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An Improved Analysis Model for the Ultimate Behavior of Unbonded Prestressed Concrete

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Abstract

An innovative analysis method is proposed in this paper for the determination of ultimate resistance of prestressed concrete beams. The proposed method can be applied to simply supported or continuous beams in a unified manner whether structure and external loads are symmetric or not. Through the iterative nonlinear strain compatibility solutions, this method can also be applied to the non-prismatic section/un-symmetrical composite structures under moving load. The conventional studies have used the failure criteria when the strain of concrete reaches 0.003. However compared with bonded case, the value of strain in the reinforcement is much smaller than bonded case, thus, unbonded prestressed cases show compressive failure mode. It is shown that the proposed method gives acceptable results within 5% error compared with the prior experimental results. It can be shown that the proposed method can reach the solution much faster than typical three-dimensional finite element analysis for the same problem. This method is applicable to the existing unbonded prestressed members where deterioration has occurred leading to the reduced ultimate resistance or safety. In all, the proposed procedure can be applied to the design and analysis of newly constructed structures, as well as the risk assessment of rehabilitated structures.

Keywords : Unbonded tendon, Strain compatibility condition, Ultimate resistance, Iterative method

1. Introduction

Post-tensioned prestressed concrete (PSC, hereafter) structures have been successfully used for more than 40 years in the United States in a wide variety of constructions, including bridge construction, tanks, office buildings, hotels, parking structures, pavement, masonry structures, seismic resistant structural walls and foundation of residential houses. The flexibility of post-tensioning allows it to match exact design requirements with few limitations. The tendons in a post-tensioned system can be unbonded or bonded. In comparison with bonded tendons, unbonded mono strand systems are widely adopted in new construction sites, in terms of slabs, beams, joists, and mat foundations. Even if unbounded tendons are lighter, more flexible, and economical than conventional ones, they have replaced for new and existing structures internally or externally, they are not dealt rigorously in their ultimate behavior.

The current ACI 318-08 equations[1)] for calculating the expected ultimate strength of unbonded tendons are mainly based on the experimental results. ACI 318-08 gives the following expression for the evaluation of ultimate strength of unbonded tendons:

$$f_{ps} = f_{pe} + 70 + \frac{f_c}{100\rho_p} \le f_{py} \text{or} f_{pe} + 420 \qquad (1)$$

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where, f_{ps} is the stress in prestressed reinforcement at nominal strength (psi), f_{pe} is the effective prestress after loss (psi), f_c is 28th day compressive strength of concrete, and f_{py} is the specified yield strength of prestressing tendons in psi.

Note that compressive strength of concrete is included in Eq. 1. Besides ACI method, there are two major approaches for the estimation of strain and stress of unbonded PSC members. These approaches, however, ignore the effect of concrete compressive strength. The first one is a simplified calculation of ultimate stress of prestressed steel based on the strain of steel in plastic deformed region[2-5]. The previous approaches[2-5] require approximate assumption for the length of tendon subjected to plastic deformation, or require an assumed model for the partial bond strength between concrete and prestressed component, and inelastic strain of concrete.

As stated above, most existing methods determine the ultimate strength of unbonded tendons by ignoring elastic deformations and only consider the contribution from the length of inelastic deformation, or require detailed three dimensional finite element modeling. In other words, most existing models have the following drawbacks:

1) Elastic deformation of unbonded prestressed tendon is not considered.

2) They are applicable only to symmetrical load, section, and boundary conditions. In other cases, computational expensive and time-consuming due to iterative three-dimensional finite element modeling is needed.

3) can be assumed that balanced or tensile failure will always occur, which could lead to the unsafe design or evaluation, especially when concrete deteriorates thus compression failure prevails consisted of highly correlated composite elements.

Due to the discrepancies between the assumed and real behavior of the failure mode of unbonded PSC beams, there are several problems when applying the current design specification to the deteriorated-unbonded PSC structures, which is damaged at the upper part of the girder or slab[6-9].

Reversely, the unbonded pressed concrete structures can be much more durable structures. By reinforcing compressive area, in which the equivalent compressive force is larger than equivalent tensile forces, the section would more endure than bonded one[10),11)].

Accordingly, the main objectives of this study are:

1) Identify strain and stress distributions of unbonded tendons, as well as deflection and ultimate resistance for varying compressive and tensile forces in an unbonded PSC section under external ultimate forces,

2) Determine ultimate bending resistance for an asymmetric load and section or a non-prismatic section, which are commonly used in structures, and

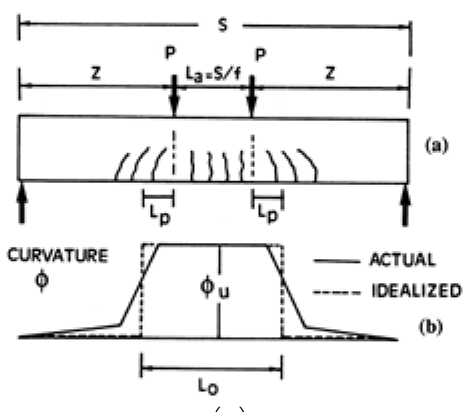
3) Validate the proposed iterative nonlinear section analysis method for unbonded PSC beams.

2. Existing methods and proposed study

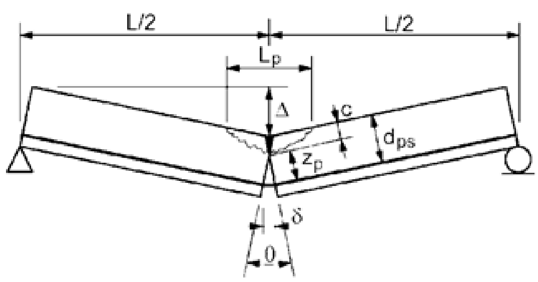
It has been noted that prior studies for the analysis of ultimate bending resistance of unbonded PSC beams can only be applied to symmetrical sections and loadings[2),3)]. Approximating and treating only plastic area 4, 5 in prismatic section under concentric loads or distributed loading conditions, the conventional methods cannot apply to moving load condition, non-prismatic section, composite structures, non-symmetric sections, and continuous beam boundary conditions. Consequently the approximation of assumptions could provide limited solutions, which are different with the real structures.

Naaman and Alkhairi[2),3)] proposed a method for analysis of beams prestressed with external or unbonded tendons. To design unbonded prestressed structures in a simple equation using strain reduction factor, this can be applied to symmetric boundary condition and loading in the prismatic section of PSC structures[1)].

Another important simplification to the existing methods was proposed by Haraji[2)] by assuming plastic deformation concentrated on the local area of the beam, which was further simplified by Carin et al.[5)] as shown in <Fig. 1> (a) and (b), respectively.



(a)



(b)

〈Fig. 1〉 An existing ultimate deformation model

As shown in <Fig. 1 (a)>, Haraji ignored small curvature variation in region of *L-Lo*, but focused on and only considered calculating the elongation of unbonded prestressed tendons in plastic deformed length of *Lo*. In a similar approximation, Carin et al ignored small strain increase in the elastic region and adopting *Lp* as:

$$f=\frac{L_p}{c}=10.5 \tag{2}$$

Where flexible length, f, is infinite for concentrated load at center point of simply supported beam, and f is 3 when two loads are at L/3 and 2L/3, and f is 6 for evenly distributed load case. $\frac{L_o}{L}$ is 1.0 for bonded tendon case. in which *Lp* is the plastic deformed length, and c represents a selected constant.

The plastic lengths used by the two approaches have some slight difference, as listed in <Table 1>.

〈Table 1〉 Comparison for the considered plastic deformed lengths

Haraji[2)]	Carin et al.[13)]
$L_p=0.5d_p+0.05Z^{*}$ $L_o=d_p\left[\frac{L}{d_p}(\frac{0.95}{f}+0.05)+1.0\right]^{*}$ $\frac{L_o}{L}=\frac{0.95}{f}+0.05+\frac{1}{L/d_p}{}^{*}$	$f=\frac{L_p}{c}=10.5^{**}$

* Following Mattock et al.[12)]
** Following Carin et al.[13)]

As compared in <Table 1>, the difference between the two previous approaches is the value of *f*. Haraji used three different values of *f* depending on the types of loading, but Carin et al suggested using the mean value of *f* as 10.5. It is well established that, in a steel member, the plastic deformed length *f* is the ratio of yielded moment(M_y) to the plastic moment(M_p):

$$f = \frac{M_y}{M_p} \tag{3}$$

The value, f in Eq. (3) depends on the geometry of the considered section. In other words, f of rectangular section is 1.5, and approximately 1.1 for I-shaped beams. The limitation of design specification show larger error if the compressive area of a section is damaged by any reasons. In fact, the compressive slab of simply supported beams may be the most vulnerable member to be attacked by chloride ion, water, live loads, temperature, and etc[14].

3. Proposed model for evaluation of ultimate strain in unbonded tendons

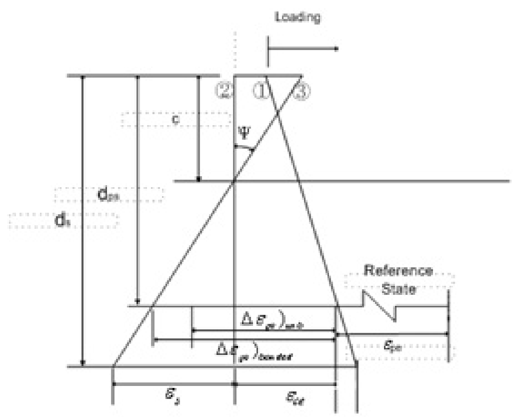
For the evaluation of ultimate bending resistance of a unbonded PC beam, the total strain or stress of tendon is generally obtained using initial stress or strain with the additional stress or strain resulted from the external loads:

$$f_{ps} = f_{pe} + \triangle f_{ps} \tag{4}$$

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{ce} + \triangle \varepsilon_{ps} \tag{5}$$

where f_{ps} is the stress of prestressing steel, f_{pe} is effective stress, $\triangle f_{ps}$ is increased stress in prestressing steel over effective value, ε_{ps} is the strain of prestressing steel, ε_{pe} is effective prestrain, ε_{ce} is precompressive strain in the concrete at the level of prestressing steel, and $\triangle \varepsilon_{ps}$ is the increased strain in prestressing steel over effective value.

The additional strain in steel due to external forces, $\triangle \varepsilon_{ps}$, can be obtained by considering strain compatibility condition as shown in <Fig. 2>.



〈Fig. 2〉 Strain compatibility condition for PSC section by bonded or unbonded tendons

In <Fig. 2>, d_s is the depth of steel in the section, d_{ps} is the depth of prestressed tendons in section, c the length of neutral axis from top of the section, Ψ is the rotated angle defined as 1/curvature of the section, while ①, ②, and ③ illustrates the initial, decompression, and ultimate strain by the increased external moment, respectively.

Shown at the lower part of the <Fig. 2>, the strain of bonded tendon is calculated by linear relationship of $\triangle \varepsilon_{ps,bonded} = \varepsilon_{cu} \cdot \frac{d_{ps} - c}{c}$, when compressive concrete strain reaches $\varepsilon_{cu} = 0.0014\sqrt{f_c^{'}}$. However unlike bonded prestressed case, due to the lack of bond between tendon and concrete, the unbonded prestressed tendons show much less strain and stress than bonded case at the ultimate state.

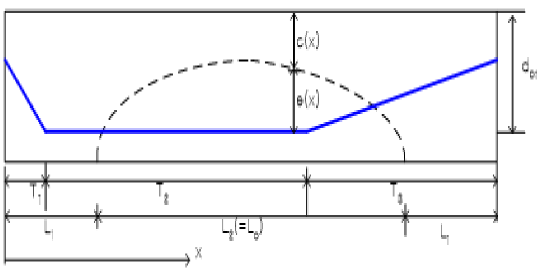
In the present study, the total strain along the entire beam length is considered for the evaluation of ultimate stress and the resistance of unbonded PC members. For the purpose, the strain of steel is integrated in a form of:

$$\triangle \varepsilon_i = \frac{M(x) \cdot e(x)}{E_c \cdot I(x)} \tag{6}$$

where $\triangle\varepsilon_i$ is the increased strain in prestressing steel over effective value of ε_{pe}, $M(x)$ is external moment along axis, $e(x)$ is the distance between prestressing steel to neutral axis, E_c is the modulus of elasticity for concrete, and $I(x)$ is the moment of inertia for cracked or uncracked section.

$$\triangle_{unbond} = \sum_{i=1}^{n}\int_0^{L_i}\triangle\varepsilon_i dx + \sum_{i=1}^{n}\int_0^{L_o}\triangle\varepsilon_i dx \qquad (7)$$

Inserting axis-dependent external moment, the location of neutral axis, and crack depth-dependent moment of inertia allow the integration of strain in the divided lengths<Fig. 3>, the sum of which is utilized for the total strain of unbonded tendon in Eq. (6).



〈Fig. 3〉 Integral contour for evaluating total strain

Hence, including small deformations gives a more exact total deformation. Moreover, dividing sections removes the limitation of the approach to be applied in asymmetric section or non-prismatic section under asymmetric or moving load cases. In a conservative approach, the authors use a criterion for the length of plastically deformed part of a beam when the depth of neutral axis (c) is smaller than the depth of a section (d_{ps}).

The additional strain in prestressing steel over the effective value ($\triangle\varepsilon_i$) is integrated over the length L_1 for the uncracked section ($c \geq d_{ps}$) and through the length (L_2), for cracked inelastic area ($c < d_{ps}$).

The difference and improvements, compared with the existing approaches, are the total elongation of unbonded prestressed tendon calculated along the entire beam. The neutral axis depth (c) is obtained by iterative evaluation of force equilibrium with nonlinear constitutive relationship for concrete, mild steel reinforcement, and prestressing tendons, which is an expansion of previous models shown in the following Eq. (8) for $\triangle\varepsilon_{ps}$, explained in <Table 2>.

〈Table 2〉 Summarizes the main discrepancies between Haraji's approach with the present study

	Haraji[2)]
$\triangle\varepsilon_{ps}$	1.Haraji[2)] $\triangle = \int \varepsilon dx = \int \Phi y dx = \int \frac{Me_0}{EI}dx = L_p^* \bullet \varepsilon_{cu}$ $\varepsilon_{cu} = \frac{\int \frac{Me_0}{EI}dx}{L_p}$ 2. $\triangle\varepsilon_{ps} = \frac{\delta}{L} = \frac{Z_p \bullet \theta}{L} = \frac{(d_{ps}-c)\bullet\theta}{L} = \frac{(d_{ps}-c)}{c}\frac{\int\Phi dx}{L}$ $\int\Phi dx = \int\frac{M}{EI}dx = \int\frac{\varepsilon}{y}dx = \theta = I_p \bullet \Phi = I_p \bullet \frac{\varepsilon_{cu}}{c}$
fps	1.$f_{ps} = f_{pe} + \Omega_u E_{ps}\varepsilon_{cu}(\frac{d_{ps}}{c} - 1)$ 2.Haraji[2)] $\triangle f_{ps} = E_{ps} \bullet \Psi \bullet \varepsilon_{cu} \bullet (\frac{d_{ps}-c}{L})$ $= 193,000 \bullet 10.5 \bullet 0.003 \bullet (\frac{d_{ps}-c}{L})$ $= 6080 \bullet (\frac{d_{ps}-c}{L})6200 \bullet (\frac{d_{ps}-c_y}{L}), MP$ 3. $f_{ps} = \frac{1}{A_{ps}} \bullet \frac{0.85\beta_1 f_c' b_w (L_0/S) d_p \varepsilon_{cu}}{\varepsilon_{ps} - \varepsilon_{pe} - L_0/S(\varepsilon_{ce} - \varepsilon_{cu})} + \frac{(A_s' - A_s)f_y + C_f)}{A_{ps}}$
Plastic length	1. Haraji[2)] $L_1 = S - L_0$ $L_2 = 2L_p$ $L_0 = d_p[\frac{L}{d_p}(\frac{0.95}{f}) + 0.05) + 1.0]$ Constant value and based on experimental results

	Proposed
$\triangle\varepsilon_{ps}$	$\triangle\varepsilon_1 = \frac{M(x)e(x)}{EI_g}$ $\triangle\varepsilon_2 = \frac{M(x)e(x)}{EI_{cr}}$ $I_\alpha = \frac{bc^3}{3} - \frac{b}{3}(c-h_f)^3 + nA_s(d-c)^2 + nA_{ps}(d-c)^3$ $Icr = \frac{bc^3}{3} + nA_s(d-c)^2 + nA_{ps}(d-c)^3$ $M(x) = P_u \bullet x$ $M(x) = \frac{P_u \bullet L}{3} or \frac{P_u \bullet L}{4}$ $\triangle\varepsilon_{ps})_{avg} = \frac{\sum\triangle\varepsilon_i \bullet L_i}{S}$
fps	$f_{ps} = E\varepsilon[Q + \frac{1-Q}{[1+(\frac{E\varepsilon}{Kf_y})^N]^{1/N}}]$
Plastic length	$L_1 = S - L_o$ $L_2 = L_o$ L_o : where c < d_p (Fig. 2) Based on strain compatibility condition

* L_p = f(time, f_c, corrosion, accidental damage)

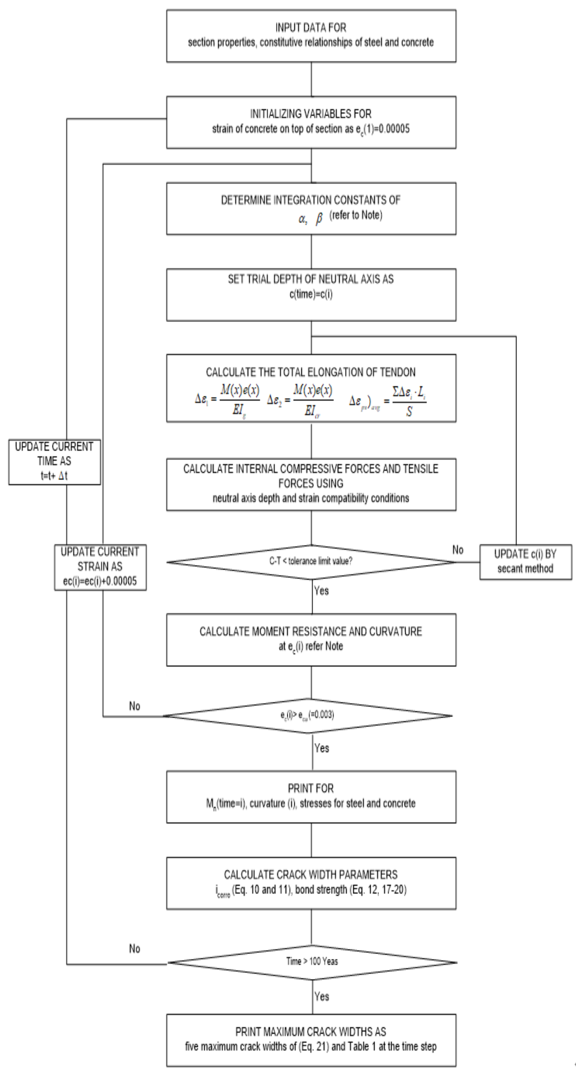
In <Table 2>, f is a coefficient dependent on the arrangement of loading. ∞ for single concentrated load, 3 for two third-point concentrated loads and 6 for uniform loading, Harajli[2]. Z is the shear span or the distance between the point of maximum moment and the point of contra-flexure as shown in <Fig. 1 (a)>.

Based on strain compatibility condition, even if the proposed equation needs nonlinear iterative analysis for longitudinal axis, it can be applied to constructed, deteriorated, and asymmetric section and loads in a unified way, which has been approximated or could not applied to deteriorated structures.

4. Procedure for calculation of the interactive nonlinear section analysis

For the calculation of a resistance of a section, constitutive relationships for prestressed and non-prestressed reinforcement, concrete are used for compressive and tensile force calculation at each increased steps of strain in the top of concrete section. The computation procedure is summarized in the flowchart of <Fig. 4>.

In <Fig. 4>, c is a neutral axis depth, ε_c is a given strain of the concrete section, $\Phi = \frac{\varepsilon_c}{c}$ is the curvature, C is a total compressive force ($\sigma_{av}bc = \alpha f_c' bc$), b is a bay of the section, T is total tensile force of a section, α represents the ratio of average stress to compressive concrete strength ($\frac{\sigma_{av}}{f_c'}$), and $\beta \bullet c$ gives a centroid of area of strain energy.



〈Fig. 4〉 Flow chart for nonlinear analysis for un-cracked / cracked section

As shown in the flowchart of proposed iterative nonlinear section analysis, L_p varies depending on the degree or type of deterioration, for example partial corrosion, fire, freeze thaw or any other accidental damages.

Intentional reinforce at the compression area in the design step or as a rehabilitation, the capacity and maintenance schedule need to be determined based on the exact prediction of ultimate strength, in which can predict the value under asymmetric load in non-prismatic sections.

5. Validation for unbonded PSC members

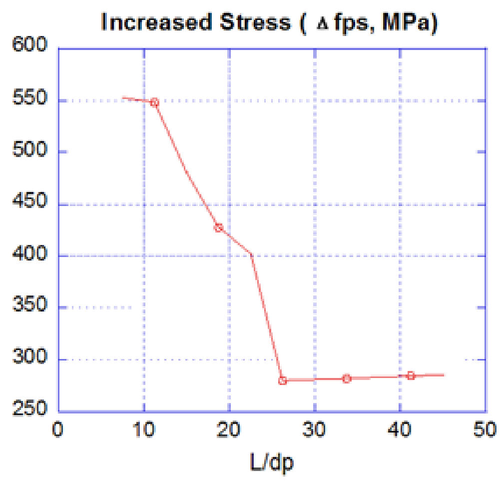
Proposed procedure was compared with the Tan et al.'s experimental results[15]. Properties of target PSC bridge is presented in <Table 3>.

〈Table 3〉 The properties of target PSC girder bridge

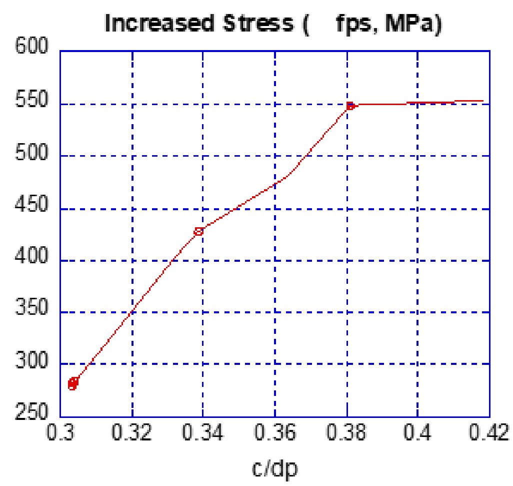
Geometric and material Property	Values
Compressive strength of concrete	$f_c = 28.7MPa$
Tensile strength of tendon	$f_{pu} = 1900MPa$
Tensile strength of steel	$f_{pe} = 0.7f_{pu}MPa$
Yield strength of tendon	$f_{py} = 1716.2MPa$
Yield strength of steel	$f_y = 530MPa$
Elasticity of tendon	$E_p = 199000MPa$
Elasticity of rebar	$E_s = 199955MPa$
Elasticity of concrete	$E_c = 29000MPa$
Area of tendon	$A_{ps} = 141.8\,mm^2$
Area of rebar	$As = 402mm^2$
Depth of rebar	$d_s = 250mm$
Depth of tendon	$d_p = 200-250mm$
Span length	$3000mm$
Height of beam	$300mm$

〈Table 4〉 Calculated results compared with experiments

Beam	d_p (mm)	f_{pe} (MPa)	f_c (MPa)	Ultimate resistance moment, M_u (kN-m)	
				Tan et al.[15]	This study
T-0	200	1296	34.6	78.02	74.79
T-1	200	1200	34.2	82.46	81.18
T-1A	250	326.8	30.4	79.91	70.03
T-1 Draped	250	288.2	32.1	76.58	69.94
T-1B	200	751.4	33.2	92.43	92.28
T-2	200	1179	28.7	81.79	81.85



(a)



(b)

〈Fig. 5〉 Stresses in tendon while varying (a) depth and (b) span length

As shown in <Table 4>, the average comparative results of ultimate resistance moment for 6 test beams are lower than 5% difference. The proposed procedures are applied for varying depth of composite girders and span length from 1.5 to 9 meters for model of Beam T-1<Table 3, 4>. While span/depth ratio increases from 7.5 to 45, the increased stress in unbonded tendon until section failed is decreased from 553.14 to 285.90MPa, depicted in <Fig. 5>.

6. Concluding remarks

The previous models have focused on the approximate evaluations of deformation of unbonded tendons at the plastically deformed location. However, the studies have ignored the distribution of strain of unbonded prestressed steel along the longitudinal axis, and the conventional models are limited to symmetric section and loading. In other words, it needs tremendous time for modeling and analysis of 3 dimensional finite element model discourages for adopting at design, which could not analyze partly damaged members, which are common in the existing structure.

The proposed model in this paper for the determination of ultimate resistance of prestressed concrete beams can be applied to simply supported or continuous beams in a unified manner whether structure and external loads is symmetric or not. Through iterative nonlinear strain compatibility solutions, it can also be applied to non-prismatic section or asymmetric composite structures under moving load. The proposed method gives acceptable results as less than 5% differences, compared with the prior experimental results. The results are also applicable to existing unbonded prestressed members where deterioration has occurred leading to the reduced ultimate resistance or safety. Consequently, the proposed procedure can be applied to the design and analysis of newly constructed structures, as well as the risk assessment of rehabilitated structures.

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