



Original Article

On the performance of heat absorption/generation and thermal stratification in mixed convective flow of an Oldroyd-B fluid

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ABSTRACT

This investigation explores the thermally stratified stretchable flow of an Oldroyd-B material bounded by a linear stretched surface. Heat transfer characteristics are addressed through thermal stratification and heat generation/absorption. Formulation is arranged for mixed convection. Application of suitable transformations provides ordinary differential systems through partial differential systems. The homotopy concept is adopted for the solution of nonlinear differential systems. The influence of several arising variables on velocity and temperature is addressed. Besides this, the rate of heat transfer is calculated and presented in tabular form. It is noticed that velocity and Nusselt number increase when the thermal buoyancy parameter is enhanced. Moreover, temperature is found to decrease for larger values of Prandtl number and heat absorption parameter. Comparative analysis for limiting study is performed and excellent agreement is found.

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1. Introduction

Recent developments in advanced technologies have pushed several researchers to report liquid flows that demand communication of numerous phenomena. The understanding of mixed convective flows has recently improved because these flows are encountered frequently in both nature and in engineering equipment, for example, in the ocean, in certain features of electronic cooling, in nuclear reactor technology, and in the movement of flow in the atmosphere [1,2]. Stretched flows with mixed convection have been reported by several investigators. For example, mixed convective stretchable flow of a radiating Maxwell liquid was reported by Hayat et al. [3,4]. Shehzad et al. [5] explored mixed convection and magnetohydrodynamic (MHD) impact in the stretchable flow of a thixotropic liquid. Simultaneous characteristics of joule heating and MHD in nonlinear dissipative mixed convective flow of micropolar liquid were elucidated by Waqas et al. [6]. Furthermore, heat transfer characteristics regarding heat absorption or generation phenomenon have significance in industrial and engineering procedures such as dilution and strengthening of copper wires, fertilization, filled bed reactors, waste stowage materials, disassociating liquids, and several metallurgical

procedures [7]. Several studies have been conducted on these phenomena [8–12].

The non-Newtonian materials are well recognized now in oil drilling, food processing biomechanics, plastic and paper making processes, and many other fields. These materials cannot be classified by one expression. Thus, several relations regarding non-Newtonian liquids have been presented [13–20]. In this work, we are interested in exploring the salient characteristics of Oldroyd-B liquid, which is a modified form of a Maxwell liquid that portrays the relaxation of stress through constant strain. However, memory impacts cannot be interpreted through Maxwell materials. Thus, to interpret the characteristics of memory and elasticity, the Oldroyd-B liquid model has been recommended. Several biological and polymeric materials are commonly used to illustrate the impacts of memory and elasticity. Moreover, this model is often used to report small relaxation/retardation times. In the absence of a retardation time factor, this model corresponds to the Maxwell liquid model. Also, the case of the classical Newtonian liquid model is recovered when relaxation/retardation factors are absent. Several investigations into the Oldroyd-B liquid model have been reported [21–26].

The abovementioned attempts clearly reveal that the aspect of thermal stratification has not been properly addressed. This aspect arises owing to the temperature difference, which gives rise to density variation in the medium. Examples of thermal stratification

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include lake thermohydraulics, geothermal systems, power plant contraction systems, geological transport, and volcanic flows. Kandasamy et al. [27] addressed the thermally stratified stretchable flow of magneto viscous nanoliquid by considering solar radiation. Cross diffusion and MHD aspects in dissipative stratified flow of viscous liquid were explored by Zaib and Shafie [28]. Hayat et al. [29,30] examined the thermal stratification characteristics in stretchable flows of Eyring–Powell and Jeffrey materials. Several investigations have already been conducted on heat transfer [31–33].

Keeping the abovementioned investigations in view, the intention of the current attempt is fourfold. First, to model and investigate the two-dimensional (2-D) flow of an Oldroyd-B liquid bounded by a linear stretchable surface; second, to report on the mixed convection aspect; third, to consider the effects of heat absorption/generation and thermal stratification; and fourth, using the homotopy scheme, to derive convergent series solutions for the velocity and temperature [34–44]. The contributions of the arising physical variables are interpreted and discussed in detail. Furthermore, the heat transfer rate has also been analyzed via numerical values.

2. Formulation

Here, the 2-D mixed convective flow of an incompressible Oldroyd-B liquid bounded by a linear stretchable surface is investigated. The sheet has linear velocity of $u_w(x) = cx$, where c denotes the stretching rate. Heat transfer in the presence of heat generation/absorption and thermal stratification is reported. The boundary layer concept is adopted for the whole treatment. With these assumptions in view, the continuity, momentum, and energy relations governing the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda_1 \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \nu \left[\frac{\partial^2 u}{\partial y^2} + \lambda_2 \left(u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} \right) \right] + g\beta_T(T - T_\infty), \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty), \tag{3}$$

subjected to the conditions

$$u = u_w(x) = cx, \quad v = 0, \quad T = T_w = T_0 + ax \text{ at } y = 0, \tag{4}$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty = T_0 + bx \text{ as } y \rightarrow \infty.$$

Here, the velocity components are denoted by (u, v) in (x,y) directions, respectively, with ν standing for kinematic viscosity, ρ for liquid density, c_p for specific heat, (λ_1, λ_2) for relaxation/retardation times, g for gravitational acceleration, β_T for thermal expansion coefficient, $\alpha = \left(\frac{k}{\rho c_p}\right)$ for the thermal diffusivity, k for the thermal conductivity, Q_0 for the uniform volumetric heat absorption/generation coefficient, (T, T_∞) for the fluid and ambient fluid temperatures, T_0 for the reference temperature, and (a, b) for the dimensional constants.

The following variables are used in order to transform Eqs. (2–4) into a system of dimensionless expressions:

$$\eta = y \sqrt{\frac{c}{\nu}}, \quad u = cx f'(\eta), \quad v = -\sqrt{c\nu} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0}. \tag{5}$$

The continuity expression is fulfilled identically and remaining expressions [i.e., (2)–(4)] have the final forms:

$$f'''' + ff'' - f'^2 + \beta_1 (2ff'f'' - f^2f''') + \beta_2 (f''^2 - ff''') + \lambda\theta = 0, \tag{6}$$

$$\theta'' + Pr(f\theta' - f'\theta) - PrSf' + Pr\delta\theta = 0, \tag{7}$$

$$f = 0, \quad f' = 1, \quad \theta = 1 - S \text{ at } \eta = 0, \tag{8}$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty.$$

Here, (β_1, β_2) represents the Deborah numbers, λ the mixed convection or thermal buoyancy parameter, Gr_x the Grashof number due to temperature, Re_x the local Reynolds number, Pr the Prandtl number, S the thermal stratified parameter, and δ the heat generation (>0) and absorption (<0). The results for viscous fluid are obtained when $\beta_1 = \beta_2 = 0$. These parameters have the following definitions:

$$\beta_1 = \lambda_1 c, \quad \beta_2 = \lambda_2 c, \quad \lambda = \frac{Gr_x}{Re_x^2} = \frac{g\beta_T a}{c^2}, \quad Gr_x = \frac{g\beta_T(T_w - T_0) x^3}{\nu^2},$$

$$Re_x = \frac{xu_w(x)}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad S = \frac{b}{a}, \quad \delta = \frac{Q_0}{\rho c_p c}. \tag{9}$$

The local Nusselt number Nu_x is expressed as:

$$Nu_x = \frac{xq_w}{k(T_w - T_0)}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}. \tag{10}$$

In non-dimensional variables, we have

$$Nu_x Re_x^{-1/2} = -\theta'(0). \tag{11}$$

3. Series solutions

To calculate series solutions of Expressions (6) and (7), we used the homotopic technique [34–44]. We have the following initial guesses and auxiliary linear operators:

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = (1 - S) \exp(-\eta), \tag{12}$$

$$L_f = f'''' - f', \quad L_\theta = \theta'' - \theta. \tag{13}$$

The abovementioned auxiliary linear operators must validate the following properties:

$$\left. \begin{aligned} L_f(C_1 + C_2 e^\eta + C_3 e^{-\eta}) &= 0, \\ L_\theta(C_4 e^\eta + C_5 e^{-\eta}) &= 0, \end{aligned} \right\} \tag{14}$$

where C_i ($i = 1 - 5$) indicate the arbitrary constants.

The corresponding problems at the zeroth order are represented as follows:

$$(1 - p)L_f[\widehat{f}(\eta; p) - f_0(\eta)] = p\hbar_f \mathbf{N}_f[\widehat{f}(\eta; p), \widehat{\theta}(\eta, p)], \tag{15}$$

$$(1 - p)L_\theta[\widehat{\theta}(\eta; p) - \theta_0(\eta)] = p\hbar_\theta \mathbf{N}_\theta[\widehat{f}(\eta; p), \widehat{\theta}(\eta, p)], \tag{16}$$

$$\begin{aligned} \widehat{f}(0; p) &= 0, \quad \widehat{f}'(0; p) = 1, \quad \widehat{f}'(\infty; p) = 0, \quad \widehat{\theta}(0, p) \\ &= 1 - S, \quad \widehat{\theta}(\infty, p) = 0, \end{aligned} \tag{17}$$

$$\begin{aligned} \mathbf{N}_f \left[\widehat{f}(\eta; p) \right] &= \frac{\partial^3 \widehat{f}(\eta; p)}{\partial \eta^3} + \widehat{f}(\eta; p) \frac{\partial^2 \widehat{f}(\eta; p)}{\partial \eta^2} - \left(\frac{\partial \widehat{f}(\eta; p)}{\partial \eta} \right)^2 \\ &+ \beta_1 \left[2\widehat{f}(\eta; p) \frac{\partial \widehat{f}(\eta; p)}{\partial \eta} \frac{\partial^2 \widehat{f}(\eta; p)}{\partial \eta^2} \right. \\ &- \left. \left(\widehat{f}(\eta; p) \right)^2 \frac{\partial^3 \widehat{f}(\eta; p)}{\partial \eta^3} \right] + \beta_2 \left[\left(\frac{\partial^2 \widehat{f}(\eta; p)}{\partial \eta^2} \right)^2 \right. \\ &- \left. \widehat{f}(\eta; p) \frac{\partial^4 \widehat{f}(\eta; p)}{\partial \eta^4} \right] + \lambda \widehat{\theta}(\eta; p), \end{aligned} \tag{18}$$

$$\begin{aligned} \mathbf{N}_\theta \left[\widehat{f}(\eta; p), \widehat{\theta}(\eta; p) \right] &= \frac{\partial^2 \widehat{\theta}(\eta; p)}{\partial \eta^2} + Pr \left(\widehat{f}(\eta; p) \frac{\partial \widehat{\theta}(\eta; p)}{\partial \eta} \right. \\ &- \left. \frac{\partial \widehat{f}(\eta; p)}{\partial \eta} \widehat{\theta}(\eta; p) \right) - PrS \frac{\partial \widehat{f}(\eta; p)}{\partial \eta} \\ &+ Pr\delta \widehat{\theta}(\eta; p). \end{aligned} \tag{19}$$

here, the embedding parameter is represented by p and the nonzero auxiliary parameters are denoted by (h_f, h_θ) .

The developed problems at the m^{th} -order deformation are formulated in the forms:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_f \mathbf{R}_f^m(\eta), \tag{20}$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_\theta \mathbf{R}_\theta^m(\eta), \tag{21}$$

$$f_m(0) = f'_m(0) = f'_m(\infty) = 0, \quad \theta_m(0) = \theta_m(\infty) = 0, \tag{22}$$

$$\begin{aligned} \mathbf{R}_f^m(\eta) &= f'''_{m-1}(\eta) + \sum_{k=0}^{m-1} (f_{m-1-k} f''_k - f'_{m-1-k} f'_k) \\ &+ \beta_1 \sum_{k=0}^{m-1} \left(2f_{m-1-k} \sum_{l=0}^k f'_{k-l} f''_l - f_{m-1-k} \sum_{l=0}^k f_{k-l} f'''_l \right) \\ &+ \beta_2 \left(f''_{m-1-k} \sum_{k=0}^{m-1} f''_k - f_{m-1-k} \sum_{k=0}^{m-1} f''''_k \right) + \lambda \theta_{m-1}(\eta), \end{aligned} \tag{23}$$

$$\begin{aligned} \mathbf{R}_\theta^m(\eta) &= \theta''_{m-1}(\eta) + Pr \sum_{k=0}^{m-1} (\theta'_{m-1-k} f_k - \theta_{m-1-k} f'_k) \\ &- PrS f'_{m-1}(\eta) + Pr\delta \theta_{m-1}(\eta), \end{aligned} \tag{24}$$

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \tag{25}$$

The general solution expressions now are:

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^\eta + C_3 e^{-\eta}, \tag{26}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^\eta + C_5 e^{-\eta}, \tag{27}$$

here, special solutions are denoted by f_m^* and θ_m^* .

4. Convergence analysis

This section aims to certify the convergence of Expressions (20) and (21) subject to the boundary conditions of Expression (22). Clearly, Expressions (20) and (21) comprise auxiliary variables (h_f, h_θ) . These variables are used to control and adjust the convergence region. Hence, we show the h curves in Fig. 1. Allowable values for established solutions are in the ranges of $-1.95 \leq h_f \leq -0.25$ and $-2.00 \leq h_\theta \leq -0.35$. Table 1 shows the convergence through numerical values. Clearly, 15th-order approximations are sufficient for the convergence of the momentum and energy expressions.

5. Discussion

In this segment, physical illustrations of distinct variables such as the velocity $f'(\eta)$ and temperature $\theta(\eta)$ distribution are presented in Figs. 2–9. The impact of β_1 on $f'(\eta)$ is illustrated in Fig. 2. Clearly, $f'(\eta)$ and the thickness of the momentum boundary layer are decaying functions of the larger Deborah number (β_1) because β_1 arises owing to the relaxation time factor. A larger β_1 corresponds to a longer relaxation time factor, and this factor resists the liquid flow owing to the lower velocity and thinner momentum layer. Moreover, an increment in β_2 provides an improvement in $f'(\eta)$, and in the thickness of the momentum boundary layer (for details, see Fig. 3). Actually, the Deborah number (β_2) is directly associated with the retardation time factor; this factor has the ability to improve the liquid velocity. It is worth mentioning that the case of Maxwell liquid flow is achieved by letting $\beta_2 = 0$. Also, the case of a viscous liquid can be obtained by setting $\beta_1 = \beta_2 = 0$. Fig. 4 shows the effects of λ on $f'(\eta)$. Here, both f' and the corresponding thickness layer increase when λ increases because λ yields higher buoyancy forces, which correspond to higher velocity. The effects of Pr on temperature (θ) are reported in Fig. 5. It should be noticed that the rise in Pr corresponds to decays in the temperature (θ) and thickness of the thermal boundary layer because Pr is the ratio of momentum to thermal diffusivity. For larger Pr , the thermal diffusivity is lower because the diffusion rate drops. This drop in the rate of diffusion acts as a representative exhibiting a diminution in temperature (θ) and thickness of thermal layer. The appropriate Prandtl numbers are reasonably important in industrial procedures because they are used to control the rate of heat transfer in the ultimate product. Fig. 6 illustrates the effect of S on $\theta(\eta)$. Here, θ is lower for higher values of S . In fact, the temperature difference steadily drops between the surface of the sheet and the ambient liquid; this reduction produces a drop in temperature (θ). The effects of $\delta > 0$ and $\delta < 0$ on temperature (θ) are illustrated in Figs. 7 and 8, respectively. Here, $\theta(\eta)$ is an increasing function of $\delta > 0$, whereas the opposite situation is noticed for $\delta < 0$. Physically, heat transfers significantly at larger δ , which corresponds to higher temperature (θ) when compared with $\delta < 0$. Thus temperature (θ) increases via $\delta > 0$ and drops for $\delta < 0$. Fig. 9 shows the effect of λ on temperature (θ). As expected, temperature (θ) and thickness of the thermal layer drop with an increase in λ . Physically, buoyancy force controls the inertial force, which improves the heat transfer rate. Consequently, the temperature (θ) drops.

Impacts of distinct variables on the Nusselt number $(Nu_x Re_x^{-1/2})$ are shown in Table 2. Clearly, Nusselt number increases with larger Pr and λ , whereas the reverse behavior is found in the cases of δ and S . Table 3 provides a comparative analysis of the results provided in two studies [45,46]. It is concluded that our series expressions are in reasonable agreement with those of Abel et al. [45] and Megahed [46]. Table 4 validates the presented analysis according to the results of the studies provided by Sadeghy et al. [47] and Mukhopadhyay [48]. In this table, it can also be noticed that the analytical expressions we present are in excellent agreement with those in

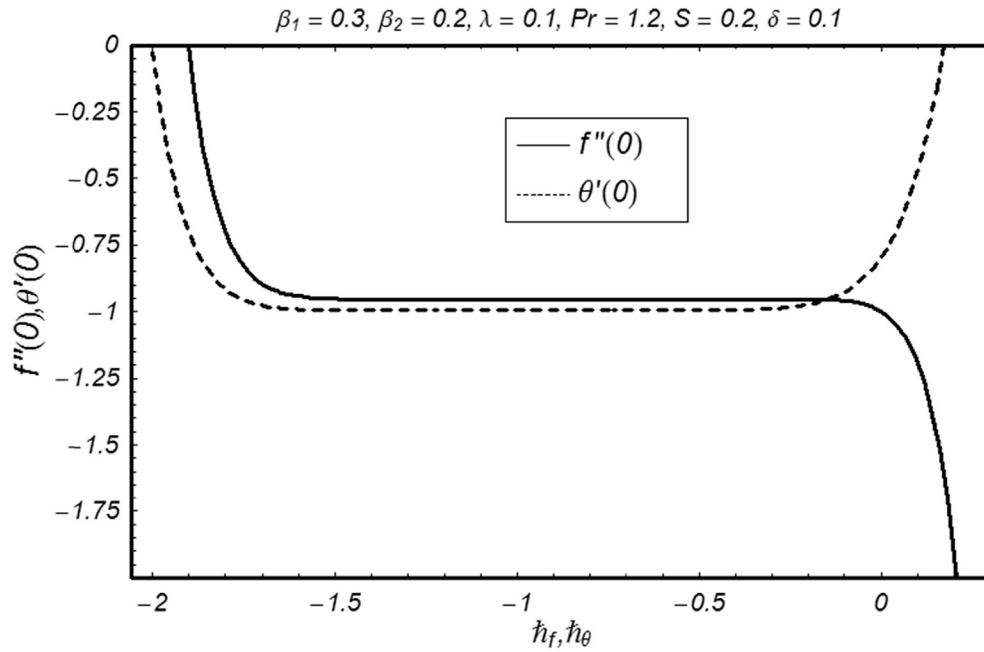


Fig. 1. h curves for f and θ .

Table 1
Homotopy solution convergence when $\beta_1 = 0.3$, $\beta_2 = S = 0.2$, $\lambda = \delta = 0.1$, and $Pr = 1.2$.

Order of approximations	$-f''(0)$	$-\theta'(0)$
1	0.9350	0.9520
5	0.9544	0.9957
10	0.9545	0.9958
15	0.9545	0.9958
20	0.9545	0.9958
25	0.9545	0.9958
30	0.9545	0.9958

the studies by Sadeghy et al. [47] and Mukhopadhyay [48]. Moreover, it should be noticed that the numerical values of $f''(0)$ are higher for larger β_1 because the Deborah number (β_1) is directly proportional to the relaxation time factor (λ_1). The value of λ_1 increases with higher β_1 . Thus, a higher relaxation time decays the liquid velocity but increases $f''(0)$.

6. Concluding remarks

In this study, computations are presented to explore the 2-D stretchable mixed convective flow of an Oldroyd-B material in the

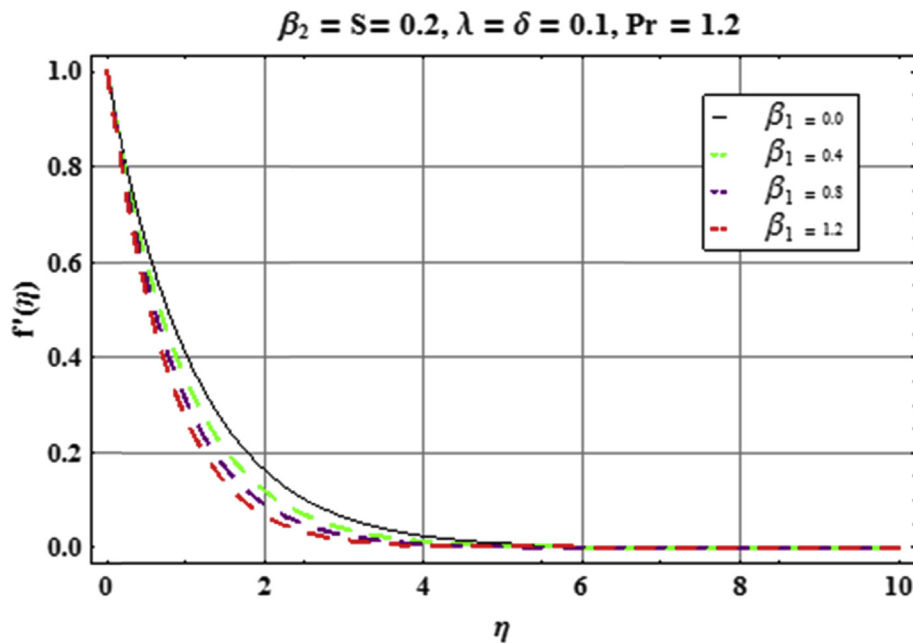


Fig. 2. f' via β_1 .

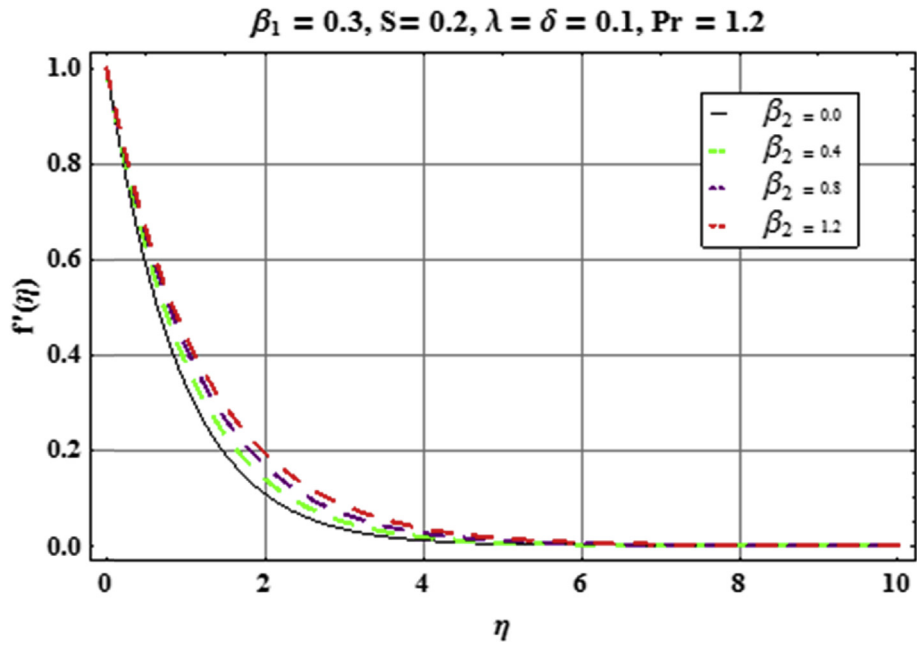


Fig. 3. f' via β_2 .

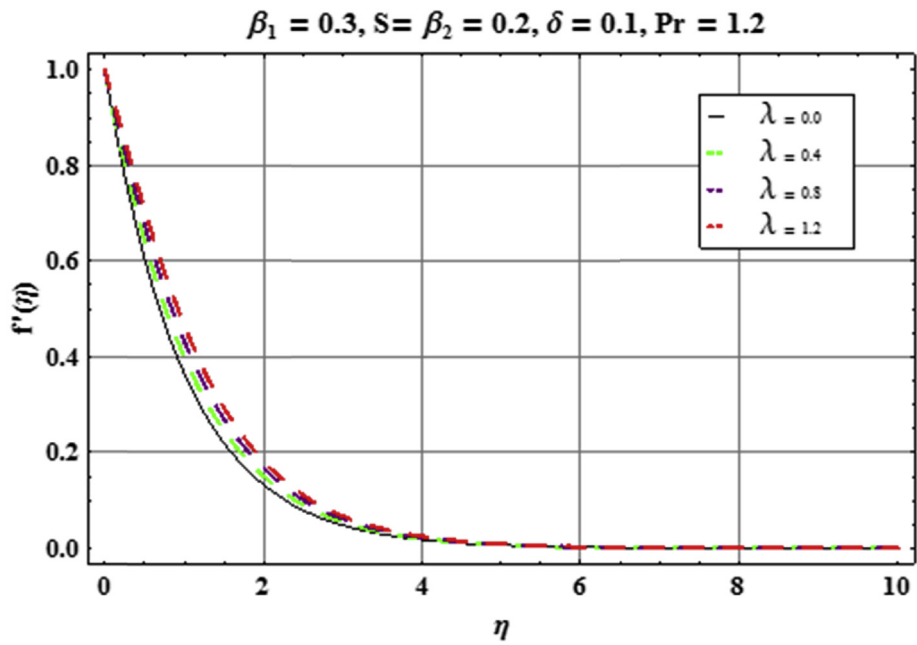


Fig. 4. f' via λ .

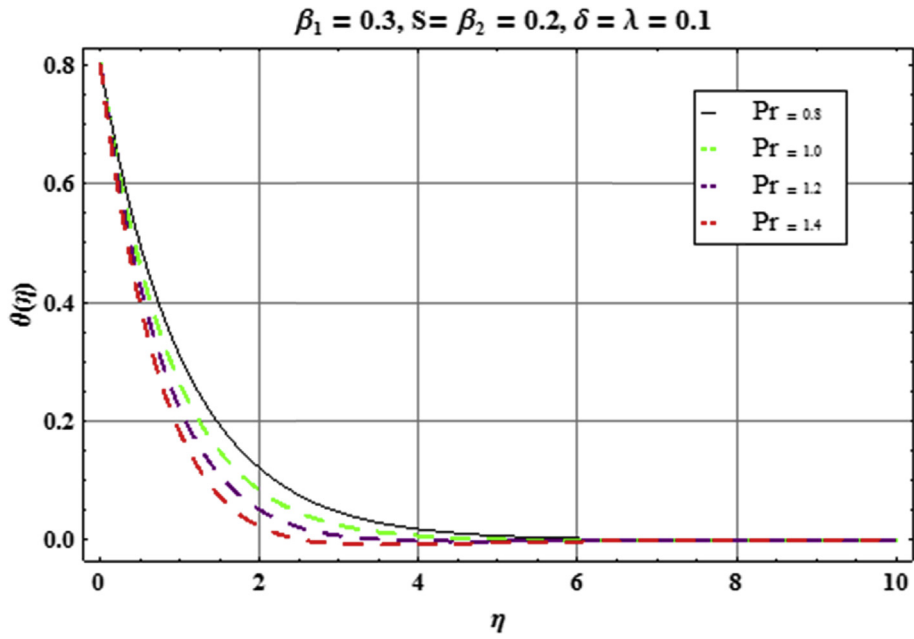


Fig. 5. θ via Pr.

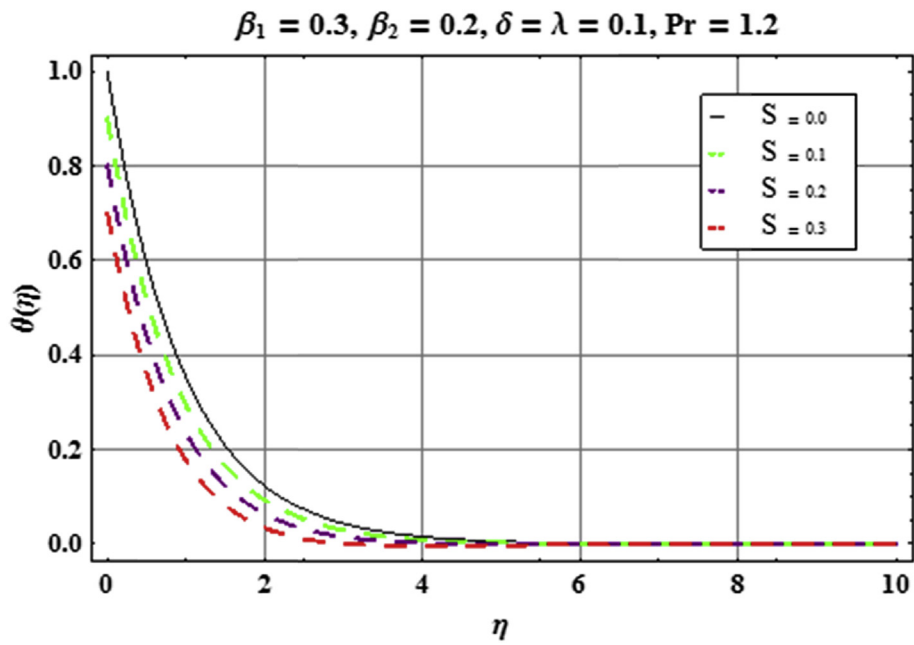


Fig. 6. θ via S.

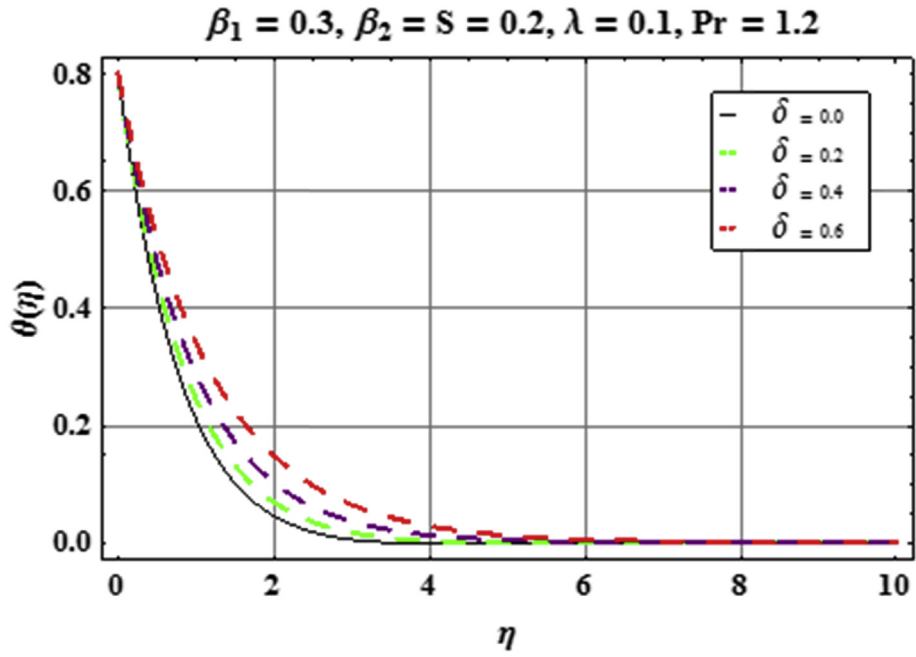


Fig. 7. θ via $\delta > 0$.

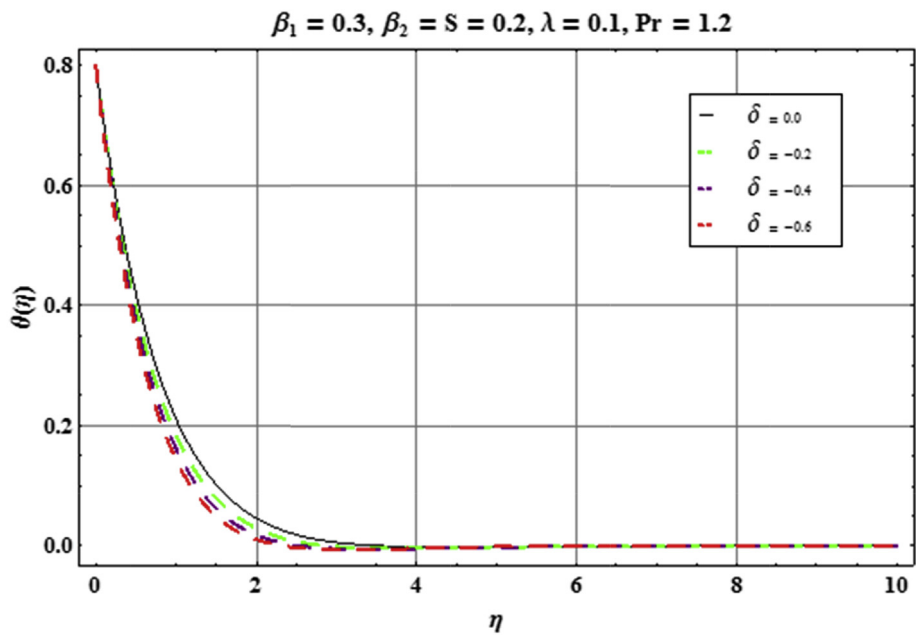


Fig. 8. θ via $\delta < 0$.

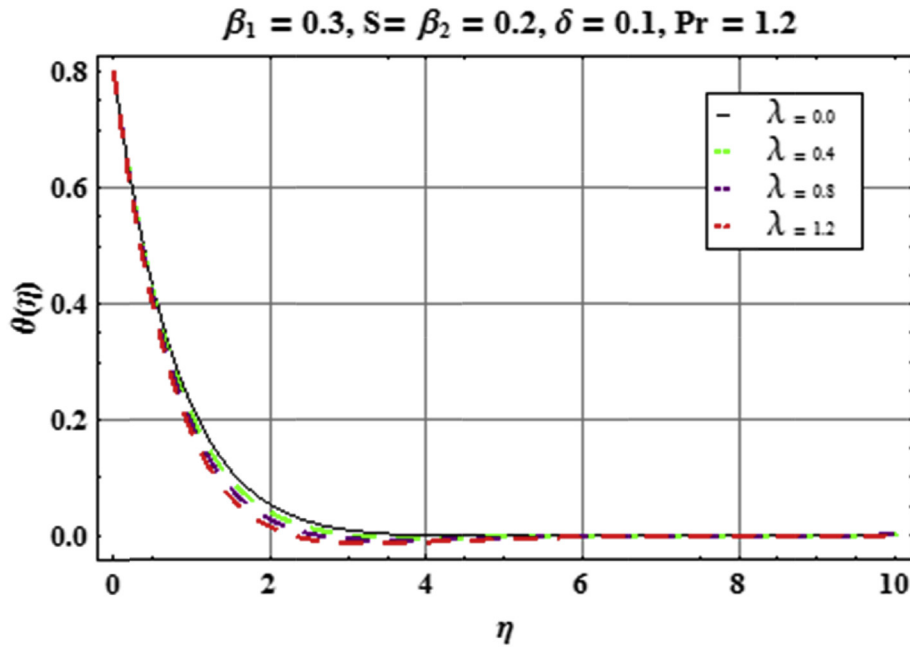


Fig. 9. θ via λ .

Table 2
Behaviors of δ , S , Pr , and λ on Nusselt number ($Nu_x Re_x^{-1/2}$) when $\beta_1 = 0.3$ and $\beta_2 = 0.2$.

δ	S	Pr	λ	$-\theta'(0)$
0.0	0.2	1.2	0.1	1.0033
0.2				0.8921
0.3				0.8147
0.1	0.0			1.0165
	0.3			0.9185
	0.6			0.8165
	0.2	1.3		1.0044
		1.4		1.0554
		1.5		1.1048
		1.2	0.0	0.9407
			0.2	0.9608
			0.4	0.9764

Table 3
Comparison of numerical values of $f''(0)$ with those in the studies by Abel et al. [45] and Megahed [46] for several values of β_1 when $\beta_2 = 0 = \lambda$.

β_1	Abel et al. [45]	Megahed [46]	Present
0.0	1.000000	0.999978	1.000000
0.2	1.051948	1.051945	1.051889
0.4	1.101850	1.101848	1.101903
0.6	1.150163	1.150160	1.150137
0.8	1.196692	1.196690	1.196711
1.2	1.285257	1.285253	1.285363
1.6	1.368641	1.368641	1.368758
2.0	1.447617	1.447616	1.447651

Table 4
Comparative analysis of $f''(0)$ with the studies by Sadeghy et al. [47] and Mukhopadhyay [48] for distinct values of β_1 when $\beta_2 = 0 = \lambda$.

β_1	Sadeghy et al. [47]	Mukhopadhyay [48]	Present
0.0	1.000000	0.9999963	1.000000
0.2	1.0594	1.051949	1.051889
0.4	1.10084	1.101851	1.101903
0.6	1.0015016	1.150162	1.150137
0.8	1.19872	1.196693	1.196711

presence of heat generation/absorption and thermal stratification. The present analysis has the following outcomes: (1) larger β_1 retards the flow, whereas the reverse situation is noticed for β_2 ; (2) thermal buoyancy or mixed convection parameter (λ) shows opposite behaviors on velocity and temperature; (3) larger Pr corresponds to a decay in the temperature profile (θ) and in the thickness of the thermal boundary layer; (4) heat transfers more rapidly in the case of the heat generation process ($\delta > 0$); however, it is absorbed when ($\delta < 0$); (5) Nusselt number increases when Pr and λ are increased, whereas it decays via larger S and δ ; and (6) the case of no heat generation/absorption can be achieved by setting $\delta = 0$.

Conflict of interest

It is declared that authors have no conflict of interest.

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