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X-stern 배열을 가진 대형급 무인잠수정의 경로점 추적

Waypoint Tracking of Large Diameter Unmanned Underwater Vehicles with X-stern Configuration

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Abstract - This paper focuses on a horizontal waypoint tracking and a speed control of large diameter unmanned underwater vehicles (LDUUVs) with X-stern configuration plane. The concerned design problem is converted into an asymptotic stabilization of the error dynamics with respect to the desired yaw angle and surge speed. It is proved that the error dynamics under the proposed control scheme based on the linear control and the feedback linearization can be considered as a cascade system; the cascade system is asymptotically stable if its nominal systems are so. This stability connection enables to separately deal with the waypoint tracking problem and the speed control one. By using the sector nonlinearity, the nominal system with nonlinearities is modeled as a polytopic linear parameter varying (LPV) system with parametric uncertainties. Then, sufficient linear matrix inequality (LMI) conditions for its asymptotic stabilizability are derived in the sense of Lyapunov stability criterion. An example is given to show the validity of the proposed methodology.

Key Words : Unmanned Underwater Vehicles (UUV), X-stern, Lyapunov, Linear Matrix Inequality (LMI), Waypoint, Asymptotic stability

1. Introduction

In general, the control surface of unmanned underwater vehicles (UUVs) fall into two configurations, the cruciform stern arrangement and X-stern one. For the waypoint tracking of UUVs with the cruciform stern, several design methodologies have been introduced in the literatures, such as sliding model control [1,2], model predictive control [3], backstepping technique [4-6], linear matrix inequality (LMI)-based design approach [7]. However, contrary to various research efforts in the case of the cruciform stern, there is a lack of control studies on UUVs with X-stern regardless of the improved control effectiveness of large diameter UUVs (LDUUVs) [8].

This paper presents an LMI approach to horizontal waypoint tracking and the surge speed control problems for a class of LDUUVs via linear and nonlinear feedback controls. The interested problem becomes an asymptotic stabilization of the error dynamics with respect to the desired yaw angle and surge speed. Contrary to the previous LMI result [7], the proposed approach is based on not the cruciform stern but X-stern. Moreover, when $u=0$, the control input, the rudder angle presented in [7] may make the given system unstable since it depends on $1/u$, where u is the surge velocity. It is proved that under the proposed control scheme, the tracking and speed control problems can be tackled separately. Based on this property, sufficient LMI conditions for asymptotic stabilization of the error dynamics are derived in the sense of Lyapunov stability criterion. Finally, simulation results are provided for showing the effectiveness of the proposed theoretical claims.

Notations: The relation $P>Q$ ($P<Q$) means that the matrix $P-Q$ is positive (negative) definite. $\lambda_{max}(A)$ ($\lambda_{min}(A)$) is the maximum (minimum) eigenvalue of matrix A. $A_{(i)}$ denotes ith row of the matrix A. col{ · } means for a matrix column with blocks given by the matrices in { · }. Sym{S} is defined as S^T+S. B_η indicates the ball $\{\eta : \|\eta\| \le \Delta_\eta\}$ with $\Delta_\eta \in R_{>0}$. I_m indicates the integer set $\{1,\cdots,m\}$.

2. Preliminaries

The mathematical modeling for the steering motion of the UUV can be achieved in the global reference frame with (x, y, ψ) and the body fixed coordinate frame with (u, v, r), where (x, y) is the inertial coordinates of the center of mass, ψ is the yaw angle, u and v mean the surge and the sway

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velocities, respectively, and r is the angular velocity in yaw (see [9-11] for more details).

Assumption 1 ([9-11]) : Assume that

i. the vehicle is the top-bottom (xy-plane) and port-starboard (xz-plane) symmetry;
ii. the vehicle is equipped with the X-stern and a propeller to provide the forward thrust;
iii. the origin of the body-fixed frame is same to the center of gravity of the vehicle;
iv. the center of gravity coincides with the center of buoyancy;
v. any damping terms greater than second-order are negligible;
vi. the heave, pitch, and roll motions can be neglected;
vii. the y-position of the center of gravity is negligible.

Under Assumption 1 and the definitions $\eta := [x\, y\, \psi]^T \in R^3$ and $\phi := [u\, v\, r]^T \in R^3$, the kinematics and dynamics of UUV are represented by

$$\dot{\eta} = J(\psi)\phi \tag{1}$$

$$M\dot{\phi} + f(\phi)\phi = \tau \tag{2}$$

where $J \in R^{3\times 3}$ is a frame transformation, $M \in R^{3\times 3}$ includes mass and hydrodynamic added mass terms and $f(\phi)\phi \in R^3$ captures Coriolis-centripetal matrices including the added mass and a damping matrix, τ is the control actuator forces with the propeller thrust ξ, and δ_i, $i \in I_4$ is the rudder angles defined as

$$\tau = \begin{bmatrix} \xi + X_{uu\delta\delta} u^2 \sum_{i=1}^{4} \delta_i^2 \\ u^2 \sum_{i=1}^{4} Y_{uu\delta_i} \delta_i \\ u^2 \sum_{i=1}^{4} N_{uu\delta_i} \delta_i \end{bmatrix} \in R^3.$$

The matrices J, M, and f are given by

$$J = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & m_3 \\ 0 & m_4 & m_5 \end{bmatrix}$$

$$f = \begin{bmatrix} -X_{uu}u & -X_{vv}v & -mv - mx_g r - X_{rr} r \\ 0 & -Y_v - Y_{v|v|}|v| & \mu - Y_r - Y_{r|r|}|r| \\ 0 & -N_v - N_{v|v|}|v| & mx_g u - N_r - N_{r|r|}|r| \end{bmatrix}.$$

in which $m_1 = m - X_{\dot{u}}$, $m_2 = m - Y_{\dot{v}}$, $m_3 = mx_g - Y_{\dot{r}}$, $m_4 = mx_g - N_{\dot{v}}$, $m_5 = I_{zz} - N_{\dot{r}}$, m is the vehicle mass, x_g is the x-position of the center of gravity, I_{zz} is the mass moment of inertia term. The related coefficients are listed in Appendix 1, which are obtained by computational fluid dynamics (CFD) and empirical formulations based on hull geometry for LIG Nex1 LDUUV.

Problem 1: Consider UUV described by (1) and (2), its desired constant forward speed $u_d \in R_{>0}$, and its desired line of sight to be the horizontal plane angle defined as

$$\psi_d = \tan^{-1}\frac{e_y}{e_x} \tag{3}$$

where $e_x := x - x_{dk}$, $e_y := y - y_{dk}$, and $(x_{dk}, y_{dk}), k \in I_{n_w}$ is the k th given waypoint. Design ξ and δ_i, $i \in I_4$ such that both $\|e_u\|$ and $\|e_\psi\|$ asymptotically converge to zero, where $e_u := u - u_d$ and $e_\psi := \psi - \psi_d$.

Remark 1: From the UUV with the X-stern (1) and (2), setting $X_{uu\delta\delta} = 0$, $Y_{uu\delta_i} = Y_{uu\delta}$, $N_{uu\delta_i} = N_{uu\delta}$, and $\delta_i = \delta$ for $i \in I_4$ all simplifies an UUV with cruciform stern.

3. Main Results

Before proceeding to our main results, the following propositions and lemma will be needed throughout the proof:

Proposition 1: Consider (1) and (2). Define $\chi := \mathrm{col}\{e_\psi, v, r\} \in R^3$ and $\delta := \mathrm{col}\{\delta_1, \delta_2, \delta_3, \delta_4\} \in R^4$, an augmented error system is described by

$$\begin{bmatrix} \dot{\chi} \\ \dot{e}_u \end{bmatrix} = \begin{bmatrix} F_1(e_\psi, e_x, e_y, v, r) & F_2(e_\psi, e_x, e_y, r, \delta) \\ F_3(v, r) & F_4(e_u, \delta) \end{bmatrix} \begin{bmatrix} \chi \\ e_u \end{bmatrix} + \begin{bmatrix} 0 \\ F_5(\delta) \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & \frac{1}{m_1} \end{bmatrix} \begin{bmatrix} \delta \\ \xi \end{bmatrix} \tag{4}$$

where

$$F_1 = E^{-1} \begin{bmatrix} \frac{u_d \sin e_\psi}{e_\psi \|\mathrm{col}\{e_x, e_y\}\|} & \frac{\cos e_\psi}{\|\mathrm{col}\{e_x, e_y\}\|} & 1 \\ 0 & Y_v + Y_{v|v|}|v| & \begin{pmatrix} -\mu_d + Y_r \\ + Y_{r|r|}|r| \end{pmatrix} \\ 0 & N_v + N_{v|v|}|v| & \begin{pmatrix} -mx_g u_d + N_r \\ + N_{r|r|}|r| \end{pmatrix} \end{bmatrix}$$

$$F_2 = E^{-1} \begin{bmatrix} \frac{\sin e_\psi}{\|\mathrm{col}\{e_x, e_y\}\|} \\ -mr + (e_u + 2u_d) \sum_{i=1}^{4} Y_{uu\delta_i}\delta_i \\ -mx_g r + (e_u + 2u_d) \sum_{i=1}^{4} N_{uu\delta_i}\delta_i \end{bmatrix}$$

$$F_3 = \frac{1}{m_1}\begin{bmatrix} 0 & X_{vv} v & mv + mx_g r + X_{rr} r \end{bmatrix}$$

$$F_4 = \frac{1}{m_1}\left(X_{uu} e_u + 2X_{uu} u_d + X_{uu\delta\delta}(e_u + 2u_d)\Sigma_{i=1}^4 \delta_i^2\right)$$

$$F_5=\frac{1}{m_1}\left(X_{uu}u_d^2+X_{uu\delta\delta}u_d^2\Sigma_{i=1}^4\delta_i^2\right)$$

$$G=E^{-1}\begin{bmatrix}0 & 0 & 0 & 0\\ u_d^2Y_{uu\delta_1} & u_d^2Y_{uu\delta_2} & u_d^2Y_{uu\delta_3} & u_d^2Y_{uu\delta_4}\\ u_d^2N_{uu\delta_1} & u_d^2N_{uu\delta_2} & u_d^2N_{uu\delta_3} & u_d^2N_{uu\delta_4}\end{bmatrix},\ E=\begin{bmatrix}1 & 0 & 0\\ 0 & m_2 & m_3\\ 0 & m_4 & m_5\end{bmatrix}.$$

Proof: Differentiating (3), substituting (1) into it, and using $\dot{e}_\psi=\dot{\psi}-\dot{\psi}_d$ and (2) yield

$$\dot{e}_\psi=\frac{u_d\mathrm{sin}e_\psi}{\|\mathrm{col}\{e_x,e_y\}\|}+\frac{\mathrm{cos}e_\psi}{\|\mathrm{col}\{e_x,e_y\}\|}v+r+\frac{\mathrm{sin}e_\psi}{\|\mathrm{col}\{e_x,e_y\}\|}e_u$$

(see [7] for more details). From (2) and e_u, we see that

$$\begin{bmatrix}\dot{v}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}Y_v+Y_{v|v|}|v| & -m(e_u+u_d)+Y_r+Y_{r|r|}|r|\\ N_v+N_{v|v|}|v| & -mx_g(e_u+u_d)+N_r+N_{r|r|}|r|\end{bmatrix}\begin{bmatrix}v\\ r\end{bmatrix}$$
$$+\begin{bmatrix}m_2 & m_3\\ m_4 & m_5\end{bmatrix}^{-1}\begin{bmatrix}\left(e_u^2+2u_de_u+u_d^2\right)\sum_{i=1}^4Y_{uu\delta_i}\delta_i\\ \left(e_u^2+2u_de_u+u_d^2\right)\sum_{i=1}^4N_{uu\delta_i}\delta_i\end{bmatrix}$$

and

$$\dot{e}_u=\frac{1}{m_1}\left(X_{uu}e_u^2+2X_{uu}e_uu_d+X_{uu}u_d^2+X_{vv}v^2+mvr+mx_gr^2\right.$$
$$\left.+X_{rr}r^2+\xi+X_{uu\delta\delta}\left(e_u^2+2u_de_u+u_d^2\right)\Sigma_{i=1}^4\delta_i^2\right).$$

Taking the change of variables, we have an augmented error system becomes (4). ■

Proposition 2: Consider (4). Let

$$\delta=K\chi \tag{5}$$

$$\xi=-m_1F_3(v,r)\chi-m_1F_4(e_u,\delta_1,\delta_2,\delta_3,\delta_4)e_u$$
$$-m_1F_5(\delta_1,\delta_2,\delta_3,\delta_4)-m_1\gamma e_u \tag{6}$$

where K is the controller gain to be determined and $\gamma\in R_{>0}$. Then the closed-loop system takes the following cascade form:

$$\Sigma_1:\ \dot{\chi}=\left(F_1(e_\psi,e_x,e_y,v,r)+GK\right)\chi+F_2(e_\psi,e_x,e_y,r)e_u \tag{7}$$

$$\Sigma_2:\ \dot{e}_u=-\gamma e_u \tag{8}$$

Proof: The proof directly follows from [Lemma 1, 7].

Proposition 3: It is true that

$$F_1(e_\psi,e_x,e_y,v,r)=\sum_{i_1}^2\sum_{i_2}^2\sum_{i_3}^2\sum_{i_4}^2\theta_{1i_1}\theta_{2i_2}\theta_{3i_3}\theta_{4i_4}A_{i_1i_2i_3i_4}$$

on $B_v\times B_r$, where θ_{1i_1}, θ_{2i_2}, θ_{3i_3}, θ_{4i_4}, and $A_{i_1i_2i_3i_4}$ are given in Appendix 2.

Proof: The proof directly follows from [Theorem 2, 7].

Lemma 1: There exists $\zeta\in R_{>0}$ such that

$$\|F_2\|\le\zeta$$

on $B_r\times B_{\delta_1}\times B_{\delta_2}\times B_{\delta_3}\times B_{\delta_4}$.

Proof: It is not hard to see that choosing

$$\zeta\ge\|E^{-1}\|\left\|\begin{bmatrix}\frac{1}{\rho}\\ m\Delta_r+|e_u+2u_d|\Sigma_{i=1}^4|Y_{uu\delta_i}|\Delta_{\delta_i}\\ mx_g\Delta_r+|e_u+2u_d|\Sigma_{i=1}^4|N_{uu\delta_i}|\Delta_{\delta_i}\end{bmatrix}\right\|$$

implies

$$\|F_2\|\le\|E^{-1}\|\left\|\begin{bmatrix}\frac{\mathrm{sin}e_\psi}{\|\mathrm{col}\{e_x,e_y\}\|}\\ -mr+(e_u+2u_d)\Sigma_{i=1}^4Y_{uu\delta_i}\delta_i\\ -mx_gr+(e_u+2u_d)\Sigma_{i=1}^4N_{uu\delta_i}\delta_i\end{bmatrix}\right\|\le\zeta$$

on B_r. ■

The following Theorem gives an LMI formulation of the Problem 1:

Theorem 1: Consider (1) and (2) under (5) and (6) and Propositions 1 and 2. Let $\Omega:=\{\mathrm{col}\{\chi,e_u\}\in R^4:\ \chi^TP\chi+\alpha e_u^2\langle 1\}$. Suppose that i) $\Omega\subset B_v\times B_r$; ii) there exist $\tilde{P}=\tilde{P}^T>0$ and $\tilde{K}$ such that

$$\mathrm{Sym}\left\{A_{i_1i_2i_3i_4}\tilde{P}+G\tilde{K}\right\}<0 \tag{9}$$

$$E_v\tilde{P}E_v^T<\Delta_v^2 \tag{10}$$

$$E_r\tilde{P}E_r^T<\Delta_r^2 \tag{11}$$

$$\begin{bmatrix}\Delta_{\delta_i}^2 & \tilde{K}_{(i)}\\ \tilde{K}_{(i)}^T & -\tilde{P}\end{bmatrix}<0 \tag{12}$$

for all $(i_1,i_2,i_3,i_4,i)\in I_2\times I_2\times I_2\times I_2\times I_4$. Then, for, $\|e_u\|$, $\|e_\psi\|$, $\|v\|$, and $\|r\|$ of (1) and (2) under (5) and (6) asymptotically converge to zero as $t\to\infty$. In this feasible case, $P=\tilde{P}^{-1}$ and $K=\tilde{K}\tilde{P}^{-1}$.

Proof: Consider a Lyapunov function

$$V(\chi,e_u)=\chi^TP\chi+\alpha e_u^2$$

for $\alpha\in R_{>0}$. Its derivative becomes

$$\dot{V}|_{(7),(8)}=\mathrm{Sym}\left\{\chi^TP\left(F_1(e_\psi,e_x,e_y,v,r)+GK\right)\chi\right\}$$
$$+2\chi^TPF_2(e_\psi,e_x,e_y,r)e_u-2\alpha\gamma e_u^2.$$

From Proposition 3, we see that

$$\dot{V}_{(7),(8)} = \sum_{i_1}^{2}\sum_{i_2}^{2}\sum_{i_3}^{2}\sum_{i_4}^{2}\theta_{1i_1}\theta_{2i_2}\theta_{3i_3}\theta_{4i_4}\mathrm{Sym}\{\chi^T P(A_{i_1i_2i_3i_4}+GK)\chi\} + 2\chi^T PF_2(e_\psi,e_x,e_y,r)e_u - 2\alpha\gamma e_u^2$$

on $B_v \times B_r$. Because it is true that by congruence transformation and definitions $\tilde{P}:=P^{-1}$ and $\tilde{K}:=KP^{-1}$, LMI (9)$\Leftrightarrow P(A_{i_1i_2i_3i_4}+GK)<0$ on $(i_1,i_2,i_3,i_4)\in I_2\times I_2\times I_2\times I_2$, there exists $Q=Q^T>0$ such that

$$\dot{V}_{(7),(8)} < -\chi^T Q\chi + 2\chi^T PF_2(e_\psi,e_x,e_y,r)e_u - 2\alpha\gamma e_u^2.$$

Also, from Lemma 1 on $B_r\times B_{\delta_1}\times B_{\delta_2}\times B_{\delta_3}\times B_{\delta_4}$, we see that

$$\dot{V}_{(7),(8)} < -\chi^T Q\chi + 2\chi^T\zeta Pe_u - 2\alpha\gamma e_u^2 = \begin{bmatrix}\chi\\ e_u\end{bmatrix}^T\begin{bmatrix}-Q & \zeta P\\ \zeta P & -2\alpha\gamma\end{bmatrix}\begin{bmatrix}\chi\\ e_u\end{bmatrix}$$

Here, it can be shown that by Schur complement,

$$\dot{V}_{(7),(8)}<0 \Leftarrow \begin{bmatrix}-Q & \zeta P\\ \zeta P & -2\alpha\gamma\end{bmatrix}<0 \Leftrightarrow \begin{cases}\frac{\zeta^2}{2\alpha\gamma}P^2 - Q < 0\\ 2\alpha\gamma > 0\}\end{cases}$$

Choosing $\alpha\in R > \frac{\zeta^2}{2\gamma\lambda_{\min}(P^{-1}QP^{-1})}$ implies that the closed-loop system (7) and (8) is asymptotically stable on $\Omega\subset B_v\times B_r\times B_{\delta_1}\times B_{\delta_2}\times B_{\delta_3}\times B_{\delta_4}$.

Now, we are in a position to derive LMI conditions to guarantee $\Omega\subset B_v\times B_r\times B_{\delta_1}\times B_{\delta_2}\times B_{\delta_3}\times B_{\delta_4}$. It can be shown that by Schur complement,

$$(\chi,e_u)\in\Omega \Rightarrow \chi^T P\chi<1 \Leftrightarrow \chi\chi^T<P^{-1}$$

and $\chi\chi^T<P^{-1}$ implies $v^2<E_vP^{-1}E_v^TEE$, $r^2<E_rP^{-1}E_r^T$, and $\delta_i^2<K_{(i)}P^{-1}K_{(i)}^T$. Thus, if LMIs (10), (11), and (12) hold, then $\Omega\subset B_v\times B_r\times B_{\delta_1}\times B_{\delta_2}\times B_{\delta_3}\times B_{\delta_4}$.

Consequently, if LMIs (9), (10), (11), and (12) hold, then for $\mathrm{col}\{\chi(t_0),e_u(t_0)\}\in\Omega$, $\|e_u\|$, $\|e_\psi\|$, v, and r of (1) and (2) under (5) and (6) asymptotically converge to zero as $t\to\infty$. Moreover, we know that the asymptotic stability of (7) is ensured if its nominal system

$$\Sigma_1': \ \dot{\chi}=(F_1(e_\psi,e_x,e_y,v,r)+GK)\chi$$

and (8) are asymptotically stable. ■

Remark 2: In [7], the control input δ, the rudder angle depends on $1/u$, and thereby, $u=0$ may make the given UUV unstable. Contrary to [7], the proposed approach does not have this problem since (5) is independent on u.

4. An Example

Consider (1) and (2) under (5) and (6). Given $u_d=3$ and

$$(x_{dk},y_{dk})=\begin{cases}(100(\sin\varrho_1(k)+1),100(\cos\varrho_1(k)+1)), & \text{if } k\le 31;\\ (100(\sin\varrho_2(k)+1),100(-\cos\varrho_2(k)+3)), & otherwise\end{cases}$$

on $k\in I_{60}$, where $\varrho_1(k)=2\pi(k-1)/30$ and $\varrho_2(k)=2\pi(k-31)/30$, our design goal is to design K in (5) so that $\|e_u\|$, $\|e_\psi\|$, $\|v\|$, and $\|r\|$ of (1) and (2) asymptotically converge to zero as $t\to\infty$. The criteria for updating the waypoint is to shift from (x_{dk},y_{dk}) to $(x_{d(k+1)},y_{d(k+1)})$ when $\|\mathrm{col}\{e_{xk},e_{yk}\}\|\le 2$. By solving LMIs (9), (10), (11), and (12) in Theorem 1 with

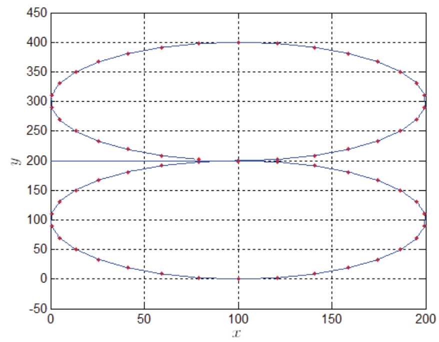

Fig. 1 The trajectories of (x,y) ('·': the desired waypoint).

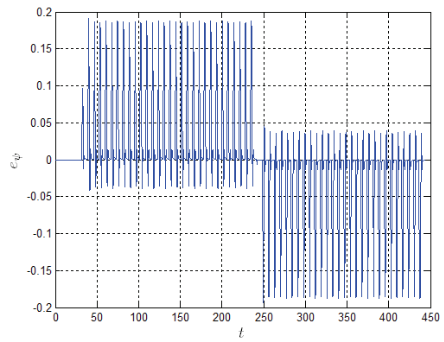

Fig. 2 The time response of e_ψ.

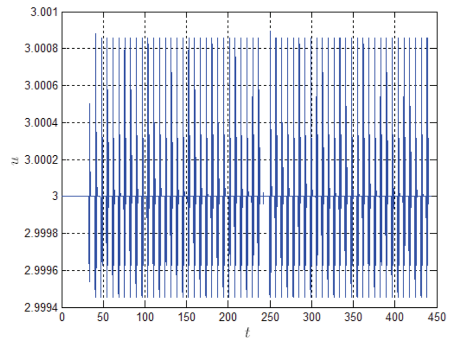

Fig. 3 The time response of u.

$\Delta_v = 10$, $\Delta_r = 1$, and $\Delta_{\delta_i} = 0.2374$, $i \in I_4$, we have

$$P = 10^4 \times \begin{bmatrix} 6.2091 & 1.2391 & 2.5489 \\ 1.2391 & 0.3603 & 0.6493 \\ 2.5489 & 0.6493 & 1.2637 \end{bmatrix},\ K = \begin{bmatrix} -51.3974 & -9.5864 & -22.3875 \\ 51.3974 & 9.5864 & 22.3875 \\ 51.3974 & 9.5864 & 22.3875 \\ -51.3974 & -9.5864 & -22.3875 \end{bmatrix}.$$

Figs. 1-10 show the simulation results when $\gamma = 10$ in (6) and $(x(0), y(0), \psi(0), u(0), v(0), r(0)) = (0, 200, 0, 3, 0, 0)$. In this simulation, the control input $|\delta_i|$ is limited to 0.2374. Figs. 1-5 demonstrate the trajectories (x, y) and the time responses of (e_ψ, u, v, r) of the UUV (1) and (2) under the proposed controller (5) and (6), respectively. Figs. 6-10 provide its control inputs. From these figures, UUV tracks (x_{dk}, y_{dk}), $k \in I_{60}$

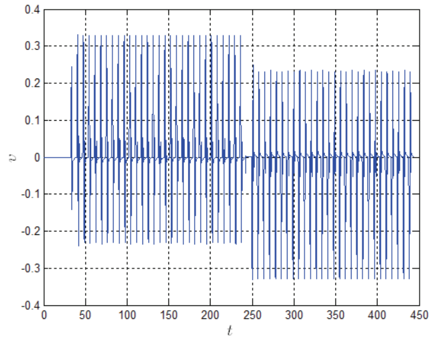

Fig. 4 The time response of v.

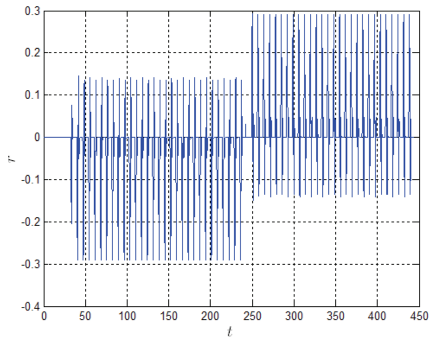

Fig. 5 The time response of r.

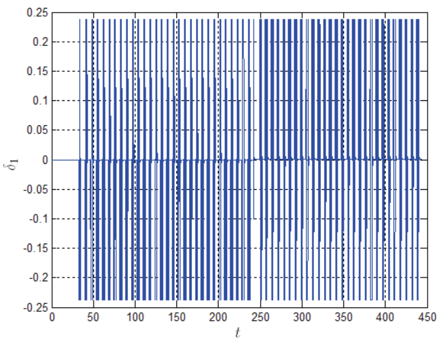

Fig. 6 The control input δ_1.

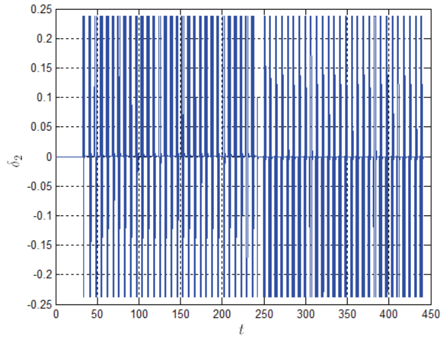

Fig. 7 The control input δ_2.

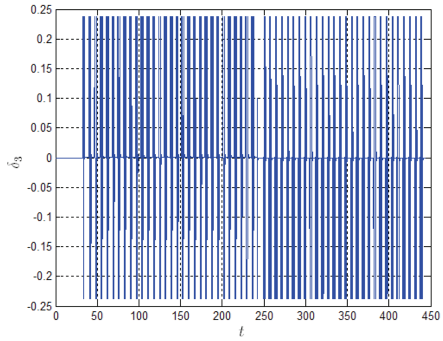

Fig. 8 The control input δ_3.

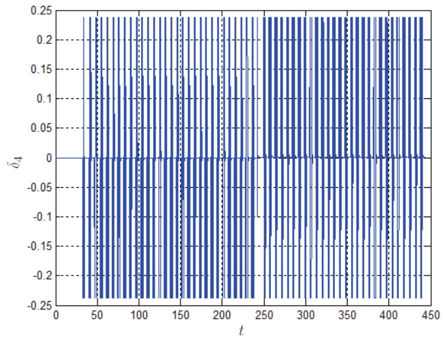

Fig. 9 The control input δ_4.

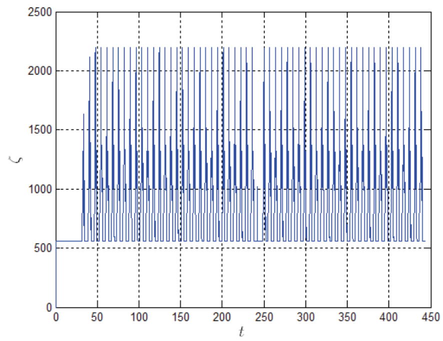

Fig. 10 The control input ζ.

successfully while keeping $u=u_d=3$ under the prosed controller (5) and (6). Moreover, we know the asymptotic convergence of $\|v\|$, $\|r\|$, and $\|e_\psi\|$.

4. Conclusions

This paper has proposed an LMI-based design approach to waypoint tracking problem for a class of large UUVs while maintaining its constant surge speed via feedback controls. The proposed results build on the X-stern configuration, rather than cruciform-stern one, and ensure its asymptotic stabilization. The validity of theoretical claims has been successfully checked by the numerical simulation.

Appendix 1

In this paper, we use hydro coefficients and parameters of LIG Nex1 LDUUV model. The coefficients are obtained by CFD and empirical formulations based on hull geometry.

$$m=7500,\ I_{zz}=27081.3,\ x_g=0,\ L=6.5,\ u_d=3,$$
$$X_{\dot{u}}=-157.7122,\ X_{uu}=-62.0022,\ X_{vv}=515.6301,$$
$$X_{rr}=-5.6269\times10^3,\ X_{uu\delta\delta}=-228.0492,\ Y_{\dot{v}}=-2.8612\times10^3,$$
$$Y_{\dot{r}}=417.3126,\ Y_v=-2.9887\times10^3,\ Y_{v|v|}=-3.1868\times10^3,$$
$$Y_r=2.7937\times10^4,\ Y_{r|r|}=-1.3259\times10^3,\ Y_{uu\delta_1}=-591.0686,$$
$$Y_{uu\delta_2}=591.0686,\ Y_{uu\delta_3}=591.0686,\ Y_{uu\delta_4}=-591.0686,$$
$$N_{\dot{v}}=417.3126,\ N_{\dot{r}}=-9.0809\times10^3,\ N_v=-1.1884\times10^4,$$
$$N_{v|v|}=6.9397\times10^3,\ N_r=-2.1449\times10^4,\ N_{r|r|}=-1.7355\times10^4,$$
$$N_{uu\delta_1}=1.4740\times10^3,\ N_{uu\delta_2}=-1.4740\times10^3,$$
$$N_{uu\delta_3}=-1.4740\times10^3,\ N_{uu\delta_4}=1.4740\times10^3.$$

Appendix 2

$$\theta_{11}\frac{\frac{\sin e_\psi}{e_\psi\|\mathrm{col}\{e_x,e_y\}\|}-a_{12}}{a_{11}-a_{12}},\theta_{12}=1-\theta_{11}$$

$$\theta_{21}=\frac{\frac{\cos e_\psi}{\|\mathrm{col}\{e_x,e_y\}\|}-a_{22}}{a_{21}-a_{22}},\ \theta_{22}=1-\theta_{21}$$

$$\theta_{31}=\frac{|v|-a_{32}}{a_{31}-a_{32}},\ \theta_{32}=1-\theta_{31},\ \theta_{41}=\frac{|r|-a_{42}}{a_{41}-a_{42}},\ \theta_{42}=1-\theta_{41}$$

$$A_{i_1i_2i_3i_4}=E^{-1}\begin{bmatrix} u_d a_{1i_1} & a_{2i_2} & 1 \\ 0 & Y_v+Y_{v|v|}a_{3i_3} & \begin{pmatrix}-\mu_d+Y_r\\+Y_{r|r|}a_{4i_4}\end{pmatrix} \\ 0 & N_v+N_{v|v|}a_{3i_3} & \begin{pmatrix}-mx_g u_d+N_r\\+N_{r|r|}a_{4i_4}\end{pmatrix}\end{bmatrix}$$

$$a_{11}=\sup_{e_\psi\in\{|e_\psi|\le\pi\}}\frac{\sin e_\psi}{e_\psi\mathrm{col}\{e_x,e_y\}},a_{12}=\inf_{e_\psi\in\{|e_\psi|\le\pi\}}\frac{\sin e_\psi}{e_\psi\mathrm{col}\{e_x,e_y\}}$$

$$a_{11}=\sup_{e_\psi\in\{|e_\psi|\le\pi\}}\frac{\cos e_\psi}{\mathrm{col}\{e_x,e_y\}},a_{12}=\inf_{e_\psi\in\{|e_\psi|\le\pi\}}\frac{\cos e_\psi}{\mathrm{col}\{e_x,e_y\}}$$

$$a_{31}=\sup_{v\in B_v}|v|,\ a_{32}=\inf_{v\in B_v}|v|$$

$$a_{41}=\sup_{r\in B_r}|r|,\ a_{42}=\inf_{r\in B_r}|r|$$

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