ISSN 1975-8359(Print) / ISSN 2287-4364(Online)
The Transactions of the Korean Institute of Electrical Engineers Vol. 66, No. 2, pp. 409~415, 2017
http://doi.org/10.5370/KIEE.2017.66.2.409

불확실 유체 역학 계수를 가진 대형급 무인잠수정의 강인 경로점 추적

Robust Waypoint Tracking of Large Diameter Unmanned Underwater Vehicles with Uncertain Hydrodynamic Coefficients

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Abstract - This paper addresses on an linear matrix inequality (LMI) formulation of the robust waypoint tracking problem of large diameter unmanned underwater vehicles (LDUUVs) in the horizontal plane. The interested design issue can be reformed as the robust asymptotic stabilization of the provided error dynamics with respect to the desired yaw angle, surge speed and attitude. Sufficient conditions for its robust asymptotic stabilizability against the hydrodynamic uncertainties are derived in the format of LMI. An example is provided to testify the validity of the proposed theoretical claims.

Key Words : Large diameter unmanned underwater vehicles (LDUUV), Uncertainty, robust, Lyapunov, Linear matrix inequality (LMI), Waypoint, Asymptotic stability.

1. Introduction

Recently, several techniques involving the control of unmanned underwater vehicles (UUVs) have been proposed in [1]-[7], which are based on their mathematical model. The mathematical model contains hydrodynamic forces and moments expressed in terms of a set of hydrodynamic coefficients [13]. These hydrodynamic coefficients are usually empirically determined, and thereby its exact estimation is virtually impossible. From this reason, the robust control of uncertain UUVs is one of the most important research issues. However, unfortunately, there is still a lack in design methods based on linear matrix inequality (LMI) for uncertain UUVs.

The objective of this study is to present an LMI-based design methodology for the horizontal waypoint tracking control of a class of large diameter UUVs (LDUUVs) with the hydrodynamic uncertainties while keeping its the desired surge speed and attitude. The proposed approach is an extension of [7] to uncertain LDUUVs. The concerned problem can be viewed as the robust asymptotic stablilization of the uncertain error dynamics in terms of the desired yaw angle, surge speed, and attitude. It is shown that two control inputs of uncertain LDUUVs, the rudder angles and the propeller thrust can be separately designed. Finally, sufficient LMI conditions for the robust asymptotic stabilization of uncertain LDUUVs are provided in the sense of Lyapunov stability criterion.

Notations : The relation $P > Q$ ($P < Q$) means that the matrix $P-Q$ is positive (negative) definite. $\lambda_{\max}(A)$ ($\lambda_{min}(A)$) is the maximum (minimum) eigenvalue of matrix A. $A_{(i)}$ denotes ith row of the matrix A. $Sym\{S\}$ is defined as S^T+S. B_η indicates the ball $\{\eta : \|\eta\| \le \Delta_\eta\}$ with $\Delta_\eta \in R_{>0}$. $\mathcal{I}_m$ indicates the integer set $\{1,\cdots,m\}$. Symbol * denotes a transposed element in a symmetric position.

2. Preliminaries

Consider a class of LDUUVs [8]-[10] with uncertain hydrodynamic coefficients described by the horizontal model

$$\dot{\eta} = J(\psi)\phi \tag{1}$$

$$M\dot{\phi} + (f(\phi) + \Delta f(\phi))\phi = \tau + \Delta\tau \tag{2}$$

where $\eta = [x\, y\, \psi]^T \in \Re^3$, $\phi = [u\, v\, r]^T \in \Re^3$, (x,y) is the inertial coordinates of the center of mass, ψ is the yaw angle, u and v are the surge and the sway velocities, respectively, and r is the angular velocity in yaw, $J \in \Re^{3\times 3}$ is a frame transformation, $M \in \Re^{3\times 3}$ includes mass and hydrodynamic added mass terms and $f(\phi)\phi \in \Re^3$ captures Coriolis-centripetal matrices including the added mass and a damping matrix, τ is the control actuator forces with the propeller thrust ξ, and δ_i, $i \in \mathcal{I}_4$ is the rudder angles, and

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Received : January 12, 2017; Accepted : January 25, 2017

Δf and $\Delta\tau$ are uncertainties. In detail, the terms J, M, f, Δf, τ, and $\Delta\tau$ are given by

$$J=\begin{bmatrix}\cos(\psi) & -\sin(\psi) & 0\\ \sin(\psi) & \cos(\psi) & 0\\ 0 & 0 & 1\end{bmatrix},\quad M=\begin{bmatrix}m_1 & 0 & 0\\ 0 & m_2 & m_3\\ 0 & m_4 & m_5\end{bmatrix}$$

$$f=\begin{bmatrix}-X_{uu}u & -X_{vv}v & -mv-mx_gr-X_{rr}r\\ 0 & -Y_v-Y_{v|v|}|v| & mu-Y_r-Y_{r|r|}|r|\\ 0 & -N_v-N_{v|v|}|v| & mx_gu-N_r-N_{r|r|}|r|\end{bmatrix}$$

$$\Delta f=\begin{bmatrix}0 & 0 & 0\\ 0 & -\Delta Y_v-\Delta Y_{v|v|}|v| & -\Delta Y_r-\Delta Y_{r|r|}|r|\\ 0 & -\Delta N_v-\Delta N_{v|v|}|v| & -\Delta N_r-\Delta N_{r|r|}|r|\end{bmatrix}$$

$$\tau=\begin{bmatrix}\xi+X_{uu\delta\delta}u^2\sum_{i=1}^4\delta_i^2\\ u^2\sum_{i=1}^4 Y_{uu\delta_i}\delta_i\\ u^2\sum_{i=1}^4 N_{uu\delta_i}\delta_i\end{bmatrix}\in\Re^3,\ \Delta\tau=\begin{bmatrix}0\\ u^2\sum_{i=1}^4\Delta Y_{uu\delta_i}\delta_i\\ u^2\sum_{i=1}^4\Delta N_{uu\delta_i}\delta_i\end{bmatrix}\in\Re^3.$$

where $m_1=m-X_{\dot u}$, $m_2=m-Y_{\dot v}$, $m_3=mx_g-Y_{\dot r}$, $m_4=mx_g-N_{\dot v}$, $m_5=I_{zz}-N_{\dot r}$, m is the vehicle mass, x_g is the x-position of the center of gravity, I_{zz} is the mass moment of inertia term, X_{uu}, X_{vv}, X_{rr}, Y_v, $Y_{v|v|}$, Y_r, $Y_{r|r|}$, N_v, $N_{v|v|}$, N_r, $N_{r|r|}$, $X_{uu\delta\delta}$, $Y_{uu\delta_i}$, and $N_{uu\delta_i}$ are the hydrodynamic coefficients, and ΔY_v, $\Delta Y_{v|v|}$, ΔY_r, $\Delta Y_{r|r|}$, ΔN_v, $\Delta N_{v|v|}$, ΔN_r, $\Delta N_{r|r|}$, $\Delta Y_{uu\delta_i}$, and $\Delta N_{uu\delta_i}$ are considered as the time-varying hydrodynamic uncertainties. The related coefficients in (1) and (2) are listed in Appendix 1.

Assumption 1: There exist the known constant $D_\star$ and the unknown time-varying function $F_\star$ satisfying $\| F_\star \| \le 1$ for $\star\in\{Y_v,\ Y_{v|v|},\ Y_r,\ Y_{r|r|},\ N_v,\ N_{v|v|},\ N_r,\ N_{r|r|},\ X_{uu\delta\delta},\ Y_{uu\delta_i},\ N_{uu\delta_i}\}$ such that

$$\Delta\star=D_\star F_\star\ \star.$$

For example, $\Delta Y_v=D_{Y_v}F_{Y_v}X_{Y_v}$.

Problem 1: Consider uncertain LDUUV (1) and (2). Suppose that the criteria for updating the waypoint is to shift from (x_{dk},y_{dk}) to $(x_{d(k+1)},y_{d(k+1)})$ when $\|[e_{xk}\ e_{yk}]^T\|\le\rho$, where (x_{dk},y_{dk}), $k\in\mathcal{I}_{n_w}$ is the kth given waypoint, $e_{xk}=x-x_{dk}$, and $e_{yk}=y-y_{dk}$. Then, given the desired constant surge velocity $u_d\in\Re_{>0}$ and the desired line of sight (LOS) to be the horizontal plane angle defined as

$$\Psi_d=\tan^{-1}\frac{e_y}{e_x} \tag{3}$$

design ξ and δ_i, $i\in\mathcal{I}_4$ such that $\|e_u\|$, $\|e_\Psi\|$, $\|v\|$, and $\|r\|$ robustly asymptotically converge to zero against the hydrodynamic uncertainties, where $e_u=u-u_d$ and $e_\psi=\psi-\psi_d$.

3. Main Results

Before proceeding to our main results, the following propositions and the lemmas will be needed throughout the proof:

Proposition 1: Consider (1) and (2). Define $\chi:=[e_\psi\ v\ r]^T\in\Re^3$, $e_{xy}:=[e_x\ e_y]^T\in\Re^2$, and $\delta:=[\delta_1\ \delta_2\ \delta_3\ \delta_4]^T\in\Re^4$. An augmented error system can be represented by

$$\begin{bmatrix}\dot\chi\\ \dot e_u\end{bmatrix}=\begin{bmatrix}F_1(\chi,e_{xy})+\Delta F_1(v,r) & F_2(e_\psi,e_{xy},r,e_u,\delta)+\Delta F_2(e_u,\delta)\\ F_3(v,r) & F_4(e_u,\delta)\end{bmatrix}\begin{bmatrix}\chi\\ e_u\end{bmatrix}+\begin{bmatrix}0\\ F_5(\delta)\end{bmatrix}+\begin{bmatrix}G+\Delta G & 0\\ 0 & \frac{1}{m_1}\end{bmatrix}\begin{bmatrix}\delta\\ \xi\end{bmatrix} \tag{4}$$

where F_1, ΔF_1, F_2, ΔF_2, F_3, F_4, F_5, G, and ΔG are given in Appendix 2.

Proof: Using $\dot e_\psi=\dot\psi-\dot\psi_d$, (1), (2), and (3), we have

$$\dot e_\psi=\frac{u_d\sin e_\psi}{\| e_{xy}\|}+\frac{\cos e_\psi}{\| e_{xy}\|}v+r+\frac{\sin e_\psi}{\| e_{xy}\|}e_u$$

(see [7] for more details). It follows from (2) and $\dot e_u=\dot u$ that

$$\begin{aligned}\begin{bmatrix}\dot v\\ \dot r\end{bmatrix}&=\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}Y_v+Y_{v|v|}|v| & -m(e_u+u_d)+Y_r+Y_{r|r|}|r|\\ N_v+N_{v|v|}|v| & -mx_g(e_u+u_d)+N_r+N_{r|r|}|r|\end{bmatrix}\begin{bmatrix}v\\ r\end{bmatrix}\\&+\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}\Delta Y_v+\Delta Y_{v|v|}|v| & \Delta Y_r+\Delta Y_{r|r|}|r|\\ \Delta N_v+\Delta N_{v|v|}|v| & \Delta N_r+\Delta N_{r|r|}|r|\end{bmatrix}\begin{bmatrix}v\\ r\end{bmatrix}\\&+\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}(e_u^2+2u_de_u+u_d^2)\sum_{i=1}^4 Y_{uu\delta_i}\delta_i\\ (e_u^2+2u_de_u+u_d^2)\sum_{i=1}^4 N_{uu\delta_i}\delta_i\end{bmatrix}\\&+\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}(e_u^2+2u_de_u+u_d^2)\sum_{i=1}^4\Delta Y_{uu\delta_i}\delta_i\\ (e_u^2+2u_de_u+u_d^2)\sum_{i=1}^4\Delta N_{uu\delta_i}\delta_i\end{bmatrix}\end{aligned}$$

and

$$\begin{aligned}\dot e_u=\frac{1}{m_1}\big(&X_{uu}e_u^2+2X_{uu}e_uu_d+X_{uu}u_d^2+X_{vv}v^2+mvr+mx_gr^2+X_{rr}r^2\\&+\xi+X_{uu\delta\delta}(e_u^2+2u_de_u+u_d^2)\Sigma_{i=1}^4\delta_i^2.\end{aligned}$$

Taking the change of variables with $[\chi,e_u]^T$ results in (4). ■

Proposition 2: Consider (4) together with

$$\delta = K\chi \tag{5}$$

$$\xi = -m_1 F_3(v,r)\chi - m_1 F_4(e_u,\delta)e_u - m_1 F_5(\delta) - m_1 \gamma e_u \tag{6}$$

where K is the controller gains to be determined and $\gamma \in \Re_{>0}$. Then the closed-loop system becomes

$$\Sigma_1 : \ \dot{\chi} = \left(F_1(\chi, e_{xy}) + \Delta F_1(v,r) + (G+\Delta G)K\right)\chi + \left(F_2(e_\psi, e_{xy}, r, e_u, \delta) + \Delta F_2(e_u,\delta)\right)e_u \tag{7}$$

$$\Sigma_2 : \ \dot{e}_u = -\gamma e_u. \tag{8}$$

Proof: By substituting (5) and (6) into (4), we have (7) and (8). ■

Proposition 3: It is true that

$$F_1(e_\psi, e_x, e_y, v, r) = \sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\sum_{i_3=1}^{2}\sum_{i_4=1}^{2} \theta_{1i_1}\theta_{2i_2}\theta_{3i_3}\theta_{4i_4} A_{i_1 i_2 i_3 i_4}$$

$$\Delta F_1(v,r) = \sum_{i_3=1}^{2}\sum_{i_4=1}^{2} \theta_{3i_3}\theta_{4i_4} D_{\Delta F_1} F_{\Delta F_1} \tilde{A}_{i_3 i_4}$$

$$\Delta G = D_{\Delta G} F_{\Delta G} \tilde{G}$$

on $B_v \times B_r$, where θ_{1i_1}, θ_{2i_2}, θ_{3i_3}, θ_{4i_4}, $A_{i_1 i_2 i_3 i_4}$, $D_{\Delta F_1}$, $F_{\Delta F_1}$, $\tilde{A}_{i_3 i_4}$, $D_{\Delta G}$, $F_{\Delta G}$, and $\tilde{G}$ are given in Appendix 3.

Proof: The proof directly follows from Assumption 1, [Theorem 2, 7], and the sector nonlinearity methodology [11]. ■

Lemma 1 [12]: Given constant matrices D and X, a time-varying matrix F satisfying $F^T(t)F(t) \le I$ for all $t \in \Re_{>0}$, and $\Lambda = \Lambda^T$ of appropriate dimensions, the following equivalence

$$\Lambda + sym\{DFX\} < 0 \Leftrightarrow \Lambda + \epsilon DD^T + \epsilon^{-1} X^T X < 0$$

holds for some $\epsilon \in \Re_{>0}$.

Lemma 2: There exists $\zeta \in \Re_{>0}$ such that

$$\left\| F_2(e_\psi, e_{xy}, r, e_u, \delta) + \Delta F_2(e_u, \delta) \right\| \le \zeta$$

on $B_r \times B_{\delta_1} \times B_{\delta_2} \times B_{\delta_3} \times B_{\delta_4}$.

Proof: Under Assumption 1, $F_2 + \Delta F_2$ satisfies

$$\begin{aligned} &\left\| F_2 + \Delta F_2 \right\| \\ &\le \left\| E^{-1} \right\| \left\| \begin{bmatrix} 1/\rho \\ m\Delta_r + \left| e_u + 2u_d \right| \sum_{i=1}^{4} \left(1 + \left| D_{Y_{uu\delta_i}} \right|\right) \left| Y_{uu\delta_i} \right| \Delta_{\delta_i} \\ mx_g \Delta_r + \left| e_u + 2u_d \right| \sum_{i=1}^{4} \left(1 + \left| D_{N_{uu\delta_i}} \right|\right) \left| N_{uu\delta_i} \right| \Delta_{\delta_i} \end{bmatrix} \right\| \end{aligned}$$

on $B_r \times B_{\delta_1} \times B_{\delta_2} \times B_{\delta_3} \times B_{\delta_4}$. ■

Theorem 1: Consider (1) and (2) under (5) and (6). Let $\Omega := \left\{ [\chi \ e_u]^T \in R^4 : \ \chi^T P \chi + \alpha e_u^2 \langle 1 \right\}$ with $P = P^T > 0$ and $\alpha \in \Re_{>0}$ Suppose that there exist $\tilde{P} = \tilde{P}^T > 0$, $\tilde{K}$, $\epsilon_1 \in \Re_{>0}$, and $\epsilon_2 \in \Re_{>0}$ such that

$$\begin{bmatrix} \begin{pmatrix} Sym\left\{A_{i_1 i_2 i_3 i_4}\tilde{P} + G\tilde{K}\right\} \\ +\epsilon_1 D_{\Delta F_1} D_{\Delta F_1}^T + \epsilon_2 D_{\Delta G} D_{\Delta G}^T \end{pmatrix} & * & * \\ \tilde{A}_{i_3 i_4}\tilde{P} & -\epsilon_1 I & * \\ \tilde{G}\tilde{K} & 0 & -\epsilon_2 I \end{bmatrix} < 0 \tag{9}$$

$$E_v \tilde{P} E_v^T < \Delta_v^2 \tag{10}$$

$$E_r \tilde{P} E_r^T < \Delta_r^2 \tag{11}$$

$$\begin{bmatrix} \Delta_{\delta_i}^2 & \tilde{K}_{(i)} \\ \tilde{K}_{(i)}^T & -\tilde{P} \end{bmatrix} < 0 \tag{12}$$

for all $(i_1, i_2, i_3, i_4, i) \in \mathcal{I}_2 \times \mathcal{I}_2 \times \mathcal{I}_2 \times \mathcal{I}_2 \times \mathcal{I}_4$, where $E_v = [0\,1\,0]$ and $E_r = [0\,0\,1]$. Then, for $C\{\chi(t_0),\ e_u(t_0)\} \in \Omega$, the closed-loop system of (1) and (2) under (5) and (6) is robustly asymptotically stable. In this feasible case, $P = \tilde{P}^{-1}$ and $K = \tilde{K}\tilde{P}^{-1}$.

Proof: Consider a Lyapunov function $V(\chi, e_u) = \chi^T P \chi + \alpha e_u^2$ for the closed-loop system (1), (2), (5), and (6). From Propositions 1, 2, and 3, we can see that

$$\begin{aligned} \dot{V}|_{(1),(2),(5),(6)} &= \mathrm{Sym}\left\{\chi^T P\left(F_1(\chi, e_{xy}) + \Delta F_1(v,r) + (G+\Delta G)K\right)\chi\right\} \\ &\quad + 2\chi^T P\left(F_2(e_\psi, e_{xy}, r, e_u, \delta) + \Delta F_2(e_u,\delta)\right)e_u - 2\alpha\gamma e_u^2 \\ &= \sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\sum_{i_3=1}^{2}\sum_{i_4=1}^{2} \theta_{1i_1}\theta_{2i_2}\theta_{3i_3}\theta_{4i_4} \\ &\quad \times \left(\mathrm{Sym}\left\{\chi^T P\left(A_{i_1 i_2 i_3 i_4} + GK\right)\chi\right\}\right. \\ &\quad \left. + \mathrm{Sym}\left\{\chi^T P\left(D_{\Delta F_1} F_{\Delta F_1} \tilde{A}_{i_3 i_4} + D_{\Delta G} F_{\Delta G} \tilde{G} K\right)\chi\right\}\right) \\ &\quad + 2\chi^T P\left(F_2(e_\psi, e_{xy}\right. \end{aligned}$$

Note that from Lemma 1,

$$\begin{aligned} &\mathrm{Sym}\left\{P\left(A_{i_1 i_2 i_3 i_4} + GK\right) + P\left(D_{\Delta F_1} F_{\Delta F_1} \tilde{A}_{i_3 i_4} + D_{\Delta G} F_{\Delta G} \tilde{G} K\right)\right\} < 0 \\ &\Leftarrow \mathrm{Sym}\left\{P\left(A_{i_1 i_2 i_3 i_4} + GK\right)\right\} + \epsilon_1 P D_{\Delta F_1} D_{\Delta F_1}^T P + \epsilon_1^{-1} \tilde{A}_{i_3 i_4}^T \tilde{A}_{i_3 i_4} \\ &\quad + \epsilon_2 P D_{\Delta G} D_{\Delta G}^T P + \epsilon_2^{-1} (\tilde{G}K)^T \tilde{G}K < 0 \\ &\Leftrightarrow \mathrm{LMI}\ (9) \end{aligned}$$

Then, by using from Lemma 2, it can be shown that there exists $Q = Q^T > 0$ such that

$$\begin{aligned} \dot{V}|_{(1),(2),(5),(6)} &< -\lambda_{\min}(Q) \|\chi\|^2 - 2\alpha\gamma e_u^2 + 2\zeta \|P\| \, \|\chi\| \, |e_u| \\ &= \begin{bmatrix} \|\chi\| \\ |e_u| \end{bmatrix}^T \begin{bmatrix} -\lambda_{\min}(Q) & \zeta\|P\| \\ \zeta\|P\| & -2\alpha\gamma \end{bmatrix} \begin{bmatrix} \|\chi\| \\ |e_u| \end{bmatrix} \end{aligned}$$

on $B_v \times B_r \times B_{\delta_1} \times B_{\delta_2} \times B_{\delta_3} \times B_{\delta_4}$. From the fact that

$$\begin{bmatrix} -\lambda_{\min}(Q) & \zeta\|P\| \\ \zeta\|P\| & -2\alpha\gamma \end{bmatrix} < 0 \Leftrightarrow \begin{cases} \dfrac{\zeta^2\|P\|^2}{2\alpha\gamma} - \lambda_{\min}(Q) < 0 \\ 2\alpha\gamma > 0 \end{cases}$$

by Schur complement, choosing $\alpha \in \Re_{> \frac{\zeta^2\|P\|^2}{2\gamma\lambda_{\min}(Q)}}$ implies that the closed-loop system (1), (2), (5), and (6) is robustly asymptotically stable. Also, it is not hard to see that by Schur complement, LMI (10), (11), (12) $\Rightarrow \Omega \subset B_v \times B_r \times B_{\delta_1} \times B_{\delta_2} \times B_{\delta_3} \times B_{\delta_4}$. Therefore, if LMIs (9), (10), (11), and (12) hold, then the robust asymptotic stability of the closed-loop system of (1), (2), (5), (6) is guaranteed for $\mathrm{C}\{\chi(t_0),\ e_u(t_0)\} \in \Omega$. ■

Remark 1: If $D_{\Delta F_1} = D_{\Delta G} =: D$ and $F_{\Delta F_1} = F_{\Delta G} =: F$, LMI (9) becomes

$$\begin{bmatrix} Sym\{A_{i_1i_2i_3i_4}\tilde{P} + G\tilde{K}\} + \epsilon D^T D & * & * \\ \tilde{A}_{i_3i_4}\tilde{P} + \tilde{G}\tilde{K} & -\epsilon I & * \end{bmatrix} < 0$$

for some $\epsilon \in \Re_{>0}$ and $(i_1, i_2, i_3, i_4) \in \mathcal{I}_2 \times \mathcal{I}_2 \times \mathcal{I}_2 \times \mathcal{I}_2$.

4. An Example

For the given $u_d = 3$, $\rho = 2$, and

$$(x_{dk}, y_{dk}) = \left(100\sin\frac{2\pi(k-1)}{60} + 100, 100\cos\frac{2\pi(k-1)}{60} + 100\right)$$

on $k \in I_{61}$, our goal is to determine K in (5) so that $\|e_u\|$, $\|e_\psi\|$, $\|v\|$, and $\|r\|$ of uncertain LDUUV (1) and (2) under (5) and (6) with $\gamma = 10$ robustly asymptotically converge to zero. Suppose that $\Delta_v = 10$, $\Delta_r = 1$, $\Delta_{\delta_i} = 0.2374$, $i \in \mathcal{I}_4$,

$$D_{\Delta F_1} = D_{\Delta G} = 0.03 \times E^{-1} \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,1\,1\,1 \end{bmatrix}$$

and

$$F_{\Delta F_1} = F_{\Delta G} = diag\{F, F, F, F, F, F, F, F\}$$

in which $\|F(t)\| < 1$. Then, solving LMIs (9), (10), (11), and (12) in Theorem 1 based on Remark 1 results in

$$P = 10^7 \times \begin{bmatrix} 8.7092 & 1.7694 & 3.7088 \\ 1.7694 & 0.3723 & 0.7709 \\ 3.7088 & 0.7709 & 1.6062 \end{bmatrix}$$

$$K = 10^3 \times \begin{bmatrix} -2.1302 & -0.4317 & -0.9118 \\ 1.7068 & 0.3450 & 0.7266 \\ 2.1445 & 0.4353 & 0.9158 \\ -2.1520 & -0.4369 & -0.9202 \end{bmatrix}.$$

Figs. 1-5 demonstrates the robust waypoint tracking results when $(x(0), y(0), \psi(0), u(0), v(0), r(0)) = (0, 200, 0, 3, 0, 0)$ and

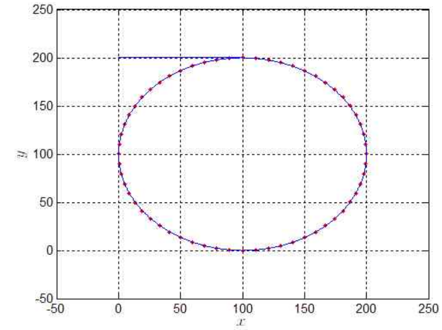

Fig. 1 The trajectories of (x, y) ('·': the desired waypoint).

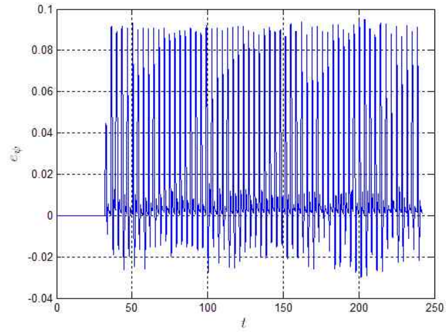

Fig. 2 The time response of e_ψ.

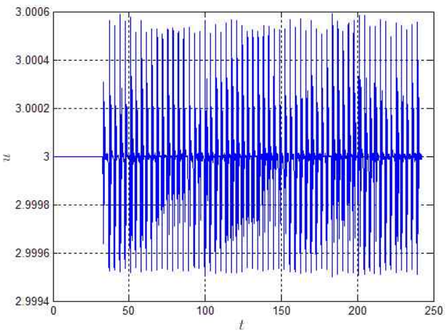

Fig. 3 The time response of u.

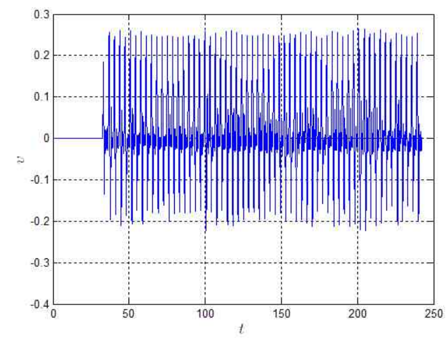

Fig. 4 The time response of v.

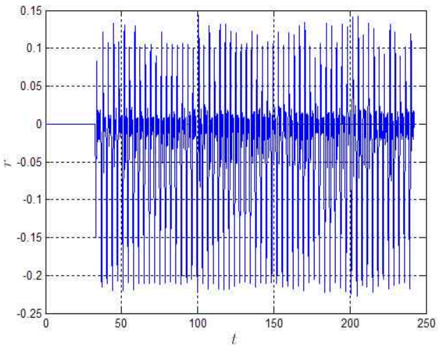

Fig. 5 The time response of r.

$|\delta_i| < 0.2374$. As shown in these figures, the LDUUV (1) and (2) under (5) and (6) successfully tracks all waypoints while keeping the desired surge speed u_d and its attitude.

4. Conclusions

This paper has proposed an LMI formulation of robust waypoint tracking problem for a class of uncertain LDUUVs while keeping its surge speed and attitude. Differently from the prior LMI approach [7], the proposed approach incorporates uncertainties on the hydrodynamic coefficients into design. The theoretical claims has been successfully verified in the given numerical example.

Appendix 1

This paper uses the following hydrodynamic coefficients, obtained by CFD and empirical formulations, of LIG Nex1 LDUUV model:

$$m = 7500,\ I_{zz} = 27081.3,\ x_g = 0,\ L = 6.5,\ u_d = 3,\ X_{\dot u} = -157.7122$$

$$X_{uu} = -62.0022,\ X_{vv} = 515.6301,\ X_{rr} = -5.6269\times 10^3$$

$$X_{uu\delta\delta} = -228.0492,\ Y_{\dot v} = -2.8612\times 10^3,\ Y_{\dot r} = 417.3126$$

$$Y_v = -2.9887\times 10^3,\ Y_{v|v|} = -3.1868\times 10^3,\ Y_r = 2.7937\times 10^4$$

$$Y_{r|r|} = -1.3259\times 10^3,\ Y_{uu\delta_1} = -591.0686,\ Y_{uu\delta_2} = 591.0686$$

$$Y_{uu\delta_3} = 591.0686,\ Y_{uu\delta_4} = -591.0686,\ N_{\dot v} = 417.3126$$

$$N_{\dot r} = -9.0809\times 10^3,\ N_v = -1.1884\times 10^4,\ N_{v|v|} = 6.9397\times 10^3$$

$$N_r = -2.1449\times 10^4,\ N_{r|r|} = -1.7355\times 10^4,\ N_{uu\delta_1} = 1.4740\times 10^3$$

$$N_{uu\delta_2} = -1.4740\times 10^3,\ N_{uu\delta_3} = -1.4740\times 10^3,$$

$$N_{uu\delta_4} = 1.4740\times 10^3.$$

Appendix 2

$$F_1 = E^{-1}\begin{bmatrix} \dfrac{u_d \mathrm{sin} e_\psi}{e_\psi \| e_{xy} \|} & \dfrac{\mathrm{cos} e_\psi}{\| e_{xy} \|} & 1 \\ 0 & Y_v + Y_{v|v|}|v| & \begin{pmatrix} -mu_d + Y_r \\ +Y_{r|r|}|r| \end{pmatrix} \\ 0 & N_v + N_{v|v|}|v| & \begin{pmatrix} -mx_g u_d + N_r \\ +N_{r|r|}|r| \end{pmatrix} \end{bmatrix}$$

$$\Delta F_1 = E^{-1}\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta Y_v + \Delta Y_{v|v|}|v| & \Delta Y_r + \Delta Y_{r|r|}|r| \\ 0 & \Delta N_v + \Delta N_{v|v|}|v| & \Delta N_r + \Delta N_{r|r|}|r| \end{bmatrix}$$

$$F_2 = E^{-1}\begin{bmatrix} \dfrac{\mathrm{sin} e_\psi}{\| e_{xy} \|} \\ -mr + (e_u + 2u_d)\sum_{i=1}^{4} Y_{uu\delta_i}\delta_i \\ -mx_g r + (e_u + 2u_d)\sum_{i=1}^{4} N_{uu\delta_i}\delta_i \end{bmatrix}$$

$$\Delta F_2 = E^{-1}\begin{bmatrix} 0 \\ (e_u + 2u_d)\sum_{i=1}^{4} \Delta Y_{uu\delta_i}\delta_i \\ (e_u + 2u_d)\sum_{i=1}^{4} \Delta N_{uu\delta_i}\delta_i \end{bmatrix}$$

$$F_3 = \frac{1}{m_1}\left[0 \quad X_{vv}v \quad mv + mx_g r + X_{rr} r\right]$$

$$F_4 = \frac{1}{m_1}\left(X_{uu}e_u + 2X_{uu}u_d + X_{uu\delta\delta}(e_u + 2u_d)\Sigma_{i=1}^4 \delta_i^2\right)$$

$$F_5 = \frac{1}{m_1}\left(X_{uu}u_d^2 + X_{uu\delta\delta}u_d^2\Sigma_{i=1}^4 \delta_i^2\right)$$

$$G = E^{-1}\begin{bmatrix} 0 & 0 & 0 & 0 \\ u_d^2 Y_{uu\delta_1} & u_d^2 Y_{uu\delta_2} & u_d^2 Y_{uu\delta_3} & u_d^2 Y_{uu\delta_4} \\ u_d^2 N_{uu\delta_1} & u_d^2 N_{uu\delta_2} & u_d^2 N_{uu\delta_3} & u_d^2 N_{uu\delta_4} \end{bmatrix}$$

$$\Delta G = E^{-1}\begin{bmatrix} 0 & 0 & 0 & 0 \\ u_d^2 \Delta Y_{uu\delta_1} & u_d^2 \Delta Y_{uu\delta_2} & u_d^2 \Delta Y_{uu\delta_3} & u_d^2 \Delta Y_{uu\delta_4} \\ u_d^2 \Delta N_{uu\delta_1} & u_d^2 \Delta N_{uu\delta_2} & u_d^2 \Delta N_{uu\delta_3} & u_d^2 \Delta N_{uu\delta_4} \end{bmatrix},\quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & m_2 & m_3 \\ 0 & m_4 & m_5 \end{bmatrix}.$$

Appendix 3

$$\theta_{11} = \frac{\dfrac{\mathrm{sin} e_\psi}{e_\psi \|e_{xy}\|} - a_{12}}{a_{11} - a_{12}},\quad \theta_{21} = \frac{\dfrac{\mathrm{cos} e_\psi}{\|e_{xy}\|} - a_{22}}{a_{21} - a_{22}},\quad \theta_{31} = \frac{|v| - a_{32}}{a_{31} - a_{32}}$$

$$\theta_{41} = \frac{|r| - a_{42}}{a_{41} - a_{42}},\quad \theta_{i2} = 1 - \theta_{i1},\ i \in \mathcal{J}_4,$$

$$a_{11} = \sup_{e_\psi \in \{|e_\psi| \le \pi\}} \frac{\mathrm{sin} e_\psi}{e_\psi \|e_{xy}\|},\ a_{12} = \inf_{e_\psi \in \{|e_\psi| \le \pi\}} \frac{\mathrm{sin} e_\psi}{e_\psi \|e_{xy}\|}$$

$$a_{21} = \sup_{e_\psi \in \{|e_\psi| \le \pi\}} \frac{\mathrm{cos} e_\psi}{\|e_{xy}\|},\ a_{22} = \inf_{e_\psi \in \{|e_\psi| \le \pi\}} \frac{\mathrm{cos} e_\psi}{\|e_{xy}\|}$$

$$a_{31} = \sup_{v \in B_v} |v|,\ a_{32} = \inf_{v \in B_v} |v|$$

$$a_{41} = \sup_{r \in B_r} |r|,\ a_{42} = \inf_{r \in B_r} |r|$$

$$A_{i_1 i_2 i_3 i_4} = E^{-1} \begin{bmatrix} u_d a_{1i_1} & a_{2i_2} & 1 \\ 0 & Y_v + Y_{v|v|} a_{3i_3} & \begin{pmatrix} -\mu_d + Y_r \\ + Y_{r|r|} a_{4i_4} \end{pmatrix} \\ 0 & N_v + N_{v|v|} a_{3i_3} & \begin{pmatrix} -m x_g u_d + N_r \\ + N_{r|r|} a_{4i_4} \end{pmatrix} \end{bmatrix}$$

$$D_{\Delta F_1} = E^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{Y_v} & D_{Y_{v|v|}} & D_{Y_r} & D_{Y_{r|r|}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{N_v} & D_{N_{v|v|}} & D_{N_r} & D_{N_{r|r|}} \end{bmatrix}$$

$$F_{\Delta F_1} = \mathrm{diag}\left\{ F_{Y_v}, F_{Y_{v|v|}}, F_{Y_r}, F_{Y_{r|r|}}, F_{N_v}, F_{N_{v|v|}}, F_{N_r}, F_{N_{r|r|}} \right\}$$

$$D_{\Delta G} = E^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D_{Y_{uu\delta_1}} & D_{Y_{uu\delta_2}} & D_{Y_{uu\delta_3}} & D_{Y_{uu\delta_4}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{N_{uu\delta_1}} & D_{N_{uu\delta_2}} & D_{N_{uu\delta_3}} & D_{N_{uu\delta_4}} \end{bmatrix}$$

$$F_{\Delta G} = \mathrm{diag}\left\{ F_{Y_{uu\delta_1}}, F_{Y_{uu\delta_2}}, F_{Y_{uu\delta_3}}, F_{Y_{uu\delta_4}}, F_{N_{uu\delta_1}}, F_{N_{uu\delta_2}}, F_{N_{uu\delta_3}}, F_{N_{uu\delta_4}} \right\}$$

$$\tilde{A}_{i_3 i_4} = \begin{bmatrix} 0 & Y_v & 0 \\ 0 & Y_{v|v|} a_{3i_3} & 0 \\ 0 & 0 & Y_r \\ 0 & 0 & Y_{r|r|} a_{4i_4} \\ 0 & N_v & 0 \\ 0 & N_{v|v|} a_{3i_3} & 0 \\ 0 & 0 & N_r \\ 0 & 0 & N_{r|r|} a_{4i_4} \end{bmatrix},\ \tilde{G} = \begin{bmatrix} u_d^2 Y_{uu\delta_1} & 0 & 0 & 0 \\ 0 & u_d^2 Y_{uu\delta_2} & 0 & 0 \\ 0 & 0 & u_d^2 Y_{uu\delta_3} & 0 \\ 0 & 0 & 0 & u_d^2 Y_{uu\delta_4} \\ u_d^2 N_{uu\delta_1} & 0 & 0 & 0 \\ 0 & u_d^2 N_{uu\delta_2} & 0 & 0 \\ 0 & 0 & u_d^2 N_{uu\delta_3} & 0 \\ 0 & 0 & 0 & u_d^2 N_{uu\delta_4} \end{bmatrix}.$$

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