Nuclear Engineering and Technology 49 (2017) 1006–1009

Contents lists available at ScienceDirect

Nuclear Engineering and Technology

journal homepage: www.elsevier.com/locate/net

Original Article

Scattering cross section for various potential systems

Myagmarjav Odsuren [a,*], Kiyoshi Katō [b], Gonchigdorj Khuukhenkhuu [a,*], Suren Davaa [a]

[a] Nuclear Research Center, School of Engineering and Applied Sciences, National University of Mongolia, Ikh Surguuuliin Street, Ulaanbaatar 210646, Mongolia

[b] Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, N10W8, Sapporo 060-0810, Japan

ARTICLE INFO

Article history:
Received 6 February 2017
Received in revised form
11 March 2017
Accepted 27 April 2017
Available online 23 May 2017

Keywords:
Complex Scaling Method
Continuum Level Density
Resonance Problems
Scattering Problems

ABSTRACT

We discuss the problems of scattering in this framework, and show that the applied method is very useful in the investigation of the effect of the resonance in the observed scattering cross sections. In this study, not only the scattering cross sections but also the decomposition of the scattering cross sections was computed for the $\alpha-\alpha$ system. To obtain the decomposition of scattering cross sections into resonance and residual continuum terms, the complex scaled orthogonality condition model and the extended completeness relation are used. Applying the present method to the $\alpha-\alpha$ and $\alpha-n$ systems, we obtained good reproduction of the observed phase shifts and cross sections. The decomposition into resonance and continuum terms makes clear that resonance contributions are dominant but continuum terms and their interference are not negligible. To understand the behavior of observed phase shifts and the shape of the cross sections, both resonance and continuum terms are calculated.

1. Introduction

Studies of scattering problems in nuclear physics have been developed using various experimental techniques and theoretical methods. One very promising method, the complex scaling method (CSM) [1], has been applied to scattering and resonance problems. This approach seems to promise to unify the description of the nuclear structure and reactions, also including nuclear data evaluation, especially for light nuclear mass systems [2–6]. In this work, we use the CSM to study scattering phase shifts. It has been shown to be possible to calculate scattering phase shifts according to the continuum level density (CLD) [7]. We develop a method of calculating the CLD to investigate the effects of the resonant states, which are related to the nuclear structures, and which are separate from the continuum states. The background contributions to the phase shifts are also considered. This method is applied to the complex scaled orthogonality condition model [4] of different scattering systems including the $\alpha-n$ and $\alpha-\alpha$ systems. The background phase shift is also obtained using the residual continuum solutions in the CSM. We discuss the problems of scattering in this framework, and show that this method is very useful in the investigation of the effect of the resonance in the observed scattering cross sections.

* Corresponding author.
E-mail addresses: odsuren@seas.num.edu.mn (M. Odsuren), khuukhenkhuu@seas.num.edu.mn (G. Khuukhenkhuu).

2. Theoretical framework

2.1. Complex scaling method

The CSM has been introduced to determine resonant states within L^2 basis functions, and is defined by the following complex-dilatation transformation for relative coordinate $\vec{r}$ and momentum $\vec{k}$

$$\vec{r} \to \vec{r} e^{i\theta}, \quad \vec{k} \to \vec{k} e^{i\theta} \tag{1}$$

where θ is a scaling angle and $0 < \theta < \theta_{\max}$. The maximum value $\theta_{\max}$is determined to keep analyticity of the potential. For example, $\theta_{\max} = \pi/4$ for a Gaussian potential. This transformation makes every branch cut to rotate by -2θ on the complex energy plane. Applying this transformation, we can write the complex-scaled Schrödinger equation as follows:

$$H^\theta \Psi^\nu_{J^\pi}(\theta) = E^\theta_\nu \Psi^\nu_{J^\pi}(\theta) \tag{2}$$

The complex-scaled Hamiltonian H^θ and wave function $\Psi^\nu_{J^\pi}(\theta)$ are defined as $U(\theta)\, HU(\theta)^{-1}$ and $U(\theta)\Psi^\nu_{J^\pi}$, respectively—see [1,2] for details.

Applying the L^2 basis function method, the radial wave function is expanded as

http://dx.doi.org/10.1016/j.net.2017.04.007

$$\Psi^{\nu}_{J^{\pi}}(\theta) = \sum_{i=1}^{N} c_i^{J^{\pi}\nu}(\theta)\phi_i(r) \tag{3}$$

where $\phi_i(r)$ is an appropriate basis function set. The expansion coefficients $c_i^{J^{\pi}\nu}$ and the complex energy eigenvalues E^{θ}_{ν} are obtained by solving the complex eigenvalue problem given in Eq. (2). The complex energies of the resonant states are obtained as $E_r = E_r^{res} - i\Gamma/2$, when $\tan^{-1}(\Gamma_r/2E_r^{res}) < 2\theta$.

To solve the eigenvalue problem of Eq. (2), we employ the Gaussian basis functions given as follows:

$$\phi_i(r) = N_l(b_i)r^l \exp\left(-\frac{1}{2b_i^2}r^2\right)Y_{lm}(\hat{r}), \tag{4}$$

where the range parameters are given by a geometric progression as $b_i = b_0\gamma^{i-1}$; $i = 1,\ldots,N$, and $N_l(b_i)$ is the normalization factor. We take $N = 60$ and employ the optimal values of b_0 and γ to obtain stationary solutions. All results are obtained with $\theta = 15^{\circ}$.

2.2. Continuum-level density and phase shift

The CLD $\Delta(E)$ is given as

$$\Delta(E) = -\frac{1}{\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[G^{+}(E) - G_0^{+}(E)\right]\right\} \tag{5}$$

where

$G^{+}(E) = (E + i\varepsilon - H)^{-1}$ and

$$G_0^{+}(E) = (E + i\varepsilon - H_0)^{-1}$$

are the full and free Green's functions, respectively. In this study, the Hamiltonian H and H_0 are transformed using the CSM.

The CLD is related to the scattering phase shift $\delta(E)$; it can be expressed in the following form in the single channel case [7]:

$$\Delta(E) = \frac{1}{\pi}\frac{d\delta(E)}{dE} \tag{6}$$

Using this relation, we can obtain the phase shift as a function of the eigenvalues in the complex scaled Hamiltonian by integrating the CLD.

When we expand the wave functions in terms of the finite number of basis states N, the discretized eigenstates are obtained with number N and the level density can be approximated as in [7]:

$$\Delta(E) \approx \Delta^N_{\theta}(E) = -\frac{1}{\pi}\mathrm{Im}\left[\sum_{b=1}^{N_b}\frac{1}{E + i0 - E_b} + \sum_{r=1}^{N_r^{\theta}}\frac{1}{E - E_r^{res} + i\Gamma_r/2} + \sum_{c=1}^{N_c^{\theta}}\frac{1}{E - \varepsilon_c^r + i\varepsilon_c^i} - \sum_{k=1}^{N}\frac{1}{E - \varepsilon_k^{0r} + i\varepsilon_k^{0i}}\right] \tag{7}$$

where $N = N_b + N_r^{\theta} + N_c^{\theta}$ is the total number of N_b (bound states), N_r^{θ} (resonance states), and N_c^{θ} (continuum states) solutions. Then, we can obtain the phase shift

$$\begin{aligned}\delta^N_{\theta}(E) = &N_b\pi + \sum_{r=1}^{N_r^{\theta}}\left\{-\cot^{-1}\left(\frac{E - E_r^{res}}{\Gamma_r/2}\right)\right\}\\ &+ \sum_{c=1}^{N_c^{\theta}}\left\{-\cot^{-1}\left(\frac{E - \varepsilon_c^r}{\varepsilon_c^i}\right)\right\}\\ &- \sum_{k=1}^{N}\left\{-\cot^{-1}\left(\frac{E - \varepsilon_k^{0r}}{\varepsilon_k^{0i}}\right)\right\}\end{aligned} \tag{8}$$

where $E \geq 0$. When we define δ_r, δ_c, and δ_k as

$$\cot\,\delta_r = \frac{E_r^{res} - E}{\Gamma_r/2}, \quad \cot\,\delta_c = \frac{\varepsilon_c^r - E}{\varepsilon_c^i}, \quad \cot\,\delta_k = \frac{\varepsilon_k^{0r} - E}{\varepsilon_k^{0i}} \tag{9}$$

respectively, we can write the phase shift as

$$\delta^N_{\theta}(E) = N_b\pi + \sum_{r=1}^{N_r^{\theta}}\delta_r + \sum_{c=1}^{N_c^{\theta}}\delta_c - \sum_{k=1}^{N}\delta_k \tag{10}$$

The geometrical indications for δ_r, δ_c, and δ_k are given for two energy cases, larger or smaller than the real parts of the eigenenergies E_r, ε_c, and ε_k, as shown in Fig. 1. The phase shift δ_r for the resonances is the angle of the r th resonant pole measured at the energy E on the real energy axis. At $E = E_r^{res}$, we have $\delta_r = \pi/2$ for every resonant pole. In addition, $\delta_r = \tan^{-1}(\Gamma_r/2E_r^{res}) > 0$ at $E = 0$ and $\delta_r = \pi$ at $E = \infty$ for each resonance. Similarly, phase shifts from continuum terms including the asymptotic part, δ_k, are given by the angles of the discretized continuum energies. At $E = \infty$, the continuum terms of the phase shifts go to $-(N_b + N_r^{\theta})\pi$ because of the relation $N = N_b + N_r^{\theta} + N_c^{\theta}$.

2.3. Cross section

The cross section is described using the calculated phase shifts; we can identify the contributions from every resonant pole and continuum term. When we concentrate our interest on the contribution from a single resonant pole and other terms that are mainly described as background phase shift, we can achieve the same results as those of Fano [8]. The total and partial reaction cross sections can be calculated using the results of the phase shifts

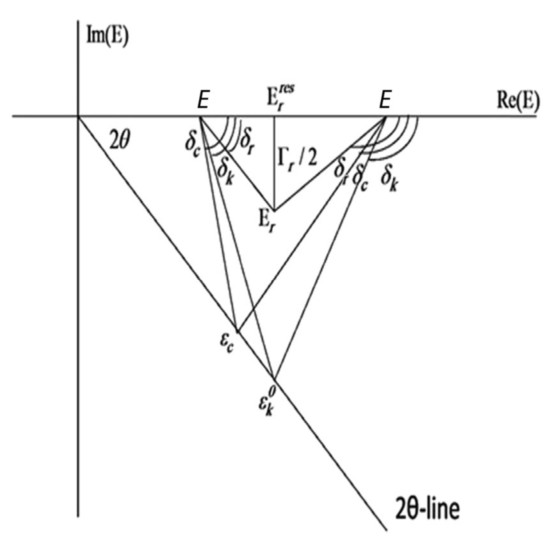

Fig. 1. Geometrical indications of phase shifts: δ_r, δ_c, and δ_k.

decomposed into the contributions of the resonance and continuum. From these results, we can investigate the contributions of resonance and continuum states in the cross sections.

The partial cross sections σ_l for each partial wave with index l can be expressed as follows:

$$\sigma_l = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l(E) \tag{11}$$

where $k^2 = 2E\mu/h^2$ and μ are the reduced mass of the system.

The phase shift $\delta_l(E)$ is expressed in the form $\delta_r + \delta_b$, where δ_r and δ_b are the single resonance and the background terms, including all other terms given in Eq. (8), respectively. The shape of the cross section can be investigated by evaluating the resonance δ_r and background δ_b phase shifts. The total cross section is expressed as

$$\sigma(E) = \sum_{l}^{\infty}\sigma_l(E) \tag{12}$$

3. Results and discussion

One of the methods to prove the nuclear structure is to use the scattering phenomena. In this part, we concern ourselves with the scattering due to the relative motion of two particles. The α–α scattering phase shifts provide a convenient test of several important properties of low-energy nuclear scattering. The scattering phase shifts of this two-body system are calculated using Eq. (8), which is derived, using the extended completeness relation, from the CLD. After the calculation of the decomposed scattering phase shifts, the partial cross sections of low-lying states are studied with the resonance and continuum contributions for the α–α system.

In Fig. 2, the scattering phase shifts of five partial waves as functions of energy in α–α are displayed.

Two different potentials [9,10], phenomenological and folding potentials, are used, for which treatment of the Pauli principle between two alphas is different. From the results shown in Fig. 2, it can be seen that both potentials are capable in the α–α system; it can also be clearly seen that the calculated scattering phase shifts are similar for the different potential sets.

The total cross sections of the α–n system are calculated in terms of the scattering phase shifts using Eq. (12), which is shown in Fig. 3. The open circles in Fig. 3 show the experimental data,

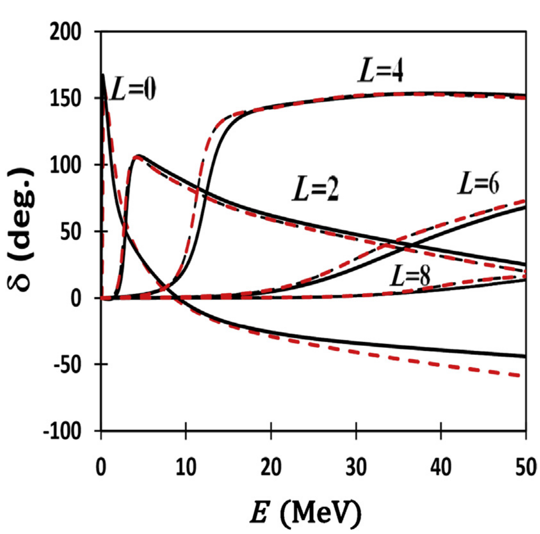

Fig. 2. Scattering phase shifts of the α–α system at the $J^\pi = 0^+, 2^+, 4^+, 6^+$, and 8^+ waves with Buck potential [9] (solid curves) and Schmid–Wildermuth [10] potential (dashed curves).

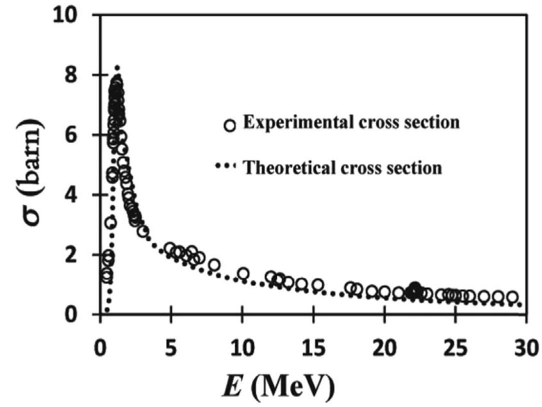

Fig. 3. Total cross section of the α–n system. Open circles display the experimental data taken from [11–14], and the dotted line shows the present results.

which were taken from [11–14]. As can be seen in Fig. 3, the theoretical total cross section is in reasonable agreement with the experimental data.

The total cross section is given by the sum of the partial ones, which are expressed as the interference of the resonance and the continuum contribution, as discussed by Fano [8], due to the relation $\delta_l = \delta_r + \delta_c$ in the phase shifts, given in Eq. (10). As can be seen in Fig. 3, there is a very sharp peak at the low energy of around 2 MeV that has a long tail distribution in the higher energies. The

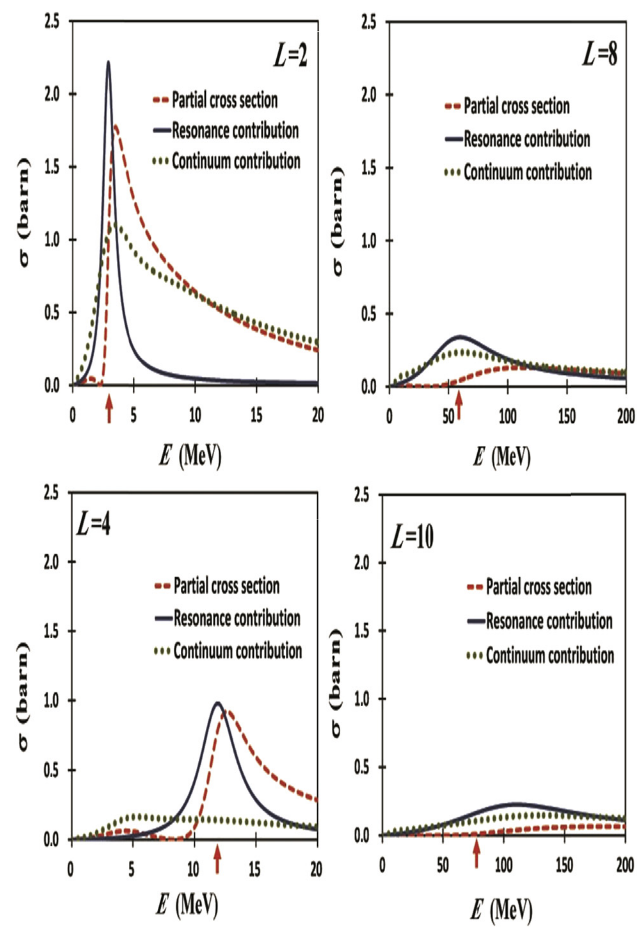

Fig. 4. Partial cross sections at the $J^\pi = 2^+, 4^+, 8^+$, and 10^+ of the α–α system. The curves, dotted lines, and dashed lines denote the results of resonance and continuum contributions of cross sections and partial cross sections, respectively. The arrow indicates the resonance energy.

low-energy cross section dominantly comes from $l = 1$ partial waves.

In Fig. 4, the partial cross sections and their decomposition into resonance and continuum terms are shown for $L = 2$, 4, 8, and 10 waves of the α–α system. The partial cross sections are calculated using Eq. (11).

The partial cross section for $L = 0$ is very sharp, like the δ function, because of the small decay width of the $L = 0$ resonance. Thus, we skipped the $L = 0$ partial cross section. For $L = 2$ and 4, resonance cross sections have shapes like that of the Breit–Wigner form. The continuum contribution of $L = 2$ is rather large, whereas this contribution is small for $L = 4$. Compared with the $L = 2$ case, the partial cross section of $L = 4$ is not so different from the resonance cross section. It is interesting that the peak energies of the partial cross-section shift fairly far from the position of the resonance energies. From Fig. 4, it can be clearly seen that the L =2 and 4 partial waves give a bell-shaped structure of the cross section. However, no bell-shaped structure of the cross sections in the $L = 8$ and 10 partial waves is observed.

The continuum contributions of the cross section exhibit almost the same behavior in both the $L = 8$ and $L = 10$ states, as shown in Fig. 4. These contributions change the form of the cross section from a symmetric Breit–Wigner shape to asymmetric peaks. Although the resonant peak of the cross section can be clearly seen in the cases of $L = 2$ and $L = 4$, for $L = 8$ and $L = 10$, the results show a mild bump in the partial cross section.

4. Summary

We have investigated the scattering phase shifts of the two-body α–α system with different effective potentials. Two different potentials were used to show that the resonance behaviors of the scattering phase shifts are similar to each other. In this study, not only the scattering cross sections but also the decomposition of the scattering cross sections was computed for the α–α system; resonance and continuum contributions were also obtained at the same time by applying the extended completeness relation.

Applying the present framework to the α–α and α–n systems, we obtained good reproduction of the observed phase shifts and cross sections. The decomposition into resonance and continuum terms makes clear that resonance contributions are dominant but continuum terms and their interference are not negligible. To understand the behavior of observed phase shifts and the shape of the cross sections, both resonance and continuum terms must necessarily be taken into account. If the continuum term is zero, the cross section exhibits a typical Breit–Wigner form. As was discussed by Fano [8], deviation from the Breit–Wigner form can be investigated by calculating the interference between the resonance and continuum terms.

Conflicts of interest

The authors have no conflicts of interest to declare.

Acknowledgments

This work was supported by funds provided by the National University of Mongolia (ARC2017) for Asia Research Center project. The numerical calculation was supported by the cluster computing system at the Nuclear Research Center, National University of Mongolia.

References

[1] Y.K. Ho, The method of complex coordinate rotation and its applications to atomic collision processes, Phys. Rep. 99 (1983) 1–68.
[2] S. Aoyama, T. Myo, K. Katō, K. Ikeda, The complex scaling method for many-body resonances and its applications to three-body resonances, Prog. Theor. Phys. 116 (2006) 1–35.
[3] R. Suzuki, A. Kruppa, B.G. Giraud, K. Katō, Continuum level density of a coupled-channel system in the complex scaling method, Prog. Theor. Phys. 119 (2008) 949–963.
[4] A.T. Kruppa, K. Katō, Resonances in complex-scaled orthogonality condition model of nuclear cluster system, Prog. Theor. Phys. 84 (1990) 1145–1159.
[5] M. Odsuren, K. Katō, G. Khuukhenkhuu, Scattering phase shifts in the complex scaling method, JETP Lett. 102 (2015) 463–466.
[6] M. Odsuren, K. Katō, M. Aikawa, Analysis of three body resonances in the complex scaled orthogonal condition model, Nucl. Data Sheets 120 (2014) 126–128.
[7] S. Shlomo, Energy level density of nuclei, Nucl. Phys. A 539 (1992) 17–36.
[8] U. Fano, Effects of configuration interaction on intensities and phase shifts, Phys. Rev. 124 (1961) 1866–1878.
[9] B. Buck, H. Friedrich, C. Wheatley, Local potential models for the scattering of complex nuclei, Nucl. Phys. A 275 (1977) 246–268.
[10] E.W. Schmid, K. Wildermuth, Phase shift calculations on α–α scattering, Nucl. Phys. A 26 (1961) 463–468.
[11] S.M. Austin, H.H. Barschall, R.E. Shamu, Scattering of neutrons by α particles, Phys. Rev. 126 (1962) 1532–1539.
[12] R.E. Shamu, J.G. Jenkin, Neutron-alpha scattering in the 20-MeV range, Phys. Rev. 135 (1964) B99–B109.
[13] F.J. Vaughn, W.L. Imhof, R.G. Johnson, M. Walt, Total neutron cross sections of helium, neon, argon, krypton, and xenon, Phys. Rev. 118 (1960) 683–686.
[14] Los Alamos Physics and Cryogenics Groups, Total neutron cross sections of the hydrogen and helium isotopes, Nucl. Phys. 12 (1959) 291–297.