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Complex Quadruplet Zero Locations from the Perturbed Values of Cross-Coupled Lumped Element

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Abstract

In this paper, complex quadruplet zeros of microwave filter systems are investigated. For the cascaded systems the chain matrices are most conveniently used to derive the voltage transfer function of Laplace transform with cascaded two-port subsystems. The convenient relations of transfer function and chain matrix are used in order to find the transmission zeros. Starting from a ladder network, we introduced a crossed-coupled lumped element, in order to show the improved response of bandpass filter. By solving the transmission zero characteristic equation derived from the cascaded subsystems, we found the zeros of filter system with externally cross-coupled lumped elements. With the cross-coupled elements of capacitors, the numerator polynomial of system transfer function is used to locate the quadruplet zeros in complex plane. When the two pairs of double are on the zeros $j\omega$-axis, with the perturbed values of element, we learned that the transition band of lowpass filter is improved. By solving the characteristic equation of cascaded transfer function, we can obtain the zeros of the cross-coupled filter system, as a result of perturbed values on lumped element.

Keywords: *Transmission zeros(TZ's), TZ characteristic equation (or TZCE), transmission poles (or reflection zeros), Cross-coupling, Dynamic quadruplet*

1. Introduction

Given a linear system, it is conventional, although not universal, to define transfer function as the s-domain ratio of the Laplace transform of output signal (response) to the Laplace transform of the input signal (source).To define the transfer function, the linear system is assumed to be a circuit where all initial conditions are zero. If a system has multiple independent sources, the transfer function for each source can be found, and the principle of superposition is used to find the response to all sources. As one of the possible forms of transfer function, that relates input quantities to output quantities, a voltage transfer function is defined.

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2. Voltage transfer function of a linear system

To define the voltage transfer function, consider a linear system with an input and an output signals and $v_0(t)$in the time domain input and out signals, with the corresponding Laplace transform pairs $\mathbf{V}_i(s)$ and $\mathbf{V}_0(s)$, respectively. The voltage transfer function of the linear system of the figure above is defined as the ratio of output to input in frequency domain [1].

$$\mathbf{H}(\mathrm{s}) = \frac{\mathbf{V}_0(\mathrm{s})}{\mathbf{V}_\mathrm{i}(\mathrm{s})} = \frac{b_0 s^m + b_0 s^{m-1} + \ldots + b_0 s^0}{a_0 s^m + a_0 s^{m-1} + \ldots + a_0 s^0} \quad (1)$$

In Equation (1), $\mathbf{H}(s)$ is rational function of complex variable *s*. The transfer function $\mathbf{H}(s)$ is the frequency-domain description of a linear time-invariant system and is a necessary function for analysis and synthesis in this domain [2]. A method for determining the transfer function of systems (filters) composed of lumped constants (those described by ordinary constant-coefficient differential equations) is investigated.

2.1 Transmission zeros

Given a voltage transfer function with the form of Equation (1), it can be expressed as

$$\mathbf{H}(\mathrm{s}) = \frac{\mathbf{V}_0(\mathrm{s})}{\mathbf{V}_\mathrm{i}(\mathrm{s})} = \frac{N(\mathrm{s})}{D(\mathrm{s})} \quad (2)$$

In Equation (2), **H**(*s*), is a rational polynomial function expressed as a polynomial quotient of two polynomials *N*(*s*), the numerator polynomial, and *D*(*s*), the denominator polynomial. After the common term cancellation, *N*(*s*) and *D*(*s*) do not have any common terms. Then **H**(*s*), *N*(*s*), and *D*(*s*) are called "of the canonical form". *Transmission zeros* (TZ's) are defined as the roots of canonical forms of the numerator polynomial of the transfer function [3]. Reflection zeros or transmission poles are defined as the roots of canonical forms of the denominator polynomial. Equating *N*(*s*) to zero, the equation,

$$N(s) = 0 \quad (3)$$

is obtained. This equation is defined as the *TZ characteristic equation* (or TZCE). The roots of Equation (3) are the *transmission zeros* (TZ's) of the system. *Transmission poles* (TP's) are defined as the roots of canonical forms of denominator polynomial of the transfer function. Equating *D(s)* to zero, the equation,

$$D(s) = 0 \quad (4)$$

is obtained. This equation is defined as the TP characteristic equation. The roots of Equation (4) are the *transmission poles* (or *reflection zeros*) of the system [4].

2.2 Chain matrix

Figure 1 represents the basic two-port building block to define chain matrix. This system should be a linear system.

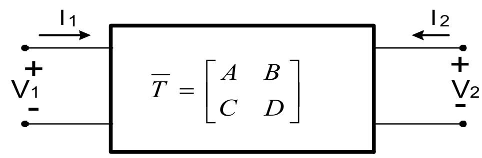

Figure 1.System to define chain matrix

The chain parameters are used to relate the voltage and current at one port to voltage and current at the other port.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (5)$$

For convenience, the chain matrix in Equation (5) is written as [5]

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (6)$$

From Equations (5) and (6), the entry A is given by

$$A = \overline{T}(1,1) = \frac{V_1}{V_1} |I_{2=0} \quad (7)$$

Equation (7) means that entry A can be calculated from the entry (1, 1) of the (2×2) chain matrix $\overline{T}$, obtained by open-circuiting port #2 [6]. The voltage transfer function can be expressed as

$$\mathrm{H(s)} = \frac{1}{A} = \frac{1}{T(1,1)} = \frac{N(s)}{D(s)} \quad (8)$$

Equation (8) tells that if the entry (1.1) of the chain matrix is known the transfer function can be obtained [7]. The chain matrix of *n* cascaded networks can be represented as the product of each of the chain matrix by

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{\mathrm{i=1}}^{\mathrm{n}} \begin{bmatrix} Ai & Bi \\ Ci & Di \end{bmatrix} = \prod_{\mathrm{i=1}}^{\mathrm{n}} \overline{T}i \quad (9)$$

In Equation (9), $\prod$ is the operator for product of *n* chain matrices. Bandpass filters can be effectively designed by adjusting the locations of *transmission zeros* (TZ's) and *transmission poles* (TP's) in the complex *s*-domain. Given a filter network, determining the TZ locations as a function of element values includes deriving the transfer function. Here, a practical method for determination of the complex TZ locations of the cross-coupled bandpass filter is discussed. This technique uses chain matrices for subsystems (discrete parts of the network), and can be extended to other types of filters with cross-coupled sections. An important result is that a complex doublet and/or quadruplet (one-, two-, or four-pairs) of TZ's are shown to result solely from the cross-coupled portion of the circuit. Modifications to the cross coupled portion have only a small effect on the TP's (otherwise known as reflection zeros). The method for determining the locus and location of TZ's for both positively and negatively cross-coupled bandpass filters will be considered below. The several closed-forms of expressions in terms of elements are obtained, and TZ's are located by solving what is called the TZ characteristic equation. This is derived by taking advantage of the bridged-T structure for the cross-coupled part.

3. The ladder network

A frequently used ladder network is composed of series-connected and parallel-connected elements as shown in Figure 2. The pattern is that every other element is alternatively in series-connected and shunt-connected as a signal travels from the source to the load. So the $S_i(i = 1 - 4)$ subsystem makes a ladder network, where the subscript i is used to indicate the system is the i-th subsystem. Subsystem S_5 is an external load connected to the ladder network. The network is an initially synthesized ladder networks without any cross-coupling. It is a four-pole (four resonators) bandpass filter [8]. Four shunt-connected LC resonators have impedances Z_2, Z_4, Z_6, and Z_8 due to the parallel LC components composed of (L_2, C_2), (L_2, C_4), (L_6, C_6), and (L_8, C_8), respectively. The impedances Z_3, Z_5, and Z_7 are due to the series-connected elements, and could be inductors and/or capacitors, respectively. The impedances Z_1, and Z_9 represent the source and load impedance of 50 Ohms, respectively [9,10].

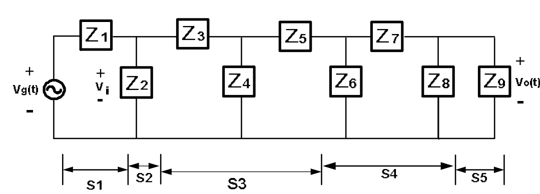

Figure 2. Ladder network without cross-coupling

In frequency domain, the generic response of the ladder network, for example, without cross-coupled element added, is shown in Figure 3. In the figure, $m = -2$ is used to indicate the slope of the attenuation of the response is -2, and f_c is used to indicate the center frequency of the filter [4].

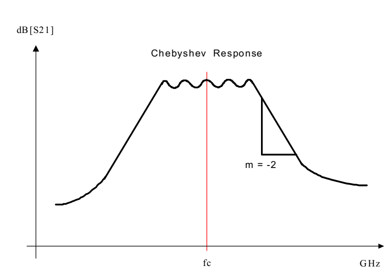

Figure 3. Response of a ladder network, without cross-coupling

The generic response of the filter, for example, with cross-coupled element added, is shown in Figure 4.

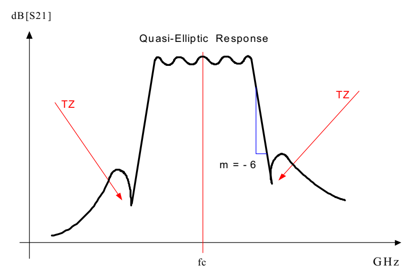

Figure 4. Improved insertion loss of a cross-coupled filter

In the figure, $m = -6$ is used to indicate the slope of the attenuation of the response is -6, and f_c is used to indicate the center frequency. Two TZ's are located at the both sides of passband. The transition slope of Figure 4 is steeper than that of Figure 3. This occurs due to the addition of a cross-coupling element between the two resonators. There are several possibilities to add cross-coupled elements for the filter network. A few possible examples are as follows:

1)Without skipping any resonators (adjacent resonators),
2) Skipping one resonator,
3) Skipping two resonators.

In this paper, 1) is considered.

4. Cross-coupled (CC) filter configuration

Locus is defined as the path of motion for dynamic TZ's as functions of cross-coupling [10]. In Figure 5, the series-connected elements are all inductors. A negatively cross-coupled filter network is obtained by using capacitor impedance for Z_{33} connected between the first and the third resonators, as shown in Figure. If the series-connected elements are all capacitors, the cross-coupled (CC) elements should be an inductor to result in the same locations for the TZ's. Here is the first case to be considered. A cross-coupled circuit, or a bridge-T circuit, is installed from the first resonator (Z_2)and the 3rd resonator (Z_{41}). The whole system is considered to be composed of five subsystems (S_1,S_2,S_3, S_4 and S_5) connected in cascade. Therefore, the *chain* (ABCD) matrix of the whole system is expressed by

$$\overline{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = T = \overline{T}_1 \cdot \overline{T}_2 \cdot \overline{T}_3 \cdot \overline{T}_4 \cdot \overline{T}_5 \qquad (10)$$

In Equation (10), each entry of five chain matrices must be expressed in terms of Laplace impedance shown in the Figure 5.

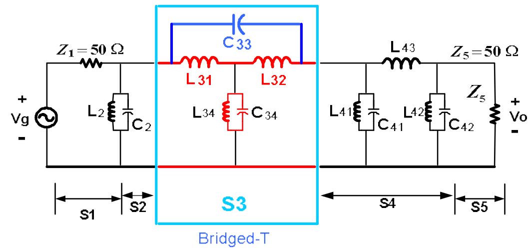

Figure 5. A negatively cross-coupled filter network

The polynomial is expressed as

$$\begin{aligned} N_s &= 50\, L_2 \cdot L_{41} \cdot L_{42} \cdot s \cdot. \\ &\quad [\, L_{31} \cdot L_{32} \cdot C_{33} \cdot L_{34} \cdot C_{34} \cdot s^4 \\ &\quad + (\, L_{31} \cdot L_{32} + L_{34} \cdot L_{31} + L_{34} \cdot L_{34})\, C_{33} \cdot s^4 \\ &\quad + L_{34}] \\ &= ks \cdot [\, a_{34}\, s^4 + a_{32}\, s^2 + a_{30}] \end{aligned} \tag{11}$$

where,

$$\begin{aligned} k &= 50\, L_2 \cdot L_{41} \cdot L_{42} \\ a_{34} &= L_{31} \cdot L_{32} \cdot C_{33} \cdot L_{34} \cdot C_{34} \\ a_{32} &= (L_{31} \cdot L_{32} + L_{34} \cdot L_{31} + L_{34} \cdot L_{34}) C_{33} \\ a_{30} &= L_{34} \end{aligned} \tag{12}$$

From the 4th degree polynomial,

$$[\, a_{34}\, s^4 + a_{32}\, s^2 + a_{30}] \tag{13}$$

four roots are obtained, with an even polynomial that produces a dynamic quadruplet of complex zeros. The quadruplet is only due to the cross-coupled subsystem, as shown in Figure 6.

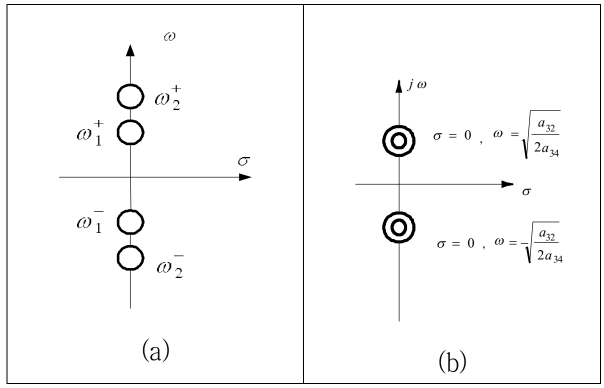

Figure6.
(a) Quadruplet zero locations in complex plane; four complex zeros on *jω-axis*
(b) Complex quadruplet zero locations: two pairs of double zeros are on *jω-axis*

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Conclusion

In this paper, an investigation of a practical method to determine *quantitatively* the locus and location of complex *transmission zeros* (TZ's) in the cross-coupled microwave filter network was presented. To take advantage of chain matrices applied to cascaded subsystem, the cross-coupled subsystem was considered. Since a filter network is two-port linear system, the transfer function was derived by taking advantage of the chain matrices applied to cascaded subsystem. The subsystem was characterized by its own chain matrix. The cascaded chain matrices represent the whole filter network. The matrix entry (1, 1) was used to find The investigation is unique in that it proved that cross-coupled filter produces complex TZ's.

Remarks:

(1) This work is the extended and advanced from the paper by K. Um, Y. S. Im, G. K. Kim, J. J. Kang of Ref.[6].

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