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# Beacon-Based Indoor Location Measurement Method to Enhanced Common Chord-Based Trilateration

Jeonghoon Kwak\* and Yunsick Sung\*

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## Abstract

To make an unmanned aerial vehicle (UAVs) fly in indoor environments, the indoor locations of the UAV are required. One of the approaches to calculate the locations of an UAV in indoor environments is enhanced trilateration using one Bluetooth-based beacon and three or more access points (APs). However, the locations of the UAV calculated by the common chord-based trilateration has errors due to the distance errors of the beacon measured at the multiple APs. This paper proposes a method that corrects the errors that occur in the process of applying the common chord-based trilateration to calculate the locations of an UAV. In the experiments, the results of measuring the locations using the proposed method in an indoor environment was compared and verified against the result of measuring the locations using the common chord-based trilateration. The proposed method improved the accuracy of location measurement by 81.2% compared to the common chord-based trilateration.

## Keywords

Beacon, Chord-Based Trilateration, Indoor Location, Trilateration, Unmanned Aerial Vehicle

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## 1. Introduction

To perform delivery, surveillance, and reconnaissance missions using an unmanned aerial vehicle (UAV) [1] in indoor environments, a method of calculating the indoor locations of the UAV is required. The location of a UAV can be determined using a camera mounted in the UAV and a three-dimensional environment [2]. However, the 3D environments must have been constructed to determine the locations of a UAV using the camera mounted in it. Therefore, a method of calculating the locations of a UAV without prior construction of the aviation environments is required.

Beacons can be used to calculate the locations of an UAV [3-6]. One such method is to calculate the locations using trilateration based on the distances from three or more access points (APs) to one beacon. It is possible to calculate the location of beacon even if it is impossible to calculate it using the common chord-based trilateration [3], but the problem is that the calculated locations are often inaccurate. Thus, a method of finding the accurate location of the beacon is necessary for cases where the location of the beacon cannot be calculated.

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This paper proposes a method of calculating the beacon in cases that is inappropriate for the formula in the process of using common chord-based trilateration [3]. The possibility of using common chord-based trilateration [3] with three distances from the APs to the beacon is determined. If it is possible to use the common chord-based trilateration [3], the locations of the beacon are calculated using common chord-based trilateration. However, a method of calculating the locations of beacon when common chord-based trilateration [3] cannot be applied is proposed. In this paper, the locations of the UAV are calculated by mounting the beacon on the UAV, without mounting the APs, given that the weight that the UAV can carry is limited.

This paper is organized as follows: Section 2 discusses related works. In Section 3, a method of calculating the location of beacon is proposed in the case common chord-based trilateration [3] is not applicable. In Section 4, the location of a UAV is measured in an indoor environment using the proposed method and the improvement is verified. In Section 5, the conclusions about the proposed method are presented.

## 2. Related Works

In an outdoor environment, the Global Positioning System (GPS) is used to measure the location of a UAV. However, other methods to measure the location of a UAV in an indoor environment are needed, because the GPS cannot measure the location in an indoor environment.

One method of calculating the indoor location of a UAV is the fingerprinting method, which is based on signals such as beacon signals that are measurable in an indoor environment [7-10]. The fingerprinting method first uses a device measuring beacon signals in an offline process, to collect signals for each position in the indoor environment and generate a signal map. During the online process, the location of the UAV in the indoor environment is measured by comparing it with beacon signals collected in the offline process. However, the drawback with the fingerprinting method is that beacon signals corresponding to each location need to be collected in advance during the offline process in order to be able to measure the location of the UAV during the online process.

Therefore, there is a need for a method to measure the location of a UAV in real time based on the placement of the beacon, without having to collect beacon signals in advance. There has been prior research on using trilateration to measure the position of a beacon based on the placement of three or more APs [3-6]. The location of the beacon is measured using the distance from three or more APs to the beacon. However, if the distance from the APs to the beacon includes an error, the trilateration method that calculates the location of the beacon based on the common chord cannot be applied, as the location of the beacon is calculated incorrectly in such a scenario.

To utilize the distance of the beacon in the trilateration method, there has been research on revising the measured distance from the APs to the beacon [11,12]. The current distance value of the beacon is revised using the previous distance value of the beacon measured from the beacon [11]. Another method is to use a pre-arranged beacon to revise the distance to the beacon, to measure its relative position [12]. However, even if the distance between the beacon and the APs are revised, there is still the possibility that these distances include errors, and therefore the location of the beacon might be measured incorrectly by common chord-based trilateration [3].

### 3. Beacon-Based Localization Approach

First, we describe situations where the use of common chord-based trilateration is possible, and where it is not. The proposed method for measuring the location of the UAV in situations where the use of common chord-based trilateration is not possible is described.

Let a beacon  $b$  be mounted on a UAV. The location of the beacon at time  $t$  is denoted by  $p_t$  and the 2D coordinates of this location are denoted by  $[x, y]$ . The positions of the corresponding APs are denoted by AP  $a_1$ , AP  $a_2$ , and AP  $a_3$ . For example, AP  $a_1$  has the 2D coordinates  $[x, y]$ . The distance measured from AP  $a_1$  to beacon  $b$  is  $d_{t,1}$ , the distance measured from AP  $a_2$  to beacon  $b$  is  $d_{t,2}$ , and the distance measured from AP  $a_3$  to beacon  $b$  is  $d_{t,3}$  at time  $t$ . Using the distance measured at each AP, a circle is generated for each AP based on the position and distance of the AP.

#### 3.1 The Case Where the Location of the Beacon Can Be Calculated Using Common Chord-Based Trilateration

Given that the common chord-based trilateration approaches sometimes cannot measure the locations as below when the approaches are utilized to UAVs. The common chord-based trilateration of the existing study [3] is not applicable if the circle consisting of the location of AP and the measured distance is overlapped or belongs to another circle consisting of other APs. For example, Fig. 1 shows the case where location measurement is possible using common chord-based trilateration.

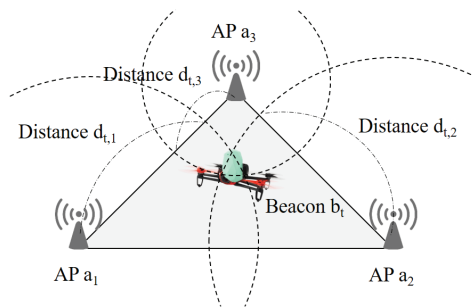


Fig. 1. UAV locations sometimes cannot be calculated.

As shown in Fig. 2, a common chord corresponding to two circles is generated when the two circles for AP  $a_1$  and AP  $a_2$  overlap. It is possible to measure the location  $p_t$  of the beacon at the intersection of the three common chords generated when the three circles overlap.

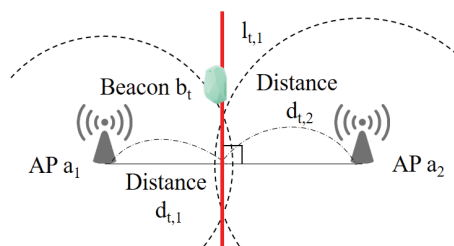
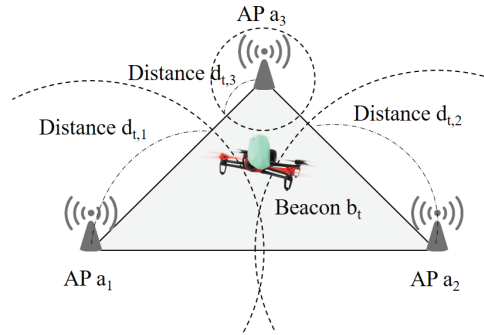


Fig. 2. Two circles are overlapped by two points.

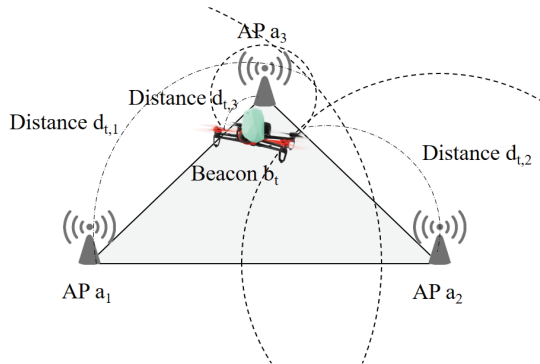
### 3.2 The Case Where the Location of the Beacon Is Incorrectly Calculated When Using Common Chord-Based Trilateration

To use common chord-based trilateration [3], a common chord needs to be generated. Two circles must overlap or meet for a common chord to be created. The location of AP in Fig. 3 is the case where the circles consisting of the APs and measured distance do not overlap.



**Fig. 3.** UAV locations sometimes cannot be calculated.

There is the case where a common chord is generated at a position different from the position where it is supposed to be generated owing to a distance error of the beacon measured at the APs. As shown in Fig. 4, although the location of the beacon is inside the triangle, it may be calculated to be outside the triangle owing to the distance error of the beacon measured at the APs.

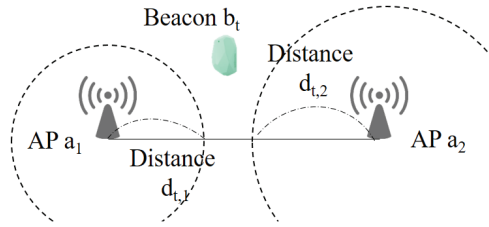


**Fig. 4.** Case where the location of the beacon is incorrect due to the distance error of the beacon measured at the APs.

If the common chord-based trilateration of the existing study [3] can be used, it should be used; otherwise, the proposed method can be used. Whether the location of the beacon can be calculated using the common chord-based trilateration of the existing study should be found out [3]. In details, the two circles in common chord-based trilateration is not overlapped as shown in Fig. 5, because of the inaccurately measured distances by Eq. (1) as below. Therefore, another approach is required to calculate the middle point between the two not overlapped circles in common chord-based trilateration.

For example, if the sum of distances  $d_{t,1}$  and  $d_{t,2}$  in Eq. (1) is smaller than the sum of the Euclidean distances of AP  $a_1$  and AP  $a_2$ , the two circles do not overlap as shown in distances  $d_{t,1}$  and  $d_{t,2}$  in Fig. 4.

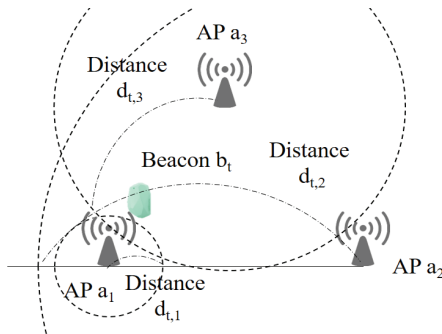
$$d_{t,1} + d_{t,2} < \sqrt{(a_{1,1} - a_{2,1})^2 + (a_{1,2} - a_{2,2})^2} \tag{1}$$



**Fig. 5.** Two circles in trilateration are not overlapped.

As shown in Eq. (2), if the distances  $d_{t,1}$  or  $d_{t,2}$  is greater than the sum of the Euclidean distances of AP  $a_1$  and AP  $a_2$ , the circle consisting of the location and distance of AP as shown in Fig. 6 exists inside the circle of another AP and the two circles do not overlap with each other. They may also overlap with other APs or go past other APs. In this case, a beacon location error occurs when the location of the beacon is inside the triangle formed by the three APs.

$$d_{t,1} > \sqrt{(a_{1,1} - a_{2,1})^2 + (a_{1,2} - a_{2,2})^2} \text{ or } d_{t,2} > \sqrt{(a_{1,1} - a_{2,1})^2 + (a_{1,2} - a_{2,2})^2} \tag{2}$$



**Fig. 6.** One circle is located in the other circle.

### 3.3 Proposed Location Calculation Method Using Common Chord

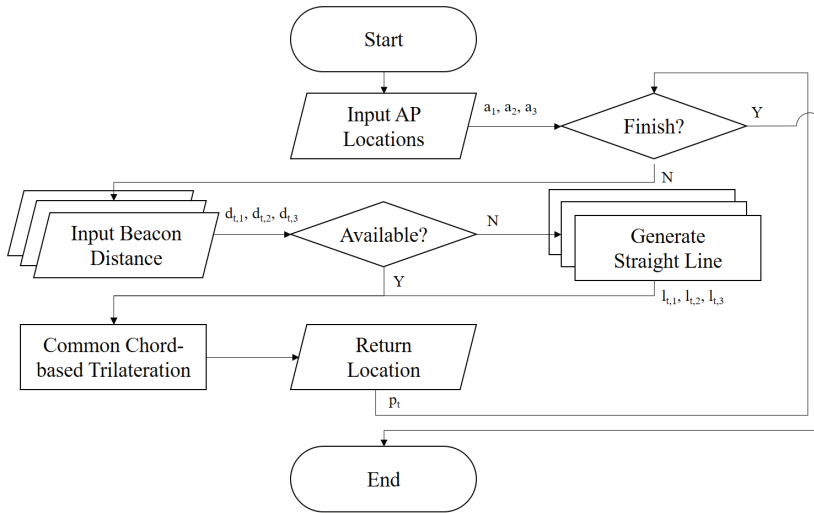
The procedure for measuring the location of the beacon, based on distance, which addresses the drawback of common chord-based trilateration [3], is shown in Fig. 7. The locations of the APs are entered in the *Input AP Locations* step. For all APs, the position  $x, y$  corresponding to each AP is entered. After all AP locations are entered, we proceed to the *End* step.

Next, whether the location calculation should be continued or not is determined in the *Finish?* step. If the number of input APs is not three or more, or if the location measurement for the beacon is not continued, the process is terminated. From here, we proceed to the *Input Beacon Distance* step to continue measuring the location of the beacon.

The distance from each AP to the beacon is measured in the *Input Beacon Distance* step. The distance from each AP to the beacon measured at time  $t$  is inputted in this step. That is, the distance  $d_{t,1}$  from AP

$a_1$  to the beacon, the distance  $d_{t,2}$  from AP  $a_2$  to the beacon, and the distance  $d_{t,3}$  from AP  $a_3$  to the beacon, are inputted.

Following this, the *Available* step checks whether the location of the beacon can be calculated by common chord-based trilateration [3]. Using the positions of AP  $a_1$ , AP  $a_2$  and AP  $a_3$ , and the corresponding distances between each of them, whether the common chords can be generated and whether the beacon signal can be revised using Eq. (1) and Eq. (2) is confirmed. For two circles, if one of Eq. (1) or Eq. (2) is satisfied, we proceed to the *Generate Straight Line* step. However, if they are all satisfied, we proceed to the *Common Chord-based Trilateration* step.



**Fig. 7.** Proposed Beacon Location Measurement Flowchart.

In the *Generate Straight Line* step, the straight lines corresponding to each pair circles are calculated. The straight lines calculated using the circles corresponding to the pairs AP  $a_1$  and AP  $a_2$ , AP  $a_2$  and AP  $a_3$ , and AP  $a_1$  and AP  $a_3$  are denoted by line  $l_{t,1}$ , line  $l_{t,2}$ , and line  $l_{t,3}$ , respectively. The method of calculating the location of the beacon based on these straight lines is as follows.

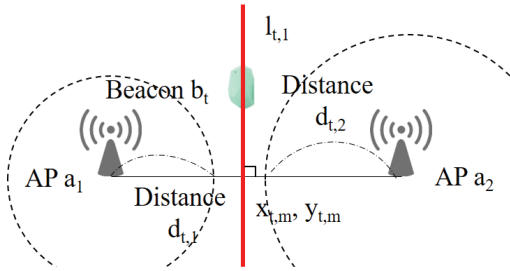
Using the straight lines calculated in each case, the location of the beacon is determined by the average of the three locations, which are the intersections of three straight lines calculated in the three cases described above.

By the proposed method, the locations are calculated by lines as Fig. 2. There are three cases. When Eq. (1) and Eq. (2) are not satisfied, the two points are located between the two circles. Therefore, it is easy to obtain the line  $l_{t,i}$  as below.

When Eq. (1) is satisfied, two circles are not overlapped. As shown in Eq. (3), the midpoint of two APs denoted by  $x_{t,m}$  and  $y_{t,m}$ , is calculated using the two distances measured by each AP. As shown in Eq. (4), the line  $l_{t,i}$  is calculated using the slope of the straight line connecting the midpoint and the APs. The line for the two circles is obtained as Fig. 8.

$$x_{t,m} = \frac{d_{t,1}}{|d_{t,1}+d_{t,2}|} \times (a_{1,1} - a_{1,2}) + a_{1,1}, y_{t,m} = \frac{d_{t,1}}{|d_{t,1}+d_{t,2}|} \times (a_{1,2} - a_{2,2}) + a_{1,2} \quad (3)$$

$$y = \left( \frac{a_{2,2}-a_{1,2}}{a_{2,1}-a_{1,1}} \times -1 \right) \times (x - x_{t,m}) + y_{t,m} \quad (4)$$



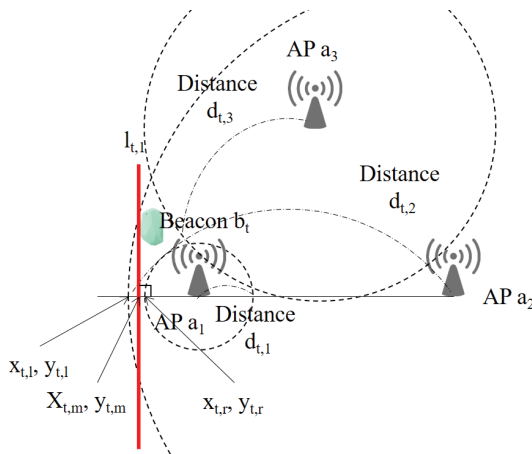
**Fig. 8.** Result of calculating the line  $l_{t,1}$  when the two circles do not overlap.

We now consider the case when the circle formed by one AP contains that of another AP, or the intersection of the two circles does not lie in between the two APs. In this case, we check whether the beacon is located in the triangle formed by the three APs. If the beacon is located inside the triangle formed by the APs, the midpoint is calculated by Eq. (3).

If the beacon is located outside the triangle formed by the APs, the midpoint is calculated by Eq. (5). A straight line is generated using the position of AP  $a_1$  and the position of AP  $a_2$ . We find the two points where the generated straight line and the circle corresponding to AP  $a_1$  meet, and the two points where the generated straight line and the circle corresponding to AP  $a_2$  meet. Between the two points where the straight line and AP  $a_1$  meet, and the two points where the straight line and AP  $a_2$  meet, we pick one point each such that these picked points are the closest pair based on Euclidean distance. These points are denoted by  $x_{t,r}, y_{t,r}$  and  $x_{t,l}, y_{t,l}$ . We now calculate  $x_{t,m}$  and  $y_{t,m}$  using  $x_{t,r}, y_{t,r}, x_{t,l}, y_{t,l}$  as shown in Eq. (5).

$$x_{t,m} = \frac{d_{t,1}}{|d_{t,1}+d_{t,2}|} \times (x_{t,r} - x_{t,l}) + x_{t,r}, y_{t,m} = \frac{d_{t,1}}{|d_{t,1}+d_{t,2}|} \times (y_{t,r} - y_{t,l}) + y_{t,r} \tag{5}$$

Among the points calculated by Eq. (3) and Eq. (5), we find a point close to the circle corresponding to AP  $a_3$ . Based on the points near the calculated circle, we calculate the line  $l_{t,1}$  by Eq. (4). The line  $l_{t,1}$  is obtained by Fig. 9. When each line is obtained, by utilizing lines, the locations of UAV can be calculated.



**Fig. 9.** Result of calculating the line  $l_{t,1}$  in case where a circle is in another circle or circles do not intersect between the APs.

Next, the location  $p_t$  of the beacon is calculated using the lines  $l_{t,1}$ ,  $l_{t,2}$ , and  $l_{t,3}$  in the *Calculate Line-based Location* step. Cross point  $c_{t,1}$  is defined as the position where the line  $l_{t,1}$  intersects the line  $l_{t,2}$ , Cross point  $c_{t,2}$  as the position where the line  $l_{t,2}$  intersects the line  $l_{t,3}$ , and Cross point  $c_{t,3}$  as the position where the line  $l_{t,3}$  intersects the line  $l_{t,1}$ . Each intersection point has x and y coordinates. The location  $p_t$  of the beacon is calculated as the average of the intersection points as shown in Eq. (6). The location  $p_t$  of the beacon is set as shown in Fig. 10. Next, the location  $p_t$  of the calculated beacon is returned in the *Return Location* step.

$$p_{t,1} = \frac{c_{t,1,1} + c_{t,2,1} + c_{t,3,1}}{3}, p_{t,2} = \frac{c_{t,1,2} + c_{t,2,2} + c_{t,3,2}}{3} \quad (6)$$

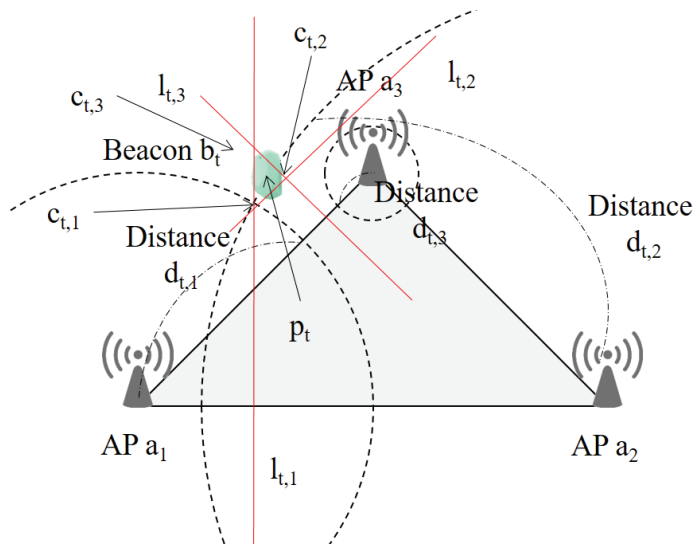
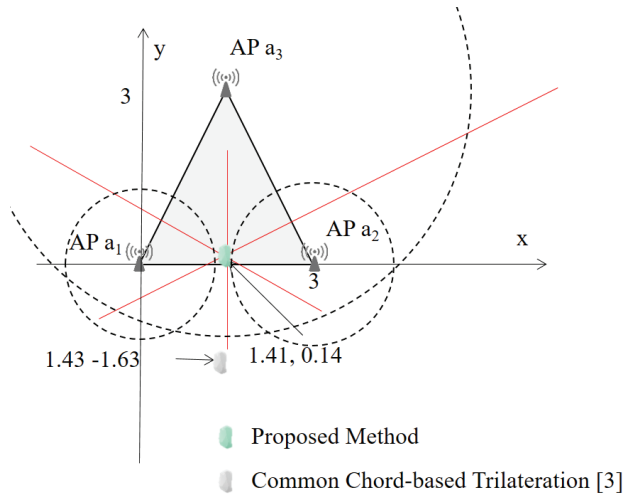


Fig. 10. Calculated result of the location of the beacon.

## 4. Experiments

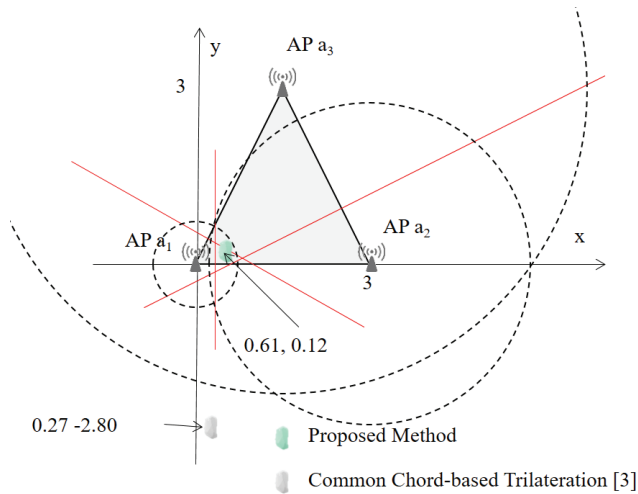
In the experiments, the locations of beacon were calculated using the beacon signals measured while the UAV was moving in the triangular formed by AP  $a_1$  at  $[0,0]$ , AP  $a_2$  at  $[3,0]$ , and AP  $a_3$  at  $[1.5, 3]$ . In Fig. 11, the distance measured at each AP satisfies Eq. (1) and Eq. (2). When the location of the beacon is in the triangle area, the location calculated by the proposed method is  $[1.43, 0.14]$  and the location calculated by common chord-based trilateration [3] is  $[1.43, -1.63]$ . In the proposed method, the line  $l_{t,1}$  was generated by Eq. (4) using  $x_{t,m}$  and  $y_{t,m}$ , the position of which is calculated by Eq. (3), and between AP  $a_1$  and AP  $a_2$  satisfying Eq. (1). Among locations calculated by Eq. (3) and Eq. (5) using the pairs AP  $a_2, AP a_3$  and AP  $a_1, AP a_3$  satisfying Eq. (2), the straight lines line  $l_{t,2}$  and line  $l_{t,3}$  are generated in the triangle by  $x_{t,m}$  and  $y_{t,m}$  whose positions are close to the circles corresponding to AP  $a_1$  and AP  $a_2$ . The intersection where the straight lines line  $l_{t,1}$ , line  $l_{t,2}$ , and line  $l_{t,3}$  meet is calculated by Eq. (6), to find the location  $p_t$  of the beacon.





**Fig. 11.** Case where two circles do not meet but are touched outside the triangle.

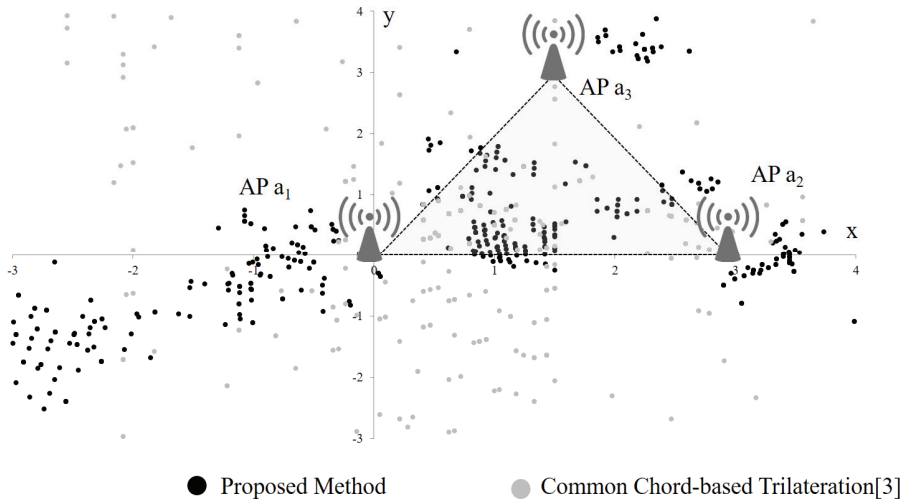
In Fig. 12, the distance measured at each AP satisfies Eq. (2). When the location of the beacon is in the triangular area, the location calculated by the proposed method is [0.61, 0.12] and the location calculated by common chord-based trilateration [3] is [0.27, -2.8]. AP  $a_1$  is contained in the circle of AP  $a_3$ . The line  $l_{i,3}$  was generated by  $x_{i,m}$  and  $y_{i,m}$  whose positions are close to the circle of AP  $a_3$  and between AP  $a_1$  and AP  $a_3$ . Based on the three lines, the location  $p_i$  of the beacon was calculated to be inside the triangle.



**Fig. 12.** Case where one circle contains another circle and is touched from outside the triangle.

The calculation result using the existing common chord-based trilateration [3] and the calculation result using the proposed method were compared as shown in Fig. 13. A total of 780 locations of beacon were measured. The locations calculated with the proposed method ranged from -6.04 to 7.6 along the x-axis, and from -4.68 to 6.35 along the y-axis. When the existing Common Chord-based trilateration [3] was used, the locations ranged from -43.08 to 13.81 along the x-axis, and from -241.76 to 21.88

along the y-axis. When the beacon was inside the triangle, 210 locations were calculated to be inside the triangle. On the other hand, the common chord-based trilateration [3] calculated 78 locations to be inside the triangle.



**Fig. 13.** The comparison between the common chord-based traditional approach [3] and the proposed method.

## 5. Conclusions

Enhanced trilateration was proposed to correct the location of the beacon. In case it was possible to use the existing common chord-based trilateration [3], the location of the beacon was calculated using the trilateration; otherwise, the location of the beacon was calculated using the distances of beacon measured at APs.

In the experiments, the proposed method and the existing common chord-based trilateration was compared and analyzed. The proposed method improved the accuracy of location measurement by 81.2%. The location was measured closer to the actual location of the user when the proposed method was used. However, it is predictable that with the proposed method, the error will increase as the triangle size increases. Although the location error of the beacon was reduced, this error occurred owing to the distance error of the beacon measured at the APs.

## Acknowledgement

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