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비결함 샘플 데이타 제어를 가지는 정적 지연 뉴럴 네트웍의 강인 상태추정

H_∞ State Estimation of Static Delayed Neural Networks with Non-fragile Sampled-data Control

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Abstract - This paper studies the state estimation problem for static neural networks with time-varying delay. Unlike other studies, the controller scheme, which involves time-varying sampling and uncertainties, is first employed to design the state estimator for delayed static neural networks. Based on Lyapunov functional approach and linear matrix inequality technique, the non-fragile sampled-data estimator is designed such that the resulting estimation error system is globally asymptotically stable with H_∞ performance. Finally, the effectiveness of the developed results is demonstrated by a numerical example.

Key Words : State estimation, Neural networks, Time−varying delay, Non−fragile sampled−data control

1. Introduction

In the past decades, static neural networks, where the neuron states are utilized as basic variables to depict the dynamical evolution rule, have been received much attention due to their successful applications in a variety of areas such as associative memory and combinatorial optimization. Some typical examples of the static neural networks are the brain-state-in-a box neural networks and the projection neural networks, etc [1], [2]. It should be pointed out that static neural networks are different from the local field neural networks where the local field states of neurons are taken as basic variables. On the other hand, time delay is often encountered in neural networks.

The existence of time delay may result in poor performance such as instability and oscillation of the underlying neural networks. Up to now, stability of static neural networks has been widely discussed and various stability conditions have been obtained in the literatures [3-6].

In practice, it is impossible or very expensive to completely acquire the state information of all neurons in neural networks due to their complicated structure. However, in some engineering applications, it is needed to know these information in advance to achieve specific objectives. Recently, some results on state estimation of static neural networks have been derived [7-11].

A lot of control methods have recently been applied to the design of a state estimator for neural networks. Nowadays, it is important to consider that the control input signals are discontinuous due to the development of high-performance computing technology and modern digital communication technique. As a result, the controller design problem using sampled-data has received much attention and many important results have been presented in recent years.

For example, the state estimation problem for neural networks with a time-varying delay via stochastic sampled-data control was studied in [12]. It is worth mentioning that, in the above mentioned papers, the communication between the neural networks and the estimator is assumed to be perfect. However, inaccuracies or uncertainties do occur in sampled-data controller implementation, and thus the ideal assumption may not be satisfied.

The controllers are very sensitive to their own uncertainties (implementation errors) and this is called fragility problem of controllers. In this regard, the non-fragile controllers have been employed for dynamical networks to tolerate some uncertainties [13-15]. H_∞ state

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estimation of static neural networks with time varying delay via non-fragile sampled-data controller has not been investigated by any researchers.

Motivated the above discussion and facts, the design problem of the state estimator for delayed static neural networks is investigated in this paper. Unlike previous studies, the states of the proposed static neural networks are estimated using the time-varying sampling with uncertainties. By constructing the augmented Lyapunov function, with the help of employing integral inequalities and convex combination method to deal with some cross terms, some sufficient conditions are derived such that the estimating error system is globally asymptotically stable with a guaranteed H_∞ performance, and the gain matrix of the state estimator can be easily obtained by solving a convex optimization problem under the constraint of LMIs. A numerical example is given to demonstrate the effectiveness of the proposed method.

Notations: Throughout this paper, I denotes the identity matrix with appropriate dimensions, R^n denotes the n dimensional Euclidean space, and $R^{m\times n}$ is the set of all m n real matrices, For symmetric matrices A and B, the notation $A>B$(respectively, $A\geq B$) means that the matrix $A-B$ is positive definite (respectively, non-negative). diag{...} denotes the block diagonal matrix.

2. Problem Statement

Consider the following static neural network with time delay and noise disturbance.

$$\begin{aligned}\dot{x}(t) &= -Ax(t)+f(Wx(t-h(t))+J)+Bw(t), \\ y(t) &= Cx(t), \\ z(t) &= Hx(t),\end{aligned} \tag{1}$$

where $x(t)=[x_1(t),x_2(t),...,x_n(t)]^T\in R^n$ is the state vector of the model, n corresponds to the number of neurons, $w(t)\in R^q$ denotes a noise disturbance belonging to $\mathcal{L}_2[0,\infty)$, $y(t)\in R^m$ is the network output measurement $A=diag\{a_1,a_2,\ldots,a_n\}>0$ is a diagonal matrix, $W=[W_{ij}]$ is the delayed connection weight matrix, $f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t))]^T\in R^n$ is the neuron activation function , $J=[J_1,J_2,...,J_n]^T$ is a constant external input vector, B,C and H are known real constant matrices with compatible dimensions, and $h(t)$ is a the time-varying delay satisfying

$$0\leq h(t)\leq h,\dot{h}(t)\leq\mu \tag{2}$$

where h and μ are known constants.

Assumption 1 Each neuron activation function $f_i(\bullet)$ is continuous and bounded, and satisfy the following condition:

$$\begin{aligned}&k_i^-\leq\frac{f_i(\xi_1)-f_i(\xi_2)}{\xi_1-\xi_2}\leq k_i^+, \\ &f_i(0)=0,\ \xi_1,\xi_2\in R,\xi_1\neq\xi_2,\ i=1,2,...,n\end{aligned} \tag{3}$$

where k_i^- and k_i^+ are known real scalar.

The following full-order observer for the neural network is proposed:

$$\begin{aligned}\dot{\hat{x}}(t) &= -A\hat{x}(t)+f(W\hat{x}(t-h(t))+J)+u(t), \\ \hat{y}(t) &= C\hat{x}(t)\end{aligned} \tag{4}$$

where $\hat{x}(t)\in R^n$ is the estimation of the neuron state $x(t)$, $y(t)\in R^m$ is the estimated output vector, and $u(t)\in R^n$ is the control input.

Define the error signal as $e(t)=x(t)-\hat{x}(t)$, and the output signal as $\bar{z}(t)=z(t)-\hat{z}(t)$. Then, the error system can be represented as

$$\begin{aligned}\dot{e}(t) &= -Ae(t)+g(We(t-h(t)))+Bw(t)-u(t), \\ \bar{z}(t) &= He(t)\end{aligned} \tag{5}$$

where $g(We(t))=f(Wx(t)+J)-f(W\hat{x}(t)+J)$.

In this paper, the non-fragile sampled-data control law is expressed as

$$\begin{aligned}u(t) &= (K+\triangle K(t_k))(y(t_k)-\hat{y}(t_k)) \\ &= (K+\triangle K(t_k))Ce(t_k), \\ &\quad t_k\leq t\leq t_{k+1},\ k=0,1,2,\ldots\end{aligned} \tag{6}$$

where K is the gain matrix of the feedback controller to be determined later, tk is the updating instant time of the Zero-Order-Hold (ZOH) and the sampling interval satisfies $t_{k+1}-t_k\leq\tau$.

Remark 1 For actual systems, parameter perturbation is unavoidable. This phenomenon may affect the stability and the performance of the systems if they do not be dealt with appropriately. Therefore, the H_∞ no non-fragile sampled-data state estimation for static neural networks is consider in this paper, which has never been considered of static neural networks.

Assumption 2 The uncertainties $\triangle K(t_k)$ represents the possible controller gain fluctuations. It is assumed that $\triangle K(t_k)$ has the following form:

$$\triangle K(t_k) = D\triangle(t_k)E \tag{7}$$

where D and E are known constant matrices with appropriate dimensions, and $\triangle(t_k)$ is unknown matrix function satisfying $\triangle^T(t_k)\triangle(t_k) \le I$.

Using input delay method [12], error system (5) can be represented as:

$$\begin{aligned} \dot{e}(t) &= -Ae(t) + g(We(t-h(t))) + Bw(t) \\ &\quad -KCe(t-\tau(t)) - D_p(t-\tau(t)), \\ p(t-\tau(t)) &= \triangle(t-\tau(t))q(t-\tau(t)) \\ q(t-\tau(t)) &= ECe(t-\tau(t)), \\ \bar{z}(t) &= He(t) \end{aligned} \tag{8}$$

where $\tau(t) = t - t_k \in [0,\tau]$ and $\dot{\tau}(t) = 1$.

The H_∞ performance state estimation problem is stated as follows. For a prescribed level 〉 0 of noise attenuation, it is to find a suitable estimator (4) such that:

1) the estimation error system (5) with $w(t)=0$ is globally asymptotically stable;
2) under the zero-initial condition

$$\| \bar{z}(t) \|_2 < \gamma \| w(t) \|_2 \tag{9}$$

holds for all non-zero $w(t) \le \mathcal{L}_2[0,\infty)$, where $\| \phi(t) \|_2 = \sqrt{\int_0^\infty \psi^T(g)\psi(t)dt}$..

3. Main Results

In this section, a design problem of state estimation for delayed static neural networks using non-fragile sampled-data controller will be investigated. The following lemmas are essential to derive the main results.

Lemma 1 [16] For any constant positive definite matrix $M \in R^{m\times n}$ and $\beta \le s \le \alpha$, the following inequalities hold

$$-(\alpha-\beta)\int_\alpha^\beta \dot{x}^T(s)M\dot{x}(s)ds \le -|x(\alpha)-x(\beta)|^T M[x(\alpha)-x(\beta)]$$

Lemma 2 (Lower bounds lemma [17]) Let $f_1, f_2, \dots$ $f_1, f_2, \dots, f_N : R^m \to R$ have positive values in an open subset D of R_m. Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\alpha_i \mid \alpha_i > 0, \Sigma\alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{ij}} \sum_{i\ne j} g_{ij}(t)$$

subject to

$$\left\{ g_{ij} : R^m \to R, g_{j,i(t)} \cong g_{i,j}(t), \begin{vmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{vmatrix} \ge 0 \right\}.$$

Lemma 3 [18] For given matrix $R>0$, the following inequality holds for all continuously differentiable function $x(t)$ in $[a,b] \in R^n$:

$$\begin{aligned} &-(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \\ &\le -[x(b)-x(a)]^T R[x(b)-x(a)] - 3\Omega^T R\Omega, \end{aligned}$$

where $\Omega = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds$.

Theorem 1 For given scalars and 〉 0, error system (8) is asymptotically stable, if there exist $3n\times 3n$ matrices $P>0$, $Q>0$, $n\times n$ matrices $R_1>0, R_2>0$ $S_1>0, S_2>0$, diagonal matrices $\Gamma = \{\gamma_1, \gamma_2, \dots, \gamma_n\} > 0$ $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_n\} > 0$, any $n\times n$ matrices T_1, $2n\times 2n$ matrix T_2, $n\times 1$ matrix G and a scalar $\epsilon > 0$ such that the following LMIs are satisfied for $\tau(t) = \{0, \tau\}$

$$\begin{aligned} \Omega(h(t)) &= [e_1 \ \ \tau(t)e_6 + (\tau-\tau(t))e_\tau]P[e_{10} \ \ e_1 - e_5]^T \\ &+ [e_{10} \ \ e_1 - e_5]P[e_1 \ \ \tau(t)e_6 + (\tau-\tau(t))e_7]^T \\ &+ [e_1 \ \ e_8]Q[e_1 \ \ e_8]^T - (1-\mu)[e_2 \ \ e_9]Q[e_2 \ \ e_9]^T \\ &+ e_1S_1e_1^T - e_3S_1e_3^T + e_1S_2e_1^T - e_5S_2e_5^T \\ &+ h^2e_{10}R_1e_{10}^T - \Pi_1\begin{bmatrix} R_1 & T_1 \\ * & R_1 \end{bmatrix}\Pi_1^T \\ &+ \tau^2e_{10}R_2e_{10}^T - \Pi_2\begin{bmatrix} P_2 & T_2 \\ * & P_2 \end{bmatrix}\Pi_2^T \\ &+ e_1W^T(K^- + K^+)\Gamma e_8^T \\ &+ e_8\Gamma^T(K^- + K^+)We_1^T \\ &- 2e_1W^TK^-\Gamma K^+We_1^T - 2e_8\Gamma e_8^T \\ &+ e_2W^T(K^- + K^+)\Lambda e_9^T \\ &+ e_9\Lambda^T(K^- + K^+)We_2^T \\ &- 2e_2W^TK^-\Lambda K^+We_2^T - 2e_9\Lambda e_9^T \\ &+ (e_1 + \alpha e_{10})\Psi + \Psi^T(e_1 + \alpha e_{10})^T \\ &+ \epsilon(e_4C^TE^TECe_4^T - e_{11}e_{11}^T) \\ &+ e_1H^THe_1^T - \gamma^2e_{12}e_{12}^T < 0 \end{aligned} \tag{10}$$

$$\begin{bmatrix} R_1 & T_1 \\ * & R_1 \end{bmatrix} \ge 0, \begin{bmatrix} R_2 & T_2 \\ * & R_2 \end{bmatrix} \ge 0 \tag{11}$$

where

$$\Pi_1 = [e_1 - e_2 \quad e_2 - e_3]$$
$$\Pi_2 = [e_1 - e_4 \quad e_1 + e_4 - 2e_6 \quad e_4 - e_5 \quad e_4 + e_5 - 2e_7]$$
$$\Psi = [-MA\ 0\ 0\ -GC\ 0\ 0\ 0\ \ 0\ M\ -M\ -MD\ MB]$$

and $e_i \in R^{(11n+q)\times n}$ $(i=1,2,...,12)$ means the block entry matrices, e.g,$e_2^T\xi(t) = e(t-h(t))$ where

$$\xi^T(t) = [e^T(t)\ e^T(t-h(t))\ e^T(t-h)\ e^T(t-\tau(t))\ e^T(t-\tau)\ \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} e(s)ds\ \ \frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)} e(s)ds\ g^T(We(t))\ \ g^T(W_e(t-h(t)))\ \dot{e}^T(t)\ \ p^T(t-\tau(t))\ \ w^T(t)].$$

Moreover, the non- fragile sampled- data controller gain is given by $K = M^{-1}G$.

Proof Choose a Lyapunov functional candidate as

$$V(t) = \sum_{i=1}^{5} V_i \tag{12}$$

where

$$V_1(t) = \begin{bmatrix} e(t) \\ \int_{t-\tau}^{t} e(s)ds \end{bmatrix}^T P \begin{bmatrix} e(t) \\ \int_{t-\tau}^{t} e(s)ds \end{bmatrix}$$
$$V_2(t) = \int_{t-h(t)}^{t} \begin{bmatrix} e(s) \\ g(We(s)) \end{bmatrix}^T Q \begin{bmatrix} e(s) \\ g(We(s)) \end{bmatrix} ds$$
$$V_3(t) = \int_{t-h}^{t} e^T(s)S_1e(s)ds + \int_{t-\tau}^{t} e^T(s)S_2e(s)ds$$
$$V_4(t) = h\int_{-h}^{0}\int_{t+\alpha}^{t} \dot{e}^T(t)R_1\dot{e}(s)dsd\alpha$$
$$V_5(t) = \tau\int_{-\tau}^{0}\int_{t+\alpha}^{t} \dot{e}^T(t)R_2\dot{e}(s)dsd\alpha$$

Taking the time-derivative of $V_i(t)\,(i=1,2,...,5)$ along the trajectory of system (8) yields

$$\dot{V}_1(t) = \begin{bmatrix} e(t) \\ \int_{t-\tau}^{t} e(s)ds \end{bmatrix}^T P \begin{bmatrix} e(t) \\ e(t)-e(t-\tau) \end{bmatrix}, \tag{13}$$

$$\dot{V}_2(t) \le \begin{bmatrix} e(t) \\ g(We(t)) \end{bmatrix}^T Q \begin{bmatrix} e(t) \\ g(We(t)) \end{bmatrix} - (1-\tau)\begin{bmatrix} e(t) \\ \int_{t-\tau}^{t} e(s)ds \end{bmatrix}^T Q \begin{bmatrix} e(t-h(t)) \\ g(We(t-h(t))) \end{bmatrix}, \tag{14}$$

$$\dot{V}_3(t) = e^T(t)S_1e(t) - e^T(t-h)S_1e(t-h) + e^T(t)S_2e(t) - e^T(t-\tau)S_2e(t-\tau) \tag{15}$$

$$\dot{V}_4(t) = h^2\dot{e}^T(t)R_1\dot{e}(t) - h\int_{t-h}^{t}\dot{e}^T(s)R_1\dot{e}(s)ds \tag{16}$$

$$\dot{V}_5(t) = \tau^2\dot{e}^T(t)R_2\dot{e}(t) - \tau\int_{t-\tau}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \tag{17}$$

Based on Lemmas 1, 2 and 3, one can obtain

$$\begin{aligned}
&-h\int_{t-h}^{t}\dot{e}^T(s)R_1\dot{e}(s)ds \\
&\le -h\int_{t-h(t)}^{t}\dot{e}^T(s)R_1\dot{e}(s)ds - h\int_{t-h}^{t-h(t)}\dot{e}^T(s)R_1\dot{e}(s)ds \\
&\le -\frac{h}{h(t)}\left(\int_{t-h(t)}^{t}\dot{e}(s)ds\right)^T R_1\left(\int_{t-h(t)}^{t}\dot{e}(s)ds\right) \\
&\quad -\left(\frac{h}{h-h(t)}\right)\left(\int_{t-h}^{t-h(t)}\dot{e}(s)ds\right)^T R_1\left(\int_{t-h}^{t-h(t)}\dot{e}(s)ds\right) \\
&\le -\begin{bmatrix} e(t)-e(t-h(t)) \\ e(t-h(t))-e(t-h) \end{bmatrix}^T \begin{bmatrix} R_1 & T_1 \\ * & R_1 \end{bmatrix} \times \begin{bmatrix} e(t)-e(t-h(t)) \\ e(t-h(t))-e(t-h) \end{bmatrix}
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
&-\tau\int_{t-\tau}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \\
&= -\tau\int_{t-\tau}^{t-\tau(t)}\dot{e}^T(s)R_2\dot{e}(s)ds - \tau\int_{t-\tau(t)}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \\
&\le -\frac{\tau}{\tau-\tau(t)}(\alpha_1^T(t)R_2\alpha_1(t) + 3\alpha_2^T(t)R_2\alpha_2(t)) \\
&\quad -\frac{\tau}{\tau(t)}(\alpha_3^T(t)R_2\alpha_3(t) + 3\alpha_4^T(t)R_2\alpha_4(t)) \\
&= -\frac{\tau}{\tau-\tau(t)}\begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \end{bmatrix}^T \overline{R_2}\begin{bmatrix} \alpha_1(t) \\ \alpha_2(t) \end{bmatrix} \\
&\quad -\frac{\tau}{\tau(t)}\begin{bmatrix} \alpha_3(t) \\ \alpha_4(t) \end{bmatrix}^T \overline{R_2}\begin{bmatrix} \alpha_3(t) \\ \alpha_4(t) \end{bmatrix} \\
&\le -\alpha^T(t)\begin{bmatrix} \overline{R_2} & T_2 \\ * & \overline{R_2} \end{bmatrix}\alpha(t)
\end{aligned} \tag{19}$$

where

$$\alpha(t) = [\alpha_1^T(t), \alpha_2^T(t), \alpha_3^T(t), \alpha_4^T(t)]^T,$$
$$\alpha_1(t) = e(t-\tau(t)) - e(t-\tau),$$
$$\alpha_2(t) = e(t-\tau(t)) + e(t-\tau) - \frac{2}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)} e(s)ds,$$
$$\alpha_3(t) = e(t) - e(t-\tau(t)),$$
$$\alpha_4(t) = e(t) + e(t-\tau(t)) - \frac{2}{\tau(t)}\int_{t-\tau(t)}^{t} e(s)ds,$$
$$\overline{R_2} = \begin{bmatrix} R_2 & 0 \\ * & 3R_2 \end{bmatrix}$$

On the other hand, according to (8), for any appropriately dimensioned matrix M, the following equation holds

$$2[e^T(t)M+\dot{e}^T(t)M][-\dot{e}(t)-Ae(t)+g(We(t-h(t))) + Bw(t)-KCe(t-\tau(t))-Dp(t-\tau(t))]=0. \tag{20}$$

Furthermore, the diagonal matrix, $\Gamma=\{\gamma_1,\gamma_2,...,\gamma_n\}>0$, $\Lambda=\{\lambda_1,\lambda_2,...,\lambda_n\}>0$, we can achieve the following inequalities,

$$\begin{aligned}&-2g^T(We(t))\Gamma g(We(t))\\&+2e^T(t)W^T(K^-+K^+)\Gamma g(We(t))\\&-2e^T(t)W^TK^-\Gamma K^+We(t)\geq 0,\end{aligned} \tag{21}$$

$$\begin{aligned}&-2g^T(We(t-h(t)))\Lambda g(We(t-h(t)))\\&+2e^T(t-h(t))W^T(K^-+K^+)\Lambda g(We(t-h(t)))\\&-2e^T(t-h(t))W^TK^-\Lambda K^+We(t-h(t))\geq 0\end{aligned} \tag{22}$$

From (7), the following inequality holds

$$p^T(t-\tau(t))p(t-\tau(t))\leq q^T(t-\tau(t))q(t-\tau(t)) \tag{23}$$

and so there exists a positive scalar ϵ_2 satisfying

$$\epsilon[q^T(t-\tau(t))q(t-\tau(t))-p^T(t-\tau(t))p(t-\tau(t))]\geq 0 \tag{24}$$

To establish the H_∞ performance for the system, we define

$$J=\int_0^\infty[\overline{z^T}(t)\bar{z}(t)-\gamma^2w^T(t)w(t)]dt \tag{25}$$

Under the zero-initial condition, it is obvious that $V(r(t))|_{t=0}=0$, for any non-zero $w(t)\in\mathcal{L}_2[0,\infty)$, we can get

$$\begin{aligned}J_\infty &\leq \int_0^\infty[\overline{z^T}(t)\bar{z}(t)-\gamma^2w^T(t)w(t)]dt\\&\quad+V(e(t))|_{t\to\infty}-V(e(t))|_{t=0}\\&\leq\int_0^\infty[\overline{z^T}(t)\bar{z}(t)-\gamma^2w^T(t)w(t)+\dot{V}(r(t))]dt\end{aligned} \tag{26}$$

Then, combined with (13)-(26) and define $MK=G$, we have

$$\begin{aligned}&\overline{z^T}(t)\bar{z}(t)-\gamma^2w^T(t)w(t)+\dot{V}(r(t))\\&\leq\xi^T(t)\Omega(h(t))\xi(t)<0\end{aligned} \tag{27}$$

which completes the proof.

Consider the particular case of the controller scheme (6) where there is in input uncertainty, estimating error system (8) reduces to

$$\begin{aligned}\dot{e}(t)&=-Ae(t)+g(We(t-h(t)))+Bw(t)\\&\quad-KCe(t-\tau(t)),\\ \bar{z}(t)&=He(t),\end{aligned} \tag{28}$$

Based on Theorem 1, the following corollary follows immediately.

Corollary 1 For given scalars μ and $\tau>0$, error system (28) is asymptotically stable, if there exist $3n\times 3n$ matrices $\overline{P}>0$, $\overline{Q}>0$, $n\times n$, matrices $R_1>0$ $R_2>0$, $S_1>0$, $S_2>0$ diagonal matrices, $\Gamma=\{\gamma_1,\gamma_2,...,\gamma_n\}>0$, $\Lambda=\{\lambda_1,\lambda_2,...,\lambda_n\}>0$, any $n\times n$ matrices T_1, $2n\times 2n$ matrix T_2 and $n\times 1$ matrix G such that the following LMIs are satisfied for $\tau(t)=\{0,\tau\}$

$$\begin{aligned}\Omega(h(t))=&[\overline{e_1}\ \ \tau(t)\overline{e_6}+(\tau-\tau(t))\overline{e_\tau}]\overline{P}[\overline{e_{10}}\ \ \overline{e_1}-\overline{e_5}]^T\\&+[\overline{e_{10}}\ \ \overline{e_1}-\overline{e_5}]\overline{P}[\overline{e_1}\ \ r(t)\overline{e_6}+(\tau-\tau(t))\overline{e_\gamma}]^T\\&+[\overline{e_1}\ \ \overline{e_8}]\overline{Q}[\overline{e_1}\ \ \overline{e_8}]^T-(1-\mu)[\overline{e_2}\ \ \overline{e_9}]\overline{Q}[\overline{e_2}\ \ \overline{e_9}]^T\\&+\overline{e_1}S_1\overline{e_1^T}-\overline{e_3}S_1\overline{e_3^T}+e_1S_2\overline{e_1^T}-\overline{e_5}S_2^-\\&+h^2\overline{e_{10}}R_1\overline{e_{10}^T}-\overline{\Pi_1}\begin{bmatrix}R_1 & T_1\\ * & R_1\end{bmatrix}\overline{\Pi_1^T}\\&+\tau^2\overline{e_{10}}R_2\overline{e_{10}^T}-\overline{\Pi_2}\begin{bmatrix}R_2 & T_2\\ * & R_2\end{bmatrix}\overline{\Pi_2^T}\\&+\overline{e_1}W^T(K^-+K^+)\Gamma\overline{e_8^T}\\&+e_8\Gamma^T(K^-+K^+)W\overline{e_1^T}\\&-2\overline{e_1}W^TK^-\Gamma K^+W\overline{e_1^T}-2\overline{e_8}\Gamma\overline{e_8^T}\\&+\overline{e_2}W^T(K^-+K^+)\Lambda\overline{e_9^T}\\&-2\overline{e_2}W^TK^-\Lambda K^+W\overline{e_2^T}-2\overline{e_9}\Lambda\overline{e_9^T}\\&+(\overline{e_1}+\alpha\overline{e_{10}})\Psi+\Psi^T(\overline{e_1}+\alpha\overline{e_{10}})^T\\&+\overline{e_1}H^TH\overline{e_1^T}-\gamma^2\overline{e_{11}}\overline{e_{11}^T}<0\end{aligned} \tag{29}$$

$$\begin{bmatrix}R_1 & T_1\\ * & R_1\end{bmatrix}\geq 0,\ \begin{bmatrix}R_2 & T_2\\ * & R_2\end{bmatrix}\geq 0 \tag{30}$$

where

$$\begin{aligned}\overline{\Pi_1}&=[\overline{e_1}-\overline{e_2}\ \ \overline{e_2}-\overline{e_3}]\\ \overline{\Pi_2}&=[\overline{e_1}-\overline{e_4}\ \ \overline{e_1}+\overline{e_4}-2\overline{e_6}\ \ \overline{e_4}-\overline{e_5}\ \ \overline{e_4}+\overline{e_5}-2\overline{e_\gamma}]\\ \overline{\Psi}&=[-MA\ \ 0\ \ 0\ \ -GC\ \ 0\ \ 0\ \ 0\\&\qquad 0\ \ M\ \ -M\ \ MB]\end{aligned}$$

and $e\overline{e_i}\in R^{(10n+1)\times n}$ $(i=1,2,...,11)$ means the block entry matrices, e.g., $\overline{e_2^T}\,\bar{\xi}(t)=e(t-h(t))$ where

$$\begin{aligned}\overline{\xi^T}(t)=&[e^T(t)\ \ e^T(t-h(t))\ \ e^T(t-h)\\&e^T(t-\tau(t))\ \ e^T(t-\tau)\\&\frac{1}{\tau(t)}\int_{t-\tau(t)}^t e(s)ds\quad\frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}e(s)ds\\&g^T(We(t))\quad g^T(We(t-h(t)))\\&\dot{e}^T(t)\quad w^T(t)].\end{aligned}$$

Moreover, the sampled-data controller gain is given by

$K = M^{-1}G.$

Remark 2 Comparing with the existing works for the state estimation of static neural networks [7-11], the results obtained in this paper have two advantages. On the one hand, the varying sampling and uncertainties are considered in the control scheme. On the other hand, the non-fragile sampled-data control designed here is robust than the one only used the sampled-data control scheme, which will be demonstrate in the following example.

4. Numerical Examples

In this section, a numerical example is given to illustrate the validity and superiority of the proposed state estimator design method.

Consider the system with the following matrix parameters:

$$A = \begin{bmatrix} 1.06 & 0 & 0 \\ 0 & 1.42 & 0 \\ 0 & 0 & 0.88 \end{bmatrix}, W = \begin{bmatrix} -0.32 & 0.85 & -1.36 \\ 1.1 & 0.41 & -0.5 \\ 0.42 & 0.82 & -0.95 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0.5 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}, J = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = [1 \ \ 0.5 \ \ 0], D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}.$$

The sector-bounded nonlinear functions are assumed to be $f(x) = \tanh(x)$, and the time varying delay is given as $h(t) = 0.5 + 0.5\sin(t)$. The initial value is taken as $e(0) = [1 \ \ 0.5 \ \ -2]^T$.
Letting $\gamma = 3, \alpha = 1, \tau = 0.24$ and $w(t) = \sin(t)e^{-10t}$, we will considering the following two cases:

Case I: For parameter uncertainties $\triangle(t_k) = \sin(t_k)$, by solving the LMIs (10) and (11) in Theorem 1, the non-fragile sampled-data controller gain can be obtained as

$$K = [2.8383 \ \ 1.6839 \ \ 0.7731]^T.$$

Under the aforementioned gain matrix, the response curves of the error systems (8) are given in Fig. 1, which shows the states tend to zero. Fig. 2 shows the response curve of the control input $u(t)$.
Case II: For the case of $\triangle(t_k) = 0$, by solving the LMIs (29) and (30) in Corollary 1, the sampled- data controller gain can be obtained as follow:

$$K = [1.3229 \ \ 1.0720 \ \ 0.6007]^T.$$

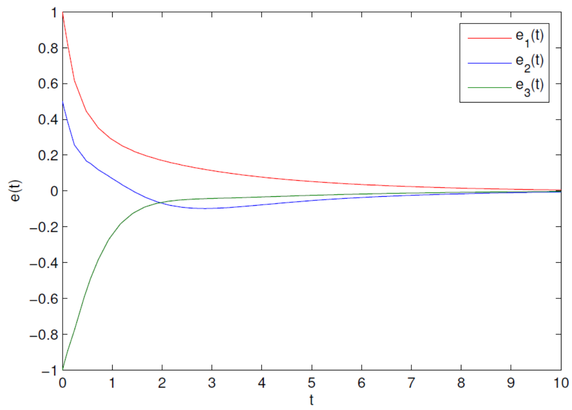

Fig. 1 State trajectory of error system with non-fragile sampled-data control
그림 1 비결함 샘플데이타 제어를 가지는 에러시스템의 상태궤적

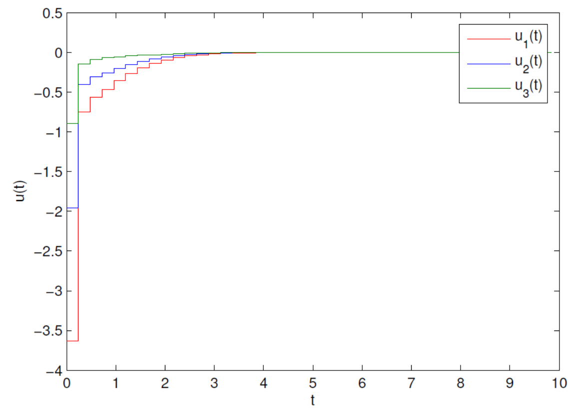

Fig. 2 Non-fragile sampled-data control input
그림 2 비결함 샘플데이타 제어입력

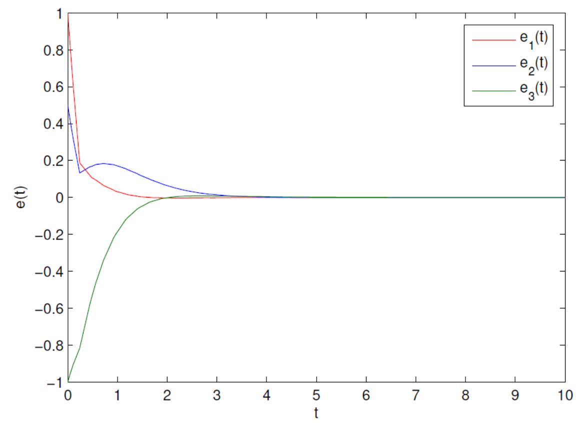

Fig. 3 State trajectories of error system under sampled-data control
그림 3 샘플데이타 제어를 가지는 에러시스템의 상태궤적

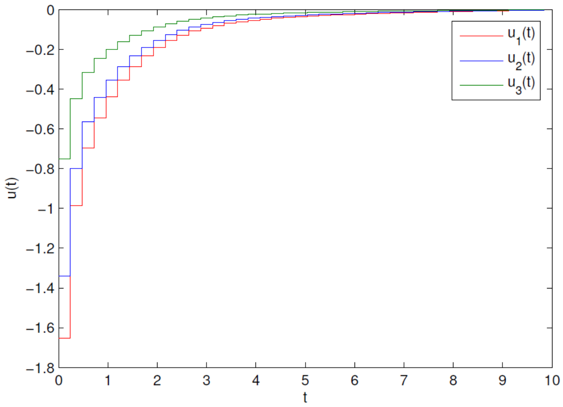

Fig. 4 Sampled-data control input
그림 4 샘플데이타 제어입력

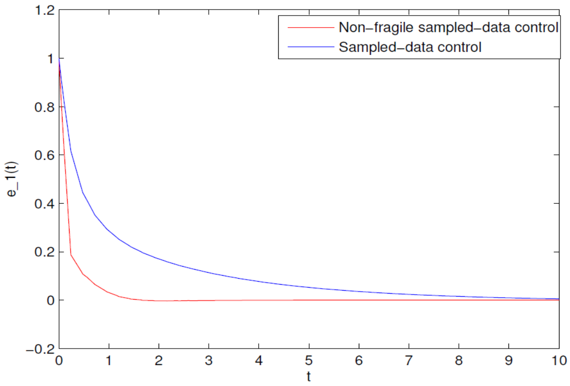

Fig. 5 The error state trajectory of $e_1(t)$
그림 5 $e_1(t)$의 상태궤적

5. Conclusions

In this paper, we have studied a H_∞ state estimation of static neural networks with time-varying delay. Based on Lyapunov functional method and LMI framework, Some sufficient conditions of H_∞ state estimation have been established. The estimator matrix gain has been obtained by solving a set of LMIs. A simulation example has been used to illustrate th effectiveness of this approach.

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