FEM Analysis of RC Deep Beam Depending on Shear-Span Ratio

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Abstract In this research, we carried out finite element analysis depends on the variations such as the strength of the main bar, concrete, shear-span ratio(a/d) and existence of shear reinforcing bar. Throughout the results of FEM analysis, we were able to figure out how each variation can effect on shear performance. As the strength of concrete increased, the maximum shear force enhancement effect of each specimen was evaluated. As a result, the shear strengthening effect was 51~97% for shear reinforced specimens, and 26~44% for non-shear reinforced specimens. As the yield strength of reinforcing bars increases, the shear reinforcement effect of the specimen the specimens without shear reinforcement were 3%~6% higher than those with shear reinforcement. Theoretical and analytical values were compared using the design equations obtained from the CEB code. As a result, the error rate was the highest at 3.64 in the S1.0-C0 series and the lowest at 1.46 in the S1.7-C1 series. Therefore, the design equation of the CEB code is estimated to underestimate the actual shear strength of deep beams that are not subjected to shear reinforcement.

Keywords: Reinforced Concrete Deep Beam, Shear Strength, Shear Behavior

1. INTRODUCTION

As the recent construction of buildings is becoming skyscraper, the design load of each floor is also required to be higher. Accordingly, development of a deep beam which has superior strength and deformation performance is required. The shear behavior of deep reinforced concrete beams has been a subject of intensive experimental studies since the 1950s. It has long been recognized that, due to their small shear-span-todepth ratios (a/d approx. 2.5), deep beams can carry significantly larger shear forces than slender beams (Liu, et al., 2016).

Reinforced-concrete deep beams are used mainly for load transfer, such as transfer girders, bent caps, and pile caps. The behavior of reinforced-concrete deep beams is different from that of slender beams because of their relatively larger magnitude of shearing and normal stresses (Alexander, 1972). Unlike slender beams, deep beams transfer shear forces to

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supports through compressive stresses rather than shear stresses. The diagonal cracks in deep beams eliminate the inclined principal tensile stresses required for beam action and lead to a redistribution of internal stresses so that the beam acts as a tied arch (William, 1970). Moreover, in order to reduce the amount of steel used and to improve the workability of member placement, the use of high strength steel is also tending to be increased. Therefore, it is necessary to develop the detailed structure and to evaluate the structural performance of the deep beam which has high-strength reinforcing bars with a yield strength of 600MPa or more. From a modeling point of view, deep beams do not obey the classical plane-sections-remainplane hypothesis, and therefore require different models than slender beams. Since the 1960s, researchers have proposed various empirical formulas and analytical models for evaluating the shear strength of deep beams (Mau, et al., 1989; Ashour, 2000; Matamoros, et al., 2003; Russo, et al., 2005; Yang, et al., 2011; Mihayloy, et al., 2013).

Also, Finite-Element Analysis(FEA) is considered as an alternative for in-depth analysis. FEA is currently the most complex and advanced approach for predicting the response of reinforced-concrete structures (Mohamed, et al., 2017). In this study, we carry out 2-D FEM analysis in order to evaluate how high-strength concrete and steel can affect the shear performance of deep beam.

| | | | | | Vertical | |
|-----------------|--------|-----------------------|----------------------|--------|-----------------|----------|
| Specimen | d (mm) | f _{ck} (MPa) | f _y (MPa) | a (mm) | reinforce rebar | a/d (mm) |
| S1.0-C0-F24-600 | 539 | 24 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F35-600 | 539 | 35 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F45-600 | 539 | 45 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F60-600 | 539 | 60 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F24-600 | 539 | 24 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F35-600 | 539 | 35 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F45-600 | 539 | 45 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F60-600 | 539 | 60 | 600 | 539 | 16-D10@100 | 1.0 |
| S1.7-C0-F24-600 | 539 | 24 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F35-600 | 539 | 35 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F45-600 | 539 | 45 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F60-600 | 539 | 60 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F24-600 | 539 | 24 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F35-600 | 539 | 35 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F45-600 | 539 | 45 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F60-600 | 539 | 60 | 600 | 916.3 | 23-D10@100 | 1.7 |
| S1.0-C0-F24-400 | 539 | 24 | 400 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F24-500 | 539 | 24 | 500 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F24-400 | 539 | 24 | 400 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F24-500 | 539 | 24 | 500 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F60-400 | 539 | 60 | 400 | 539 | 16-D10@100 | 1.0 |
| S1.0-C0-F60-500 | 539 | 60 | 500 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F60-400 | 539 | 60 | 400 | 539 | 16-D10@100 | 1.0 |
| S1.0-C1-F60-500 | 539 | 60 | 500 | 539 | 16-D10@100 | 1.0 |
| S1.7-C0-F24-400 | 539 | 24 | 400 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F24-500 | 539 | 24 | 500 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F24-400 | 539 | 24 | 400 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F24-500 | 539 | 24 | 500 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F60-400 | 539 | 60 | 400 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C0-F60-500 | 539 | 60 | 500 | 916.3 | 23-D10@100 | 1.7 |
| S1.7-C1-F60-400 | 539 | 60 | 400 | 916.3 | 23-D10@100 | 1.7 |
| | | | | | | |

Table 1. Detail of specimens

S1.7-C1-F60-500

539

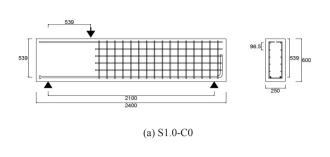
60

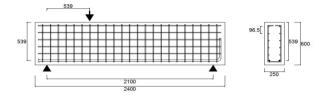
500

916.3

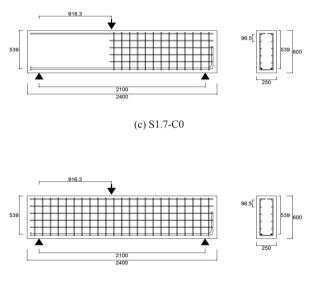
23-D10@100

1.7









(d) S1.7-C1

Figure 1. Detailed view of the specimens

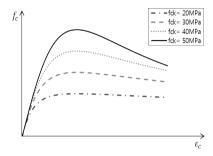
2. ANALYSIS PROCESS

2.1 Design of speciment

The number of a specimen is 22 in the variation of the yield strength of main bar (f_y , f_y : 400MPa, 500MPa, 600Mpa), and concrete compressive strength (f_{ck} , f_{ck} : 24MPa, 35MPa, 45MPa, 60MPa), shear-span ratio (a/d, a/d: 1.0, 1.7), and the existence of shear reinforcing bar (Table 1). Specimens are designed to be 250mm × 600mm for a section and 2400mm for length and 2100mm as their clear span. D22 for bottom bar, D13 for horizontal shear reinforcing bar, and vertical shear reinforcing bar of D10 is used.

2.2 Finite element model

VecTor 2, the analysis program, is used. This program is using nonlinear FEM analysis that is based on Modified compression field theory. The stress-strain curve analysis model of concrete used the Eq. (1). proposed by popovics. the Popovics model is used until before it reaches the ultimate compression strength and then Modifed Park-Kent is used after ultimate compression strength. Popovics (1973) presented stress-strain curves for a range of normal strength concretes.



(a) Concrete stress-strain response model

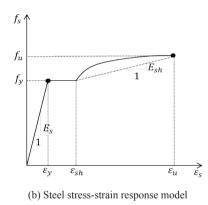


Figure 2. Stress-strain response model

These curves reflect the greater stiffness and linearity of the ascending branch and the reduced ductility of concretes as the peak compressive stress increases. Park, Priestly, and Gill (1982) modified a stress-strain curve proposed by Kent and Park to account for the enhancement of concrete strength and ductility due to confinement. The stress-strain curve was utilized to compute the flexural strength of reinforced concrete columns confined by transverse hoop reinforcement.

$$f_{ci} = -\left(\frac{\varepsilon_{ci}}{\varepsilon_p}\right) f_{ck} \frac{n}{n - 1\left(\frac{\varepsilon_{ci}}{\varepsilon_p}\right)} \quad for \ \varepsilon_{ci} < 0 \tag{1}$$

Where f_p = corresponding to the peak compressive stress, ε_p = less compressive than the strain, n = the curve fitting parameter.

For analysis model of steel bar, 3 parted line model is used for the stress-strain curve (Fig. 2). Until steel bar reaches a yielding point, it shows the straight line. And, until it reaches failure point, the shape of line behaves to make linear or non-linear line according to the parameters of hardening phenomena. The tension and fs are determined by Eq. (2)

$$f_{S} = \begin{bmatrix} E_{S}\varepsilon_{S} & for \varepsilon_{S} < \varepsilon_{y} \\ f_{y} & for \varepsilon_{y} < \varepsilon_{S} \le \varepsilon_{sh} \\ f_{u} + (f_{y} - f_{u}) \left(\frac{\varepsilon_{u} - \varepsilon_{s}}{\varepsilon_{u} - \varepsilon_{sh}}\right)^{p} & for \varepsilon_{sh} < \varepsilon_{s} \le \varepsilon_{u} \\ 0 & for \varepsilon_{u} < \varepsilon_{s} \end{bmatrix}$$
(2)

Where ε_s is the reinforcement strain, ε_y is the yield strain, ε sh is the strain at the onset of the strain hardening, ε_u is the ultimate strain, P is the strain-hardening parameter

3. RESULT

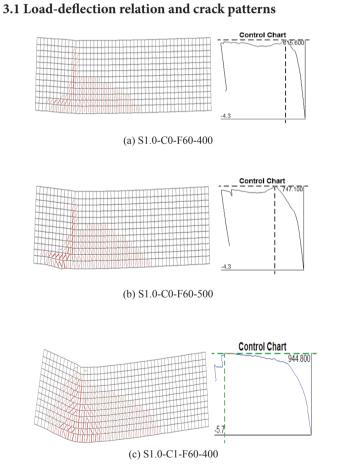


Figure 3. Finite element analysis result of S1.0-fy series

The Fig. 3 shows the finite-element analysis result of S1.0- $C0-f_y$ series. The figures on the left side show the specimen of crack and deformation when the analysis subject reached the maximum load, and the figures on the right side are the load-displacement graph by analysis.

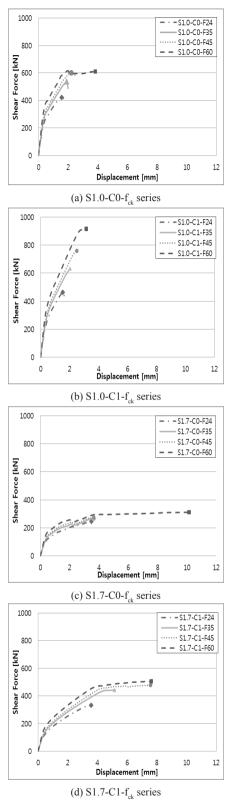


Figure 4. .Shear force-displacement relationship of fck series

The shear force-displacement curve by FEM analysis is shown on the Fig. 4. As shown in Figs. 4 (a) and (b), in the case of the S1.0 series, the shear force of the specimen without shear reinforcement increased by 44% in maxi-mum as the concrete strength increased.

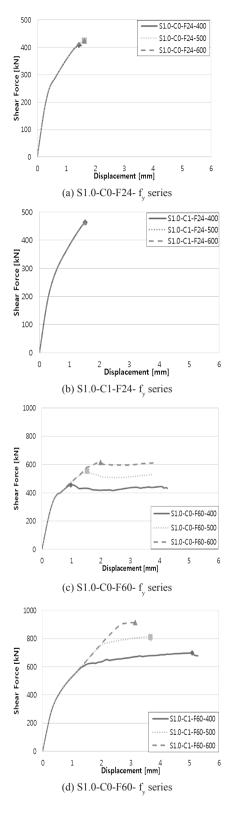


Figure 5. Shear force-displacement relationship of S1.0 series

And, shear force of the specimen with shear reinforcing bar has increased by 97% in maximum. In the case of S1.7 series, maximum 26% and 51% increase of specimen without shear reinforcement and specimen with shear reinforcement, respectively. Therefore, it can be seen that the effect of increasing the shear strength with increasing the strength of the concrete is higher when specimens with shear reinforce bar.

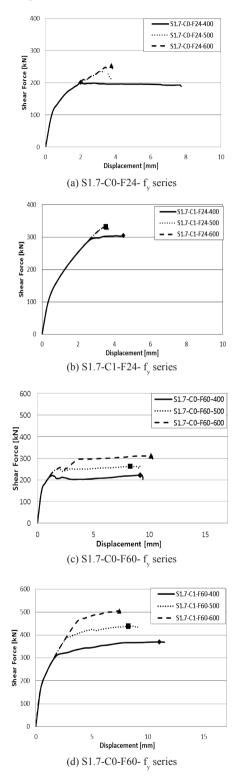


Figure 6. Shear force-displacement relationship of S1.7 series

Shear force-displacement curve of the specimen with a shearspan ratio of 1.0 when yielding strength is increased is shown in Fig. 5. As shown in Figs. 5 (a) and (b), the maximum shear force of the specimens with the concrete strength of 24 MPa were not increased with increasing the strength of the reinforcing bars. On the other hand, the S1.0-C0-F60- f_y without shear reinforcing bar have in-crease in its shear force by 19%, 37% as the strength of steel is increased by 100MPa, and 200MPa. And for the S1.0-C0-F60- f_y with shear reinforcing bar also have increased by 16%, and 31%.

Therefore, the higher the strength of the concrete, the more the shear force is affected by the strength of the reinforcing bar. And enhancement effect on shear strength according to increase in strength of steel seemed to be higher when the shear reinforcing bar is absent. As shown in Fig. 6, the S1.7 series with a shear span ratio of 1.7 showed an increase similar to that of the S1.0 series.

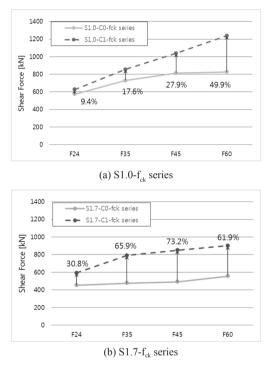


Figure 7. Shear force relationship with concrete strength of specimens

Figure 7 shows the shear force according to concrete strength as a line graph with depending on whether or not of shear reinforcement. For the S1.0 series with a shear span ratio of 1.0, the shear reinforcement effect was the largest at 49.9% when the concrete strength was 60 MPa. Also, as the strength of concrete increased, the shear reinforcement effect also increased proportionally. For the s1.7 series, the shear force of the specimen with shear reinforcement (c1 series) was on average, about 58% higher than that of c0 series. And as the strength of concrete increases, the shear reinforcement effect tends to increase. However, when the strength of concrete exceeds 45 MPa the effect of shear reinforcement is rather lowered.

Therefore, the effect of shear reinforcement with according to increasing concrete strength was more effective when the shear span ratio was small. However, the average shear reinforcement effect was higher when the shear span ratio was larger. It is judged that the larger the shear span ratio, the greater the reinforcing effect because the shear reinforcement area is relatively wide.

3.2 Comparison between theoretical and Analysis strength

The strength of the deep beam was estimated using the practical design method of the CEB code. The CEB code is designed considering the distribution of the nonlinear strain of the deep beam section. The shear strength V_c of the concrete is calculated as shown in Eq. (3). And V_c should not exceed $0.29\sqrt{f_{ck}}b_wd$ too.

$$V_{c} = (0.16\sqrt{f_{ck}} + 17.6\rho_{w}\frac{V_{u}d}{M_{u}})b_{w}d \quad (kN)$$
(3)

The shear strength V_s of the shear reinforcement bar is calculated as shown in Eq. (4). Where s is the spacing of shear reinforcement in a direction parallel to the longitudinal reinforcement, s_h is the distance in the vertical direction to the longitudinal reinforcement, A_v is the spacing of vertical reinforcement roots in s, and A_{vh} is the cross-sectional area of horizontal reinforcement rods. s and s_h should be less than d/5 and also not more than 300mm, The minimum cross-sectional areas of A_v and A_{vh} are 0.0025 $b_w s$ and 0.0015 $b_w s$, respectively.

$$V_s = [\frac{A_v}{s}(\frac{1+l_n/d}{12}) + \frac{A_{vh}}{s_h}(\frac{11-l_n/d}{12})]f_yd \quad (kN) \quad (4)$$

Based on the above equations, the shear strength of all specimens is shown in comparison with the analytical history as shown in Table 2. The analytical values of all specimens were larger than the theoretical values. This indicates that the theoretical formula underestimates the analytical value. For the specimen with high strength steel ($f_y = 600$ MPa), the error rate was larger when the shear span ratio was small (S1.0 series) and when no shear reinforcement was applied (C0 series). When the strength of reinforcing bars was lower than 600MPa, the specimens with the strength of concrete of 24MPa showed an average error rate of 9% higher than the specimens with the strength of 60MPa.

4. CONCLUSIONS

In this study, we carry out 2-D FEM analysis in order to evaluate how high-strength concrete and steel can affect on the shear performance of deep beam when there is the variation of a shear-span ratio (1.0, 1.7), shear reinforcing bar, the strength of the main bar(400MPa~ 600MPa) and concrete(24MPa ~ 60MPa).

As a result, enhancement effect on the shear strength of specimens which has shear reinforcing bar was $26\% \sim 41\%$, and specimens which have not shear reinforcing bar appeared $51\% \sim 97\%$ each. Enhancement effect on the shear strength of the subject with a shear-span ratio of 1.0 is measured to be $31\% \sim 37\%$ when the strength of steel is increased. That is, the shear

Table 2. Shear strength of deep beam (CEB code)

| Specimen | V _{n_T} (kN) | V _{n_A} (kN) | V _{n_A} / V _{n_T} |
|-----------------|--------------------------|--------------------------|--|
| S1.0-C0-F24-600 | 135.57 | 266.84 | 3.56 |
| S1.0-C0-F35-600 | 152.64 | 275.18 | 3.82 |
| S1.0-C0-F45-600 | 175.02 | 312.03 | 3.80 |
| S1.0-C0-F60-600 | 267.90 | 332.42 | 3.38 |
| S1.0-C1-F24-600 | 289.83 | 442.68 | 1.70 |
| S1.0-C1-F35-600 | 306.91 | 476.62 | 2.14 |
| S1.0-C1-F45-600 | 329.28 | 505.18 | 2.46 |
| S1.0-C1-F60-600 | 119.25 | 409.00 | 2.74 |
| S1.7-C0-F24-600 | 119.25 | 424.17 | 2.24 |
| S1.7-C0-F35-600 | 222.09 | 424.24 | 1.97 |
| S1.7-C0-F45-600 | 247.80 | 463.98 | 1.80 |
| S1.7-C0-F60-600 | 180.63 | 443.63 | 1.78 |
| S1.7-C1-F24-600 | 180.63 | 530.58 | 1.24 |
| S1.7-C1-F35-600 | 283.47 | 699.15 | 1.53 |
| S1.7-C1-F45-600 | 309.18 | 811.85 | 1.55 |
| S1.7-C1-F60-600 | 113.64 | 201.71 | 1.53 |
| S1.0-C0-F24-400 | 113.64 | 236.77 | 3.43 |
| S1.0-C0-F24-500 | 216.48 | 304.98 | 3.56 |
| S1.0-C1-F24-400 | 242.19 | 332.08 | 1.91 |
| S1.0-C1-F24-500 | 175.02 | 221.93 | 1.87 |
| S1.0-C0-F60-400 | 175.02 | 262.98 | 2.46 |
| S1.0-C0-F60-500 | 277.86 | 369.77 | 2.94 |
| S1.0-C1-F60-400 | 303.57 | 438.20 | 2.47 |
| S1.0-C1-F60-500 | 135.57 | 266.84 | 2.63 |
| S1.7-C0-F24-400 | 152.64 | 275.18 | 1.78 |
| S1.7-C0-F24-500 | 175.02 | 312.03 | 2.08 |
| S1.7-C1-F24-400 | 267.90 | 332.42 | 1.41 |
| S1.7-C1-F24-500 | 289.83 | 442.68 | 1.37 |
| S1.7-C0-F60-400 | 306.91 | 476.62 | 1.27 |
| S1.7-C0-F60-500 | 329.28 | 505.18 | 1.50 |
| S1.7-C1-F60-400 | 119.25 | 409.00 | 1.33 |
| S1.7-C1-F60-500 | 119.25 | 424.17 | 1.44 |

strength enhancement effect of reinforced concrete was $25 \sim 53\%$ higher than that of a specimen without shear reinforcement. Therefore, it can be seen that the effect of increasing the shear strength with increasing the strength of the concrete is higher when specimens with shear reinforcing bar.

As the strength of reinforcing bars increased, the shear strength enhancement effect was $3\% \sim 6\%$ higher for specimens without shear reinforcement than for specimens with shear reinforcement. The ratio of the error rate to the analytical value of the theoretical shear strength using the design equation obtained from the CEB code was evaluated to be 2.21. And the error rate was about 72% higher than that at 1.7 when shear span ratio was 1.0 at the same reinforcement strength. The largest error rate was 3.13 in the S1.0-C0 series, which means that the theoretical formula underestimates the shear strength of the concrete.

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