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합성된 리아프노프 함수법을 통한 샘플링 된 데이터 제어 시스템의 향상된 안정화 조건 및 제어기 설계

Improved Stability and Stabilization for Sampled-data Control System via Augmented Lyapunov-Krasovskii Functional

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Abstract - This paper investigates improved stability and stabilization criteria for sampled-data control systems. By using a suitable and newly constructed augmented Lyapunov-Krasovskii functional and some recent mathematical techniques such as auxiliary function-based integral inequalities, sufficient conditions for stability and stabilization conditions are derived within the framework of linear matrix inequalities(LMI) form. The superiority and validity of the proposed results are illustrated by three numerical examples.

Key Words : Sampled-data control system, Stability and stabilization, Lyapunov method, LMI

1. Introduction

Due to the development of digital devices and network communications, digital controllers have widely been used in real systems. These controllers can be modeled by the sampled-data control systems [1]. A sampled-data control signal is decided only at the instant of sampling. The schematic sampled-data control systems are shown in Fig. 1. The sampler and the zero-order hold(ZOH) devices are located between the plant and the controller. In Fig. 1, the output signals of the plant will be transmitted to the controller inputs as discrete signals through the sampler and then the controller discrete output signals will be communicated in the plant inputs as discontinuous signals through ZOH. Thus, a sampled-data controller design method for the systems is more complicated process than the general controller design method. Several researches were dealt with the problem of sampled-data control systems [2-9]. An refined input delay approach using the Lyapunov-Krasovskii theory was investigated in [9] new constructions of Lyapunov functional for sampled-data control systems were introduced in [5]. I-O approach via Wirtinger-type inequality has proposed in [6]. Recently, a novel analysis of asynchronous sampled-data systems was proposed in [8]. In [7], application to electric power markets for the sampled-data control systems was considered. In this paper, the stabilization and stability analysis of sampled-data control will be studied by considering the digital input signal as a time-varying delay signal like other papers [5,6,9].

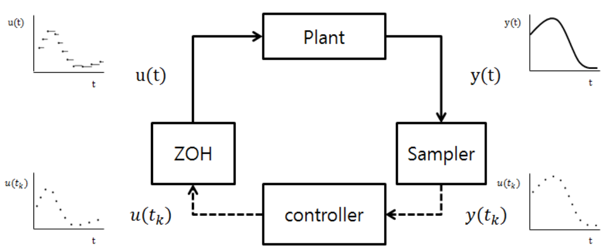

그림 1 연속 시간 시스템의 표본 추출 과정의 개략적 구조.
Fig. 1 The schematic structure of sampling process of a continuous-time system.

One of the purposes of delay-dependent stability analysis for the systems is to find maximum upper bounds of time-delay which guarantee the asymptotic stability of the systems. Many researchers have made various attempts

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to find less conservativeness of stability criteria for time-delay systems [10-12]. Jensen inequality [11] has been extensively used for analysis of many systems with time-delays since it is a key role to derive a stability condition when estimating the time-derivative of Lyapunov-Krasovskii functionals. In [12], Wirtinger-based integral inequality was introduced. Compared to Jensen inequality, it have had extra terms which can help to obtain much tighter bounds. Very recently, auxiliary function-based integral inequalities [13] for quadratic functions were developed via some auxiliary functions. It can lead to less conservative conditions than Wirtinger-based integral inequality. In this paper, auxiliary function-based integral inequalities are used to obtain stability condition and controller gain.

With motivations discussed above works, this paper focused on stability analysis and controller design for sampled-data control systems. Firstly, in Theorem 1, a stability condition will be proposed by using the appropriate Lyapunov-Krasovskii functionals with auxiliary function-based integral inequalities. Secondly, based on the result of Theorem 1, a controller design method will be introduced in Theorem 2. Three numerical examples are included to show the effectiveness of the proposed theorems.

Notations. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. $diag\{\cdots\}$ denotes the block diagonal matrix. The symmetric term in a matrices and in quadratic forms will be denoted by * (This is used if necessary.). For two symmetric matrices A and B, $A\rangle(\geq)B$ means that $A-B$ is (semi-) positive definite. I_n denotes the $n\times n$ identity matrix. 0_n and $0_{n\times m}$ are denote the $n\times n$ zero matrix and $n\times m$ zero matrix, respectively. If the context allows it, the dimensions of these matrices are often omitted. For a given matrix $B\in\mathbb{R}^{n\times n}$, we define $B^{\perp}\in\mathbb{R}^{n\times(n-r)}$ as the right orthogonal complement of B by $BB^{\perp}=0$. $Sym\{A\}$ denotes the sum of "A" and symmetric matrix of "A^T". $X_{[f(t)]}\in R^{m\times n}$ means that the elements of the matrix $X_{[f(t)]}$ includes the value of $f(t)$; e.g., $X_{[f_0]}\equiv X_{[f(t)=f_0]}$.

2. Problem statements and Preliminaries

Consider the linear system described by

$$\dot{x}(t)=Ax(t)+Bu(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the vector of control signal, $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times m}$ are constant system matrices.

The control signal is assumed to be generated by a ZOH function with a sequence of hold times $0=t_0\langle t_1\langle\ldots\langle t_k\langle\ldots\langle\lim_{k\to\infty}t_k=\infty$. Then, a state feedback controller can be transformed as

$$u(t)\xrightarrow{ZOH}u(t_k)=Kx(t_k),\qquad t_k\leq t<t_{k+1}. \tag{2}$$

By defining $h(t)=t-t_k$, a closed loop linear sampled-data control system can be represented as

$$\dot{x}(t)=Ax(t)+BKx(t-h(t)),\qquad t_k\leq t<t_{k+1}. \tag{3}$$

Thus, the sampled-data control systems can be treated as the time-varying delayed system. Moreover, it should be noted that the time delay $h(t)$ is a time-varying delay continuous function satisfying $0\leq h(t)=t-t_k\leq h_M$ and $\dot{h}(t)=1$ for $t\neq t_k$.

The purposes of this paper are to derive less conservative delay-dependent stability condition and design the sampled-data controller $u(t)=Kx(t_k)=Kx(t-h(t))$ to stabilize the system. To derive main results, the following fact and lemmas are introduced.

Fact 1. (Schur complement) [14] Given constant matrices $\Sigma_1,\Sigma_2,\Sigma_3$ with $\Sigma_1=\Sigma_1^T$ and $\Sigma_2=\Sigma_2^T$, then $\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$ if and only if

$$\begin{bmatrix}\Sigma_1 & \Sigma_3^T\\ \Sigma_3 & -\Sigma_2\end{bmatrix}<0\ \text{ or }\ \begin{bmatrix}-\Sigma_2 & \Sigma_3\\ \Sigma_3^T & \Sigma_1\end{bmatrix}<0.$$

Lemma 1. (Auxiliary function-based integral inequalities) [13] *For a positive matrix R, an integrable function $\{w(u)|u\in[a,b]\}$, the following inequalities hold:*

$$(i)\ \int_a^b\dot{w}^T(u)R\dot{w}(u)du \geq \frac{1}{b-a}\left(\Omega_1^TR\Omega_1+3\Omega_2^TR\Omega_2+5\Omega_3^TR\Omega_3\right),$$

$$(ii)\int_a^b\int_s^b\dot{w}^T(u)R\dot{w}(u)duds\geq 2\Theta_1^TR\Theta_1+4\Theta_2^TR\Theta_2,$$

where

$$\Omega_1 = \int_a^b \dot{w}(u)du,$$
$$\Omega_2 = \int_a^b w(u)du - \frac{2}{b-a}\int_a^b\int_u^b w(r)drdu,$$
$$\Omega_3 = \int_a^b w(u)du - \frac{6}{b-a}\int_a^b\int_u^b w(r)drdu + \frac{12}{(b-a)^2}\int_a^b\int_u^b\int_r^b w(v)dvdrdu,$$
$$\Theta_1 = \frac{1}{b-a}\int_a^b\int_s^b \dot{w}(u)duds,$$
$$\Theta_2 = -\frac{1}{b-a}\int_a^b\int_s^b w(u)duds + \frac{3}{(b-a)^2}\int_a^b\int_s^b\int_u^b w(v)dvduds.$$

Lemma 2. (Finsler's lemma) [15] *Let $\xi \in \mathbb{R}^n$, $M = M^T \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$ such that rank(B)<n. The following statements are equivalent:*

$i)$ $\xi^T M\xi < 0,\ \forall\ B\xi = 0,\ \xi \neq 0,$

$ii)$ $B^{\perp^T} MB^{\perp} < 0,$

$iii)$ $\exists\ F \in \mathbb{R}^{n\times m} : M + B^T F^T + FB < 0.$

Lemma 3. (Reciprocal convexity lemma) [10] *For a scalar α in the interval (0,1), a given positive matrix $T_1, T_2 \in \mathbb{R}^{n\times n}$, any matrix $X \in \mathbb{R}^{n\times n}$ and two vectors $\xi_1, \xi_2 \in \mathbb{R}^n$ satisfying that $\begin{bmatrix} T_1 & X \\ * & T_2 \end{bmatrix} > 0$, then, the following inequality holds :*

$$\frac{1}{\alpha}\xi_1^T T_1 \xi_1 + \frac{1}{1-\alpha}\xi_2^T T_2 \xi_2 \geq \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}^T \begin{bmatrix} T_1 & X \\ * & T_2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}.$$

3. Main result

This section consists of two subsections. The goal of the first subsection is to find the stability condition for linear systems with time-delay. The second subsection describes how to design a controller for the system (3).

3.1 Stability condition

In this subsection, when the given K, a delay dependent stability criterion for the system (3) is expressed as follows

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) \tag{4}$$

where $A_d = BK$.

For simplicity of matrix and vector representation, block entry matrices are defined as $e_i \in \mathbb{R}^{15n\times n}$ $(i = 1,\ldots,15)$ which will be used. For example, $e_1 = [I_n, 0_{n\times 14n}]^T$ and $e_{10} = [0_{n\times 9n}, I_n, 0_{n\times 5n}]^T$. The other notations are defined as

$$\begin{aligned}\xi(t) = [&x^T(t), x^T(t-h(t)), x^T(t-h_M), \dot{x}^T(t-h_M), \\ &\int_{t-h(t)}^t x^T(s)ds, \int_{t-h_M}^{t-h(t)} x^T(s)ds, \\ &\frac{1}{h(t)}\int_{t-h(t)}^t x^T(s)ds, \frac{1}{h_M-h(t)}\int_{t-h_M}^{t-h(t)} x^T(s)ds, \\ &\frac{1}{h(t)}\int_{t-h(t)}^t\int_s^t x^T(u)duds, \\ &\frac{1}{h_M-h(t)}\int_{t-h_M}^{t-h(t)}\int_s^{t-h(t)} x^T(u)duds, \\ &\frac{1}{h^2(t)}\int_{t-h(t)}^t\int_s^t\int_u^t x^T(v)dvduds, \\ &\frac{1}{(h_M-h(t))^2}\int_{t-h_M}^{t-h(t)}\int_s^{t-h(t)}\int_u^{t-h(t)} x^T(v)dvduds, \\ &\frac{1}{h^2(t)}\int_{t-h(t)}^t\int_s^t x^T(u)duds, \\ &\frac{1}{(h_M-h(t))^2}\int_{t-h_M}^{t-h(t)}\int_s^{t-h(t)} x^T(u)duds, \dot{x}^T(t)]^T,\end{aligned}$$

$$\Xi_1 = \begin{bmatrix} e_5^T \\ e_1^T - e_2^T \end{bmatrix},\ \Xi_2 = \begin{bmatrix} e_5^T - 2e_9^T \\ -e_1^T - e_2^T + 2e_7^T \end{bmatrix},$$
$$\Xi_2 = \begin{bmatrix} e_5^T - 6e_9^T + 12e_{11}^T \\ e_1^T - e_2^T + 6e_7^T - 12e_{13}^T \end{bmatrix},$$
$$\Xi_4 = \begin{bmatrix} e_6^T \\ e_2^T - e_3^T \end{bmatrix},\ \Xi_5 = \begin{bmatrix} e_6^T - 2e_{10}^T \\ -e_2^T - e_3^T + 2e_8^T \end{bmatrix},$$
$$\Xi_6 = \begin{bmatrix} e_6^T - 6e_{10}^T + 12e_{12}^T \\ e_2^T - e_3^T + 6e_8^T - 12e_{14}^T \end{bmatrix},$$
$$\Omega_1 = e_1^T - e_2^T,\quad \Omega_2 = -e_1^T - e_2^T + 2e_7^T,$$
$$\Omega_3 = e_1^T - e_2^T + 6e_7^T - 12e_{13}^T,\ \Omega_4 = e_2^T - e_3^T,$$
$$\Omega_5 = -e_2^T - e_3^T + 2e_8^T,\quad \Omega_6 = e_2^T - e_3^T + 6e_8^T - 12e_{14}^T.$$
$$\Xi = [\Xi_1^T, \Xi_2^T, \ldots, \Xi_6^T]^T,\quad \Omega = [\Omega_1^T, \Omega_2^T, \ldots, \Omega_6^T]^T,$$
$$\Phi_{2[h(t)]} = h(t)e_9^T + (h_M - h(t))(e_5^T + e_{10}^T),$$

$$\Psi_\alpha = Sym\left\{\begin{bmatrix} e_{15}^T \\ e_4^T \\ e_1^T - e_3^T \\ h_M e_1^T - e_5^T - e_6^T \end{bmatrix}^T P \begin{bmatrix} e_1^T \\ e_3^T \\ e_5^T + e_6^T \\ 0 \end{bmatrix}\right\}$$
$$+ \begin{bmatrix} e_1^T \\ e_{15}^T \end{bmatrix}^T Q \begin{bmatrix} e_1^T \\ e_{15}^T \end{bmatrix} - \begin{bmatrix} e_3^T \\ e_4^T \end{bmatrix}^T Q \begin{bmatrix} e_3^T \\ e_4^T \end{bmatrix}$$
$$+ \begin{bmatrix} e_1^T \\ e_{15}^T \end{bmatrix}^T h_M Z \begin{bmatrix} e_1^T \\ e_{15}^T \end{bmatrix} - \frac{1}{h_M}\{\Xi^T \hat{Z} \Xi\}$$
$$+ \begin{bmatrix} e_1^T \\ 0 \\ 0 \end{bmatrix}^T G \begin{bmatrix} e_1^T \\ 0 \\ 0 \end{bmatrix} + Sym\left\{\begin{bmatrix} 0 \\ e_{15}^T \\ e_1^T \end{bmatrix}^T G \begin{bmatrix} e_5^T \\ -e_5^T \\ 0 \end{bmatrix}\right\}$$
$$- sym\{Y_1^T \Omega_1 + 3Y_2^T \Omega_2 + 5Y_3^T \Omega_3\},$$

$$\Psi_R = \frac{h_M^2}{2} e_{15}(R_1 + R_2)e_{15}^T - 2(e_1^T - e_7^T)^T R_1 (e_1^T - e_7^T)$$
$$- \Omega^T \hat{R} \Omega + \Omega_1^T R_1 \Omega_1 + \Omega_2^T 3R_1 \Omega_2 + \Omega_3^T 5R_1 \Omega_3$$
$$+ \Omega_4^T R_2 \Omega_4 + \Omega_5^T 3R_2 \Omega_5 + \Omega_6^T 5R_2 \Omega_6$$
$$- 2(e_2^T - e_8^T)^T R_1 (e_2^T - e_8^T) - 2(e_2^T - e_7^T)^T R_2 (e_2^T - e_7^T)$$
$$- 2(e_3^T - e_8^T)^T R_2 (e_3^T - e_8^T)$$
$$- 4\left(\frac{1}{2}e_1^T + e_7^T - 3e_{13}^T\right)^T R_1 \left(\frac{1}{2}e_1^T + e_7^T - 3e_{13}^T\right)$$
$$- 4\left(\frac{1}{2}e_2^T + e_8^T - 3e_{14}^T\right)^T R_1 \left(\frac{1}{2}e_2^T + e_8^T - 3e_{14}^T\right)$$
$$- 4\left(\frac{1}{2}e_2^T + e_7^T - 3e_{13}^T\right)^T R_2 \left(\frac{1}{2}e_2^T + e_7^T - 3e_{13}^T\right)$$
$$- 4\left(\frac{1}{2}e_3^T + e_8^T - 3e_{14}^T\right)^T R_2 \left(\frac{1}{2}e_3^T + e_8^T - 3e_{14}^T\right),$$

$$\Theta_{[h(t)]} = Sym\left\{\begin{bmatrix} 0 \\ e_{15}^T \\ e_1^T \end{bmatrix}^T G \begin{bmatrix} 0 \\ h(t)e_1^T \\ h(t)e_9^T \end{bmatrix}\right\}$$
$$+ Sym\left\{\begin{bmatrix} e_{15}^T \\ e_4^T \\ e_1^T - e_3^T \\ h_M e_1^T - e_5^T - e_6^T \end{bmatrix}^T P \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Phi_{2[h(t)]} \end{bmatrix}\right\}$$
$$+ (h_M - h(t))x^T(t-h(t))Dx(t-h(t))$$
$$- h(t)x^T(t-h(t))Dx(t-h(t)),$$
$$\Psi = \Psi_\alpha + \Psi_R, \quad N = Ae_1^T + BKe_2^T - e_{15}^T.$$

Now, following theorem is given as a stability criterion for the system (4).

Theorem 1. *Given a scalar* $h_M > 0$ *and gain K, the system (4) is asymptotically stable if there exist positive definite matrices* $P \in \mathbb{R}^{4n \times 4n}$, $Q, Z \in \mathbb{R}^{2n \times 2n}$, $G \in \mathbb{R}^{3n \times 3n}$, R_1, R_2, $H, D \in \mathbb{R}^{n \times n}$, *any matrices* $C \in \mathbb{R}^{6n \times 6n}$, $S \in \mathbb{R}^{3n \times 3n}$, $Y_i \in \mathbb{R}^{n \times 15n}(i = 1,2,3)$ *satisfying the following conditions hold:*

$$\hat{R} = \begin{bmatrix} diag\{R_1, 3R_1, 5R_1\} & S \\ * & diag\{R_2, 3R_2, 5R_2\} \end{bmatrix} > 0, \tag{5}$$

$$\hat{Z} = \begin{bmatrix} diag\{Z_1, 3Z_1, 5Z_1\} & C \\ * & diag\{Z_1, 3Z_1, 5Z_1\} \end{bmatrix} > 0, \tag{6}$$

$$N^{\perp T}(\Theta_{[0]} + \Psi)N^{\perp} < 0, \tag{7}$$

$$\begin{bmatrix} N^{\perp T}(\Theta_{[h_M]} + \Psi)N^{\perp} & N^{\perp T}Y_1^T & N^{\perp T}Y_2^T & N^{\perp T}Y_3^T \\ * & -\frac{1}{h_M}H & 0 & 0 \\ * & * & -\frac{1}{3h_M}H & 0 \\ * & * & * & -\frac{1}{5h_M}H \end{bmatrix} < 0. \tag{8}$$

Proof. Let us consider the following Lyapunov-Krasovskii functional candidate:

$$V(t) = \sum_{i=1}^{8} V_i(t) \tag{9}$$

where

$$V_1(t) = \begin{bmatrix} x(t) \\ x(t-h_M) \\ \int_{t-h_M}^{t} x(s)ds \\ \int_{t-h_M}^{t}\int_{s}^{t} x(u)duds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ x(t-h_M) \\ \int_{t-h_M}^{t} x(s)ds \\ \int_{t-h_M}^{t}\int_{s}^{t} x(u)duds \end{bmatrix},$$
$$V_2(t) = \int_{t-h_M}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T Q \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds,$$
$$V_3(t) = \int_{t-h_M}^{t}\int_{s}^{t} \begin{bmatrix} x(u) \\ \dot{x}(u) \end{bmatrix}^T Z \begin{bmatrix} x(u) \\ \dot{x}(u) \end{bmatrix} duds,$$
$$V_4(t) = \int_{t-h_M}^{t}\int_{s}^{t}\int_{u}^{t} \dot{x}^T(v)R_1\dot{x}(v)dvduds,$$
$$V_5(t) = \int_{t-h_M}^{t}\int_{t-h_M}^{s}\int_{u}^{t} \dot{x}^T(v)R_2\dot{x}(v)dvduds,$$
$$V_6(t) = \int_{t-h(t)}^{t} \begin{bmatrix} x(s) \\ \int_{s}^{t}\dot{x}(u)du \\ \int_{s}^{t} x(u)du \end{bmatrix}^T G \begin{bmatrix} x(s) \\ \int_{s}^{t}\dot{x}(u)du \\ \int_{s}^{t} x(u)du \end{bmatrix} ds,$$
$$V_7(t) = (h_M - h(t))\int_{t-h(t)}^{t} \dot{x}^T(s)H\dot{x}(t)ds,$$
$$V_8(t) = h(t)(h_M - h(t))x^T(t-h(t))Dx(t-h(t)).$$

The time-derivatives of $\dot{V}_i(t)(i=1,\ldots,5)$ are written by

$$
\begin{aligned}
&\dot{V}_1(t)\\
&= Sym\left\{\begin{bmatrix}\dot{x}(t)\\ \dot{x}(t-h_M)\\ x(t)-x(t-h_M)\\ h_M x(t)-\int_{t-h_M}^{t}x(s)ds\end{bmatrix}^T P\begin{bmatrix}x(t)\\ x(t-h_M)\\ \int_{t-h_M}^{t}x(s)ds\\ \int_{t-h_M}^{t}\int_{s}^{t}x(u)duds\end{bmatrix}\right\}\\
&=\xi^T(t)\left(Sym\left\{\begin{bmatrix}e_{15}^T\\ e_4^T\\ e_1^T-e_3^T\\ h_M e_1^T-e_5^T-e_6^T\end{bmatrix}^T P\begin{bmatrix}e_1^T\\ e_3^T\\ e_5^T+e_6^T\\ \Phi_{2[h(t)]}\end{bmatrix}\right\}\right)\xi(t), \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_2(t) &= \begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^T Q\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}-\begin{bmatrix}x(t-h_M)\\ \dot{x}(t-h_M)\end{bmatrix}^T Q\begin{bmatrix}x(t-h_M)\\ \dot{x}(t-h_M)\end{bmatrix}\\
&=\xi^T(t)\left(\begin{bmatrix}e_1^T\\ e_{15}^T\end{bmatrix}^T Q\begin{bmatrix}e_1^T\\ e_{15}^T\end{bmatrix}-\begin{bmatrix}e_3^T\\ e_4^T\end{bmatrix}^T Q\begin{bmatrix}e_3^T\\ e_4^T\end{bmatrix}\right)\xi(t), \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) &= \begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^T h_M Z\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}-\int_{t-h(t)}^{t}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T Z\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&-\int_{t-h_M}^{t-h(t)}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T Z\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_4(t) &= \frac{h_M^2}{2}\dot{x}^T(t)R_1\dot{x}(t)-\int_{t-h(t)}^{t}\int_{s}^{t}\dot{x}^T(u)R_1\dot{x}(u)duds\\
&-\int_{t-h_M}^{t-h(t)}\int_{s}^{t-h(t)}\dot{x}^T(u)R_1\dot{x}(u)duds \qquad (13)\\
&-(h_M-h(t))\int_{t-h(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds,
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_5(t) &= \frac{h_M^2}{2}\dot{x}^T(t)R_2\dot{x}(t)-\int_{t}^{t-h(t)}\int_{u}^{t-h(t)}\dot{x}^T(u)R_2\dot{x}(u)duds\\
&-\int_{t-h(t)}^{t-h_M}\int_{u}^{t-h_M}\dot{x}^T(u)R_2\dot{x}(u)duds \qquad (14)\\
&-h(t)\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)ds.
\end{aligned}
$$

Considering $\dot{h}(t)=1$ leads to the $\dot{V}_i(t)(i=6,7,8)$ as

$$
\begin{aligned}
\dot{V}_6(t) &= \begin{bmatrix}x(t)\\ 0\\ 0\end{bmatrix}^T G\begin{bmatrix}x(t)\\ 0\\ 0\end{bmatrix}\\
&-(1-\dot{h}(t))\begin{bmatrix}x(t-h(t))\\ \int_{t-h(t)}^{t}\dot{x}(s)ds\\ \int_{t-h(t)}^{t}x(s)ds\end{bmatrix}^T G\begin{bmatrix}x(t-h(t))\\ \int_{t-h(t)}^{t}\dot{x}(s)ds\\ \int_{t-h(t)}^{t}x(s)ds\end{bmatrix}\\
&+2\int_{t-h(t)}^{t}\begin{bmatrix}0\\ \dot{x}(t)\\ x(t)\end{bmatrix}^T G\begin{bmatrix}x(s)\\ \int_{s}^{t}\dot{x}(u)du\\ \int_{s}^{t}x(u)du\end{bmatrix}ds\\
&=\xi^T(t)\left\{\begin{bmatrix}e_1^T\\ 0\\ 0\end{bmatrix}^T G\begin{bmatrix}e_1^T\\ 0\\ 0\end{bmatrix}+Sym\left\{\begin{bmatrix}0\\ e_{15}^T\\ e_1^T\end{bmatrix}^T G\begin{bmatrix}e_5^T\\ h(t)e_1^T-e_5^T\\ h(t)e_9^T\end{bmatrix}\right\}\right\}\xi(t) \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_7(t) &= -\dot{h}(t)\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds-(h_M-h(t))\\
&-(h_M-h(t))\dot{x}^T(t)H\dot{x}(t)\\
&+(h_M-h(t))(1-\dot{h}(t))\dot{x}^T(t-h(t))H\dot{x}(t-h(t))\\
&=-\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds-(h_M-h(t))\dot{x}^T(t)H\dot{x}(t), \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_8(t) &= \left\{\dot{h}(t)(h_M-h(t))+h(t)(-\dot{h}(t))\right\}\\
&\times x^T(t-h(t))Dx(t-h(t))\\
&=(h_M-h(t))x^T(t-h(t))Dx(t-h(t)) \qquad (17)\\
&-h(t)x^T(t-h(t))Dx(t-h(t)).
\end{aligned}
$$

By applying Lemma 1, Eqs. (12)-(14) are rewritten by

$$
\begin{aligned}
&-\int_{t-h(t)}^{t}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T Z\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&\le-\frac{1}{h(t)}\xi^T(t)\left\{\Xi_1^T Z\Xi_1+\Xi_2^T 3Z\Xi_2+\Xi_3^T 5Z\Xi_3\right\}\xi(t), \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{t-h_M}^{t-h(t)}\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}^T Z\begin{bmatrix}x(s)\\ \dot{x}(s)\end{bmatrix}ds\\
&\le-\frac{1}{h_M-h(t)}\xi^T(t)\left\{\Xi_4^T Z\Xi_4+\Xi_5^T 3Z\Xi_5+\Xi_6^T 5Z\Xi_6\right\}\xi(t), \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{t-h(t)}^{t}\int_{u}^{t}\dot{x}^T(s)R_1\dot{x}(s)dsdu\\
&\le-\xi^T(t)\Big\{2\left(e_1^T-e_7^T\right)^T R_1\left(e_1^T-e_7^T\right) \qquad (20)\\
&+4\left(\frac{1}{2}e_1^T+e_7^T-3e_{13}^T\right)^T R_1\left(\frac{1}{2}e_1^T+e_7^T-3e_{13}^T\right)\Big\}\xi(t),
\end{aligned}
$$

$$-\int_{t-h_M}^{t-h(t)}\int_{u}^{t-h(t)}\dot{x}^T(s)R_1\dot{x}(s)dsdu$$
$$\le -\xi^T(t)\Big\{2\big(e_2^T-e_8^T\big)^TR_1\big(e_2^T-e_8^T\big)$$
$$+4\Big(\frac{1}{2}e_2^T+e_8^T-3e_{14}^T\Big)^TR_1\Big(\frac{1}{2}e_2^T+e_8^T-3e_{14}^T\Big)\Big\}\xi(t), \quad (21)$$

$$-\int_{t}^{t-h(t)}\int_{u}^{t-h(t)}\dot{x}^T(s)R_2\dot{x}(s)dsdu$$
$$\le -\xi^T(t)\Big\{2\big(e_2^T-e_7^T\big)^TR_2\big(e_2^T-e_7^T\big)$$
$$+4\Big(\frac{1}{2}e_2^T+e_7^T-3e_{13}^T\Big)^TR_2\Big(\frac{1}{2}e_2^T+e_7^T-3e_{13}^T\Big)\Big\}\xi(t), \quad (22)$$

$$-\int_{t-h(t)}^{t-h_M}\int_{u}^{t-h_M}\dot{x}^T(s)R_2\dot{x}(s)dsdu$$
$$\le -\xi^T(t)\Big\{2\big(e_3^T-e_8^T\big)^TR_2\big(e_3^T-e_8^T\big)$$
$$+4\Big(\frac{1}{2}e_3^T+e_8^T-3e_{14}^T\Big)^TR_2\Big(\frac{1}{2}e_3^T+e_8^T-3e_{14}^T\Big)\Big\}\xi(t), \quad (23)$$

$$-(h_M-h(t))\int_{t-h(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds$$
$$\le -\Big(\frac{h_M}{h(t)}-1\Big)\xi^T(t)\big\{\Omega_1^TR_1\Omega_1+\Omega_2^T3R_1\Omega_2+\Omega_3^T5R_1\Omega_3\big\}\xi(t), \quad (24)$$

$$-h(t)\int_{t-h(t)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds\le$$
$$-\Big(\frac{h_M}{h_M-h(t)}-1\Big)\xi^T(t)\big\{\Omega_4^TR_2\Omega_4+\Omega_5^T3R_2\Omega_5+\Omega_6^T5R_2\Omega_6\big\}\xi(t), \quad (25)$$

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds$$
$$\le -\frac{1}{h(t)}\xi^T(t)\big\{\Omega_1^TH\Omega_1+\Omega_2^T3H\Omega_2+\Omega_3^T5H\Omega_3\big\}\xi(t), \quad (26)$$

Let define $\alpha=\dfrac{h(t)}{h_M}$. Using Lemma 3, the following Eq. (27) can be obtained from Eqs. (18) and (19).

$$-\frac{1}{h_M}\Big(\frac{1}{\alpha}\xi^T(t)\big\{\Xi_1^TZ\Xi_1+\Xi_2^T3Z\Xi_2+\Xi_3^T5Z\Xi_3\big\}\xi(t)$$
$$+\frac{1}{1-\alpha}\xi^T(t)\big\{\Xi_1^TZ\Xi_1+\Xi_2^T3Z\Xi_2+\Xi_3^T5Z\Xi_3\big\}\xi(t)\Big)$$
$$\le -\frac{1}{h_M}\xi^T(t)\big\{\Xi^T\hat{Z}\Xi\big\}\xi(t), \quad (27)$$

In a similar way, the following Eq. (28) can be obtained from Eqs. (24) and (25)

$$-\frac{1}{\alpha}\xi^T(t)\big\{\Omega_1^TR_1\Omega_1+\Omega_2^T3R_1\Omega_2+\Omega_3^T5R_1\Omega_3\big\}\xi(t)$$
$$-\frac{1}{1-\alpha}\xi^T(t)\big\{\Omega_4^TR_2\Omega_4+\Omega_5^T3R_2\Omega_5+\Omega_6^T5R_2\Omega_6\big\}\xi(t)$$
$$+\xi^T(t)\big\{\Omega_1^TR_1\Omega_1+\Omega_2^T3R_1\Omega_2+\Omega_3^T5R_1\Omega_3\big\}\xi(t)$$
$$+\xi^T(t)\big\{\Omega_4^TR_2\Omega_4+\Omega_5^T3R_2\Omega_5+\Omega_6^T5R_2\Omega_6\big\}\xi(t)$$
$$\le -\xi^T(t)\big\{\Omega^T\hat{R}\Omega\big\}\xi(t)$$
$$+\xi^T(t)\big\{\Omega_1^TR_1\Omega_1+\Omega_2^T3R_1\Omega_2+\Omega_3^T5R_1\Omega_3\big\}\xi(t)$$
$$+\xi^T(t)\big\{\Omega_4^TR_2\Omega_4+\Omega_5^T3R_2\Omega_5+\Omega_6^T5R_2\Omega_6\big\}\xi(t). \quad (28)$$

At this time, it should be noted that the LMIs (5) and (6) hold to Eqs. (27) and (28).

According to the paper [12], for any matrices $Y_i\in\mathbb{R}^{n\times 15n}(i=1,2,3)$, it holds

$$\frac{1}{h(t)}(H\Omega_i-h(t)Y_i)^TH^{-1}(H\Omega_i-h(t)Y_i)\ge 0\ (i=1,2,3),$$

which satisfy the following inequality

$$-\frac{1}{h(t)}\Omega_i^TH\Omega_i\le -Y_i^T\Omega_i-\Omega_i^TY_i+h(t)\Omega_i^TH^{-1}\Omega_i.$$

Therefore, from (26), we can obtain the following inequality

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)H\dot{x}(s)ds$$
$$\le\ -\xi^T(t)\big\{sym\{Y_1^T\Omega_1+3Y_2^T\Omega_2+5Y_3^T\Omega_3\} \quad (29)$$
$$+h(t)(Y_1^TH^{-1}Y_1+Y_2^T3H^{-1}Y_2+Y_2^T5H^{-1}Y_2)\big\}\xi(t).$$

By combining Eqs. (11)-(29), a upper bound of the $\dot{V}(t)$ has

$$\dot{V}(t)\le\zeta^T(t)(\Theta_{[h(t)]}+h(t)\Gamma+\Psi)\zeta(t), \quad (30)$$

where $\Gamma=\sum_{i=1}^{3}(2i-1)Y_i^TH^{-1}Y_i$.

From (30), the stability condition for system (4) can be derived as

$$\Theta_{[h(t)]}+h(t)\Gamma+\Psi<0. \quad (31)$$

Since the condition (31) is affine convex in $h(t)\in[0,h_M]$, Eq. (30) is equivalent to

$$\Theta_{[h(t)=0]}+\Psi<0, \quad (32)$$

$$\Theta_{[h(t)=h_M]}+h_M\Gamma+\Psi<0. \quad (33)$$

Considering the following zero equation $N\xi(t)=0$, where $N=Ae_1^T+A_de_2^T-e_{15}^T$, and Lemma 2, Eqs. (32) and (33) are equivalent to the follow as

$$N^{\perp T}(\Theta_{[h(t)=0]}+\Psi)N^{\perp}<0, \quad (34)$$

$$N^{\perp T}(\Theta_{[h(t)=h_M]}+h_M\Gamma+\Psi)N^{\perp}<0. \quad (35)$$

By using Fact 1, Eq. (35) can be equivalent to the LMI (8). Therefore, if the LMIs (7) and (8) hold, then the system (4) is asymptotically stable. □

3.2 Sampled-data controller design

In this subsection, the controller design for system (3) will be derived based on Theorem 1. Now, the following theorem is given as a stabilization condition for the system (3).

Theorem 2. *For given scalar $h_M>0$ and $\alpha>0$, the system (3) with the state feedback controller $u(t)=Kx(t-h(t))$ is asymptotic stable if there exist positive definite matrices $\hat{P}\in\mathbb{R}^{4n\times 4n}$, $\hat{Q}\in\mathbb{R}^{2n\times 2n}$, $\hat{Z}\in\mathbb{R}^{2n\times 2n}$, $\hat{G}\in\mathbb{R}^{3n\times 3n}$, $\hat{R}_1$, $\hat{R}_2$, $\hat{H}$, $\hat{D}\in\mathbb{R}^{n\times n}$ and any matrices $X\in\mathbb{R}^{n\times n}$, $Y\in\mathbb{R}^{m\times n}$ and $\hat{Y}_i\in\mathbb{R}^{n\times 15n}(i=1,2,3)$ satisfying the LMIs (5), (6) and*

$$\hat{\Theta}_{[h(t)=0]}+\hat{\Psi}+Sym\{(e_1+\alpha e_{15})\hat{N}\}<0, \quad (36)$$

$$\begin{bmatrix} \begin{pmatrix} \hat{\Theta}_{[h(t)=h_M]}+\hat{\Psi} \\ +Sym\{(e_1+\alpha e_{15})\hat{N}\} \end{pmatrix} & \hat{Y}_1^T & \hat{Y}_2^T & \hat{Y}_3^T \\ * & -\frac{1}{h_M}\hat{H} & 0 & 0 \\ * & * & -\frac{1}{3h_M}\hat{H} & 0 \\ * & * & * & -\frac{1}{5h_M}\hat{H} \end{bmatrix}<0. \quad (37)$$

Then, the gain K can be calculated by YX^{-1}.

Proof. By Lemma 2, the conditions (34) and (35) can be rewritten by

$$(\Theta_{[h(t)=0]}+\Psi)+N^TM^T+MN<0, \quad (38)$$

$$(\Theta_{[h(t)=h_M]}+h_M\Gamma+\Psi)+N^TM^T+MN<0 \quad (39)$$

for any matrix M.

Here, to design the controller gain K, the matrix M would be defined as $(e_1U+\alpha e_{15}U)$, where any matrix $U\in\mathbb{R}^{n\times n}$ and the positive scalar α. Let us define $U^{-1}=X$, and then, pre-and post-multiplying both side of Eqs. (38) and (39) with $\overline{X}_{15}$ where $\overline{X}_i=diag\{\underbrace{X,\ldots,X}_{i\ factors}\}$, and defining $KX=Y$ lead to

$$\hat{\Theta}_{[h(t)=0]}+\hat{\Psi}+Sym\{(e_1+\alpha e_{15})\hat{N}\}<0, \quad (40)$$

$$\hat{\Theta}_{[h(t)=h_M]}+h_M\hat{\Gamma}+\hat{\Psi}+Sym\{(e_1+\alpha e_{15})\hat{N}\}<0, \quad (41)$$

where

$$\overline{X}_{15}^T\Psi\overline{X}_{15}=\hat{\Psi},\ \overline{X}_{15}^T\Gamma\overline{X}_{15}=\hat{\Gamma},\ \overline{X}_{15}^T\Theta_{[h(t)]}\overline{X}_{15}=\hat{\Theta}_{[h(t)]},$$
$$\hat{N}=AXe_1^T+BYe_2^T-Xe_{15}^T.$$

After this, there is need of replacing $\overline{X_4^T}P\overline{X_4}$, $\overline{X_2^T}Q\overline{X_2}$, $\overline{X_2^T}Z\overline{X_2}$, XR_1X, XR_2X, $\overline{X_3^T}G\overline{X_3}$, XHX, XDX and $XY_i(i=1,2,3)$ with $\hat{P}$, $\hat{Q}$, $\hat{Z}$, $\hat{R}_1$, $\hat{R}_2$, $\hat{G}$, $\hat{H}$, $\hat{D}$ and $\hat{Y}_i(i=1,2,3)$, respectively. Finally, by Fact 1, the condition (41) is equivalent to the LMI (37). Therefore, if the LMIs (36) and (37) hold, then the system (3) is asymptotically stable under the gain $K=YX^{-1}$. □

Remark 1. In the field of delay-dependent stability analysis for the system (4), to the best of the authors's knowledge, Jensen inequality proposed in [11] and Wirtinger-based integral inequalities in [12] are the most useful lemmas of the existing methods to reduce less conservatism of stability criteria. By constructing the different Lyapunov-Krasovskii functionals and using auxiliary function-based integral inequalities [13], further improved results will be obtained in comparison with the previous results.

4. Numerical examples

In this section, three numerical examples will be shown to illustrate the effectiveness and superiority of the proposed results in Theorems 1 and 2.

Example 1. Consider the system (4) with following parameters:

$$A=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix},\ B=\begin{bmatrix}0\\ -0.1\end{bmatrix},\ K=\begin{bmatrix}3.75\\ 11.5\end{bmatrix}^T.$$

For this example, by applying Theorem 1(Th. 1) to the system (4), maximum allowable delay bounds h_M= 1.725 can be obtained. In Table 1, the comparisons with the proposed results are listed. Therefore, the superiority of the proposed result in Theorem 1 can be confirmed.

표 1 시간 지연 최대 허용 값 h_M (예제 1).
Table 1 Allowable upper bound h_M (Example 1).

Methods	[5]	[6]	[7]	[8]	Th. 1	Th. bounds
h_M	1.695	1.695	1.72	1.721	1.725	1.7294

Example 2. Also, let us consider the system (4) with the IEEE 4-machine 11-bus system (see Fig. 2 in [15]) and take 4-Generators as the balance node, using model analysis method so that the state matrix A and the time-delay matrix A_d are as follows

$$A=\begin{bmatrix}0 & 0 & 0 & 376.9 & 0 & 0\\ 0 & 0 & 0 & 0 & 376.9 & 0\\ 0 & 0 & 0 & 0 & 0 & 376.9\\ -0.073 & 0.065 & 0.004 & -0.73 & 0.272 & 0.076\\ 0.058 & -0.087 & 0.009 & 1.160 & -0.343 & -0.134\\ 0.008 & 0.011 & -0.082 & -0.02 & 0.047 & -0.554\end{bmatrix},$$

$$A_d=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.234 & -0.839 & 0.010\\ 0 & -0.0011 & 0.001 & 0.348 & -1.362 & -0.138\\ 0 & 0.001 & 0 & 0.049 & -0.29 & -0.638\end{bmatrix}.$$

By applying Theorem 1 with above parameters to the system (4), maximum allowable delay bounds h=0.603 can be obtained. In Figs. 3 and 4, the trajectories of $x(t)$and $x(t_k)$ in system (4) are shown, respectively.

Example 3. Finally, let us consider the system (3) with uncertainties and following system parameters:

$$A=\begin{bmatrix}1 & 0.5\\ g_1 & -1\end{bmatrix},\ B=\begin{bmatrix}1+g_2\\ -1\end{bmatrix}.$$

where $\|g_1\|\le 0.1$, $\|g_2\|\le 0.3$. It can be confirmed that the system (3) with uncertainties is stable by utilizing the sampled-data control (2). Maximum allowable delay bounds h_M=0.753 and the controller gain

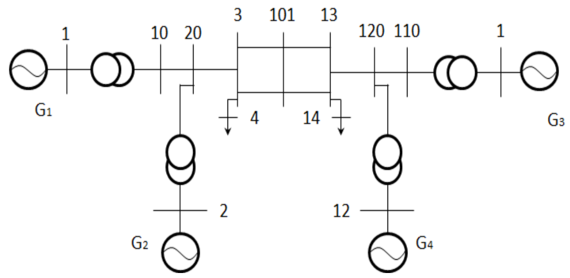

그림 2 IEEE 4-machine 11-bus 시스템 계략도 (예제 2).
Fig. 2 IEEE 4-machine 11-bus system scheme (Example 2).

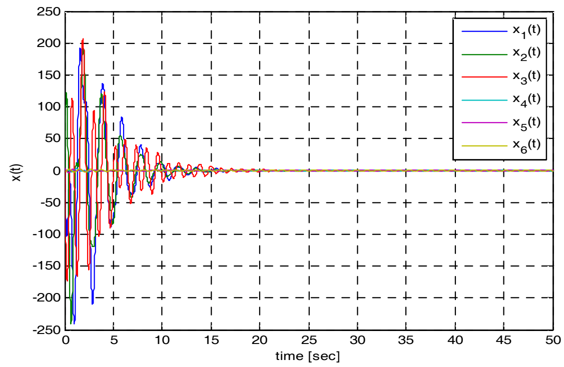

그림 3 최대 시간 지연 h_M=0.603일 때 시스템 4의 $x(t)$궤적 (예제 2).
Fig. 3 The trajectories of $x(t)$ in system (4) with upper bounds h_M=0.603 (Example 2).

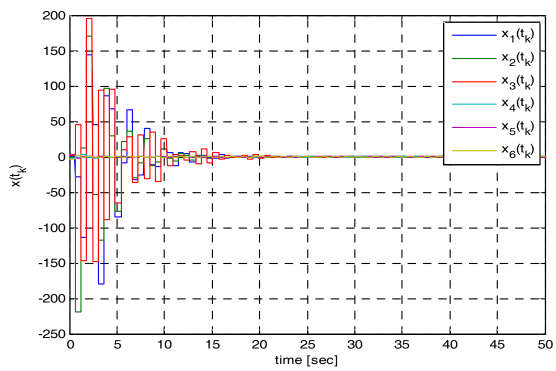

그림 4 최대 시간 지연 h_M=0.603일 때 시스템 4의 $x(t_k)$ 궤적 (예제 2).
Fig. 4 The trajectories of $x(t_k)$ in system (4) with upper bounds h_M=0.603 (Example 2).

K=[-2.5547, -0.6350] can be obtained by Theorem 2 with α=1.2. And, the results obtained by the work [16] are the bounds h_M=0.35 and the controller gain K=[-2.6884, -0.6649]. The trajectories of $x(t)$ and $Kx(t_k)$ in system (3) are shown in Figs. 5-6, respectively. Also, by applying Theorem 1 with the controller gain of the proposed result in [16] and above parameters to the system (4), maximum allowable delay bounds h_M=0.6495 can be obtained

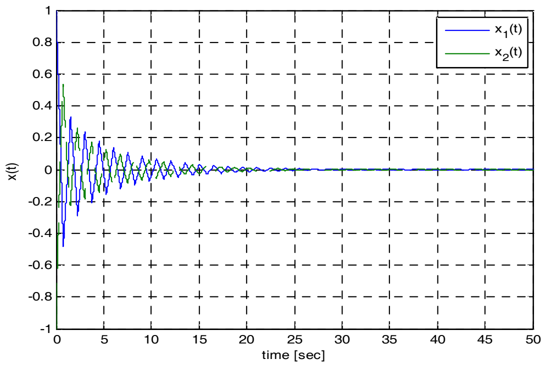

그림 5 h_M=0.753과 K=[-2.5547, -0.6350]를 고려한 시스템 3의 $Kx(t_k)$ 궤적 (α=1.2, 예제 3).

Fig. 5 The trajectories of $Kx(t_k)$ in system (3) with h_M=0.753 and K=[-2.5547, -0.6350] (α=1.2, Example 3).

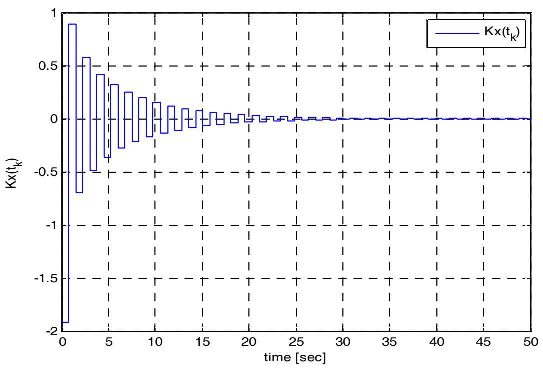

그림 6 h_M=0.753과 K=[-2.5547, -0.6350]를 고려한 시스템 3의 $Kx(t_k)$ 궤적 (예제 3).

Fig. 6 The trajectories of $Kx(t_k)$ in system (3) with h_M=0.753 and K=[-2.5547, -0.6350] (Example 3).

표 2 시간 지연 최대 허용 값 h_M (예제 3).

Table 2 Allowable upper bound h_M (Example 3).

Methods	[16]	[17]	[18]	[19]	Th. 1
h_M	0.35	0.4476	0.602	0.6409	0.6495

and the comparison of the results obtained by Theorem 1 with the previous results are conducted in Table 2.

5. Conclusions

In this paper, the stability criteria and controller design for the sampled-data control systems were proposed. Firstly, by constructing the augmented Lyapunov-Krasovskii functionals and utilizing mathematic technique such as auxiliary function-based integral inequalities, Finsler's lemma and reciprocal convexity lemma, the stability conditions were introduced in Theorem 1. Secondly, based on the result of Theorem 1, the stabilization criterion for sampled-data control systems was proposed in Theorem 2. The examples show the reduction of the conservatism compared with previous research. By using auxiliary function-based integral inequalities instead of the other mathematic lemmas such as Wirtinger-based integral inequalities and Jensen inequlities, the range of allow sampling periods has been extended.

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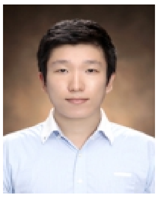

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