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ON PSEUDO SEMICONFORMALLY SYMMETRIC MANIFOLDS

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Abstract. In this paper, a type of Riemannian manifold (namely, pseudo semiconformally symmetric manifold) is introduced. Also the several geometric properties of such a manifold is investigated. Finally the existence of such a manifold is ensured by a proper example.

1. Introduction

As a special subgroup of the conformal transformation group, Ishii [10] introduced the notion of conharmonic transformation under which a harmonic function transforms into a harmonic function. In [10] the conharmonic curvature tensor H^i_{jkl} of type (1,3) on a Riemannian manifold (M^n, g) of dimension $n \geq 4$ was defined as follows:

$$H^i_{jkl} = R^i_{jkl} - \frac{1}{n-2}(g_{jk}r^i_l - \delta^i_k r_{jl} + \delta^i_l r_{jk} - g_{jl}r^i_k), \tag{1.1}$$

which remains invariant under conharmonic transformation, where R and r are the Riemannian curvature and Ricci curvature tensors respectively.

In [22] Shaikh and Hui showed that the conharmonic curvature tensor satisfies the symmetries and skew symmetries properties of the Riemannian curvature tensor as well as cyclic ones. The conharmonic curvature tensor has many applications in the theory of general relativity. In [1] Abdussattar investigated its physical significance in the theory of general relativity. This tensor has also been studied by Siddiqui and Ahsan [23]; Ghosh, De and Taleshian [9] and many others. In [11] the author introduces a type of curvature-like tensor called semiconformal curvature tensor such that its (1,3) components remain invariant under conharmonic transformation. More precisely, the semiconformal curvature tensor P^i_{jkl} of type (1,3) on a Riemannian manifold (M^n, g) is

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defined as follows:

$$P^i_{jkl} = -(n-2)bC^i_{jkl} + [a + (n-2)b]H^i_{jkl}, \tag{1.2}$$

where a, b are constants not simultaneously zero and C^i_{jkl} is the conformal curvature tensor of type (1,3). Note that the conformal curvature tensor C^i_{jkl} of type (1,3) remains invariant under conformal transformation and that such a tensor is traceless. The conformal curvature tensor C^i_{jkl} of type (1,3) is defined as follows:

(1.3)

$$C^i_{jkl} = R^i_{jkl} - \frac{1}{n-2}(g_{jk}r^i_l - \delta^i_k r_{jl} + \delta^i_l r_{jk} - g_{jl}r^i_k) + \frac{s}{(n-1)(n-2)}(\delta^i_l g_{jk} - \delta^i_k g_{jl}),$$

where s is the scalar curvature.

In particular, if $a = 1$ and $b = -\frac{1}{n-2}$, then the semiconformal curvature tensor reduces to conformal curvature tensor whereas for $a = 1$ and $b = 0$, such a tensor turns into conharmonic curvature tensor. The semiconformal curvature tensor P_{ijkl} of type (0,4) possesses the several symmetric and skew symmetric properties as well as the cyclic ones. For instance, it is easy to see that the semiconformal curvature tensor P_{ijkl} of type (0.4) holds

$$P_{ijkl} = -P_{jikl} = -P_{ijlk} = P_{klij}$$

and

$$P_{ijkl} + P_{kijl} + P_{jkil} = 0. \tag{1.4}$$

P_{ijkl} belongs to a class of tensors named generalized curvature tensors and denoted with K_{ijkl}. They were introduced by Kobayashi and Nomizu [12] and satisfy properties (1.4).

In [3] Chaki introduced a type of Riemannian manifold (M^n, g) whose curvature tensor R_{ijkl} of type (0,4) satisfies the condition

$$R_{ijkl;m} = 2A_m R_{ijkl} + A_i R_{mjkl} + A_j R_{imkl} + A_k R_{ijml} + A_l R_{ijkm},$$

where A is a nonzero 1-form and the semicolon denotes the covariant differentiation with respect to the metric tensor g. Such a manifold is called a pseudo symmetric manifold. This manifold has received a great deal of attention and is studied in considerable detail by many authors [3, 4, 5, 6, 8, 18]. Motivated by the above studies, in the present paper, we introduce a pseudo semiconformally symmetric manifold. A Riemannian manifold (M^n, g) of dimension $n \geq 4$ (this condition is assumed throughout the paper as for $n \leq 3$, the conformal curvature tensor vanishes) is said to be pseudo semiconformally symmetric if its pseudo semiconformal curvature tensor P_{ijkl} of type (0,4) satisfies the relation

$$P_{ijkl;m} = 2A_m P_{ijkl} + A_i P_{mjkl} + A_j P_{imkl} + A_k P_{ijml} + A_l P_{ijkm}, \tag{1.5}$$

where A is an associated 1-form which is not zero.

If a generalized curvature tensor K_{ijkl} satisfies the condition

$$K_{ijkl;m} = 2A_m K_{ijkl} + A_i K_{mjkl} + A_j K_{imkl} + A_k K_{ijml} + A_l K_{ijkm},$$

then the manifold is named pseudo-K symmetric and denoted with $(PKS)_n$ [7]. Some properties of $(PKS)_n$ manifolds were studied in [13] and [15].

The purpose of this paper is to investigate the various properties of pseudo semiconformally symmetric manifold on which some geometric conditions are imposed.

2. Pseudo semiconformally symmetric manifolds

Let (M^n, g) be a Riemannian manifold. The semiconformal curvature tensor P^i_{jkl} of (M^n, g) is said to be harmonic if the divergence of the semiconformal curvature tensor P^i_{jkl} vanishes, i.e.,

$$P^h_{jkl;h} = 0. \tag{2.6}$$

Notice that in this paper, we adopt the Einstein convention (that is, when an index variable appears once in an upper and once in a lower position in a term, it implies summation of that term over all the values of the index). Now we can state the following:

Theorem 2.1. *Let (M^n, g) be a Riemannian manifold with harmonic semiconformal curvature tensor. If the constant $[a + (n-2)b]$ in (1.2) is nonzero, then the scalar curvature s of (M^n, g) is constant.*

Proof. By virtue of the second Bianchi identity, we have

$$R^m_{jkl;m} = r_{jk;l} - r_{jl;k} \tag{2.7}$$

and then

$$r^k_{l;k} = \frac{1}{2} s_{;l}. \tag{2.8}$$

Taking account of (1.1), (1.2) and (1.3) we obtain from (2.7) and (2.8)

$$\begin{aligned} P^m_{jkl;m} = & -(n-2)bC^m_{jkl;m} + [a + (n-2)b]H^m_{jkl;m} \\ = & -(n-2)b(\frac{n-3}{n-2})[r_{jk;l} - r_{jl;k} - \frac{s_{;l}}{2(n-1)} g_{jk} + \frac{s_{;k}}{2(n-1)} g_{jl}] \\ & + [a + (n-2)b][(\frac{n-3}{n-2})(r_{jk;l} - r_{jl;k}) - \frac{1}{2(n-2)}(g_{jk}s_{;l} - g_{jl}s_{;k})]. \end{aligned} \tag{2.9}$$

Using the condition (2.6) and multiplying (2.9) by g^{jk} we get from (2.8)

$$\begin{aligned} 0 = & -(n-2)b(\frac{n-3}{n-2})[s_{;l} - r^k_{l;k} - \frac{s_{;l}}{2(n-1)} n + \frac{s_{;l}}{2(n-1)}] \\ & + [a + (n-2)b][(\frac{n-3}{n-2})(s_{;l} - r^k_{l;k}) - \frac{1}{2(n-2)}(ns_{;l} - s_{;l})] \\ = & [a + (n-2)b](\frac{-s_{;l}}{n-2}), \end{aligned}$$

which yields from $[a + (n-2)b] \neq 0$ that the scalar curvature s is constant. This completes the proof. □

Note that Theorem 2.1 is a particular case of the following result concerning harmonic generalized curvature tensors, i.e., generalized curvature tensors with the property $K^m_{jkl;m} = 0$ (see [14, Prop. 4.6], [16, Theorem 2.2] and [17, Theorem 3.7]): Let (M^n, g) be an n-dimensional Riemannian manifold having a generalized curvature tensor with the property

$$K^m_{jkl;m} = cR^m_{jkl;m} + d[s_{;l}g_{jk} - s_{;k}g_{jl}],$$

where c and d are constants. If $K^m_{jkl;m} = 0$ and the condition $d \neq \frac{c}{2(n-1)}$ is satisfied, then the scalar curvature is a covariant constant $s_{;j} = 0$.

Concerning pseudo semiconformally symmetric manifold, we have:

Theorem 2.2. *Let (M^n, g) be a pseudo semiconformally symmetric manifold with harmonic semiconformal curvature tensor. If the constant $[a + (n-2)b]$ in (1.2) is nonzero, then the scalar curvature s of (M^n, g) is zero.*

Proof. By virtue of (1.5) and (2.6), we have

$$0 = 2A_m P^m_{jkl} + A^m P_{mjkl} + A_j P^m_{mkl} + A_k P^m_{jml} + A_l P^m_{jkm}. \tag{2.10}$$

Multiplying (2.10) by g^{jk}, we have from (1.1), (1.2) and (1.3)

$$\begin{aligned} 0 &= 2A_l[a + (n-2)b](\frac{-s}{n-2}) + A_l[a + (n-2)b](\frac{-s}{n-2}) \\ &\quad - A_l[a + (n-2)b](\frac{-s}{n-2}) + A_l[a + (n-2)b](\frac{-s}{n-2}n) \\ &= 2A_l[a + (n-2)b](\frac{-s}{n-2}) + A_l[a + (n-2)b](\frac{-s}{n-2}n) \\ &= -A_l[a + (n-2)b](\frac{n+2}{n-2})s, \end{aligned}$$

which yields from $[a + (n-2)b] \neq 0$ and $A \neq 0$ that the scalar curvature s of (M^n, g) vanishes. This completes the proof. □

Theorem 2.3. *Let (M^n, g) be a pseudo semiconformally symmetric manifold. If both the scalar curvature s of (M^n, g) and the constant $[a+(n-2)b]$ in (1.2) are not zero, then the associated 1-form A in (1.5) is closed.*

Proof. Multiplying (1.5) by g^{il} and then multiplying the relation obtained thus by g^{jk}, we have

$$\begin{aligned} &[a + (n-2)b](\frac{-s_{;m}}{n-2})n \\ &= 2A_m[a + (n-2)b](\frac{-s}{n-2})n + A_m[a + (n-2)b](\frac{-s}{n-2}) \\ &\quad + A_m[a + (n-2)b](\frac{-s}{n-2}) + A_m[a + (n-2)b](\frac{-s}{n-2}) \\ &\quad + A_m[a + (n-2)b](\frac{-s}{n-2}). \end{aligned} \tag{2.11}$$

By virtue of $[a + (n - 2)b] \neq 0$, we have from (2.11)

$$s_{;m} = \frac{2(n+2)}{n} A_m s. \tag{2.12}$$

Taking the covariant derivative of (2.12), we get

$$\begin{aligned} s_{;mt} &= \frac{2(n+2)}{n}[A_{m;t}s + A_m s_{;t}] \\ &= \frac{2(n+2)}{n}[A_{m;t}s + \frac{2(n+2)}{n} A_m A_t s] \end{aligned} \tag{2.13}$$

because of (2.12).

Therefore it follows from (2.13) that

$$0 = s_{;mt} - s_{;tm} = \frac{2(n+2)}{n} s[A_{m;t} - A_{t;m}],$$

which yields from $s \neq 0$ that

$$A_{m;t} - A_{t;m} = 0,$$

showing that the associated 1-form A is closed. This completes the proof. □

A Riemannian manifold (M^n, g) is said to be recurrent if its curvature tensor R_{ijkl} of type (0.4) satisfies the condition

$$R_{ijkl;m} = B_m R_{ijkl}, \tag{2.14}$$

where the associated 1-form B is nonzero. Now we can state:

Theorem 2.4. *Let (M^n, g) be a pseudo semiconformally symmetric manifold with $[a+(n-2)b] \neq 0$ and $s \neq 0$. If the manifold is recurrent, then the associated 1-forms A in* (1.5) *and B in* (2.14) *satisfy the relation $A = \frac{n}{2(n+2)} B$.*

Proof. Taking the covariant derivative of (1.2), we have from (1.1) and (1.3)

$$\begin{aligned} P_{ijkl;m} = &- (n-2)bC_{ijkl;m} + [a + (n-2)b]H_{ijkl;m} \\ = &- (n-2)b[\frac{s_{;m}}{(n-1)(n-2)}(g_{il}g_{jk} - g_{ik}g_{jl})] \\ &+ a[R_{ijkl;m} - \frac{1}{(n-2)}(g_{jk}r_{il;m} - g_{ik}r_{jl;m} + g_{il}r_{jk;m} - g_{jl}r_{ik;m})]. \end{aligned} \tag{2.15}$$

Multiplying (2.14) by g^{il} and then multiplying the relation obtained thus by g^{jk}, we have

$$r_{jk;m} = B_m r_{jk} \tag{2.16}$$

and then

$$s_{;m} = B_m s. \tag{2.17}$$

Taking account of (2.14), (2.16) and (2.17) we have from (2.15)

$$P_{ijkl;m} = -\frac{B_m sb}{(n-1)}(g_{il}g_{jk} - g_{ik}g_{jl})$$

(2.18) $$+a[B_m R_{ijkl} - \frac{1}{(n-2)}(g_{jk}B_m r_{il} - g_{ik}B_m r_{jl} + g_{il}B_m r_{jk} - g_{jl}B_m r_{ik})].$$

Multiplying (2.18) by g^{il} and then multiplying the relation obtained thus by g^{jk}, we get

(2.19) $$g^{il}g^{jk}P_{ijkl;m} = -\frac{n}{n-2}B_m s[a+(n-2)b].$$

On the other hand, multiplying (1.5) by g^{il} and then multiplying the relation obtained thus by g^{jk}, we get

(2.20) $$g^{il}g^{jk}P_{ijkl;m} = A_m[a+(n-2)b]sn\frac{-2(n+2)}{n(n-2)}.$$

Taking account of $[a+(n-2)b] \neq 0$ and $s \neq 0$, we have from (2.19) and (2.20)

$$B_m = \frac{2(n+2)}{n}A_m.$$

This completes the proof. □

A Riemannian manifold (M^n, g) is said to be Einstein if its Ricci tensor r is proportional to the metric tensor g (that is, $r = \frac{s}{n}g$). Note that in this case, its scalar curvature s is constant under $n \geq 3$ [2].

Now we have the following.

Lemma 2.5. *Let (M^n, g) be a pseudo semiconformally symmetric manifold with $[a+(n-2)b] \neq 0$. If the manifold is Einstein, then its Ricci tensor vanishes.*

Proof. By virtue of (1.1), (1.2) and (1.3), we have

(2.21)

$$P_{ijkl} = aR_{ijkl} - \frac{a}{n-2}(g_{il}r_{jk} - g_{ik}r_{jl} + r_{il}g_{jk} - r_{ik}g_{jl}) - \frac{bs}{n-1}(g_{il}g_{jk} - g_{ik}g_{jl}).$$

From $r_{ij} = \frac{s}{n}g_{ij}$ it follows that $s_{;i} = 0$ and thus $r_{ij;k} = 0$ and from (2.9) it is inferred that $P^m_{jkl;m} = 0$. From Theorem 2.2 it is $s = 0$ and consequently $r_{ij} = 0$. Moreover it follows from (2.21) that $P_{ijkl} = aR_{ijkl}$, $P_{ijkl;m} = aR_{ijkl;m}$. □

Theorem 2.6. *Let (M^n, g) be a pseudo semiconformally symmetric manifold with $[a+(n-2)b] \neq 0$ and $a \neq 0$. If the manifold is Einstein, then the manifold is pseudo symmetric.*

Proof. Taking account of Lemma (2.5) and (2.21), the relation (1.5) leads to

$$aR_{ijkl;m} = 2A_m aR_{ijkl} + A_i aR_{mjkl} + A_j aR_{imkl} + A_k aR_{ijml} + A_l aR_{ijkm}.$$

Since $a \neq 0$, the last relation reduces to

$$R_{ijkl;m} = 2A_m R_{ijkl} + A_i R_{mjkl} + A_j R_{imkl} + A_k R_{ijml} + A_l R_{ijkm},$$

showing that the manifold is pseudo symmetric. □

Theorem 2.7. *Let* (M^n, g) *be a pseudo semiconformally symmetric manifold with* $[a+(n-2)b] \neq 0$. *If the manifold admits a parallel vector field* V, *then we have either* $V^m A_m = 0$ *or* $s = 0$.

Proof. From the Ricci identity and a parallel vector field V, it follows

$$0 = V^t_{;jk} - V^t_{;kj} = V^m R^t_{mjk}. \tag{2.22}$$

Taking the covariant derivative of (2.22), we obtain

$$V^m R^t_{mjk;l} = 0.$$

Multiplying the last relation by g_{ti} we have

$$V^m R_{imjk;l} = 0$$

or equivalently

$$V^m R_{jkim;l} = 0. \tag{2.23}$$

Taking account of the second Bianchi identity, we get from (2.23)

$$V^m R_{jkli;m} + V^m R_{jkml;i} = 0$$

or equivalently

$$V^m R_{jkli;m} - V^m R_{jklm;i} = 0,$$

which reduces to

$$V^m R_{jkli;m} = 0 \tag{2.24}$$

because of (2.23).

Multiplying (2.24) by g^{ji} and then multiplying the relation obtained thus by g^{kl}, we have

$$V^m r_{kl;m} = 0 \tag{2.25}$$

and then

$$V^m s_{;m} = 0. \tag{2.26}$$

From (2.21), (2.24), (2.25) and (2.26), it follows that

$$V^m P_{ijkl;m} = 0$$

or equivalently

$$V^m[2A_m P_{ijkl} + A_i P_{mjkl} + A_j P_{imkl} + A_k P_{ijml} + A_l P_{ijkm}] = 0. \tag{2.27}$$

Multiplying (2.27) by g^{il} and then multiplying the relation obtained thus by g^{jk}, we get from (2.21)

$$V^m[A_m(\frac{-s}{n-2})(2n+4)][a+(n-2)b] = 0,$$

which leads to either $V^m A_m = 0$ or $s = 0$. This completes the proof. □

Note that results (2.22), (2.23), (2.24), (2.25) and (2.26) are essentially contained in Lemma 1.5 of [21].

We immediately have the following:

Corollary 2.8. *Let (M^n, g) be a pseudo semiconformally symmetric manifold with $[a + (n-2)b] \neq 0$. If the covariant derivative of its associated 1-form A vanishes, then the scalar curvature s of (M^n, g) vanishes.*

Proof. Multiplying g^{im} to the given condition $A_{i;l} = 0$, we have

$$A^m_{;l} = 0.$$

Now it follows from Theorem 2.7 that we have either $A^m A_m = 0$ or $s = 0$, which leads to

$$s = 0$$

because of $A \neq 0$. This completes the proof. □

Now we will provide a proper example of a pseudo semiconformally symmetric manifold.

Example. Let (R^n_+, g) be a Riemannian manifold given by

$$R^n_+ = \{(x^1, x^2, \ldots, x^n) | x^i > 0, \ i = 1, 2, \ldots, n\}$$

and

$$g = f(dx^1)^2 + \delta_{\alpha\beta} dx^\alpha dx^\beta + 2dx^1 dx^n,$$

where Greek indices α and β run over the range $2, 3, \ldots, n-1$, and

$$f = (E_{\alpha\beta} + \delta_{\alpha\beta}) x^\alpha x^\beta e^{(x^1)^2}.$$

Here $\delta_{\alpha\beta}$ is the Kronecker delta, and $E_{\alpha\beta}$ is constant and satisfies the relations $E_{\alpha\beta} = 0$ if $\alpha \neq \beta$; $E_{\alpha\beta} =$ constant $(\neq 0)$ if $\alpha = \beta$; $|E_{\alpha\beta}| < 1$; $\sum_{\alpha=2}^{n-1} E_{\alpha\alpha} = 0$. This kind of metric was appeared in [19, 20]. In the metric described as above, the only nonvanishing components of Christoffel symbols, the curvature tensors and the Ricci tensors are, according to [19, 20]

$$\Gamma^\beta_{11} = -\frac{1}{2} E^{\alpha\beta} f_{,\alpha}, \ \Gamma^n_{11} = \frac{1}{2} f_{,1}, \ \Gamma^n_{1\alpha} = \frac{1}{2} f_{,\alpha},$$

$$R_{1\alpha\beta 1} = \frac{1}{2} f_{,\alpha\beta}, \ r_{11} = \frac{1}{2} \delta^{\alpha\beta} f_{,\alpha\beta}, \tag{2.28}$$

where the comma denotes the partial differentiation with respect to the coordinates. It is easy to see that the relations

$$f_{,\alpha\beta} = 2(E_{\alpha\beta} + \delta_{\alpha\beta}) e^{(x^1)^2}$$

and

$$\delta^{\alpha\beta} f_{,\alpha\beta} = 2(n-2) e^{(x^1)^2} \tag{2.29}$$

hold. From (2.28) and (2.29), it follows that the only nonzero components for the curvature tensor R_{ijkl} and the Ricci tensor r_{jk} are

$$R_{1\alpha\alpha 1} = \frac{1}{2} f_{,\alpha\alpha} = (E_{\alpha\alpha} + 1) e^{(x^1)^2}$$

and

$$r_{11} = \frac{1}{2} f_{,\alpha\beta}\delta^{\alpha\beta} = (n-2)e^{(x^1)^2}. \tag{2.30}$$

From the given metric g, we obtain $g_{ni} = g_{in} = 0$ for $i \neq 1$, which yields $g^{11} = 0$. Therefore the scalar curvature s of (M^n, g) vanishes because $s = g^{ij}r_{ij} = g^{11}r_{11} = 0$. Hence by considering the results mentioned above and (2.21), we find the only nonzero components for the semiconformal curvature tensor P_{ijkl} as

$$P_{1\alpha\alpha 1} = a[R_{1\alpha\alpha 1} - \frac{1}{n-2}(g_{\alpha\alpha}r_{11})] = aE_{\alpha\alpha}e^{(x^1)^2}. \tag{2.31}$$

In this case, it follows from (2.31) that the only nonzero components of the covariant derivative of P_{ijkl} are

$$P_{1\alpha\alpha 1;1} = 2x^1 aE_{\alpha\alpha}e^{(x^1)^2} = 2x^1 P_{1\alpha\alpha 1}. \tag{2.32}$$

Let us consider the associated 1-form A as $A_i = \frac{x^1}{2}$ for $i = 1$ and 0 otherwise. To verify the relation (1.5), it is sufficient to prove the relation

$$P_{1\alpha\alpha 1;1} = 4A_1 P_{1\alpha\alpha 1}.$$

Taking account of the definition of the associated 1-form A and (2.32), it is easy to see that the last relation holds. The other components of each term of (1.5) vanishes identically and hence the relation (1.5) holds.

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