Korean J. Math. **25** (2017), No. 4, pp. 483–494
https://doi.org/10.11568/kjm.2017.25.4.483

ON SINGULAR INTEGRAL OPERATORS INVOLVING POWER NONLINEARITY

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Abstract. In the current manuscript, we investigate the pointwise convergence of the singular integral operators involving power nonlinearity given in the following form:

$$T_\lambda(f;x)=\int_a^b \sum_{m=1}^{n} f^m(t)K_{\lambda,m}(x,t)dt,\ \lambda\in\Lambda,\ x\in(a,b),$$

where Λ is an index set consisting of the non-negative real numbers, and $n\geq 1$ is a finite natural number, at $\mu-$generalized Lebesgue points of integrable function $f\in L_1(a,b)$. Here, f^m denotes $m-th$ power of the function f and (a,b) stands for arbitrary bounded interval in $\mathbb{R}$ or $\mathbb{R}$ itself. We also handled the indicated problem under the assumption $f\in L_1(\mathbb{R})$.

1. Introduction

In [17], Taberski presented and proved some theorems about the pointwise approximation properties of functions f in $L_1\langle-\pi,\pi\rangle$ using a family of convolution type of linear singular integral operators depending

Received June 6, 2017. Revised November 5, 2017. Accepted November 8, 2017.
2010 Mathematics Subject Classification: 41A35, 41A25, 47G10, 47A58.
Key words and phrases: Pointwise convergence, Nonlinear integral operators, $\mu-$generalized Lebesgue point.
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on two parameters of the form:

$$(1.1)\qquad L_{\lambda}\left(f;x\right)=\int_{-\pi}^{\pi}f\left(t\right)K_{\lambda}\left(t-x\right)dt,\ x\in\left\langle -\pi,\pi\right\rangle,\ \lambda\in\Omega\subset\mathbb{R}_{0}^{+},$$

where $\langle -\pi,\pi\rangle$ stands for closed, semi-closed or open interval and $K_{\lambda}(t)$ is the kernel satisfying suitable properties. After this study, the pointwise convergence of the operators of type (1.1) was investigated by Gadjiev [9] and Rydzewska [14] at generalized Lebesgue points and $\mu-$Lebesgue points of functions $f\in L_{1}\left\langle -\pi,\pi\right\rangle$, respectively. Later on, Karsli and Ibikli [10] generalized some results in [9], [14] and [17], by handling the operators of type (1.1) in the space $L_{1}\left\langle a,b\right\rangle$. For some studies on linear singular integral operators in miscellaneous settings, the reader may see also [3], [11] and [7].

Musielak [12] investigated the approximation properties of nonlinear integral operators in the following setting:

$$(1.2)\qquad T_{w}f(y)=\int_{G}K_{w}(x-y;f(x))dx,\ y\in G,\ w\in\Lambda,$$

where G is a locally compact Abelian group enriched with Haar measure, and Λ is a non-empty index set with desired topology, by reconciling K_{w} with the Lipschitz condition on its second variable. Then, Musielak [13] advanced his previous sounding analysis [12] by considering the indicated operators in generalized Orlicz spaces. Afterwards, Swiderski and Wachnicki [16] investigated the pointwise convergence of the operators of type (1.2) in accordance with two parameters at Lebesgue points of the functions $f\in L_{p}\left(G\right)$. For some studies related to mentioned studies above, we refer the reader to see [1], [2], [4], [5], [6], [8] and [19].

Let $L_{1}\left(a,b\right)$ be the space of all measurable functions f for which $|f|$ is integrable on the arbitrary bounded intervals (a,b) of $\mathbb{R}$. Similarly, let $L_{1}\left(\mathbb{R}\right)$ be the space of all measurable functions f for which $|f|$ is integrable on $\mathbb{R}$. The norms on these spaces are given by $\|f\|_{L_{1}(a,b)}=\int\limits_{a}^{b}\left|f\left(t\right)\right|dt$ and $\|f\|_{L_{1}(\mathbb{R})}=\int\limits_{\mathbb{R}}\left|f\left(t\right)\right|dt$ (see, for example, [18]). The current manuscript can be seen as a generalization and a continuation of [1]. The main aim of this paper is to obtain pointwise convergence of singular integral operators involving power nonlinearity in the following form:

$$T_\lambda(f;x) = \int_a^b \sum_{m=1}^{n} f^m(t) K_{\lambda,m}(x,t)dt,\ \lambda \in \Lambda,\ x \in (a,b)\,, \tag{1.3}$$

where Λ is an index set consisting of the non-negative real numbers with accumulation point λ_0 (or $\lambda_0 = \infty$) and $n \geq 1$ is a finite natural number, to the function $f \in L_1(a,b)$ at its $\mu-$generalized Lebesgue points as λ tends to λ_0. On the other hand, we also handled the indicated problem under the assumption $f \in L_1(\mathbb{R})$. Here, f^m denotes $m-th$ power of the function and (a,b) denotes arbitrary bounded interval in $\mathbb{R}$ or $\mathbb{R}$ itself. Note that taking $n = 1$ transforms the operators of type (1.3) into linear form. Also, the rate of convergences are discussed. The operators of type (1.3) are the special type of nonlinear integral operators building the bridge between summation type operators and integral type operators. Therefore, doing a research on their approximation properties is remarkable from the theoretical point of view.

The paper is organized as follows: In Section 2, we present preliminary concepts. In Section 3, we prove main results. In Section 4, we deal with the rates of pointwise convergences.

2. Preliminaries

Now, we start by stating the following definition which is the heart of the matter.

Definition 2.1. Let δ_1 be a given fixed positive real number and $\delta_1 > h > 0$. A point $x_0 \in (a,b)$ (or $x_0 \in \mathbb{R}$) is called $\mu-$generalized Lebesgue point of the function $f \in L_1(a,b)$ (or $f \in L_1(\mathbb{R})$), if the following relation:

$$\lim_{h\to 0} \frac{1}{\mu(h)} \int_0^h |f(t+x_0) - f(x_0)|\, dt = 0$$

(also, when the integral is taken from $-h$ to 0) holds, where $\mu : \mathbb{R} \to \mathbb{R}$ is an increasing and absolutely continuous function on $[0,\delta_1]$ with $\mu(0) = 0$.

Remark 2.2. For the ideas and concepts used in Definition 2.1, we refer the reader to see [9], [10] and [14]. On the other hand, if we take $\mu(t) = t$, then we obtain the well-known definition of Lebesgue point.

Now, in view of Definition 1 in [1], we may express the following definition which can be seen as a generalization of the properties of usual approximate identities, such as well-known Gauss-Weierstrass kernel.

DEFINITION 2.3. (Class A) Let Λ be an index set consisting of the non-negative real numbers with accumulation point λ_0 (or $\lambda_0 = \infty$). Keeping in mind that the natural number n is finite, for $m = 1, ..., n$, a family $\{K_{\lambda,m}\}_{\lambda\in\Lambda}$ consisting of the globally integrable functions $K_{\lambda,m}(x,t) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_0^+$, is called Class A, if the following conditions hold there:

a. For any fixed $x \in \mathbb{R}$ and for $m = 1, ..., n$,

$$\lim_{\lambda\to\lambda_0} \left| \int_{\mathbb{R}} K_{\lambda,m}(x,t)dt - C_m \right| = 0,$$

where $C_m > 0$ are certain real numbers.

b. For any fixed $x \in \mathbb{R}$, for $m = 1, ..., n$ and for every $\xi > 0$,

$$\lim_{\lambda\to\lambda_0} \left[\sup_{|x-t|\geq\xi} K_{\lambda,m}(x,t) \right] = 0.$$

c. For any fixed $x \in \mathbb{R}$, for $m = 1, ..., n$ and for every $\xi > 0$,

$$\lim_{\lambda\to\lambda_0} \left[\int_{|x-t|\geq\xi} K_{\lambda,m}(x,t)dt \right] = 0.$$

d. Let δ_0 satisfying $0 < \delta_0 \leq \delta_1$ be a given fixed real number such that $K_{\lambda,m}(x,t)$ is non-decreasing on $[x - \delta_0, x]$ and non-increasing on $[x, x + \delta_0]$ with respect to t, for any fixed λ and $x \in \mathbb{R}$ and for $m = 1, ..., n$.

Throughout this paper $K_{\lambda,m}$ belongs to Class A.

3. Main Results

Our first main theorem in which (a, b) denotes arbitrary bounded interval in $\mathbb{R}$ is as follows:

THEOREM 3.1. *If $x_0 \in (a,b)$ is a $\mu-$generalized Lebesgue point of function $f \in L_1(a,b)$ and f is bounded on (a,b), then*

$$\lim_{\lambda\to\lambda_0} T_\lambda(f;x_0) = \sum_{m=1}^{n} C_m f^m(x_0)$$

on any set Z on which the function

$$\sum_{m=1}^{n} \int_{x_0-\delta}^{x_0+\delta} K_{\lambda,m}(x_0,t) \left| \{\mu(|x_0-t|)\}_t^{'} \right| dt$$

is bounded as λ tends to λ_0.

Proof. We start by setting $I_\lambda = \left| T_\lambda(f;x_0) - \sum\limits_{m=1}^{n} C_m f^m(x_0) \right|$.

Next, we define a function $g^m : \mathbb{R} \to \mathbb{R}$ $(m = 1, ..., n)$ such that

$$g^m(t) = \begin{cases} f^m(t), & t \in (a,b), \\ 0, & t \in \mathbb{R}\setminus(a,b). \end{cases}$$

By condition (a), we can write

$$\begin{aligned} I_\lambda &\leq \int_a^b \sum_{m=1}^{n} |f^m(t) - f^m(x_0)| \, K_{\lambda,m}(x_0,t)dt \\ &\quad + \sum_{m=1}^{n} |f^m(x_0)| \int_{\mathbb{R}\setminus(a,b)} K_{\lambda,m}(x_0,t)dt \\ &\quad + \sum_{m=1}^{n} |f^m(x_0)| \left| \int_{\mathbb{R}} K_{\lambda,m}(x_0,t)dt - C_m \right| \\ &= I_1 + I_2 + I_3. \end{aligned}$$

It is sufficient to show that the terms on the right hand side of the last inequality tend to zero as λ tends to λ_0. By conditions (c) and (a), I_2 and I_3 tend to zero as λ tends to λ_0, respectively.

Now, we consider I_1. For a fixed real number δ satisfying $0 < \delta < \delta_0$, we split the integral I_1 into four terms as follows:

$$\begin{aligned} I_1 &= \left[\int\limits_a^{x_0-\delta} + \int\limits_{x_0-\delta}^{x_0} + \int\limits_{x_0}^{x_0+\delta} + \int\limits_{x_0+\delta}^{b} \right] \sum_{m=1}^{n} \left| f^m(t) - f^m(x_0) \right| K_{\lambda,m}(x_0,t)dt \\ &= I_{11} + I_{12} + I_{13} + I_{14}. \end{aligned}$$

Since n is finite, we can write I_{11} in the following form:

$$I_{11} = \sum_{m=1}^{n} \int\limits_a^{x_0-\delta} \left| f^m(t) - f^m(x_0) \right| K_{\lambda,m}(x_0,t)dt.$$

From the condition (d), we have

$$\begin{aligned} I_{11} &\leq \sum_{m=1}^{n} K_{\lambda,m}(x_0, x_0-\delta) \left\{ \int\limits_a^{x_0-\delta} \left| f^m(t) \right| dt + \int\limits_a^{x_0-\delta} \left| f^m(x_0) \right| dt \right\} \\ &\leq \sum_{m=1}^{n} K_{\lambda,m}(x_0, x_0-\delta) \left\{ \left\| f^m \right\|_{L_1(a,b)} + \left| f^m(x_0) \right| (b-a) \right\}. \end{aligned}$$

Using same method, we can estimate I_{14}. Using condition (d), we shall write

$$\begin{aligned} I_{14} &\leq \sum_{m=1}^{n} K_{\lambda,m}(x_0, x_0+\delta) \left\{ \int\limits_{x_0+\delta}^{b} \left| f^m(t) \right| dt + \int\limits_{x_0+\delta}^{b} \left| f^m(x_0) \right| dt \right\} \\ &\leq \sum_{m=1}^{n} K_{\lambda,m}(x_0, x_0+\delta) \left\{ \left\| f^m \right\|_{L_1(a,b)} + \left| f^m(x_0) \right| (b-a) \right\}. \end{aligned}$$

By condition (b) of class A and boundedness of f, $I_{11} \to 0$ and $I_{14} \to 0$ as λ tends to λ_0.

On the other hand, since $x_0 \in (a,b)$ is a $\mu-$generalized Lebesgue point of f, for every $\varepsilon > 0$, there exists a number $\delta > 0$ such that

$$\int\limits_{x_0}^{x_0+h} \left| f(t) - f(x_0) \right| dt < \varepsilon \mu(h) \tag{3.1}$$

and

$$\int_{x_0-h}^{x_0} |f(t)-f(x_0)|\,dt < \varepsilon\mu(h) \tag{3.2}$$

hold for every h satisfying $0<h\leq\delta$.

Now, we define a new function with respect to relation (3.1) as follows:

$$F(t)=\int_{x_0}^{t} |f(u)-f(x_0)|\,du.$$

Then, for $t-x_0\leq\delta$, we have

$$|F(t)|\leq\varepsilon\mu(t-x_0).$$

To go further, we need the following identity (see, for example, [15]):

$$(q^m-r^m)=(q-r)\left(q^{m-1}+q^{m-2}r+...+r^{m-1}\right),$$

where $q,r\in\mathbb{R}$. Using this identity and depending on supremum value of f on overall (a,b), we see that there are finite real numbers $B_m>0$ such that the inequality

$$|f^m(t)-f^m(x_0)|\leq B_m\,|f(t)-f(x_0)|\,,$$

where $m=1,...,n$, holds. Therefore, we can estimate I_{13} as follows:

$$\begin{aligned} |I_{13}| &= \sum_{m=1}^{n}\left|\int_{x_0}^{x_0+\delta} |f^m(t)-f^m(x_0)|\,K_{\lambda,m}(x_0,t)dt\right| \\ &\leq \sum_{m=1}^{n} B_m\int_{x_0}^{x_0+\delta} K_{\lambda,m}(x_0,t)\,|dF(t)|\,. \end{aligned}$$

Applying integration by parts leads to the following result:

$$|I_{13}|\leq\sum_{m=1}^{n}B_m\left\{F(x_0+\delta)K_{\lambda,m}(x_0,x_0+\delta)+\int_{x_0}^{x_0+\delta}F(t)d_t\left(-K_{\lambda,m}(x_0,t)\right)\right\}.$$

Since $K_{\lambda,m}$ is non-increasing on $[x_0, x_0+\delta]$, $-K_{\lambda,m}$ is non-decreasing there. Hence, its differential is positive. Then, we can write

$$|I_{13}| \leq \sum_{m=1}^{n} B_m \left\{ \varepsilon\mu(\delta)K_{\lambda,m}(x_0, x_0+\delta) + \varepsilon \int\limits_{x_0}^{x_0+\delta} \mu(t-x_0)d_t\left(-K_{\lambda,m}(x_0,t)\right) \right\}.$$

Using integration by parts again, we get the following inequality:

$$|I_{13}| \leq \varepsilon \sum_{m=1}^{n} B_m \int\limits_{x_0}^{x_0+\delta} K_{\lambda,m}(x_0,t) \left| \{\mu(t-x_0)\}_t^{'} \right| dt.$$

In view of relation (3.2), we obtain the following inequality for the integral I_{12} :

$$|I_{12}| \leq \varepsilon \sum_{m=1}^{n} B_m \int\limits_{x_0-\delta}^{x_0} K_{\lambda,m}(x_0,t) \left| \{\mu(x_0-t)\}_t^{'} \right| dt.$$

Setting $B = \max\{B_1, ..., B_n\}$ and combining corresponding findings for the integrals I_{12} and I_{13}, we get

$$|I_{12}| + |I_{13}| \leq \varepsilon B \sum_{m=1}^{n} \int\limits_{x_0-\delta}^{x_0+\delta} K_{\lambda,m}(x_0,t) \left| \{\mu(|x_0-t|)\}_t^{'} \right| dt.$$

Since the right hand side of the above inequality is bounded by the hypothesis, the assertion follows, that is

$$\lim_{\lambda\to\lambda_0} T_\lambda(f;x_0) = \sum_{m=1}^{n} C_m f^m(x_0).$$

Thus, the proof is completed. □

In the following theorem, we suppose that $(a,b) = (-\infty,\infty)$.

THEOREM 3.2. *If $x_0 \in \mathbb{R}$ is a $\mu-$generalized Lebesgue point of function $f \in L_1(\mathbb{R})$ and f is bounded on $\mathbb{R}$, then*

$$\lim_{\lambda\to\lambda_0} T_\lambda(f;x_0) = \sum_{m=1}^{n} C_m f^m(x_0)$$

on any set Z on which the function

$$\sum_{m=1}^{n}\int\limits_{x_0-\delta}^{x_0+\delta} K_{\lambda,m}(x_0,t)\left|\{\mu(|x_0-t|)\}_t^{'}\right| dt$$

is bounded as λ tends to λ_0.

Proof. Set $J_\lambda = \left|T_\lambda(f;x_0) - \sum\limits_{m=1}^{n} C_m f^m(x_0)\right|$. Following the similar strategy as in the previous proof, we have

$$\begin{aligned} J_\lambda \leq & \sum_{m=1}^{n} K_{\lambda,m}(x_0,x_0-\delta)\left\|f^m\right\|_{L_1(\mathbb{R})} + \sum_{m=1}^{n}\left|f^m(x_0)\right| \int\limits_{-\infty}^{x_0-\delta} K_{\lambda,m}(x_0,t)dt \\ & + \sum_{m=1}^{n} K_{\lambda,m}(x_0,x_0+\delta)\left\|f^m\right\|_{L_1(\mathbb{R})} + \sum_{m=1}^{n}\left|f^m(x_0)\right| \int\limits_{x_0+\delta}^{\infty} K_{\lambda,m}(x_0,t)dt \\ & + \sum_{m=1}^{n}\left|f^m(x_0)\right|\left|\int\limits_{-\infty}^{\infty} K_{\lambda,m}(x_0,t)dt - C_m\right| \\ & + \varepsilon B^{*}\sum_{m=1}^{n}\int\limits_{x_0-\delta}^{x_0+\delta} K_{\lambda,m}(x_0,t)\left|\{\mu(|x_0-t|)\}_t^{'}\right| dt, \end{aligned}$$

where the number B^* corresponds to number B in previous proof having similar construction process. Now, the result is obvious by the hypotheses. This completes the proof. □

4. Rate of Convergence

Theorem 4.1. *Suppose that the hypotheses of Theorem 3.1 are satisfied. Let*

$$\Delta(\lambda,\delta) = \sum_{m=1}^{n}\int\limits_{x_0-\delta}^{x_0+\delta} K_{\lambda,m}(x_0,t)\left|\{\mu(|x_0-t|)\}_t^{'}\right| dt,$$

where $0<\delta<\delta_0$ and $m=1,...,n$, and the following conditions are satisfied:

(i) $\Delta(\lambda,\delta)\to 0$ *as* λ *tends to* λ_0, *for some* $\delta>0$.

(ii) *For every* $\xi>0$,

$$\sup_{|x_0-t|\geq\xi} K_{\lambda,m}(x_0,t)=o(\Delta(\lambda,\delta))$$

as λ *tends to* λ_0.

(iii) *For every* $\xi>0$,

$$\int\limits_{|x_0-t|\geq\xi} K_{\lambda,m}(x_0,t)dt=o(\Delta(\lambda,\delta))$$

as λ *tends to* λ_0.

(iv) $\left|\int\limits_{\mathbb{R}} K_{\lambda,m}(x_0,t)dt-C_m\right|=o(\Delta(\lambda,\delta))$ *as* λ *tends to* λ_0.

Then, at each $\mu-$*generalized Lebesgue point of function* $f\in L_1\left(a,b\right)$, *we have*

$$\left|T_\lambda\left(f;x_0\right)-\sum_{m=1}^{n}C_m f^m(x_0)\right|=o(\Delta(\lambda,\delta)).$$

Proof. The result is obvious by the hypotheses of Theorem 3.1 and Class A conditions. Thus, the proof is omitted. □

REMARK 4.2. The above result is also valid for the case $f\in L_1\left(\mathbb{R}\right)$ under the hypotheses of Theorem 3.2.

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