

Method Based on Sparse Signal Decomposition for Harmonic and Inter-harmonic Analysis of Power System

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Abstract – Harmonic/inter-harmonic detection and analysis is an important issue in power system signal processing. This paper proposes a fast algorithm based on matching pursuit (MP) sparse signal decomposition, which can be employed to extract the harmonic or inter-harmonic components of a distorted electric voltage/current signal. In the MP iterations, the method extracts harmonic/inter-harmonic components in order according to the spectrum peak. The Fast Fourier Transform (FFT) and nonlinear optimization techniques are used in the decomposition to realize fast and accurate estimation of the parameters. First, the frequency estimation value corresponding to the maxim spectrum peak in the present residual is obtained, and the phase corresponding to this frequency is searched in discrete sinusoids dictionary. Then the frequency and phase estimations are taken as initial values of the unknown parameters for Nelder-Mead to acquire the optimized parameters. Finally, the duration time of the disturbance is determined by comparing the inner products, and the amplitude is achieved according to the matching expression of the harmonic or inter-harmonic. Simulations and actual signal tests are performed to illustrate the effectiveness and feasibility of the proposed method.

Keywords: Matching pursuit algorithm, Harmonic, Inter-harmonic, Sinusoids dictionary, Fast Fourier Transform (FFT), Sparse decomposition, Optimal computation

1. Introduction

The increasing applications of nonlinear loads such as motor, transformer, arc furnace and domestic appliances, along with power electronic equipment in the power system result in a large amount of harmonic and inter-harmonic distortions. The presence of these distortions can cause adverse problems or effects such as voltage fluctuation, noise and vibration of motor, the decrease of power transmission efficiency, sub-synchronous oscillation of generator unit, malfunction of relay protection equipment [1-2]. Therefore, the analysis of harmonic and inter-harmonic in the electric signal is very important before taking actions to eliminate or suppress the problems mentioned above.

Different techniques have been developed in recent years for harmonic analysis, including Fast Fourier Transform (FFT) [3], short-time Fourier transform (STFT) [4], wavelet transform (WT) [5], Kalman filters [6], Prony method [7], Hilbert-Huang transform (HHT) [8], estimation of signal parameters via rotational invariance technique (ESPRIT) [9], independent component analysis (ICA) [10] and multiple signal classification (MUSIC) [11], etc.

The methods proposed above mostly assume that only

harmonics are present and the periodicity intervals are fixed. The literatures about the detection of transient harmonics or inter-harmonics are relatively small, and the discussion is incomplete. [12-13] analyzed the case that containing 5th transient harmonic only, and didn't provide phase detection results. The method in [14] can calculate the complexity of power harmonics and measure the frequency and power of transient harmonics, but didn't give the quantitative results of the parameters, and the selection rules of parameter need further study. In [15], a method for measurement of transient harmonics and inter-harmonics was proposed, but the simulation signal was quite different from real power signal. The method in [16] can detect the amplitude, frequency and duration time of the transient harmonics, but can't analyze phase information.

With the increasing diversity and complexity of the loads and power supply mode in power system, the existing methods need to be improved, and at the same time new detection techniques need to be explored also. In recent years, the sparse signal decomposition method based on overcomplete dictionary was applied to power system signal processing [17-19]. Gabor time-frequency atom method was used to analyze time-varying inter-harmonic signals, but the computation speed is not acceptable for practice [18]. The matching pursuit method based on Particle Swarm Optimization (PSO) was used to realize fast analysis of unsteady inter-harmonics, but the algorithm is limited to synchronous sampling [19].

This paper presents a method for extracting harmonics and inter-harmonics from power system signal using sparse

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signal decomposition technique with redundant dictionary. The proposed method extracts the parameters of harmonic and inter-harmonic components according to the spectrum energy by using MP iterative decomposition with over-complete sinusoids dictionary. FFT, unconstrained non-linear programming algorithm and the recursion based inner product computation are used to improve the speed and accuracy of the algorithm. To demonstrate the effectiveness of the technique, the proposed method is tested with different types of signals. The results show that the method is insensitive to spectral leakage, and is also proven to be capable of achieving better accuracy under noisy conditions comparing with other methods.

2. MP Decomposition Based on Redundant Dictionary

2.1 Sparse representation based on redundant dictionary

In recent years, sparse representation has been successfully used in many signal processing applications, including denoising, compression and feature extraction of various types of signals such as electrocardiosignal, radar signal and image signal, etc.

Redundant dictionary is constructed by function waveforms with a wide range of time-frequency distribution, such waveforms in the dictionary are called time-frequency atoms. The dictionary should contain a very large number of distinct structures for any phenomena or be constructed based on the prior information about time-frequency characteristics of the signals of interest, so that we could potentially identify or discriminate different phenomena that composing a signal. In this case, the dictionary is called redundant or over-complete, which can provide better representation of a composite signal with both stationary and non-stationary components by using hybrid elementary waveforms from different analytical functions.

For a predefined dictionary, an $N \times 1$ signal f can be expressed as:

$$f = \psi\alpha = \sum_{n=0}^{S-1} \alpha_n \psi_n \quad (1)$$

$$\psi = [\psi_0, \psi_1, \psi_2, \dots, \psi_{S-1}], \quad \alpha = [\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_{S-1}]^T$$

where ψ_n are the time-frequency atoms in an $N \times S$ dictionary matrix ψ , and α_n are the sparse transformed coefficients in $S \times 1$ column vector α . Unlike other time-frequency analysis techniques using a single or fixed basis matrix to express any signal, such as Fourier and wavelet, the sparse representation based on redundant dictionary can select adaptively the elements of the dictionary depending upon the local properties of signal f .

The sparse decomposition based on redundant dictionary

can realize flexible decomposition of signals whose localizations in time and frequency vary widely, and the goal of which is to represent signal by least atoms with small enough error. Since the atoms selected have the most matching structure with the signal, the method can be used to realize features extraction and parameters detection of electrical signal. It is clear that the construction of suitable dictionary and the way to find out appropriate atoms are the two key problems to be solved.

2.2 MP decomposition

For the input signal f expressed as (1), the sparse coefficients vector α can be estimated by solving the following optimization problem:

$$\min_{\alpha} \|\alpha\|_0 \quad \text{subject to} \quad f = \psi\alpha \quad (2)$$

The above optimization problem can be solved using matching pursuit (MP), a nonlinear iterative greedy algorithms proposed by Mallat and Zhang [20].

Defining a dictionary $D = \{g_{\gamma}\}_{\gamma \in \Gamma}$, g denotes the atoms in the dictionary, and Γ is the set of all possible parameters γ that define g , and $\|g_{\gamma}\| = 1$. MP decomposes f by projecting it on an atom of D that matches f almost at best. Let $g_{\gamma_0} \in D$, then f can be decomposed into

$$f = \langle f, g_{\gamma_0} \rangle g_{\gamma_0} + R \quad (3)$$

where R is the residual after approximating f in the direction of g_{γ_0} . Since g_{γ_0} is orthogonal to R , hence

$$\|f\|^2 = |\langle f, g_{\gamma_0} \rangle|^2 + \|R\|^2 \quad (4)$$

To minimize $\|R\|$, g_{γ_0} should be selected to maximize $|\langle f, g_{\gamma_0} \rangle|$.

MP algorithm then decomposes the residue R following the above process, and this procedure is repeated each time on the following obtained residue. Thus f can be approximated by the sum of orthogonal projections of the atoms in D . The decomposition process can be described as follows:

Let R^n be the n th order residue ($n \geq 0$), and $R^0 = f$. An atom $g_{\gamma_n} \in D$ that closely matches the residue R^n is selected in each step, and satisfies

$$g_{\gamma_n} = \arg \max_{\gamma \in \Gamma} \left| \langle R^n, g_{\gamma} \rangle \right| \quad (5)$$

The residue R^n can be decomposed into

$$R^n = \langle R^n, g_{\gamma_n} \rangle g_{\gamma_n} + R^{n+1} \quad (6)$$

Carrying out this decomposition process iteratively up to order M , and the signal f can be expressed as

$$f = \sum_{n=0}^{M-1} \langle R^n, g_{\gamma_n} \rangle g_{\gamma_n} + R^M \quad (7)$$

where $\sum_{n=0}^{M-1} \langle R^n, g_{\gamma_n} \rangle g_{\gamma_n}$ is the M -term approximation of f , R^M is the M th order residue. The iteration stops when the specified residual threshold or a fixed number of steps is achieved.

As we can see, MP algorithm obtains M atoms up to step M , and the signal f can be represented as the linear combination of these atoms, the decomposition coefficients $\alpha_n = \langle R^n, g_{\gamma_n} \rangle, n=0,1,2,\dots,M-1$.

2.3 Sinusoids dictionary

The time-frequency characteristic of atoms in the dictionary that used to decompose the signal f should adapt to the local structure of f . The power system harmonics and inter-harmonics are a series of sinusoidal waveforms, so sinusoids dictionary is used in this paper, the complex expression of the sinusoids atom is:

$$g_{\gamma}(t) = Ke^{j\omega t} \quad (8)$$

and the real sinusoids atom can be expressed as:

$$g_{\gamma}(t) = K \cos(\omega t + \varphi) \quad (9)$$

where K is the normalized amplitude of the atom, ω denotes the frequency, φ denotes the phase, and $\gamma = (\omega, \varphi)$ is the index of the parameters set.

The dictionary D may have an infinite number of elements if the parameters of the atom are successive, in order to implement MP efficiently, the parameters of the atom should be discretized, that is to construct a discretized sub-dictionary $D_{\alpha} = \{g_{\gamma} | \gamma \in \Gamma_{\alpha}\} \in D$, Γ_{α} is the discrete subset of Γ , and then we translate (5) into:

$$g_{\gamma_m} = \arg \max_{\gamma \in \Gamma_{\alpha}} \left| \langle R^n, g_{\gamma} \rangle \right|. \quad (10)$$

Referring to the discretization method of Gabor atom suggested in [20] to construct a sub-dictionary D_{α} , and the atoms of sinusoids dictionary can be discretized as:

$$\gamma = (\omega, \varphi) = (k2^{-j} \Delta \omega, i \Delta \varphi) \quad (11)$$

where $\Delta \varphi = \pi/6$, $\Delta \omega = \pi$, $0 \leq k < 2^{j+1}$, $0 \leq i \leq 12$, $0 < j \leq \log_2 N$, and N is the sampling number.

3. Harmonic/inter-harmonic Detection Based on FMP

The number of atoms is still very large in the discretized sinusoids dictionary, which will leads to high computation complexity of MP. And also much error will be introduced

inevitably for the discontinuity of the parameters. Here, a Fast Matching Pursuit (FMP) method is proposed to realize rapid calculation and achieve more precise matching of parameters.

3.1 Preliminary estimation of matching parameters

Let f_s denote the sampling frequency, and N be the signal length. Firstly, FFT is used to find the initial guess frequency of the component, the spectrum λ of R^n with the maximum amplitude is obtained by FFT in each iteration, and the corresponding frequency ω_{λ} is computed as:

$$\omega_{\lambda} = 2\pi(\lambda - 1)f_s / N \quad (\text{rad/s}) \quad (12)$$

After obtaining ω_{λ} , the phase φ_{λ} corresponding to ω_{λ} that satisfies (10) will be searched by traversing the discretized dictionary in terms of $i \Delta \varphi$ ($\Delta \varphi = \pi/6, 0 \leq i \leq 12$). Here we only get the estimate values of frequency and phase, subsequently, optimization calculation method will be used to correct or adjust the value of ω_{λ} and φ_{λ} , so as to decrease or eliminate the errors induced by spectrum leakage and discontinuity of parameters.

3.2 Parameters optimization

In every iteration, searching the atoms satisfying (10) in the sub-dictionary D_{α} can be dealt with as a nonlinear programming problem, defining the objective function as:

$$F(\omega, \varphi) = -|\langle R^n, g_{\gamma} \rangle| \quad (13)$$

Nelder-Mead algorithm is a nonlinear optimal method based on simplex, which uses direct search strategy without calculating derivative of the objective function and also have nothing to do with Hessian matrix operation. The method is especially suitable for solving the extreme value problem with not many variables.

Taking the estimation values of ω_{λ} and φ_{λ} as the initial solution of the Nelder-Mead algorithm and solving the minimum problem without constrains as:

$$\min F(\omega, \varphi) \quad (14)$$

Firstly, for the 2 variables ω and φ of the objective function (13), constructing a simplex of 2-dimensional space that containing 3 points based on $(\omega_{\lambda}, \varphi_{\lambda})$, and every point is a 2-dimensional vector, the 3 points are $(\omega_{\lambda}, \varphi_{\lambda})$, $(\omega_{\lambda_1}, \varphi_{\lambda_1})$ and $(\omega_{\lambda_2}, \varphi_{\lambda_2})$ respectively, where $\omega_{\lambda_1} = (1 + \Delta)\omega_{\lambda}$, $\varphi_{\lambda_1} = (1 + \Delta)\varphi_{\lambda}$ and $\Delta = 0.005$. Then calculating and comparing the function values at these 3 points, and update the simplex by reflection, expansion, and contraction at each stage [21]. The action contracts the simplex towards the lowest point, and will eventually obtaining the minimum or the optimal point $(\omega_{\lambda_m}, \varphi_{\lambda_m})$ as the solution of (14). $(\omega_{\lambda_m}, \varphi_{\lambda_m})$ is the final optimal atomic

parameters matching with the residual R^n in MP, so that the real atom can be written as:

$$g_{\gamma_n}(t) = K \cos(\omega_{\lambda_n} t + \varphi_{\lambda_n}) \quad (15)$$

3.3 Searching for temporal supports

The disturbance duration should be determined for the measurement of transient harmonic. We denote t_s and t_e as the start and end time of the disturbance respectively, and the corresponding sample sequences are k_s and k_e , with $t_s = k_s/f_s$ and $t_e = k_e/f_s$. Thus, the atoms expression with temporal supports can be expressed as:

$$g_{\gamma}(t) = K \cos(\omega t + \varphi) [u(t - t_s) - u(t - t_e)] \quad (16)$$

with K such that $\|g_{\gamma}(t)\| = 1$, where $u(t)$ is the unit step function. The discrete version of (16) is:

$$g_{\gamma}(k) = K \cos\left(\frac{\omega k}{f_s} + \varphi\right) [u(k - k_s) - u(k - k_e)] \quad (17)$$

where $k = 1, 2, \dots, N$, and $k_s \leq k_e$, and the vector form of (17) can be represented as:

$$g_{\gamma} = [g_{\gamma}(1), g_{\gamma}(2), \dots, g_{\gamma}(k_s), \dots, g_{\gamma}(N)]^T \quad (18)$$

In order to determine the value of k_s and k_e , we need to traverse all the possible values. N multiplications of real numbers are needed if we compute the inner product $\langle R^n, g_{\gamma} \rangle$ directly for every group of k_s and k_e , which is time consuming. We suppose that $\gamma_a = (\omega, \varphi, k_s, k_e)$ and $\gamma_b = (\omega, \varphi, k_s, k_e - 1)$, and the atom corresponding to γ_a is:

$$g_{\gamma_a} = [g_{\gamma_a}(1), g_{\gamma_a}(2), \dots, g_{\gamma_a}(k_e), \dots, g_{\gamma_a}(N)]^T \quad (19)$$

then the unit-norm atom corresponding to γ_b can be given by:

$$g_{\gamma_b} = \frac{g_{\gamma_a} - [0, 0, \dots, g_{\gamma_a}(k_e), \dots, 0]^T}{\sqrt{1 - g_{\gamma_a}(k_e)^2}} \quad (20)$$

and further we have:

$$\begin{aligned} \langle R^n, g_{\gamma_b} \rangle &= \left\langle R^n, \frac{g_{\gamma_a} - [0, 0, \dots, g_{\gamma_a}(k_e), \dots, 0]^T}{\sqrt{1 - g_{\gamma_a}(k_e)^2}} \right\rangle \\ &= \frac{\langle R^n, g_{\gamma_a} \rangle - R^n(k_e) \cdot g_{\gamma_a}(k_e)}{\sqrt{1 - g_{\gamma_a}(k_e)^2}} \end{aligned} \quad (21)$$

Formula (21) shows that if we have known g_{γ_a} and $\langle R^n, g_{\gamma_a} \rangle$, then $\langle R^n, g_{\gamma_b} \rangle$ can be calculated recursively

with very low computational complexity. Thus, we can recursively obtain all of the inner products corresponding to all values of k_s and k_e quickly. Finally, (k_{sn}, k_{en}) that gives the largest inner product is used as the best temporal support, and we obtain:

$$g_{\gamma_n}(t) = K \cos(\omega_{\lambda_n} t + \varphi_{\lambda_n}) [u(t - t_{sn}) - u(t - t_{en})] \quad (22)$$

Hence the amplitude of the harmonic or inter-harmonic can be achieved by:

$$A_n = \max(\langle R^n, g_{\gamma_n} \rangle) \quad (23)$$

The harmonic/inter-harmonic extracted at the n th iteration of MP can be written as:

$$g_{\gamma_n}(t) = A_n \cos(\omega_{\lambda_n} t + \varphi_{\lambda_n}) [u(t - t_{sn}) - u(t - t_{en})] \quad (24)$$

3.4 Disturbances extraction steps of FMP method

The steps of FMP are as follows:

- The frequency estimation ω_{λ} corresponding to the maximum peak of the spectrum of R^n is obtained using FFT.
 - With the frequency ω_{λ} obtained in step (a), search φ_{λ} that satisfying (10) in the discrete sub-dictionary D_{α} according to $i\Delta\varphi$.
 - Take $(\omega_{\lambda}, \varphi_{\lambda})$ obtained in (a) and (b) as the initial value, compute the optimized frequency ω_{λ_n} and phase φ_{λ_n} using Nelder-Mead algorithm.
 - Find t_{sn} and t_{en} using the simplified inner product method based on recursion showed in (21).
 - Compute the expression $\alpha_n g_{\gamma_n} = \langle R^n, g_{\gamma_n} \rangle g_{\gamma_n}$ that matching with the harmonic or inter-harmonic disturbance, and get the amplitude A_n of the disturbance.
- Stop the process when the norm of residual R^n is lower than a specified threshold or the maximum number of iteration is reached, otherwise repeat (a) to (e) iteratively.

4. Case Studies and Experimental Results

This section presents the results of simulated and actual signal case studies carried out using MATLAB to evaluate the performance of the proposed scheme. The following cases are considered. To simplify the magnitude values, per unit (pu) value is used.

4.1 Comparison with other methods

FFT is widely employed for harmonic analysis because of its faster computational capability. However, for the presence of inter-harmonic and the fluctuation of grid frequency, it is hard to realize strict synchronous sampling, and which leads to spectrum leakage and picket fence

effect, so the direct application of FFT for harmonic analysis is inaccurate and infeasible in this situation. Researches show that the improved window and interpolation FFT algorithms can reduce the undesirable errors caused by the spectral leakage and picket fence effect, and realize high precision harmonic detection.

But, for the intrinsic performance limitations of FFT, FFT based algorithms can not be used to analyze non-stationary signals with time-frequency localization properties. For example, if there are transient harmonics or inter-harmonics, other methods should be introduced, because the existing FFT based methods cannot solve it with satisfaction. Now, a series of other methods have been proposed to analyze harmonic and inter-harmonic, here the performance of the proposed method is compared with some of the methods available in the recent literatures.

4.1.1 Analysis of steady harmonic and inter-harmonic

The same model and sampling parameter settings as [22] are used here and the sampling rate f_s is 6400Hz, the signal length N is 2048. To analyze the behavior of the method in the presence of both harmonics and inter-harmonics, a voltage signal containing the fundamental component plus the fifth and seventh harmonic components, along with the inter-harmonic of frequency 170 Hz presented in [22] is used here, and the signal is mathematically represented by:

$$f(t) = \cos(2\pi \times 50t) + 0.2 \cos(2\pi \times 170t) + 0.12 \cos(2\pi \times 250t) + 0.08 \cos(2\pi \times 350t) \quad (25)$$

Noise is unwanted electrical signals with broadband spectral content typically lower than 200 kHz superimposed upon the power system voltage or current. The frequency range and magnitude level of noise depend on the source that produces the noise and the system characteristics. Typical magnitude of noise is less than 1% of the voltage or current magnitude, thus the corresponding signal-to-noise ratio (SNR) is more than 40dB. To study the effect of noise on the performance of the proposed method, Gaussian white noise is added to signal $f(t)$ shown in (25), and signal with lower SNR of 30dB is used here.

In the same simulation conditions, the results of FMP method is compared with the results of 3 other methods including Root-MUSIC, Beamspace Root-MUSIC and TLS-ESPRIT mentioned in literature [22]. Table 1 and 2 gives the relative errors of frequency and amplitude testing results of the 4 different methods respectively.

Table 1. Comparison of relative errors of frequency (%)

Frequency	FMP	Beamspace Root-MUSIC	Root-MUSIC	TLS-ESPRIT
50 Hz	0.0016	0.0140	0.0240	0.0500
170 Hz	0.0017	0.0141	0.0106	0.0229
250 Hz	0.0026	0.0084	0.0348	0.0584
350 Hz	0.0004	0.0074	0.0140	0.0657

Table 2. Comparison of relative errors of amplitude (%)

Amplitude	FMP	Beamspace Root-MUSIC	Root-MUSIC	TLS-ESPRIT
1 pu	0.03	0.10	1.00	0.40
0.2 pu	0.25	1.50	4.00	2.50
0.12 pu	0.33	1.67	7.50	2.50
0.08 pu	0.63	1.25	3.75	3.75

It is found that the test results of the proposed FMP method are more close to the preset values compared with other methods, and the frequency and amplitude relative errors of FMP are one order of magnitude lower than the other three methods. In addition, the SNR of the reconstructed signal is more than 50dB if we reconstruct the components extracted from the original signal, which is well above 30 dB. Therefore, we may consider that the proposed method can detect the harmonic and inter-harmonic parameters accurately and isolate the noise efficiently.

4.1.2 Analysis of transient harmonic

In order to verify the detection performance of the proposed method for transient harmonic, the same sampling setting ($f_s=3200\text{Hz}$, $N=1280$) and signal model that contains fundamental, transient harmonics of 3rd and 7th as literature [16] are used here. Table 3 shows a comparative estimation of parameters for 3rd and 7th harmonic components respectively using both FMP and LMD methods. From these figures, it is verified that proposed FMP method estimates more accurately as compared to LMD method.

Table 3. Comparison of parameter estimation results

Harmonic	Parameters	Set values	LMD method	FMP method
3 rd	A (pu)	0.5	0.4985	0.5000
	f (Hz)	150	150.1000	149.9991
	t_s (s)	0.12	0.1206	0.1197
	t_e (s)	0.24	0.2417	0.2400
7 th	A (pu)	0.3	0.2993	0.3000
	f (Hz)	350	349.7000	350.0000
	t_s (s)	0.12	0.1205	0.1197
	t_e (s)	0.24	0.2418	0.2400

4.2 Analysis with fundamental frequency variation

For performance evaluation of the proposed method at off-nominal frequency, a variation of ± 0.5 Hz is considered here. A sampling frequency of 3200Hz and a SNR of 40dB are considered, simulation was performed on records with 2048 samples. The signal used for estimation, besides the fundamental frequency, contains common harmonics of the 2nd, 3rd, 4th, 5th and inter-harmonics of 239Hz and 256.7Hz, and the amplitude of even harmonics and inter-harmonics are very close to the noise. This kind of signal is mathematically represented by:

$$f(t) = \cos(2\pi f_1 t + 10^\circ) + 0.02 \cos(2\pi f_2 t) + 0.14 \cos(2\pi f_3 t + 57^\circ) + 0.035 \cos(2\pi f_4 t + 83^\circ) + 0.1 \cos(2\pi f_5 t) + 0.04 \cos(2\pi \times 239 t + 105^\circ) + 0.015 \cos(2\pi \times 256.7 t + 35^\circ) + \delta(t) \quad (26)$$

where $\delta(t)$ is Gaussian white noise.

Table 4 shows the relative frequency estimation errors of all components of the signal at different power frequency variations. Table 5 and 6 show the amplitude and phase estimation errors of fundamental, 2nd harmonic and inter-harmonic of 256.7Hz respectively. The estimation results of the 2nd harmonic and 256.7Hz inter-harmonic are chosen because of their weak amplitude close to noise.

From Table 4, it can be seen that the relative frequency errors of fundamental, harmonics and inter-harmonics are all less than 0.02%, and for most cases the errors are less than 0.005%. Table 3 shows that the proposed method possesses good frequency detection accuracy with typical noise amplitude of 1% of the signal magnitude when the power frequency varies from 49.5Hz to 50.5Hz.

Table 5 and 6 show that for fundamental, the relative estimation errors of amplitude is less than 0.1%, and the absolute estimation errors of phase is less than 0.2°. Though the inter-harmonic frequency of 256.7Hz is very close to the frequency of 5th harmonic, and the amplitude of which is commensurate with noise, the proposed method detected the amplitude and the phase information with relatively small errors.

The fundamental frequency and its corresponding

Table 4. Relative estimation errors of frequency using FMP (%)

	49.5 Hz	49.7 Hz	49.9 Hz	50.1 Hz	50.3 Hz	50.5 Hz
1 st	0.0010	0.0014	0.0020	0.0024	0.0020	0.0016
2 nd	0.0174	0.0006	0.0141	0.0056	0.0147	0.0034
3 rd	0.0041	0.0011	0.0027	0.0062	0.0019	0.0014
4 th	0.0012	0.0007	0.0038	0.0014	0.0041	0.0018
5 th	0.0031	0.0016	0.0084	0.0022	0.0005	0.0086
239Hz	0.0003	0.0008	0.0003	0.0012	0.0001	0.0014
256.7Hz	0.0075	0.0069	0.0086	0.0046	0.0119	0.0110

Table 5. Relative estimation errors of amplitude for fundamental, 2nd harmonic and inter-harmonic of 256.7 Hz at different fundamental frequency (%)

	49.5 Hz	49.7 Hz	49.9 Hz	50.1 Hz	50.3 Hz	50.5 Hz
1 st	0.07	0.08	0.07	0.02	0.08	0.05
2 nd	0.5	2.0	1.0	1.5	0.5	1.0
256.7 Hz	1.33	1.33	1.33	2.00	2.67	4.00

Table 6. Absolute estimation errors of phase for fundamental, 2nd harmonic and inter-harmonic of 256.7Hz at different fundamental frequency (°)

	49.5 Hz	49.7 Hz	49.9 Hz	50.1 Hz	50.3 Hz	50.5 Hz
1 st	0.1426	-0.0220	0.1591	0.0415	-0.1910	0.0073
2 nd	1.5450	-0.5853	-2.2733	-0.4636	1.3644	0.6940
256.7 Hz	0.6611	0.5961	2.4172	1.0356	3.8145	2.8373

harmonic frequency in (26) are detected accurately by using FMP method, and the weak inter-harmonic components close to the 5th harmonic are distinguished also. The results demonstrate that the proposed method can effectively overcome the impact of the variation of the system frequency, and the relative errors of frequency have very small change with system frequency variation.

4.3 Analysis with complicated harmonic and inter-harmonic

For the complicated electric power signal that containing both steady and transient harmonic and inter-harmonic, the signal to be analyzed is modeled as:

$$f(t) = \cos(2\pi \times 50 t + 60^\circ) + 0.15 \cos(2\pi \times 150 t) \cdot [u(t - 0.12813) - u(t - 0.32)] + 0.12 \cos(2\pi \times 250 t) \cdot [u(t) - u(t - 0.09375)] + 0.1 \cos(2\pi \times 350 t) + 0.05 \cos(2\pi \times 27 t + 150^\circ) + 0.015 \cos(2\pi \times 326 t) + 0.02 \cos(2\pi \times 100 t + 90^\circ) + 0.08 \cos(2\pi \times 550 t) \cdot [u(t - 0.14375) - u(t - 0.22594)] + 0.017 \cos(2\pi \times 170 t) \cdot [u(t - 0.0625) - u(t - 0.32)] \quad (27)$$

The sampling frequency of 3200 Hz is used here, and $N=1024$. The waveforms extracted from $f(t)$ by using FMP are showed in Fig. 1. Table 7 shows the parameter estimation results of the extracted waveforms.

In Fig. 1, u_1 is the fundamental component, and $u_2 \sim u_9$ are the steady or transient harmonic/inter-harmonic disturbance waveforms extracted from $f(t)$. It can be found

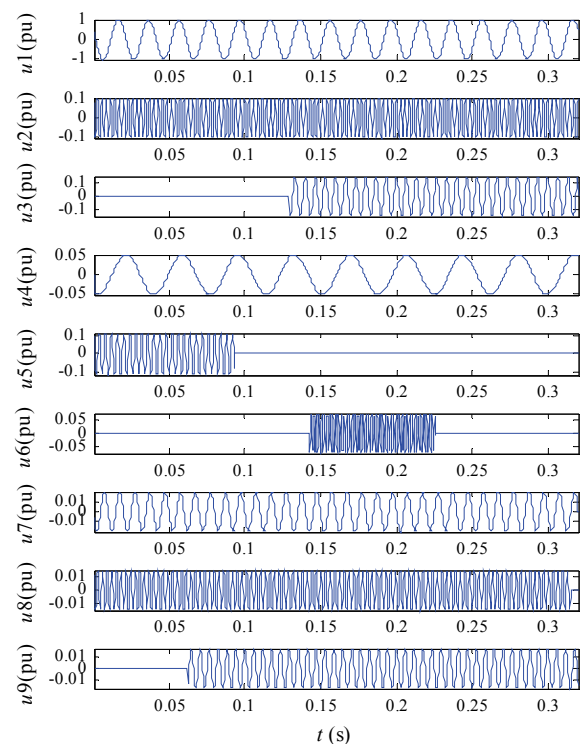


Fig. 1. Extracted waveforms from simulated signal $f(t)$

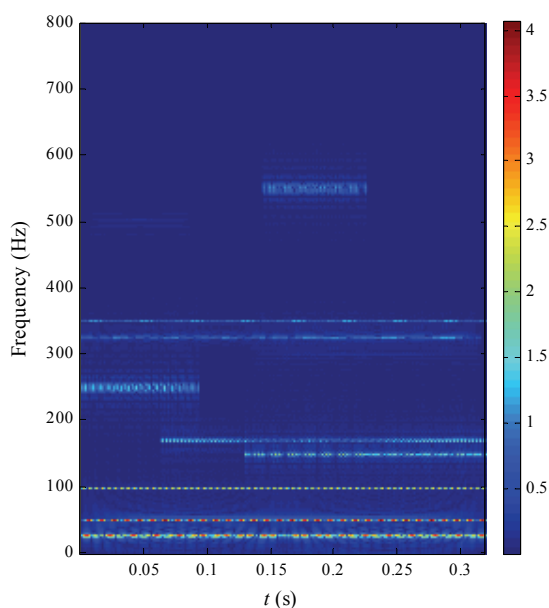


Fig. 2. Wigner-Ville distribution of the atoms corresponding to coherent components of the signal

Table 7. Parameter estimation results of the extracted waveforms

	amplitude (pu)	frequency (Hz)	phase (°)	t_s (s)	t_e (s)
u_1	0.9995	49.9980	60.1455	0.00031	0.32031
u_2	0.1007	349.9863	-2.8017	0.00031	0.32031
u_3	0.1508	149.9522	—	0.12906	0.32000
u_4	0.0502	26.9959	150.3442	0.00063	0.32031
u_5	0.1196	250.0866	0.8882	0.00031	0.09375
u_6	0.0788	549.9785	—	0.14219	0.22625
u_7	0.0201	100.0201	95.1301	0.00031	0.31969
u_8	0.0147	326.0609	0.2229	0.00031	0.31531
u_9	0.0171	170.0642	—	0.06219	0.32031

from expression (27) and Table 7 that the proposed method can estimate the amplitude, frequency, phase, start and end time of the disturbances with high precision from complicated multi-frequency electric power signal.

Fig. 2 shows the Wigner-Ville distribution of the atoms that best matching the signal structures extracted by FMP. It is clearly visible from Fig. 2 that there is no cross-term interference in the time-frequency distribution, thus avoiding the spectrum aliasing, and we can distinguish the information of frequency and duration time of each component clearly from the distribution diagram.

Furthermore, the following two cases are also considered. Case 1: adding white noise to the signal (27) and the SNR of 40dB is considered. Case 2: the fundamental frequency in (27) varies from 49.5Hz to 50.5Hz are simulated. Similar extraction effectiveness as Table 7 can be obtained, so the results are not shown here for simplification.

4.4 Results and analysis of actual power system data

In this section, the decompositions of actual voltage

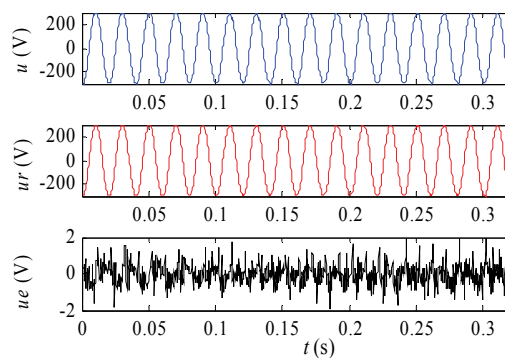


Fig. 3. Actual voltage signal u , the reconstructed signal u_r and residual signal u_e

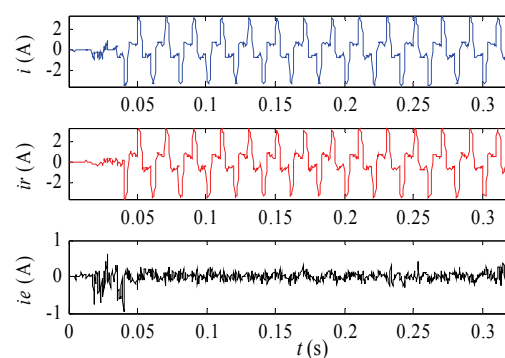


Fig. 4. Actual current signal i , the reconstructed signal i_r and residual signal i_e

and current signals are carried out using the proposed method. The USB data acquisition card is used to acquire the voltage from the sensors, the Hall voltage sensor (400V/5V) and Hall current sensor (5A/5V) are used to collect the operating voltage and current of five personal computers respectively. The sampling frequency of 3200 Hz is also used here, and the signals to be analyzed have $N = 1024$ samples (16 cycles, 0.32 s).

In Fig. 3, u is the steady state voltage with harmonics acquired when computers are running, u_r is the reconstructed voltage signal containing extracted components after 15 iterations by FMP method, and u_e is the residual voltage that containing system noise and decomposing errors. In Fig. 4, i is the initial unsteady state current with serious distortion, i_r is the reconstructed current signal containing extracted components after 15 iterations, and i_e is the residual current signal that containing system noise and decomposing errors. The parameters of the main components extracted from u and i are shown in Table 8 and Table 9 respectively.

It can be seen from the waveform of u in Fig. 3 that the voltage distortion is not obvious, so we can preliminary deduce that the amplitude of harmonic or inter-harmonic is small. As it can be seen from Table 8, many harmonics are detected, but the main components are 3rd and 9th harmonics, and the amplitudes of the two harmonics are 3.4% and 1.6% of the fundamental component respectively.

Table 8. Detection results of the actual voltage

	amplitude (V)	RMS (V)	frequency (Hz)	t_s (s)	t_e (s)
u_1	310.41	219.49	50.02	0.000	0.320
u_2	10.70	7.57	150.07	0.000	0.320
u_3	5.04	3.57	450.21	0.000	0.318
u_4	1.53	1.08	750.36	0.002	0.320
u_5	1.19	0.84	550.27	0.002	0.315
u_6	1.22	0.86	950.52	0.000	0.318
u_7	1.17	0.83	350.22	0.000	0.313
u_8	1.00	0.71	650.26	0.002	0.313

Table 9. Detection results of the actual current

	amplitude (A)	RMS (A)	frequency (Hz)	t_s (s)	t_e (s)
i_1	1.915	1.354	49.96	0.040	0.314
i_2	1.316	0.931	149.99	0.040	0.319
i_3	0.460	0.325	250.00	0.041	0.320
i_4	0.210	0.149	499.82	0.024	0.310
i_5	0.150	0.106	350.26	0.040	0.316
i_6	0.125	0.088	550.10	0.037	0.315
i_7	0.093	0.066	649.74	0.035	0.318
i_8	0.079	0.056	949.72	0.042	0.319
i_9	0.076	0.054	750.10	0.040	0.320
i_{10}	0.061	0.043	100.41	0.019	0.319
i_{11}	0.071	0.051	199.67	0.032	0.317
i_{12}	0.051	0.036	299.72	0.018	0.315
i_{13}	0.039	0.027	850.29	0.025	0.320
i_{14}	0.041	0.029	400.01	0.037	0.320
i_{15}	0.032	0.023	1249.98	0.043	0.312

The amplitudes of the other components extracted from u are all less than 0.5% of the fundamental component. Table 8 only shows the detection results of the components whose amplitudes are above 1V.

It is apparent from Fig. 4 that the distortion of current i is serious, so we can deduce that the harmonic or inter-harmonic disturbances should be accounted for a large proportion. As shown in Table 9, a large amount of harmonic currents are detected in i , of which 3rd harmonic contributes the maximum proportion, the amplitude of 3rd harmonic is 68.7% of the fundamental component, and the 5th, 9th, 7th and 11th harmonics are 24%, 11%, 7.8% and 6.5% of the fundamental component respectively. The smaller harmonics and inter-harmonics are not listed in this table. It can be seen that the content of harmonics is very high, which leads to serious current distortion. At the same time, the method detected the start and end time of the harmonic current, which is approximately coincident with the real time of the original current i .

From the analysis of the waveforms and data, it can be concluded that the proposed technique is effective in detecting steady and unsteady harmonics/inter-harmonics of field or practical power system signals.

4.5 Execution time

Often, the execution time of the algorithm is an

important indicator. As an example, using Matlab software running on a PC with Intel Pentium Dual-core E6700 3.2GHz CPU, the decomposition carried out 9 iterations for the signal in section 4.3 was performed within an average time of 0.62 s, and the average execution time of each iteration is about 0.069 s. The decomposition carried out 15 iterations for the actual current signal in section 4.4 was performed within an average time of 1.02 s, and the average execution time of each iteration is about 0.068 s. Therefore, the time cost of the proposed method is appropriate for computer-based harmonic analyses.

5. Conclusion

In this paper, an attempt is made to detect the harmonic and inter-harmonic disturbances from power system signals using the FMP method. The method presents fast detection of harmonics and inter-harmonics from electrical signals. Different simulation cases and the actual power system voltage and current signals are tested and analyzed, for all these cases, the method is shown to be efficient, and presenting the following advantages: (1) It has clear physical significance. (2) It can detect the amplitude, frequency, phase and duration time of harmonic or inter-harmonic. (3) It can distinguish the inter-harmonic frequency adjacent to fundamental or harmonic frequency. (4) The detection performance is almost not affected by fundamental frequency variation. (5) It has better anti-noise performance. Finally, we think the method provides an alternative way to harmonic and inter-harmonic analysis, and is suitable for the application of off-line fault signal analysis or power quality analysis etc.

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