \tag{3.18}$$

$$\phi(\eta,\tau) = A_8 e^{-A_1\eta} + A_9 e^{-A_3\eta} + \varepsilon e^{\omega\tau}\left[A_{10}e^{-A_2\eta} + A_{11}e^{-A_5\eta}\right] \tag{3.19}$$

Keeping in view the assumptions made earlier, solutions to the fluid velocity, temperature and concentration profiles in the case of PHF are obtained and presented in the following form

$$U(\eta,\tau) = B_7 e^{-A_1\eta} + B_8 e^{-A_3\eta} + B_9 e^{-A_6\eta} + \varepsilon e^{\omega\tau}\left[B_{10}e^{-A_2\eta} + B_{11}e^{-A_5\eta} + B_{12}e^{-A_7\eta}\right] \tag{3.20}$$

$$\theta(\eta,\tau) = B_1 e^{-A_1\eta} + \varepsilon e^{\omega\tau}\left[B_2 e^{-A_2\eta}\right] \tag{3.21}$$

$$\phi(\eta,\tau) = B_3 e^{-A_1\eta} + B_4 e^{-A_3\eta} + \varepsilon e^{\omega\tau}\left[B_5 e^{-A_2\eta} + B_6 e^{-A_5\eta}\right] \tag{3.22}$$

3.1. **Skin friction.** From the velocity field, the skin friction at the plate can be obtained as

$$Cf_{\text{PST}} = -\left[A_1A_{12} + A_3A_{13} + A_6A_{14}\right] - \varepsilon e^{\omega\tau}\left[A_2A_{15} + A_5A_{16} + A_7A_{17}\right] \tag{3.23}$$

$$Cf_{\text{PHF}} = -\left[A_1B_7 + A_3B_8 + A_6B_9\right] - \varepsilon e^{\omega\tau}\left[A_2B_{10} + A_5B_{11} + A_7B_{12}\right] \tag{3.24}$$

3.2. **Nusselt number.** From the temperature field, the heat transfer coefficient can be obtained as

$$Nu_{\text{PST}} = A_1 + \varepsilon e^{\omega\tau}A_2 \tag{3.25}$$

$$Nu_{\text{PHF}} = A_1B_1 + \varepsilon e^{\omega\tau}A_2B_2 \tag{3.26}$$

3.3. **Sherwood number.** From the concentration field, the mass transfer coefficient can be obtained as

$$Sh_{\text{PST}} = A_1A_8 + A_3A_9 + \varepsilon e^{\omega\tau}\left[A_2A_{10} + A_5A_{11}\right] \tag{3.27}$$

$$Sh_{\text{PHF}} = A_1B_3 + A_3B_4 + \varepsilon e^{\omega\tau}\left[A_2B_5 + A_5B_6\right] \tag{3.28}$$

4. GRAPHICAL RESULTS AND DISCUSSION

In order to investigate the influence of various physical parameters such as Soret effect S_r, magnetic parameter M, permeability parameter K, heat source parameter Q, Prandtl number P_r, chemical reaction K_r, mass diffusion parameter S_c, time τ and buoyancy parameter N on the flow-field, fluid velocity U, temperature θ and concentration ϕ have been studied analytically and computed results of the analytical solutions from equations (3.17) to (3.22) are displayed graphically from Figs.2 to 19 for various values of these physical parameters. The numerical values of skin friction, Nusselt number and Sherwood number computed from analytical solutions, presented by equations (3.23) to (3.28) are presented in tabular form in tables 1 to 6 for various values of different physical parameters. In the present study following default parameter values are adopted for computations: $S_r = 5.0,\ N = 2.0,\ \tau = 1.0,\ S_c = 0.22,\ Q = 5.0,\ K = 0.5,\ K_r = 0.5,\ P_r = 0.71,\ M = 2.0,\ \omega = 0.1$ and $\varepsilon = 0.001$. Therefore all the graphs and tables correspond to these values unless specifically indicated on the appropriate graph or table.

It is observed that from Figs. 2 to 19 fluid velocity U, temperature θ and concentration ϕ attain a distinctive maximum value near the surface of the plate and then decrease properly on increasing boundary layer coordinate η to approach free stream value in both PST and PHF cases. Fig. 2 depicts the influence of buoyancy parameter N on the velocity U of the flow field. The buoyancy parameter defines the ratio of Solutal Grashof number to thermal Grashof number. It is worth mentioning that β_T is a positive quantity, while β_c can take positive or negative values. Therefore, the thermal and concentration buoyancy forces act in the same direction, or they assist each other in driving the flow, in the case $N > 0$ and they oppose each other when $N < 0$. The case $N = 0$ corresponds to the situation when there is no buoyancy force effect from mass diffusion. It is evident from Figs 2. Velocity U increases on increasing N throughout the boundary layer region for both PST and PHF.

Fig. 3 depicts the influence of porosity parameter K on the velocity U of the flow field. It is evident from Fig 3 velocity U increases on increasing Kthroughout the boundary layer region for both PST and PHF. Fig.4 depicts the effect of magnetic parameter Mon the velocity U of the flow field. The Magnetic parameter $M$is found to decrease the velocity U of the flow field at all points.

Figs. 5 to 7, shows the plot of fluid velocity U, temperature θ and concentration ϕ of the flow field against different values of Prandtl number P_r taking other parameters are constant in both cases PST and PHF. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The values of the Prandtl number are chosen for air $(P_r = 0.71)$, electrolytic solution $(P_r = 1.00)$, water $(P_r = 7.00)$ and water at 4^0C $(P_r = 11.40)$. It is evident from Figs. 5 to 7, velocity U and concentration ϕ increases on increasing Prandtl number P_r in a region near to the plate and they decreases on increasing P_r in the region away from the plate for PST whereas temperature θ decreases on increasing P_r throughout the boundary layer region for PST. It is observed that the velocity U, temperature θ and concentration ϕ decreases on increasing Prandtl number P_r at all points of the flow field for PHF. Thus higher Prandtl number leads to faster cooling of the plate.

It is observed that from Figs. 8 to 10, both velocity U and concentration ϕ increases on increasing heat source parameter Q in a region near to the plate and they decreases on increasing Q in the region away from the plate whereas temperature θ decreases on increasing Q throughout the boundary layer region for PST. It is observed that the velocity U, temperature θ and concentration ϕ decreases on increasing heat absorption parameter Q at all points of the flow field for PHF. It is also noted from Figs. 11 to 13 that the fluid velocity U, temperature θ and concentration ϕ attain their steady state for large time τ for both PST and PHF.

Figs. 14 and 15, depict effect of Soret number S_r on the fluid velocity U and concentration distribution ϕ of the flow field in both cases. The velocity U and species concentration ϕ are found to increases on increasing Soret number S_r for both PST and PHF.

It is observed that from Figs. 16 and 17 the fluid velocity U and species concentration ϕ decreases on increasing chemical reaction parameter K_r.This implies that the chemical reaction parameter retard the fluid velocity and concentration throughout the boundary layer region for both PST and PHF. It is observed from Figs. 18 and 19 that the fluid velocity U and species concentration ϕ increases on increasing Schmidt number S_c in a region near to the plate and then decreases on increasing S_c in the region away from the plate for both PST and PHF. This implies that the flow field surfers a decrease in fluid velocity U and concentration ϕ in presence of heavier diffusing species.

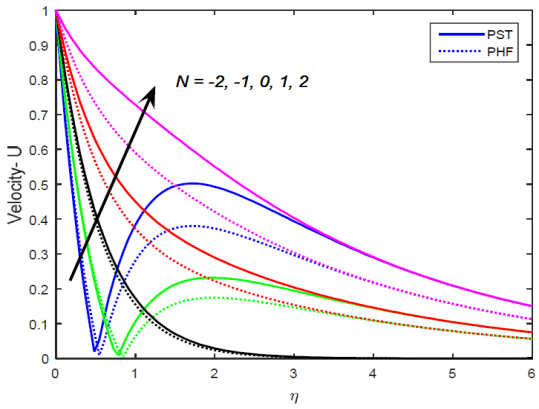

FIGURE 2. Influence of N on U.

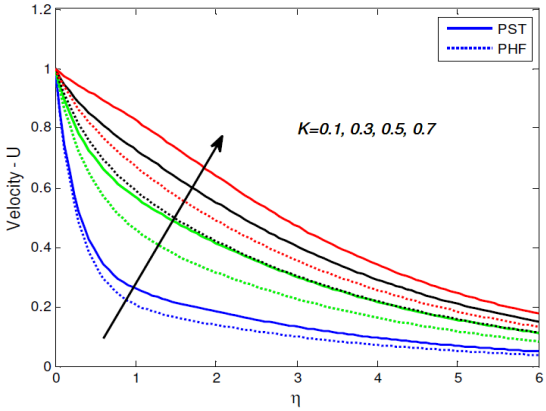

FIGURE 3. Influence of K on U.

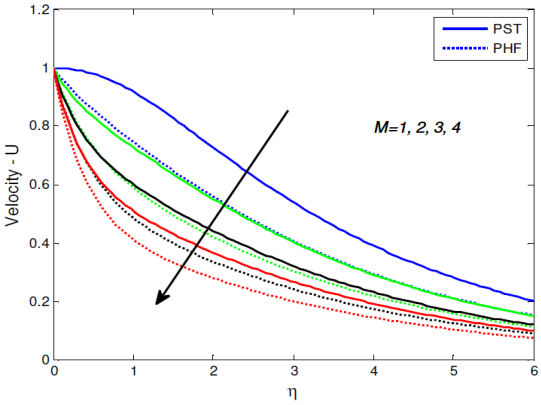

FIGURE 4. Influence of M on U.

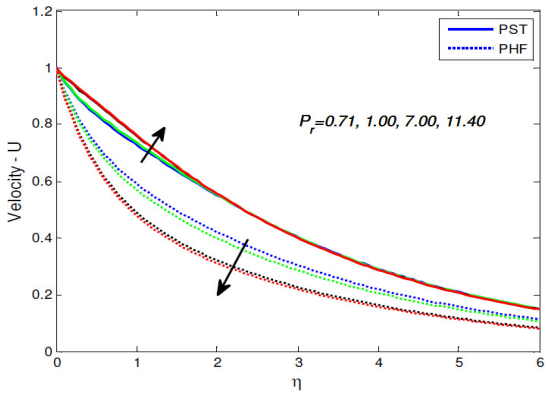

FIGURE 5. Influence of P_r on U.

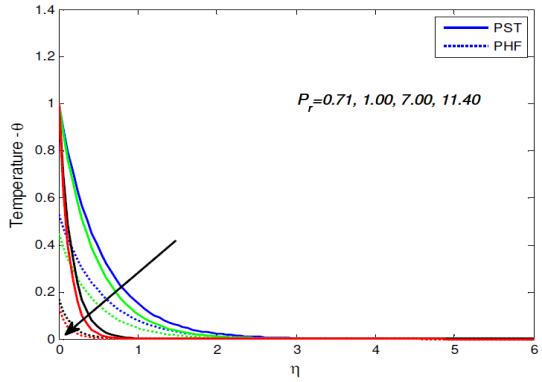

FIGURE 6. Influence of P_r on θ.

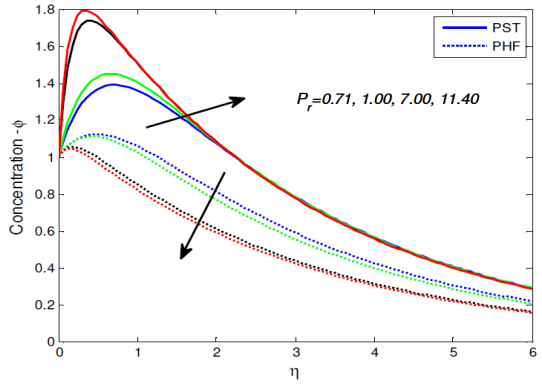

FIGURE 7. Influence of P_r on ϕ.

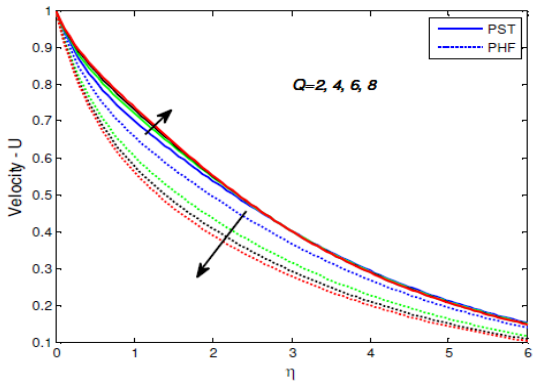

FIGURE 8. Influence of Q on U.

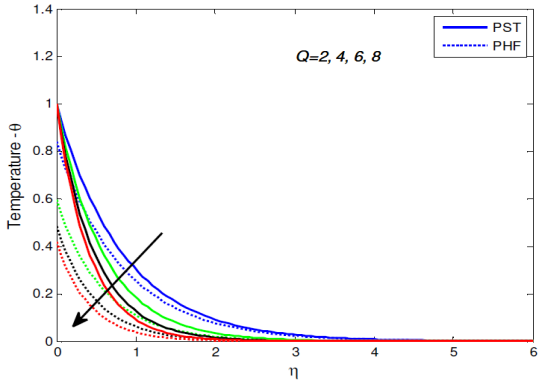

FIGURE 9. Influence of Q on θ.

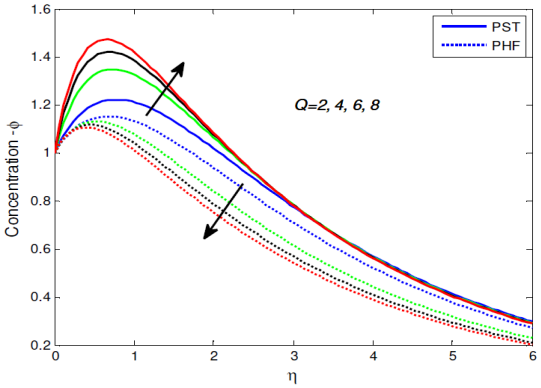

FIGURE 10. Influence of Q on ϕ.

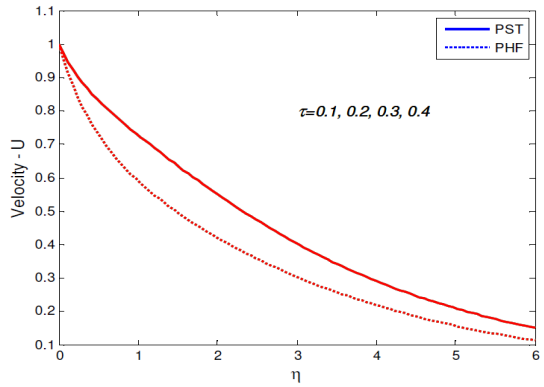

FIGURE 11. Influence of τ on U.

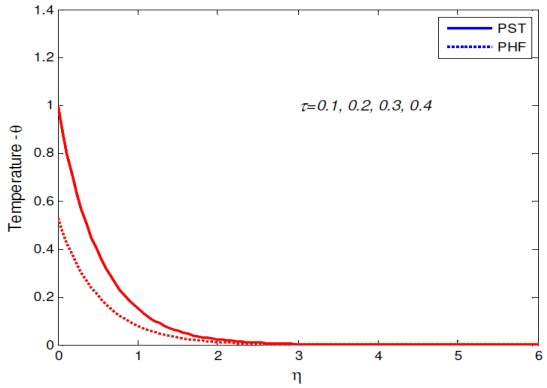

FIGURE 12. Influence of τ on θ.

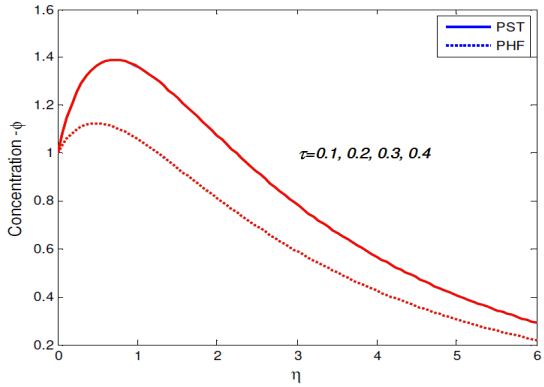

FIGURE 13. Influence of τ on ϕ.

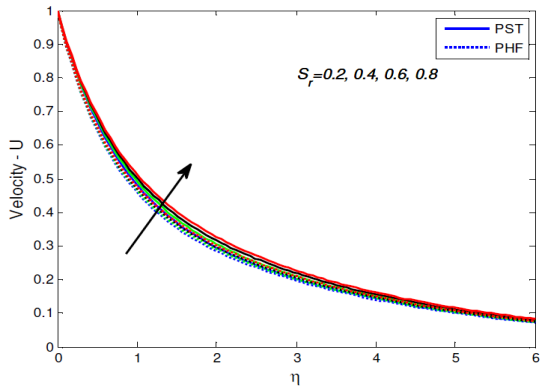

FIGURE 14. Influence of S_r on U.

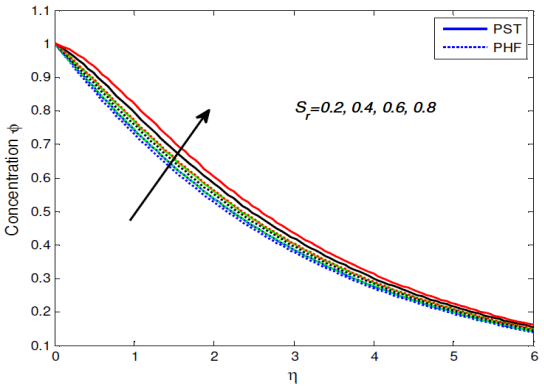

FIGURE 15. Influence of S_r on ϕ.

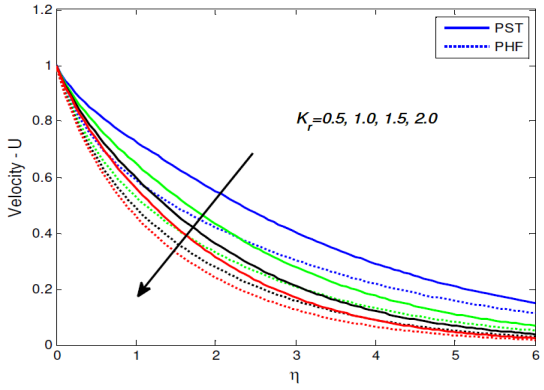

FIGURE 16. Influence of K_r on U.

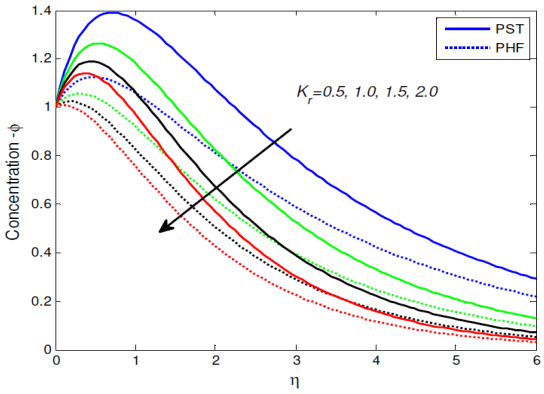

FIGURE 17. Influence of K_r on ϕ.

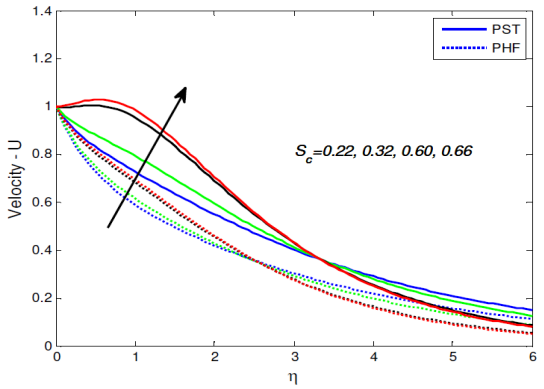

FIGURE 18. Influence of S_c on U.

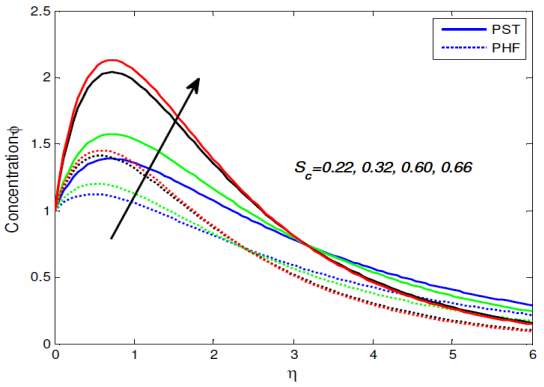

FIGURE 19. Influence of S_c on ϕ.

TABLE 1. Skin friction C_f when $S_r = 5.0$, $M = 2.0$, $N = 2.0$, $K_r = 0.5$, $K = 0.5$, $S_c = 0.22$, $N_r = 1.0$.

P_r	Q	τ	Skin friction C_f	
			PST	PHF
0.71	5.0	1.0	0.4940	0.7977
1.00	5.0	1.0	0.4708	0.8415
7.00	5.0	1.0	0.3464	1.0070
11.40	5.0	1.0	0.3212	1.0327
0.71	2.0	1.0	0.5518	0.6466
0.71	4.0	1.0	0.5088	0.7660
0.71	6.0	1.0	0.4817	0.8217
0.71	8.0	1.0	0.4621	0.8563
0.71	5.0	0.1	0.4941	0.7979
0.71	5.0	0.2	0.4941	0.7978
0.71	5.0	0.3	0.4941	0.7978
0.71	5.0	0.4	0.4941	0.7978

TABLE 2. Skin friction C_f when $P_r = 0.71$, $Q = 5.0$, $\tau = 1.0$, $N = 2.0$, $K = 0.5$, $M = 2.0$, $N_r = 1.0$.

S_r	S_c	K_r	Skin friction C_f	
			PST	PHF
0.2	0.22	0.5	0.8680	0.9963
0.4	0.22	0.5	0.8524	0.9880
0.6	0.22	0.5	0.8368	0.9797
0.8	0.22	0.5	0.8212	0.9714
5.0	0.22	0.5	0.4940	0.7977
5.0	0.32	0.5	0.3739	0.7455
5.0	0.60	0.5	0.0704	0.6071
5.0	0.66	0.5	0.0105	0.5792
5.0	0.22	0.5	0.4940	0.7977
5.0	0.22	1.0	0.5849	0.8684
5.0	0.22	1.5	0.6465	0.9167
5.0	0.22	2.0	0.6938	0.9540

TABLE 3. Skin friction C_f when $P_r = 0.71$, $Q = 5.0$, $\tau = 1.0$, $S_r = 5.0$, $S_c = 0.22$, $K_r = 0.5$, $N_r = 1.0$.

M	N	K	Skin friction C_f	
			PST	PHF
1.0	2.0	0.5	0.0122	0.3640
2.0	2.0	0.5	0.4940	0.7977
3.0	2.0	0.5	0.8798	1.1504
4.0	2.0	0.5	1.2058	1.4516
2.0	0.2	0.5	1.6174	1.7567
2.0	0.4	0.5	1.4926	1.6501
2.0	0.6	0.5	1.3678	1.5436
2.0	0.8	0.5	1.2429	1.4370
2.0	2.0	0.1	2.5756	2.7450
2.0	2.0	0.3	0.9940	1.2555
2.0	2.0	0.5	0.4940	0.7977
2.0	2.0	0.7	0.2340	0.5628

TABLE 4. Nusselt number N_u.

P_r	Q	τ	Nusselt number N_u	
			PST	PHF
0.71	5.0	1.0	1.8862	1.0011
1.00	5.0	1.0	2.2386	1.0011
7.00	5.0	1.0	5.9227	1.0011
11.40	5.0	1.0	7.5583	1.0011
0.71	2.0	1.0	1.1930	1.0011
0.71	4.0	1.0	1.6871	1.0011
0.71	6.0	1.0	2.0663	1.0011
0.71	8.0	1.0	2.3859	1.0011
0.71	5.0	0.1	1.8861	1.0010
0.71	5.0	0.2	1.8861	1.0010
0.71	5.0	0.3	1.8861	1.0010
0.71	5.0	0.4	1.8861	1.0010

TABLE 5. Sherwood number S_h when $S_c = 0.22$, $S_r = 5.0$, $K_r = 0.5$, $N_r = 1.0$.

P_r	Q	τ	Sherwood number S_h	
			PST	PHF
0.71	5.0	1.0	1.4322	0.6043
1.00	5.0	1.0	1.8123	0.6269
7.00	5.0	1.0	5.8370	0.7107
11.40	5.0	1.0	7.5583	7.5416
0.71	2.0	1.0	0.6945	0.5294
0.71	4.0	1.0	1.2186	0.5881
0.71	6.0	1.0	1.6261	0.6167
0.71	8.0	1.0	1.9718	0.6346
0.71	5.0	0.1	1.4321	0.6043
0.71	5.0	0.2	1.4321	0.6043
0.71	5.0	0.3	1.4321	0.6043
0.71	5.0	0.4	1.4321	0.6043

TABLE 6. Sherwood number S_h when $P_r = 0.22$, $Q = 5.0$, $\tau = 0.5$, $N_r = 1.0$.

S_c	S_r	K_r	Sherwood number S_h	
			PST	PHF
0.22	5.0	0.5	1.4322	0.6043
0.32	5.0	0.5	2.0890	0.9208
0.60	5.0	0.5	3.8358	1.7785
0.66	5.0	0.5	4.1950	1.9565
0.22	0.2	0.5	0.2615	0.2946
0.22	0.4	0.5	0.1909	0.2572
0.22	0.6	0.5	0.1204	0.2197
0.22	0.8	0.5	0.0498	0.1822
0.22	5.0	0.5	1.4322	0.6043
0.22	5.0	1.0	1.1917	0.4121
0.22	5.0	1.5	1.0150	0.2688
0.22	5.0	2.0	0.8705	0.1504

Tables 1 to 3. Represents the numerical values of skin friction coefficient C_f for different values of Prandtl number P_r, heat source parameter Q, Soret number S_r, Schmidt number S_c, chemical reaction parameter K_r, Magnetic parameter M, buoyancy parameter N, permeability parameter K and time τ. Skin friction coefficient C_f increases on increasing K_r and M whereas C_f decreases on increasing S_r, S_c, N and K for both PST and PHF. It is observed that skin friction coefficient C_f decreases for PST and increases for PHF on increasing P_r and

Q. It is also noted that the skin friction coefficient attain their steady state for large time τ for both PST and PHF.

Table 4, Represents the numerical values of heat transfer coefficient N_u for different values of Prandtl number P_r, heat source parameter Q and time τ. Nusselt number N_u increases on increasing P_r and Q. It is clear that the heat transfer coefficient attains their steady state for large time τ for both PST and PHF. Also the value of N_u is least for Mercury and highest for Water at 4^0C.

Tables 5 to 6, Represents the numerical values of mass transfer coefficient S_h for different values of Prandtl number P_r, heat source parameter Q, Schmidt number S_c, Soret number S_r, chemical reaction parameter K_r and time τ. Sherwood number S_h increases on increasing P_r, Q and S_c whereas Sherwood number S_h decreases on increasing S_r and K_r for both PST and PHF. It is noted that the Sherwood number attain their steady state for large time τ for both PST and PHF. Also the value of S_h is least for Hydrogen and highest for Propyl benzene.

5. CONCLUSIONS

An investigation on the influence of Soret and chemical reaction effects on the MHD natural convection flow of a viscous, incompressible, electrically conducting and radiating fluid past an impulsively moving vertical plate embedded in a fluid saturated porous medium is carried out. Important findings are as follows:

- Buoyancy parameter, permeability parameter and Soret number tends to accelerate the fluid velocity throughout the boundary layer region for both PST and PHF. Magnetic parameter and chemical reaction parameter tends to decelerate the fluid velocity throughout the boundary layer region for both PST and PHF.
- Schmidt number tends to accelerate the fluid velocity in a region close to the plate whereas it has a reverse effect in the region away from the plate. Prandtl number and heat source parameter are tends to accelerate the fluid velocity in a region close to the plate whereas they have a reverse effect on the fluid velocity in the region away from the plate for PST. Also it is observed that Prandtl number and heat source parameter tends to decelerate the fluid velocity for PHF throughout the boundary layer region.
- Prandtl number and heat source parameter tends to retard fluid temperature throughout the boundary layer region for both PST and PHF.
- Prandtl number and heat source parameter tends to accelerate the species concentration in a region close to the plate whereas they have a reverse effect on the species concentration in the region away from the plate for PST. Prandtl number and heat source parameter tends to retard the species concentration throughout the boundary layer region for PHF.
- Schmidt number tends to accelerate the species concentration in a region close to the plate whereas it has a reverse effect on the species concentration in the region away from the plate for both PST and PHF.
- Soret number, Schmidt number, buoyancy parameter and permeability parameter tends to retard skin friction coefficient and there is an enhancement in skin friction coefficient

on increasing chemical reaction parameter, magnetic parameter for both PST and PHF. But skin friction coefficient decreases on increasing Prandtl number and heat source parameter for PST whereas they have a reverse effect for PHF throughout the boundary layer region.

- Prandtl number and heat source parameter have tendency to increase the heat transfer coefficient.
- Prandtl number, heat source parameter and Schmidt number have tendency to accelerate the mass transfer coefficient. Soret number and chemical reaction parameter retard the mass transfer coefficient.
- The fluid velocity, temperature, concentration, skin friction coefficient, heat transfer coefficient and mass transfer coefficient are attain their steady state with the progress of time throughout the boundary layer region for both PST and PHF.

APPENDIX

$A_1 = \sqrt{P_r Q},\ A_2 = \sqrt{P_r(Q+\omega)},\ A_3 = \sqrt{S_c K_r},\ A_4 = S_c S_r,\ A_5 = \sqrt{S_c(K_r+\omega)},$
$A_6 = \sqrt{M+\frac{1}{K}},\ A_7 = \sqrt{M+\frac{1}{K}+\omega},\ A_8 = \frac{A_1^2 A_4}{A_3^2-A_1^2},\ A_9 = 1-A_8,\ A_{10} = \frac{A_2^2 A_4}{A_5^2-A_2^2},$
$A_{11} = 1-A_{10},\ A_{12} = \frac{1+NA_8}{A_6^2-A_1^2},\ A_{13} = \frac{NA_9}{A_6^2-A_3^2}, A_{14} = 1-(A_{12}+A_{13}),$
$A_{15} = \frac{1+NA_{10}}{A_7^2-A_2^2},\ A_{16} = \frac{NA_{11}}{A_7^2-A_5^2}\ ,\ A_{17} = -(A_{15}+A_{16}),$
$B_1 = \frac{1}{A_1},\ B_2 = \frac{1}{A_2},\ B_3 = \frac{A_1 A_4}{A_3^2-A_1^2},\ B_4 = 1-B_3,\ B_5 = \frac{A_2 A_4}{A_5^2-A_2^2},\ B_6 = 1-B_5,$
$B_7 = \frac{B_1+NB_3}{A_6^2-A_1^2},\ B_8 = \frac{NB_4}{A_6^2-A_3^2},\ B_9 = 1-(B_7+B_8)\,,\ B_{10} = \frac{B_2+NB_5}{A_7^2-A_2^2},\ B_{11} = \frac{NB_6}{A_7^2-A_5^2},$
$B_{12} = -(B_{10}+B_{11}).$

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REFERENCES

[1] A. J. EDE, *Advances in heat transfer*, Academic press, New York, **4** (1967), 1–64.
[2] B. Gebhart, *Advances in heat transfer*, Academic press, New York, **l9** (1973), 273–348.
[3] Y. Jaluria, *Natural convection heat and mass transfer*, Oxford, Pergamon press, 1980.
[4] G. D. Raithby and K. G. T. Hollands, *Natural convection in handbook of heat transfer fundamentals*, New York, McGraw Hill, 2nd edn, 1985.
[5] N. Ahmed and H. K. Sarmah, *Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate*, Journal of Applied Mathematics and Mechanics, **5** (2009), 87–98.
[6] N. Ahmed, *Soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate*, Journal of Heat Transfer, **134** (2012), 1–9.
[7] R. Muthucumaraswamy and P. Ganesan, *Radiation effects on flow past an impulsively started infinite vertical plate with variable temperature*, International Journal of Applied Mechanics and Engineering, **8** (2003), 125–129.
[8] A. J. Chamkha, *Transient MHD free convection from a porous medium supported by a surface*, Fluid - Particle Separation Journal, **10** (1997), 101–107.

[9] M. Venkateswarlu, G. V. Ramana Reddy and D.V. Lakshmi, *Diffusion-thermo effects on MHD flow past an infinite vertical porous plate in the presence of radiation and chemical reaction*, International Journal of Mathematical Archive, **4** (2013), 39–51.

[10] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Lakshmi, *Thermal diffusion and radiation effects on unsteady MHD free convection heat and mass transfer flow past a linearly accelerated vertical porous plate with variable temperature and mass diffusion*, Journal of the Korea Society for Industrial and Applied Mathematics, **18** (2014), 257–268.

[11] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Lakshmi, *Radiation effects on MHD boundary layer flow of liquid metal over a porous stretching surface in porous medium with heat generation*, Journal of the Korea Society for Industrial and Applied Mathematics, **19** (2015), 83–102.

[12] M. Venkateswarlu and P. Padma, *Unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction*, Procedia Engineering, **127** (2015), 791–799.

[13] A. Postelnicu, *Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects*, Heat and Mass Transfer, **43** (2007), 595–602.

[14] A. Raptis and N. Kafousias, *Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field*, International Journal of Energy Research, **6** (1982), 241–245.

[15] T. K. Aldoss, M. A. Al-Nimr, M. A. Jarrah and B. J. Al-Shaer, *Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium*, J. Heat and Transfer, **28** (1995), 635–645.

[16] O. D. Makinde, *On MHD boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux*, International Journal of Numerical Methods for Heat & Fluid Flow, **19** (2009), 546–554.

[17] A. J. Chamkha, *MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction*, International Journal of Heat and Mass Transfer, **30** (2003), 413–422.

[18] A. A. Afify, *The effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field*, Canadian Journal of Physics, **82** (2004), 447–458.

[19] F. S. Ibrahim, A. M. Elaiw and A. A. Bakr, *Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction,* Communications in Nonlinear Science and Numerical Simulation **13** (2008), 1056–1066.

[20] A. A. Bakr, *Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference*, Communications in Nonlinear Science and Numerical Simulation **16** (2011), 698–710.

[21] A. J. Chamkha, R. A. Mohamed and S. E. Ahmed, *Unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid with Joule heating, chemical reaction and radiation effects*, Meccanica, **46** (2011), 399–411.

[22] K. Bhattacharyya and G. C. Layek, *Similarity solution of MHD boundary layer flow with diffusion and chemical reaction over a porous flat plate with suction/blowing*, Meccanica, **47**(2012), 1043–1048.

[23] P. M. Kishore, N. V. R. V. P. S. Rao, V. K. Varma and S. Venkataramana, *The effects of radiation and chemical reaction on unsteady MHD free convection flow of viscous fluid past an exponentially accelerated vertical plate*, International Journal of Mathematics and Mathematical Sciences **4** (2013), 300–317.

[24] M. Venkateswarlu, G. V. Ramana Reddy and D. V. Lakshmi, *Effects of chemical reaction and heat generation on MHD boundary layer flow of a moving vertical plate with suction and dissipation*, Engineering International, **1** (2013), 27–38.

[25] M. Venkateswarlu and D. Venkata Lakshmi, *Dufour effect on radiative MHD flow of a viscous fluid in a parallel porous plate channel under the influence of slip condition*, Journal of Advances in Science and Technology, **12** (2016), 614–620.

[26] M. Venkateswarlu, D. Venkata Lakshmi and K. Nagamalleswara Rao, *Soret, hall current, rotation, chemical reaction and thermal radiation effects on unsteady MHD heat and mass transfer natural convection flow past*

an accelerated vertical plate, Journal of the Korea Society for Industrial and Applied Mathematics, **20** (2016), 203–224.

[27] J. Manjula, P. Padma, M. Gnaneswara Reddy and M. Venakateswarlu, *Influence of thermal radiation and chemical reaction on MHD flow, heat and mass transfer over a stretching surface*, Procedia Engineering, **127** (2015), 1315–1322.

[28] M. Turkyilmazoglu and I. pop, *Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate*, International Journal of Heat and Mass Transfer, **55** (2012), 7635–7644.

[29] E. Magyari and A. Pantokratoras, *Note on the effect of thermal radiation in the linearized Rosseland approximation on the heat transfer characteristics of various boundary layer flows*, International Communications in Heat and Mass Transfer, **38** (2011), 554–556.

[30] N. Siva Kumar, Rushi Kumar and A. G. Vijaya Kumar, *Thermal diffusion and chemical reaction effects on unsteady flow past a vertical porous plate with heat source dependent in slip flow regime*, Journal of Naval Architecture and Marine Engineering, **13** (2016), 51–62.

[31] M. Venkateswarlu, D. Venkata Lakshmi and G. Darmaiah, *Influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction*, Journal of the Korea Society for Industrial and Applied Mathematics, **20** (2016), 333–354.

[32] M. Venkateswarlu and D. Venkata Lakshmi, *Slip velocity distribution on MHD oscillatory heat and mass transfer flow of a viscous fluid in a parallel plate channel*, Journal of Bangladesh Mathematical Society, **36** (2016), 93–114.