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Joint Estimation of Near-Field Source Parameters and Array Response

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Abstract
Near-field source localization algorithms are very sensitive to sensor gain/phase response errors, and so it is important to calibrate the errors. We took into consideration the uniform linear array and are proposing a blind calibration algorithm that can estimate the directions-of-arrival and range parameters of incident signals and sensor gain/phase responses jointly, without the need for any reference source. They are estimated separately by using an iterative approach, but without the need for good initial guesses. The ambiguities in the estimations of 2-D electric angles and sensor gain/phase responses are also analyzed in this paper. We show that the ambiguities can be remedied by assuming that two sensor phase responses of the array have been previously calibrated. The behavior of the proposed method is illustrated through simulation experiments. The simulation results show that the convergent rate is fast and that the convergent precision is high.

Keywords
Array Calibration, Gain/Phase Response, Near-Field Source Localization

1. Introduction

Source localization has been widely used in radars, sonars, sensor networks, and microphone arrays. Various algorithms have been proposed for bearing estimation of multiple far-field sources where the propagating waves are considered to be plane waves at the sensor array [1,2]. However, when the source is located in the near field of the array (Fresnel region of the array), the wavefront emitted by the source is spherical, rather than planar, at each sensor. Then, the estimated algorithms of the far-field directions-of-arrivals (DOAs) are not applicable. In this situation, more sophisticated algorithms have to be derived for estimating the bearing and range to localize the near-field source [3-6].

Recently, numerous methods have been developed for near-field source localization, such as the maximum likelihood method [4], the two-dimensional (2-D) MUSIC method [5], and the higher-order ESPRIT method [6]. However, most of these algorithms either require multi-dimensional search or suffer from pairing problems or poor resolution due to heavy aperture loss. An estimation algorithm for three-dimensional frequency, DOA, and range of near-field sources was proposed in [7], which used a second-order statistics (SOS) matrix to estimate the frequency and DOA of each source. A new method based on third-order cyclic moment is presented in [8] for the joint estimation of DOAs and range of

near-field sources. Some papers have proposed mixed source classification and a localization algorithm, which can estimate both near-field and far-field incident source parameters [9-12].

However, these algorithms are very sensitive to sensor errors, such as position, orientation, and gain/phase response errors. In a practical system, the performance of algorithms is severely degraded and even out of work due to unavoidable perturbations in array manifold At the same time, the identifiably issue has not been studied completely in the existing literature. For these problems, this paper proposes an improved blind calibration algorithm that estimates near-field source parameters and sensor gain/phase responses separately and that obtains solutions iteratively but without the need for good initial guesses. The uniqueness of the estimations of near-field source parameters and sensor gain/phase responses is also discussed. We prove that the ambiguity in the estimations of near-field source parameters can be described with two unknown phase rotations. The phase rotations also deviate the estimation of the sensor phase response. We also show that the unknown phase rotations can be estimated and removed if we assume two sensor phase responses of the array have been calibrated. The simulation results show that this method works well.

2. Problem Formulation

2.1 Ideal Signal Model

Suppose that the K near-field narrowband and independent radiating sources impinge on a $2M+1$ -element uniform linear array (ULA) with inter-element spacing d, which is shown in Fig. 1. Let the array center be the phase reference point. The resulting array manifold of the m-th sensor with regard to the k-th source signal equals [13,14]:

$$a_k^m(\omega_k, \varphi_k) = e^{j(m\omega_k + m^2\varphi_k)}, -M \leq m \leq M \tag{1}$$

where, ω_k and φ_k are called 2-D electrical angles, which are connected to the physical parameters θ_k and r_k by:

$$\begin{aligned} \omega_k &= -2\pi d \sin(\theta_k) / \lambda \\ \varphi_k &= \pi d^2 \cos^2(\theta_k) / (\lambda r_k) \end{aligned} \tag{2}$$

The received signal of the m-th sensor can be written as:

$$\mathbf{r}_m(t) = \sum_{k=1}^{K} a_k^m(\omega_k, \varphi_k) s_k(t) + n_m(t) \tag{3}$$

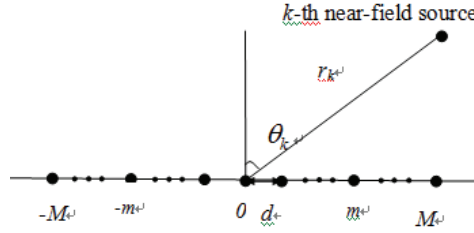$$s_k(t) = \sigma_s e^{j(2\pi ft + \gamma_k)} \tag{4}$$

Fig. 1. Uniform linear array configuration.

where, K represents the number of incident sources, $s_k(t)$ is the narrow band source signal with a carrier frequency f, γ symbolizes the phase of incident signal, σ_s denotes the amplitude of incident signal, and $n_m(t)$ denotes a zero-mean spatio-temporally uncorrelated random noise process with power σ_n^2.

2.2 Problem Model

In reality, array sensors usually have non-ideal gain/phase responses. They affect the received signal by affecting the array manifold $\mathbf{a}$. When the uniform linear array is subject to a priori unknown non-idealities in the gain or phase response, the signal received by the m-th sensor becomes:

$$\mathbf{r}_m(t) = \sum_{k=1}^{K} \Gamma_m a_k^m s_k(t) + n_m(t) \tag{5}$$

where, $\Gamma_m = g_m e^{j(\beta_m)}$ and g_m, β_m represent the gain and phase response of the m-th sensor respectively. The entire array's $(2M+1) \times 1$ received data can be written in matrix multiplication form as:

$$\mathbf{z}(t) = \mathbf{\Gamma}(\mathbf{g},\boldsymbol{\beta}) \mathbf{A}(\boldsymbol{\omega},\boldsymbol{\varphi}) \mathbf{s}(t) + \mathbf{n}(t) \tag{6}$$

where,

$$\mathbf{z}(t) = \left[r_{-M}(t), \ldots, r_0(t), \ldots, r_M(t) \right]^T \tag{7}$$

$$\mathbf{s}(t) = \left[s_1(t) s_2(t) \cdots s_K(t) \right]^T \tag{8}$$

$$\mathbf{n}(t) = \left[n_{-M}(t), \ldots, n_0(t), \ldots, n_M(t) \right]^T \tag{9}$$

$\mathbf{A}(\boldsymbol{\omega},\boldsymbol{\varphi})$ is an $(2M+1) \times K$ array manifold matrix:

$$\mathbf{A}(\boldsymbol{\omega},\boldsymbol{\varphi}) = \left[\mathbf{a}(\omega_1,\varphi_1), \mathbf{a}(\omega_2,\varphi_2), \ldots, \mathbf{a}(\omega_K,\varphi_K) \right] \tag{10}$$

where, $\boldsymbol{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \cdots & \omega_K \end{bmatrix}$, $\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_K \end{bmatrix}$ contain $2K$ electric angles, and $\mathbf{a}(\omega_k,\varphi_k)$ is the array steering vector with regard to the k-th incident source:

$$\mathbf{a}\left(\omega_k,\varphi_k\right)=\left[a_k^{-M},a_k^{-M+1},\ldots,a_k^{M}\right]^{T} \tag{11}$$

where, a_k^m is the array manifold of the m-th sensor given in (1). $\mathbf{\Gamma}\left(\mathbf{g},\mathbf{\beta}\right)$ is an $(2M+1)\times(2M+1)$ gain/phase response diagonal matrix of the entire array:

$$\mathbf{\Gamma}\left(\mathbf{g},\mathbf{\beta}\right)=\mathrm{diag}\{g_{-M}e^{j\beta_{-M}},\ldots,g_0e^{j\beta_0},\ldots,g_Me^{j\beta_M}\} \tag{12}$$

where, $\mathbf{g}=[g_{-M},\ldots,g_0,\ldots,g_M]$ and $\mathbf{\beta}=[\beta_{-M},\ldots,\beta_0,\ldots,\beta_M]$ contain $2(2M+1)$ non-ideal parameters. The present blind calibration method aims to estimate all of the aforementioned $2(2M+1)$ non-ideal parameters $\mathbf{g},\mathbf{\beta}$ contained in the matrix $\mathbf{\Gamma}\left(\mathbf{g},\mathbf{\beta}\right)$ plus 2-D electric angles $\mathbf{\omega},\mathbf{\varphi}$ of the K incident sources contained in the matrix $\mathbf{A}\left(\mathbf{\omega},\mathbf{\varphi}\right)$, given the collected $(2M+1)\times N$ data $\mathbf{Z}=\left[\mathbf{z}(T_s),\mathbf{z}(2T_s),\ldots,\mathbf{z}(NT_s)\right]$, where, N denotes the number of snapshots and T_s symbolizes the prior known time-sampling period.

Computing the covariance matrix of $\mathbf{z}(t)$:

$$\mathbf{R}=E\left\{\mathbf{z}(t)\mathbf{z}^H(t)\right\}=\mathbf{\Gamma}\mathbf{A}\mathbf{R}_S\mathbf{A}^H\mathbf{\Gamma}^H+\sigma_n^2\mathbf{I} \tag{13}$$

where $\mathbf{R}_S$ is the covariance matrix of the incident sources. The eigen-decomposition of the covariance matrix $\mathbf{R}$ yields:

$$\mathbf{R}=\mathbf{E}_S\mathbf{\Lambda}_S\mathbf{E}_S^H+\mathbf{E}_N\mathbf{\Lambda}_N\mathbf{E}_N^H \tag{14}$$

where, $\mathbf{E}_S=[\mathbf{e}_1,\mathbf{e}_2,\cdots,\mathbf{e}_K]$ contains the corresponding eigenvectors of $\mathbf{\Lambda}_S=diag[\lambda_1,\lambda_2,\cdots,\lambda_K]$ spanning the signal subspace of $\mathbf{R}$. Similarly, $\mathbf{E}_N=[\mathbf{e}_{K+1},\mathbf{e}_{K+2},\cdots,\mathbf{e}_{2M+1}]$ contains the responding eigenvectors of $\mathbf{\Lambda}_N=diag[\lambda_{K+1},\lambda_{K+2},\cdots,\lambda_{2M+1}]$ spanning the noise subspace of $\mathbf{R}$. It has been shown that $\mathbf{\Gamma}\mathbf{A}$ and $\mathbf{E}_S$ span the same subspace $\mathrm{range}\left(\mathbf{\Gamma}\mathbf{A}\right)=\mathrm{range}\left(\mathbf{E}_S\right)$, that is:

$$\mathbf{E}_S\mathbf{E}_S^H\mathbf{\Gamma}\mathbf{A}=\mathbf{\Gamma}\mathbf{A} \tag{15}$$

3. Proposed Method

In most of the existing calibration algorithms, near-field source parameters and sensor gain/phase responses are estimated jointly. Thus, the uniqueness of the estimations of near-field source parameters and sensor gain/phase responses is an important problem that needs to be addressed. In reality, the estimations of near-field source parameters and sensor gain/phase responses may exist ambiguity. This ambiguity problem has not yet been resolved in the literature. In this section, we first discuss the uniqueness of the estimations of near-field source parameters and sensor gain/phase responses and then

an iterative algorithm for the joint estimation of near-field source parameters and sensor gain/phase responses is proposed.

3.1 Uniqueness of Near-Field Source Parameters and Sensor Gain/Phase Responses

The equation we want to solve is:

$$\text{range}(\mathbf{\Gamma A}) = \text{range}(\mathbf{E}_S) \tag{16}$$

where, $\mathbf{\Gamma}$ and $\mathbf{A}$ are the sensor gain/phase response diagonal matrix and the array manifold matrix, which are defined in Subsection 2.2. It implies that if $\mathbf{\Gamma}_1 \neq \mathbf{\Gamma}_2$ and $\mathbf{A}_1 \neq \mathbf{A}_2$ exist and satisfy:

$$\text{range}(\mathbf{\Gamma}_1 \mathbf{A}_1) = \text{range}(\mathbf{\Gamma}_2 \mathbf{A}_2) = \text{range}(\mathbf{E}_S) \tag{17}$$

then, $\{\mathbf{\Gamma}_1 \mathbf{A}_1\}$ and $\{\mathbf{\Gamma}_2 \mathbf{A}_2\}$ are both the possible estimated results, where $\mathbf{\Gamma}_1 = \mathbf{\Gamma}(\mathbf{g}_1, \mathbf{\beta}_1)$, $\mathbf{\Gamma}_2 = \mathbf{\Gamma}(\mathbf{g}_2, \mathbf{\beta}_2)$, $\mathbf{A}_1 = \mathbf{A}(\mathbf{\omega}_1, \mathbf{\varphi}_1)$ and $\mathbf{A}_2 = \mathbf{A}(\mathbf{\omega}_2, \mathbf{\varphi}_2)$. It has been proved that if the array manifold matrix $\mathbf{A}$ is given, then the sensor gain and phase response matrix $\mathbf{\Gamma}$ constrained (17) is unique. However, when $\mathbf{A}$ and $\mathbf{\Gamma}$ are both unknown, we can prove that there are many such matrices that satisfy condition (17), then the parameter estimation exists ambiguity. Next, we discuss the ambiguous relationship between the estimated parameters. Eq. (17) implies that there exists a unique nonsingular matrix $\mathbf{W}$, such that:

$$\mathbf{\Gamma}_1 \mathbf{A}_1 \mathbf{W} = \mathbf{\Gamma}_2 \mathbf{A}_2 \tag{18}$$

If (18) is simplified, we get:

$$\mathbf{\Gamma}' \mathbf{A}_1 \mathbf{W} = \mathbf{A}_2 \tag{19}$$

where, $\mathbf{\Gamma}' = \mathbf{\Gamma}_1 \mathbf{\Gamma}_2^{-1}$ is a diagonal matrix. The m-th row of (19) can be transformed into $K-1$ equations:

$$\frac{\mathbf{A}_1^m \mathbf{w}_1}{\mathbf{A}_2^{m1}} = \frac{\mathbf{A}_1^m \mathbf{w}_k}{\mathbf{A}_2^{mk}}, k = 2, \ldots, K \tag{20}$$

where, $\mathbf{A}_1^m$ denotes the m-th row vector of $\mathbf{A}_1$, $\mathbf{w}_k$ denotes the k-th column vector of $\mathbf{W}$, and $\mathbf{A}_2^{mk}$ denotes the m-th row and the k-th column element of $\mathbf{A}_2$. Eq. (20) can be equivalently written as:

$$c_m \mathbf{A}_1^m \mathbf{w}_1 - \mathbf{A}_1^m \mathbf{w}_k = 0, k = 2, \ldots, K \tag{21}$$

where,

$$c_m = \frac{\mathbf{A}_2^{mk}}{\mathbf{A}_2^{m1}} = \mathbf{a}_m \left(\omega_{2,k} - \omega_{2,1}, \varphi_{2,k} - \varphi_{2,1} \right) \tag{22}$$

where, $\mathbf{a}$ has the same form with the column vector of $\mathbf{A}$, $\mathbf{a}_m$ denotes the m-th element of $\mathbf{a}$, and $\omega_{a,b}\left(\varphi_{a,b}\right)$ denotes the b-th element of $\boldsymbol{\omega}_a\left(\boldsymbol{\varphi}_a\right)$ for $a = 1, 2$. For $m = -M, -M+1, \ldots, M$, we obtained:

$$\mathbf{CA}_1\mathbf{w}_1 - \mathbf{A}_1\mathbf{w}_k = 0, k = 2, \ldots, K \tag{23}$$

where,

$$\begin{aligned} \mathbf{C} &= diag\left\{c_1, c_2, \ldots, c_M\right\} \\ &= diag\left\{\mathbf{a}\left(\omega_{2,k} - \omega_{2,1}, \varphi_{2,k} - \varphi_{2,1}\right)\right\} \end{aligned} \tag{24}$$

Then, Eq. (23) can be written in the form of the matrix multiplication:

$$\begin{bmatrix} \mathbf{CA}_1 & \mathbf{A}_1 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ -\mathbf{w}_k \end{bmatrix} = 0, k = 2, \ldots, K \tag{25}$$

where, $\mathbf{CA}_1 = \mathbf{A}\left(\boldsymbol{\omega}_1 + \omega_{2,k} - \omega_{2,1}, \boldsymbol{\varphi}_1 + \varphi_{2,k} - \varphi_{2,1}\right)$. We defined the composed matrix $\mathbf{B}$ as:

$$\begin{aligned} \mathbf{B} &= \begin{bmatrix} \mathbf{CA}_1 & \mathbf{A}_1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}\left(\boldsymbol{\omega}_1 + \omega_{2,k} - \omega_{2,1}, \boldsymbol{\varphi}_1 + \varphi_{2,k} - \varphi_{2,1}\right) & \mathbf{A}\left(\boldsymbol{\omega}_1, \boldsymbol{\varphi}_1\right) \end{bmatrix} \end{aligned} \tag{26}$$

If $\mathbf{w}_1$, $\mathbf{w}_k$ have a non-zero solution, the composed matrix $\mathbf{B}$ must be rank deficient. In the case of $(2M+1) \geq 2K$, the composed matrix $\mathbf{B}$ is a square or tall matrix. For the special form of $\mathbf{A}$, the rank deficiency of $\mathbf{B}$ implies that $\left\{\boldsymbol{\omega}_1 + \omega_{2,k} - \omega_{2,1}, \boldsymbol{\omega}_1\right\}$ and $\left\{\boldsymbol{\varphi}_1 + \varphi_{2,k} - \varphi_{2,1}, \boldsymbol{\varphi}_1\right\}$ must contain repeated elements and the repeated elements in $\boldsymbol{\omega}_1$ and $\boldsymbol{\varphi}_1$ are related to the identical source. We assumed that the elements in $\boldsymbol{\omega}_1\left(\boldsymbol{\varphi}_1\right)$ are different. Therefore, there exist two elements in $\boldsymbol{\omega}_1\left(\boldsymbol{\varphi}_1\right)$, denoted as $\left\{\omega_{1,m}, \omega_{1,n}\right\}\left(\left\{\varphi_{1,m}, \varphi_{1,n}\right\}\right)$, which satisfy:

$$\omega_{2,k} - \omega_{2,1} = \omega_{1,n} - \omega_{1,m} \tag{27}$$

$$\varphi_{2,k} - \varphi_{2,1} = \varphi_{1,n} - \varphi_{1,m} \tag{28}$$

Define

$$\Delta\omega_k = \omega_{2,k} - \omega_{2,1}, \qquad \Delta\varphi_k = \varphi_{2,k} - \varphi_{2,1} \tag{29}$$

for $k = 2, \ldots, K$. The set of differences can be effectively presented as $\left\{\Delta\omega_2, \ldots, \Delta\omega_K\right\}$ and $\left\{\Delta\varphi_2, \ldots, \Delta\varphi_K\right\}$. By substituting the set of differences into (25), we have:

$$\begin{bmatrix} \mathbf{A}\left(\boldsymbol{\omega}_1 + \Delta\omega_k, \boldsymbol{\varphi}_1 + \Delta\varphi_k\right) & \mathbf{A}\left(\boldsymbol{\omega}_1, \boldsymbol{\varphi}_1\right) \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ -\mathbf{w}_k \end{bmatrix} = 0$$
$$k = 2, \ldots, K \tag{30}$$

To obtain a solution for $\mathbf{W}$, Eq. (30) needs to be solved for $k = 2,\ldots,K$. Moreover, the solution of $\mathbf{w}_1$ of all equations must be identical. This implies that there exists an element in $\boldsymbol{\omega}_1\left(\boldsymbol{\varphi}_1\right)$ that when added with $\Delta\omega_k\left(\Delta\varphi_k\right)$ is still an element of $\boldsymbol{\omega}_1\left(\boldsymbol{\varphi}_1\right)$. Suppose that this element is the first element, for example: $\omega_{1,1}\left(\varphi_{1,1}\right)$. By applying $\left(\Delta\varphi_2,\ldots,\Delta\varphi_K\right)$ in sequel we can obtain $\omega_{1,2},\ldots,\omega_{1,K}\left(\varphi_{1,2},\ldots,\varphi_{1,K}\right)$. That is: $\omega_{1,1} + \Delta\omega_k = \omega_{1,k}$, $\varphi_{1,1} + \Delta\varphi_k = \varphi_{1,k}$, $k = 2,\ldots,K$. Hence:

$$\boldsymbol{\omega}_2 - \boldsymbol{\omega}_1 = \varepsilon, \; \boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_1 = \delta \tag{31}$$

where, $\varepsilon = \omega_{2,1} - \omega_{1,1}$ and $\delta = \varphi_{2,1} - \varphi_{1,1}$ are unknown constants. This indicates that the ambiguities in the estimations of near-field source parameters can be characterized by two unknown additive scalars. Note that these two unknown additive scalars also deviate the estimation of the sensor phase response. Set $\mathbf{G} = diag\left\{\mathbf{a}\left(\varepsilon,\delta\right)\right\}$, where:

$$\mathbf{a}(\varepsilon,\delta) = [e^{j((-M)\varepsilon + (-M)^2\delta)},\ldots,1,\ldots,e^{j((M)\varepsilon + (M)^2\delta)}] \tag{32}$$

From the expression of the array manifold matrix $\mathbf{A}\left(\boldsymbol{\omega},\boldsymbol{\varphi}\right)$ given in (10), we can obtain:

$$\mathbf{A}_2 = \mathbf{A}\left(\boldsymbol{\omega}_2,\boldsymbol{\varphi}_2\right) = \mathbf{G}\mathbf{A}\left(\boldsymbol{\omega}_1,\boldsymbol{\varphi}_1\right) = \mathbf{G}\mathbf{A}_1 \tag{33}$$

We can see from (33) that the diagonal matrix $\mathbf{G}$ is the deviation that the array manifold matrix caused to the array gain/phase response matrix. Thus, from (32) we can see that the ambiguity in the estimation of near-field source parameters deviates sensor phase responses because $\mathbf{G}$ only contains the phase factor and the ambiguous error of the m-th sensor phase response is:

$$\Delta\beta_m = \varepsilon m + \delta m^2 \tag{34}$$

We assumed that the m-th sensor real phase response was β_m and that the estimation of the m-th sensor phase response would be:

$$\hat{\beta}_m = \beta_m + \varepsilon m + \delta m^2 \tag{35}$$

Hence, the unknown scalars introduce undistinguishable errors in the estimations of near-field source parameters and sensor phase response. From (32) we noted that the ambiguity only deviates the estimation of the phase response and that the solution of the gain response is unique. It is not possible to obtain a unique solution for the sensor phase response solely based on (16). An extra constraint must be applied to remedy the ambiguity. To solve this problem, we set the phase response of the two sensors of the array to be previously calibrated. We assumed that the m_1-th and m_2-th sensor phase responses were calibrated, that is:

$$\beta_{m_1} = 0, \; \beta_{m_2} = 0 \tag{36}$$

According to (35), we have:

$$\hat{\beta}_{m_1} = \varepsilon(m_1) + \delta(m_1)^2 \tag{37}$$

$$\hat{\beta}_{m_2} = \varepsilon(m_2) + \delta(m_2)^2 \tag{38}$$

From (37) and (38), we can obtain:

$$\varepsilon = \left(m_2^2 \hat{\beta}_{m_1} - m_1^2 \hat{\beta}_{m_2}\right) \Big/ \left(m_2^2 m_1 - m_1^2 m_2\right) \tag{39}$$

$$\delta = \left(m_2 \hat{\beta}_{m_1} - m_1 \hat{\beta}_{m_2}\right) \Big/ \left(m_2 m_1^2 - m_1 m_2^2\right) \tag{40}$$

By substituting (39) and (40) into (35), we were able to remove the ambiguity in the estimation of the m-th sensor phase response. Then, we obtained the unique estimations of sensor gain/phase response.

3.2 Algorithm

The proposed iterative algorithm flow can be summarized as detailed below.

1. Initialization: Initialize the gain/phase response diagonal matrix $\hat{\Gamma}_0$ using the nominal gain/phase responses. Estimate the near-field source parameters $(\hat{\omega}_0, \hat{\varphi}_0)$ using the regular near-field source parameters estimation algorithms based on the initial value of gain/phase response diagonal matrix $\hat{\Gamma}_0$.

2. Gain/phase response estimation: Apply the latest estimates of near-field source parameters $(\hat{\omega}_i, \hat{\varphi}_i)$ to $\mathbf{E}_S \mathbf{E}_S^H \Gamma \mathbf{A} = \Gamma \mathbf{A}$, which can be rewritten as $\sum_{k=1}^{K} diag\left\{\hat{\mathbf{a}}_i^H\right\} \mathbf{E}_S \mathbf{E}_S^H diag\left\{\hat{\mathbf{a}}_i\right\} \mathbf{v} = K\mathbf{v}$, where, i denotes the ith estimated value and $\mathbf{v} = diag\left\{\Gamma\right\}$. Hence, $\mathbf{v}$ can be estimated as the eigenvector of $\mathbf{W} = \sum_{k=1}^{K} diag\left\{\hat{\mathbf{a}}_i^H\right\} \mathbf{E}_S \mathbf{E}_S^H diag\left\{\hat{\mathbf{a}}_i\right\}$, which is associated with the largest eigenvalue. Then, the gain/phase response estimation $\hat{\mathbf{v}}_i$ can be obtained.

3. Remove the ambiguity in the estimation of sensor phase response: We assumed that the phase response of two sensors was already calibrated. As such, the ambiguity ε and δ could be estimated by (39) and (40) and removed from $\hat{\mathbf{v}}_i$. Then the new estimated gain/phase response diagonal matrix $\hat{\Gamma}_i$ can be obtained.

4. Near-field source parameters re-estimation: Based on the latest estimation of gain/phase response diagonal matrix $\hat{\Gamma}_i$, re-estimate near-field source parameters $(\hat{\omega}_i, \hat{\varphi}_i)$ using the regular near-field source parameters estimation algorithms.

5. Determination of convergence: Perform the second, third, and the fourth step iteratively until some convergence criterion is achieved. The criteria to judge the convergence is calculated as:

$$P_i = \sum_{k=1}^{K} \hat{\mathbf{v}}_i^H \, diag\left\{\hat{\mathbf{a}}_i^H\right\} \mathbf{E}_N \mathbf{E}_N^H diag\left\{\hat{\mathbf{a}}_i^H\right\} \hat{\mathbf{v}}_i \tag{41}$$

If $P_{i-1} - P_i > \varepsilon$, (where ε is set a small positive value), set $i = i + 1$, and go back to the second step. If $P_{i-1} - P_i \le \varepsilon$, then break, and $(\hat{\boldsymbol{\omega}}_i, \hat{\boldsymbol{\varphi}}_i)$, $\hat{\boldsymbol{\Gamma}}_i$ are the final estimates.

4. Simulation Results

We considered a uniform linear array of $L = 2M + 1 = 9$ with an inter-element spacing of $d = \lambda / 4$. If two uncorrelated narrow-band near-field incident sources from $(\theta_1 = 10/180 \times \pi, r_1 = 1.5\lambda)$ and $(\theta_2 = 20/180 \times \pi, r_2 = 2.0\lambda)$ impinge on the array, then the non-ideal gain/phase responses are $\mathbf{g} = [1.2, 1.15, 0.85, 1.25, 1, 0.7, 0.9, 1.1, 0.8]$ $\boldsymbol{\varphi} = [0.0, 0.3, 0.2, -0.3, 0.0, 0.5, -0.6, -0.2, 0.0]$ (in radian). One thousand Monte Carlo simulations were performed at different SNRs (from 5 to 40 dB) with 1,024 snapshots. The estimation performance was measured by the root mean square error (RMSE), which defined as:

$$RMSE(\theta) = \sqrt{\frac{1}{L}\sum_{l=1}^{L}(\hat{\theta}_l - \theta)^2}, \quad RMSE(r) = \sqrt{\frac{1}{L}\sum_{l=1}^{L}(\frac{\hat{r}_l - r}{\lambda})^2}$$

where, θ, r denotes the true value of the near-field source parameters and $\hat{\theta}, \hat{r}$ denotes the estimates.

Figs. 2 and 3 show the RMSEs of bearing θ_k and range r_k estimations (of the two near-field sources) using the proposed calibration method under the non-ideal array manifold, where they are all displayed versus SNR. It can be seen that the proposed calibration method can dramatically improve the estimation precision compared to the algorithms without calibration.
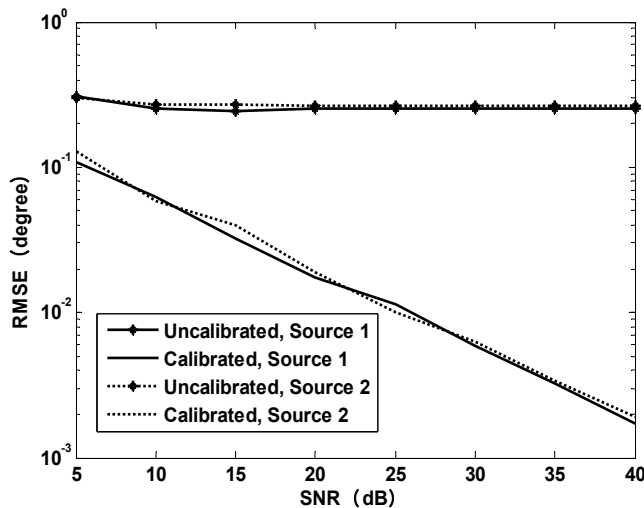

Fig. 2. Root mean square errors (RMSEs) of bearing estimates versus SNR.
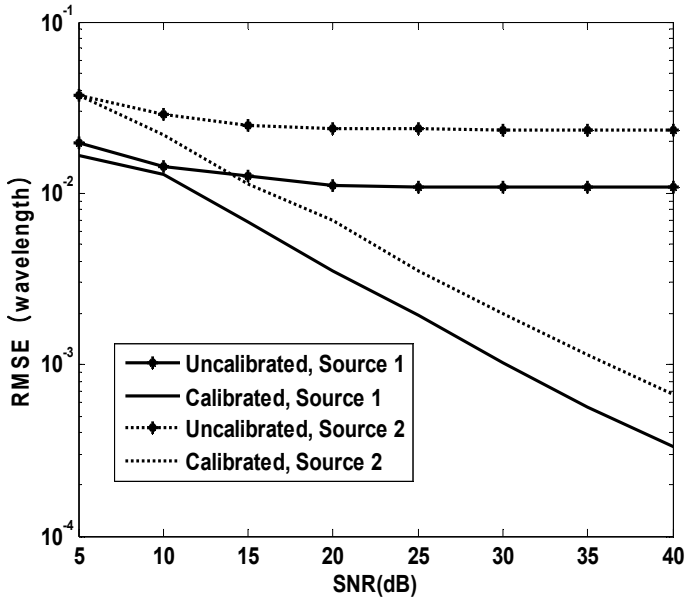
Fig. 3. Root mean square errors (RMSEs) of range estimates versus SNR.

Figs. 4 and 5 show the convergent properties of the gain/phase response estimation, where the SNR is set to 20 dB. It can be observed that the iterative procedure converges after 15 iterations. This demonstrates that the proposed algorithm provides substantial performance and a fast convergence speed. The true values and estimates of the array gain/phase responses are shown in Table 1, where the SNR is set to 10 dB.
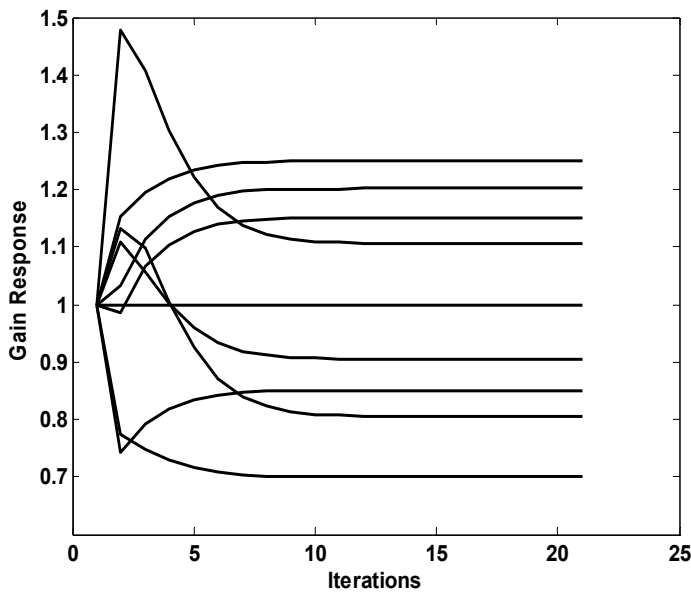

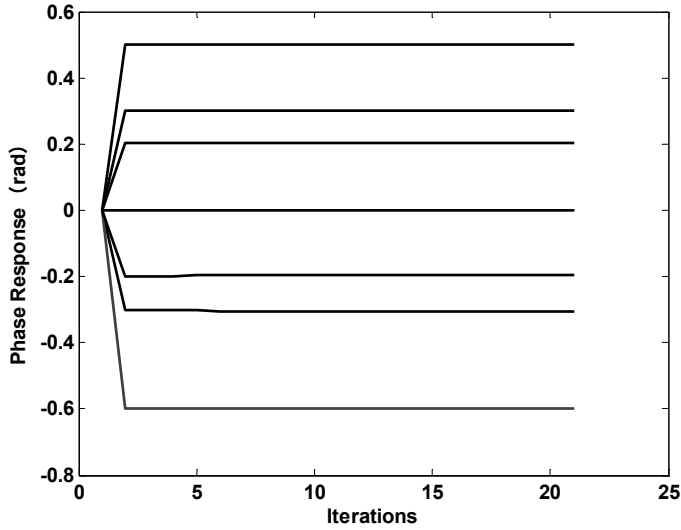**Fig. 4.** The convergence property of the estimates of gain response, SNR=20 dB.

Fig. 5. The convergence property of the estimates of phase response, SNR=20 dB.

Table 1. The real value and estimates of gain/phase response (SNR=10 dB)

Array	Gain response		Phase response (rad)	
	Real value	Estimate	Real value	Estimate
-4	1.2	1.2052	0.0	0.0
-3	1.15	1.1459	0.3	0.2911
-2	0.85	0.8494	0.2	0.1988
-1	1.25	1.2577	-0.3	-0.3089
0	1	1	0.0	0.0
1	0.7	0.7064	0.5	0.4932
2	0.9	0.9135	-0.6	-0.5984
3	1.1	1.1118	-0.2	-0.1912
4	0.8	0.8141	0.0	0.0

5. Conclusions

In this paper, we proposed an improved blind calibration algorithm, which estimates the near-field source parameters and sensor gain/phase response separately. Our experimental results show that this method provides fast convergent speed and high convergent precision. The applicability of the proposed algorithm is speaker localization using microphone arrays, guidance (homing) systems, radars, sonars, electronic surveillance, and seismic exploration applications.

Acknowledgement

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