Simultaneous Information and Power Transfer for Multi-antenna Primary-Secondary Cooperation in Cognitive Radio Networks

Zhi Hui Liu, Wen Jun Xu, Sheng Yu Li, Cheng Zhi Long, and Jia Ru Lin

In this paper, cognitive radio and simultaneous wireless information and power transfer (SWIPT) are effectively combined to design a spectrum-efficient and energyefficient transmission paradigm. Specifically, a novel SWIPT-based primary-secondary cooperation model is proposed to increase the transmission rate of energy/spectrum constrained users. In the proposed model, a multi-antenna secondary user conducts simultaneous energy harvesting and information forwarding by means of power splitting (PS), and tries to maximize its own transmission rate under the premise of successfully assisting the data delivery of the primary user. After the problem formulation, joint power splitting and beamforming optimization algorithms for decode-andforward and amplify-and-forward modes are presented, in which we obtain the optimal PS factor and beamforming vectors using a golden search method and dual methods. Simulation results show that the proposed SWIPTbased primary-secondary cooperation schemes can obtain a much higher level of performance than that of non-SWIPT cooperation and non-cooperation schemes.

Keywords: Simultaneous information and power transfer, Primary-secondary cooperation, Multi-antenna.

I. Introduction

Spectrum and energy are two high-profile scarce resources in wireless communication networks. In particular, in wireless adhoc networks, many smart nodes, always equipped with limited energy, are badly in need of extra energy supply to live longer, and meanwhile require more available bandwidth to improve their communication capability. Fortunately, cognitive radio (CR) and energy harvesting (EH) have been developed to settle these problems [1], [2]. CR enables unlicensed users, that is secondary users (SUs) to share the spectrum owned by authorized users, that is primary users (PUs). As a promising transmission paradigm, primary-secondary cooperation (PSC) allows SUs to share authorized spectrum with PUs under the premise of assisting PUs to improve their data transmission, resulting in a win-win solution for both PUs and SUs, and has thus attracted the attention of many researchers [3]. Meanwhile, simultaneous wireless information and power transfer (SWIPT), which combines EH from radio frequency (RF) signals and information transmitting together on the same wireless device has brought about significant convenience, and has been one of the most important green communication methods [4].

Joint PSC and SWIPT can improve the spectrum and energy scarcity in traditional networks. Therefore, this paper investigates the resource allocation problem for a SWIPTenabled multi-antenna PSC in CR networks. Specifically, a multi-antenna SU assists a PU in data delivery, and in return, is awarded the use of a licensed spectrum. Meanwhile, the SU conducts power splitting (PS) for simultaneous energy

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harvesting and information forwarding. Aiming at maximizing its own transmission rate, the SU also needs to design its beamforming vector. Thus, the joint power splitting and beamforming optimization (JPSBO) algorithms for decodeand-forward (DF) and amplify-and-forward (AF) modes are designed.

The studies in [5]-[7] are researches most related to our paper. The authors in [5] studied an AF relaying protocol for a three-node multi-antenna cooperation model, in which the relay simultaneously harvests energy from information transmitted by the source and a dedicated energy signal transmitted by the destination. However, PSC is not considered, and owing to the separation of EH and information forward (IF), the PS is also removed from consideration. In [6], a twoway relay model is proposed in which an SU harvests energy from two PUs, helps the PUs exchange their data using the AF protocol, and transmits its own data by sharing spectrum with the PUs. However, multi-antenna relaying is not considered in this paper. In [7], the authors studied a SWIPT-enabled multiantenna PSC model similar to our own. Their study is limited to AF mode, and the centralized beamforming policy is employed at the SU, requiring vast information exchanges and more complex signal processing. In addition, the proposed algorithm is based on a one-dimension (1-D) exhaustive search, and a huge computation complexity making it inapplicable in practice. By contrast, both AF and DF modes are studied in our research, and a distributed beamforming strategy is adopted to reduce the information exchange. Meanwhile, we also contribute to the design of a low-complexity algorithm by proving the quasi-convexity/quasi-concavity of the objective function, and then exploit a bisection search and/or Golden search to quickly achieve a globally optimal solution.

The contents of this paper are as follows. Section II summarizes the related studies regarding cognitive cooperation and SWIPT in cooperation networks. In Section III, the system model and problem formulation are introduced. The algorithm design is presented in Section IV, followed by a performance analysis in Section V. Finally, Section VI provides some concluding remarks regarding this research.

II. Related Works

Apart from the works mentioned above [5]–[7], the related works mainly center on the following two aspects: researches on non-SWIPT PSC [8]–[13], and researches on SWIPT in traditional relay networks [14]–[19]. In [8] and [9], one PU selects one or more SUs to assist in its data delivery. In [10] and [11], the authors studied resource allocation for network-coded-based cognitive cooperative networks. The study in [12] further analyzes the physical security in PSC. In addition, our previous

paper [13] proposed a network-coded PSC for OFDM-based cognitive multicast networks.

On the other hand, researches on SWIPT in traditional relay networks involve a DF relay, an AF relay, a compress-andforward relay, and a compute-and-forward relay [14]-[19]. In [14], multiple source-destination pairs communicate with the help of an EH relay, and the PS and power allocation are jointly conducted during the relay to improve the energy utilization. Chen and others [15] adopted game theory to study SWIPT for relay interference channels with multiple source-destination pairs communicating through their dedicated EH relays. A multi-antenna relay is considered in [16]-[19]. In [16], the authors studied the antenna selection and PS optimization for multiple multi-antenna relay models, whereas the authors of [17] analyzed a transceiver design for multiple input multiple output interference channels. These researches are further extended to a full-duplex relay in [18], and the impact of relaying strategies described in [19].

In this paper, SWIPT in a cognitive cooperation network is considered, and resource allocation at a multi-antenna SU for SWIPT-enabled PSC is investigated. Specifically, the SU maximizes its own transmission rate under the premise that the target transmission rate of the PU is satisfied. Thus, a JPSBO problem is formulated for DF and AF modes. Both transmission rate maximization problems are transformed into a contrapositive power minimization problem, which can be effectively solved using the dual method. Moreover, in DF mode, the optimal PS ratio can be obtained directly, and beamforming vectors are derived through a bisection method and by solving the quadratic equations. While in AF mode, the optimal PS ratio and beamforming vector are derived by jointly utilizing the fix point iteration, bisection search, and golden ratio methods. The contributions of this paper are summarized as follows.

- The SWIPT-enabled multi-antenna PSC model is established for both AF and DF relaying modes, in which a multiantenna SU harvests energy from a PU, and uses the harvested energy to conduct information forwarding for the PU and its own data transmission simultaneously.
- A unified problem formulation for both DF and AF modes is built, which aims at maximizing the transmission rate of the SU subject to the target transmission rate requirement of the PU.
- Bisection searching and/or Golden searching-based lowcomplexity algorithms are designed for the JPSBO problems in DF and AF modes, respectively, by analyzing the monotonicity and quasi-convexity/quasi-concavity of the objective functions.

Notations: Throughout this paper, the vectors and matrices are represented by boldface lowercase and uppercase letters,

respectively. In addition, $\|\cdot\|$, $(\cdot)^{T}$, $(\cdot)^{-1}$, $(\cdot)^{H}$, and \odot represent the Frobenius norm, transpose, inverse, Hermitian transpose, and element-wise product operations of the vectors or matrices, respectively. Moreover, $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \Theta)$ indicates that \mathbf{x} is a complex Gaussian entry with a mean value of \mathbf{m} and covariance matrix of Θ ; \emptyset is an empty set; $\mathbf{A} \succeq \mathbf{0}$ indicates that \mathbf{A} is positive semi-definite; and diag(\mathbf{x}) is a diagonal matrix with *k*-th diagonal element equaling to the *k*-th element of \mathbf{x} .

III. System Model and Problem Formulation

1. System Model

The communication system considered consists of a primary transmitter (PT), a primary receiver (PR), a secondary transmitter (ST), and a secondary receiver (SR). The PT, PR, and SR are equipped with one antenna, whereas the ST is equipped with *N* antennas. The ST harvests RF signal energy from the PT, and transmits both the received signal and its own signal to the PR and SR, simultaneously. As shown in Fig. 1, $\mathbf{h}_{p,s}$, $\mathbf{h}_{s,p}$, and $\mathbf{h}_{s,s} \in \mathbb{C}^{N \times 1}$ denote the transmission channel from the PT to ST, from the ST to PR, and from the ST to SR, respectively, and $h_{p,p}$ denotes the transmission channel from the PT to PR.

2. Transmission Model

A. Phase I

Each transmission consists of two phases. In phase I, a PT broadcasts its information x_1 to both a PR and ST. As a result, the received signals at the PR and ST are

$$y_{\text{PR},1} = h_{\text{p},\text{p}} \sqrt{P_1 x_1 + n_1^p}$$
 and (1)

$$\mathbf{y}_{\mathrm{ST},1} = \mathbf{h}_{\mathrm{p},\mathrm{s}} \sqrt{P_1} x_1 + \mathbf{n}_1^{\mathrm{s}}, \qquad (2)$$

where P_1 is the transmit power at PT, and $n_1^p \sim C\mathcal{N}(0, \sigma_p^2)$



Fig. 1. System model of multi-antenna primary-secondary cooperation.

ETRI Journal, Volume 38, Number 5, October 2016 http://dx.doi.org/10.4218/etrij.16.0115.0561 and $\mathbf{n}_1^s \sim C\mathcal{N}(\mathbf{0}, \sigma_s^2 \mathbf{I})$ are the received noise at the PR and ST, respectively.

A portion of the received signal at the ST is used for energy harvesting, and the left part is for IF by adopting the DF/AF protocol. Denote $\beta \in [0, 1]$ as the PS ratio. The finally received signal and harvested energy can then be calculated as

$$\mathbf{r}_{\rm ST} = \sqrt{\beta} \mathbf{y}_{\rm ST,1} + \mathbf{n}_1^{\prime} = \sqrt{\beta} \mathbf{h}_{\rm p,s} \sqrt{P_1} x_1 + \sqrt{\beta} \mathbf{n}_1^{\prime s} + \mathbf{n}_1^{\prime} \quad \text{and} \quad (3)$$

$$Q(\beta) = \eta (1 - \beta) (P_1 || \mathbf{h}_{p,s} ||^2 + \sigma_s^2 N), \qquad (4)$$

where $\mathbf{n}_1^I \sim \mathcal{CN}(\mathbf{0}, \sigma_I^2 \mathbf{I})$ is the additive noise caused from power splitting at the ST, and $\eta \in [0, 1]$ is the energy conversion efficiency. Therefore, using MMSE, the transmission rate of the PT at the first hop is

$$R_{\rm P,1} = 0.5\Delta f \log_2(1 + \gamma_{\rm P,1}^{\rm DF})$$
 and (5)

$$\gamma_{\mathrm{P},\mathrm{l}}^{\mathrm{DF}} = \frac{\beta P_{\mathrm{l}} ||\mathbf{h}_{\mathrm{p},\mathrm{s}}||^{2}}{\beta \sigma_{\mathrm{s}}^{2} + \sigma_{\mathrm{l}}^{2}},\tag{6}$$

where $\gamma_{P,1}^{DF}$ is the achievable transmission signal to interference plus noise power ratio (SINR) at the SR.

B. Phase II in the Case of DF

During phase II, the received information for the IF and its own information of the ST are processed by $N \times 1$ dimensional encoding vectors \mathbf{w}_1 and \mathbf{w}_2 , respectively. As a consequence, the transmitted signal at the ST is $\mathbf{s} = \mathbf{w}_1 x_1 + \mathbf{w}_2 x_2 \in \mathbb{C}^{N \times 1}$. The information received at the PR is

$$y_{\text{PR},2} = \mathbf{h}_{s,p}^{\text{H}} \mathbf{s} + n_2^{p} = \mathbf{h}_{s,p}^{\text{H}} (\mathbf{w}_1 x_1 + \mathbf{w}_2 x_2) + n_2^{p} , \qquad (7)$$

where $n_2^p \sim C\mathcal{N}(0, \sigma_p^2)$ is the received noise at the PR. Thus, with the maximum ratio combining, the transmission rate of the PU's data at the second hop is formulated as

$$R_{\rm P,2}^{\rm DF} = 0.5\Delta f \log_2(1 + \gamma_{\rm P,2}^{\rm DF}), \qquad (8)$$

$$\gamma_{p,2}^{\text{DF}} = \frac{P_{l}|h_{p,p}|^{2}}{\sigma_{p}^{2} + \tilde{\gamma}_{p,2}^{\text{DF}}}, \text{ and }$$
(9)

$$\tilde{\gamma}_{P,2}^{\text{DF}} = \frac{|\mathbf{h}_{s,p}^{\text{H}} \mathbf{w}_{1}|^{2}}{|\mathbf{h}_{s,p}^{\text{H}} \mathbf{w}_{2}|^{2} + \sigma_{p}^{2}},$$
(10)

where Δf is the spectrum bandwidth.

In addition, the received information at the SR is

$$y_{\mathrm{SR},2} = \mathbf{h}_{\mathrm{s},s}^{\mathrm{H}} \mathbf{s} + n_{2}^{s} = \mathbf{h}_{\mathrm{s},s}^{\mathrm{H}} \left(\mathbf{w}_{1} x_{1} + \mathbf{w}_{2} x_{2} \right) + n_{2}^{s} , \qquad (11)$$

where $n_2^s \sim CN(0, \sigma_r^2)$ is the received noise at the SR. The achievable transmission rate of the SR is formulated as

$$R_{\rm S}^{\rm DF} = 0.5\Delta f \log_2(1+\gamma_{\rm S}^{\rm DF}) \quad \text{and} \tag{12}$$

$$\gamma_{\rm S}^{\rm DF} = \frac{|{\bf h}_{\rm ss}^{\rm H} {\bf w}_2|^2}{|{\bf h}_{\rm ss}^{\rm H} {\bf w}_1|^2 + \sigma_r^2},$$
(13)

where $\gamma_{\rm S}^{\rm DF}$ is achievable transmission SINR at the SR.

Finally, the power consumed at the ST is

$$P_c^{\rm DF} = \|\mathbf{w}_1\|^2 + \|\mathbf{w}_2\|^2 - Q(\beta).$$
(14)

C. Phase II for AF

In the case of AF, the transmitted signal at the ST is $\mathbf{s} = \mathbf{w}_1 \odot \mathbf{r}_{ST} + \mathbf{w}_2 x_2 \in \mathbb{C}^{N \times 1}$. The received signal at the PR is

$$y_{\text{PR},2} = \mathbf{h}_{s,p}^{\text{H}} \mathbf{s} + n_{2}^{p} = \sqrt{\beta} P_{1} \mathbf{h}_{s,p}^{\text{H}} (\mathbf{w}_{1} \odot \mathbf{h}_{p,s} x_{1}) + n_{2}^{p} + \sqrt{\beta} \mathbf{h}_{s,p}^{\text{H}} (\mathbf{w}_{1} \odot \mathbf{n}_{1}^{s}) + \mathbf{h}_{s,p}^{\text{H}} (\mathbf{w}_{1} \odot \mathbf{n}_{1}^{s}) + \mathbf{h}_{s,p}^{\text{H}} \mathbf{w}_{2} x_{2}.$$
(15)

Therefore, the transmission rate of the PU at the second hop is

$$R_{\rm P,2}^{\rm AF} = \frac{1}{2} \Delta f \log_2(1 + \gamma_{\rm P,2}^{\rm AF}), \qquad (16)$$

$$\gamma_{P,2}^{AF} = \frac{P_{||}h_{p,p}|^2}{\sigma_p^2} + \tilde{\gamma}_{P,2}^{AF}$$
, and (17)

$$\tilde{\gamma}_{\mathrm{P},2}^{\mathrm{AF}} = \frac{\beta P_{\mathrm{I}} |\mathbf{h}_{\mathrm{s},p}^{\mathrm{H}} \mathbf{B}_{2} \mathbf{w}_{\mathrm{I}}|^{2}}{(\beta \sigma_{\mathrm{s}}^{2} + \sigma_{\mathrm{I}}^{2}) |\mathbf{h}_{\mathrm{s},p}^{\mathrm{H}} \mathbf{w}_{\mathrm{I}}|^{2} + |\mathbf{h}_{\mathrm{s},p}^{\mathrm{H}} \mathbf{w}_{2}|^{2} + \sigma_{p}^{2}}, \qquad (18)$$

where $\mathbf{B}_2 = \text{diag}\{\mathbf{h}_{p,s}\}$.

The received information at the SR is

$$y_{\text{SR},2} = \mathbf{h}_{s,s}^{\text{H}} \mathbf{s} + n_{2}^{s} = \sqrt{\beta P_{1}} \mathbf{h}_{s,s}^{\text{H}} \mathbf{w}_{1} \odot \mathbf{h}_{p,s} x_{1} + \sqrt{\beta} \mathbf{h}_{s,s}^{\text{H}} (\mathbf{w}_{1} \odot \mathbf{n}_{1}^{s}) + \mathbf{h}_{s,s}^{\text{H}} (\mathbf{w}_{1} \odot \mathbf{n}_{1}^{l}) + \mathbf{h}_{s,s}^{\text{H}} \mathbf{w}_{2} x_{2} + n_{2}^{s}.$$
(19)

In addition, the achievable transmission rate of the SR is calculated as

$$R_{\rm S}^{\rm AF} = 0.5\Delta f \log_2(1+\gamma_{\rm S}^{\rm AF}) \quad \text{and} \tag{20}$$

$$\gamma_{\rm S}^{\rm AF} = \frac{|\mathbf{h}_{\rm s,s}^{\rm H} \mathbf{w}_2|^2}{(\beta \sigma_{\rm s}^2 + \sigma_{\rm I}^2) |\mathbf{h}_{\rm s,s}^{\rm H} \mathbf{w}_1|^2 + \beta P_1 |\mathbf{h}_{\rm s,s}^{\rm H} \mathbf{B}_2 \mathbf{w}_1|^2 + \sigma_{\rm r}^2} \,.$$
(21)

The consumed power at the ST is

$$P_{c}^{AF} = E\left[\|\mathbf{w}_{1} \odot \mathbf{r}_{ST}\|^{2} + \|\mathbf{w}_{2}x_{2}\|^{2} \right] - Q(\beta)$$

= $\beta P_{1} \|\mathbf{B}_{2}\mathbf{w}_{1}\|^{2} + (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) \|\mathbf{w}_{1}\|^{2} + \|\mathbf{w}_{2}\|^{2} - Q(\beta).$ (22)

3. Problem Formulation

Aiming at maximizing the achievable transmission rate at the SR, the investigated problem can be formulated as follows.

$$\begin{split} \mathcal{OP}_{1|u} : & \max_{\{\mathbf{w}_1, \mathbf{w}_2, \beta\}} & \gamma_{\mathrm{S}}^u, \\ \mathrm{s.t.} & \mathcal{C}_1 : \gamma_{\mathrm{P}, 1}^{\tilde{u}} \geq z_{\mathrm{p, th}}, \\ & \mathcal{C}_2 : \gamma_{\mathrm{P}, 2}^u \geq z_{\mathrm{p, th}}, \text{ and} \\ & \mathcal{C}_3 : P_c^u \leq P_2, \end{split}$$

where $u \in \{\text{DF}, \text{AF}\}$, and $\tilde{u} = \text{DF}$ if u equals DF; otherwise, $\tilde{u} = \emptyset$. In addition, $z_{\text{p,th}} = 2^{2R_{\text{p,th}}/\Delta f} - 1$ with $R_{\text{p,th}}$ indicating the target transmission rate threshold of the PR. Constraints C_1 and C_2 indicate that the achievable transmission rates for two hops should satisfy the QoS requirement of the PR, and constraint C_3 denotes the maximal transmit power budget constraint, with P_2 being the maximal available transmit power at the ST that it owns before harvesting energy.

IV. Algorithm Design

1. Problem Solving in the Case of DF

Analyzing $\mathcal{OP}_{1|DF}$, it is easy to find the following: 1) From constraint C_1 , $\beta \ge z_{p,th} \sigma_I^2 / (P_1 || \mathbf{h}_{p,s} ||^2 - z_{p,th} \sigma_s^2)$ if $P_1 || \mathbf{h}_{p,s} ||^2 > z_{p,th} \sigma_s^2$; otherwise, the problem cannot be solved. 2) From constraint C_3 , $||\mathbf{w}_1||^2 + ||\mathbf{w}_2||^2 \le \eta(1-\beta)$ $(P_1 || \mathbf{h}_{p,s} ||^2 + N\sigma_s^2) + P_2$. Therefore, the available transmit power decreases as β increases, and the maximum available power is achieved at

$$\beta^* = \frac{z_{\text{p,th}} \sigma_I^2}{P_1 \| \mathbf{h}_{\text{p,s}} \|^2 - z_{\text{p,th}} \sigma_s^2}.$$
 (23)

The problem $\mathcal{OP}_{1|DF}$ can then be simplified as

$$\mathcal{OP}_{2|\text{DF}} : \max_{\{\mathbf{w}_{1}, \mathbf{w}_{2}\}} \frac{|\mathbf{h}_{s,s}^{\text{H}} \mathbf{w}_{2}|^{2}}{|\mathbf{h}_{s,s}^{\text{H}} \mathbf{w}_{1}|^{2} + \sigma_{r}^{2}},$$

s.t. $\mathcal{C}_{2}' : \frac{|\mathbf{h}_{s,p}^{\text{H}} \mathbf{w}_{1}|^{2}}{|\mathbf{h}_{s,p}^{\text{H}} \mathbf{w}_{2}|^{2} + \sigma_{p}^{2}} \ge \tilde{z}_{p,\text{th}}$ and
 $\mathcal{C}_{3}' : ||\mathbf{w}_{1}||^{2} + ||\mathbf{w}_{2}||^{2} \le P_{2} + Q(\beta^{*}),$

where $\tilde{z}_{p,th} = z_{p,th} - P_1 |h_{p,p}|^2 / \sigma_p^2$.

To solve the problem $\mathcal{OP}_{2|DF}$, the following contrapositive problem $\mathcal{OP}_{3|DF}$ is proposed, in which we fix the objective transmission SINR of the SR and try to minimize the total power consumption.

$$\begin{aligned} \mathcal{OP}_{3|\mathrm{DF}} &: f(z_{\mathrm{s,th}}) = \min_{\{\mathbf{w}_{1},\mathbf{w}_{2}\}} \|\mathbf{w}_{1}\|^{2} + \|\mathbf{w}_{2}\|^{2}, \\ \mathrm{s.t.} \ \mathcal{C}_{2}' &: |\mathbf{h}_{\mathrm{s,p}}^{\mathrm{H}}\mathbf{w}_{1}|^{2} \geq \tilde{z}_{\mathrm{p,th}} (|\mathbf{h}_{\mathrm{s,p}}^{\mathrm{H}}\mathbf{w}_{2}|^{2} + \sigma_{p}^{2}) \\ \mathcal{C}_{3}' &: |\mathbf{h}_{\mathrm{s,s}}^{\mathrm{H}}\mathbf{w}_{2}|^{2} \geq z_{\mathrm{s,th}} (|\mathbf{h}_{\mathrm{s,s}}^{\mathrm{H}}\mathbf{w}_{1}|^{2} + \sigma_{r}^{2}). \end{aligned}$$

The problem $\mathcal{OP}_{3|\text{DF}}$ can be solved with the dual method. After achieving the minimum power $f(z_{s,th})$, it is compared with the available transmit power $P_2 + Q(\beta^2)$, and the optimal $z_{s,th}$ is found using bisection search methods [20].

The dual method used to solve $\mathcal{OP}_{3|DF}$ is briefly explained as follows. First, Lagrangian multipliers λ_1 and λ_2 are defined for constraints \mathcal{C}'_2 and \mathcal{C}'_3 , respectively, and the Lagrangian function is

$$L = \|\mathbf{w}_{1}\|^{2} + \|\mathbf{w}_{2}\|^{2} - \lambda_{1} [|\mathbf{h}_{s,p}^{H}\mathbf{w}_{1}|^{2} - \tilde{z}_{p,th} (|\mathbf{h}_{s,p}^{H}\mathbf{w}_{2}|^{2} + \sigma_{p}^{2})] - \lambda_{2} [|\mathbf{h}_{s,s}^{H}\mathbf{w}_{2}|^{2} - z_{s,th} (|\mathbf{h}_{s,s}^{H}\mathbf{w}_{1}|^{2} + \sigma_{r}^{2})] = \mathbf{w}_{1}^{H}\mathbf{A}_{1}\mathbf{w}_{1} + \mathbf{w}_{2}^{H}\mathbf{A}_{2}\mathbf{w}_{2} + \tilde{z}_{p,th}\sigma_{p}^{2}\lambda_{1} + z_{s,th}\sigma_{r}^{2}\lambda_{2},$$
(24)

where $\mathbf{A}_1 = \mathbf{I} - \lambda_1 \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} + \lambda_2 z_{s,th} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H}$ and $\mathbf{A}_2 = \mathbf{I} - \lambda_2 \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H} + \lambda_1 \tilde{z}_{p,th} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H}$.

Its dual problem is presented as follows:

$$\begin{split} \mathcal{F}_{\mathrm{DF}} &: \max_{\{\lambda_1,\lambda_2\}} \tilde{z}_{\mathrm{p,th}} \sigma_p^2 \lambda_1 + z_{\mathrm{s,th}} \sigma_r^2 \lambda_2, \\ \mathrm{s.t.} & \mathbf{I} + \lambda_2 z_{\mathrm{s,th}} \mathbf{h}_{\mathrm{s,s}} \mathbf{h}_{\mathrm{s,s}}^{\mathrm{H}} \succeq \lambda_1 \mathbf{h}_{\mathrm{s,p}} \mathbf{h}_{\mathrm{s,p}}^{\mathrm{H}}, \\ & \mathbf{I} + \lambda_1 \tilde{z}_{\mathrm{p,th}} \mathbf{h}_{\mathrm{s,p}} \mathbf{h}_{\mathrm{s,p}}^{\mathrm{H}} \succeq \lambda_2 \mathbf{h}_{\mathrm{s,s}} \mathbf{h}_{\mathrm{s,s}}^{\mathrm{H}}. \end{split}$$

Using the matrix inversion lemma, it is easy to prove that the optimal λ_1 and λ_2 should satisfy

$$\lambda_{1} = \frac{1 + \lambda_{2} z_{s,th} \| \mathbf{h}_{s,s} \|^{2}}{\| \mathbf{h}_{s,p} \|^{2} \left[1 + \lambda_{2} z_{s,th} \| \mathbf{h}_{s,s} \|^{2} (1 - \rho) \right]}, \text{ and } (25)$$

$$\lambda_{2} = \frac{1 + \lambda_{1} \tilde{z}_{p,th} \| \mathbf{h}_{s,p} \|^{2}}{\| \mathbf{h}_{s,s} \|^{2} \left[1 + \lambda_{1} \tilde{z}_{p,th} \| \mathbf{h}_{s,p} \|^{2} (1 - \rho) \right]},$$
 (26)

where $\rho = \frac{|\mathbf{h}_{s,s}^{H}\mathbf{h}_{s,p}|^{2}}{\|\mathbf{h}_{s,s}\|^{2}\|\mathbf{h}_{s,p}\|^{2}}$.

Substituting (25) into (26), the optimal λ_2 can be obtained by solving a quadratic equation. Then, using (25), the optimal λ_1 can also be obtained. Finally, according to constraints C_2' and C_3' , we compute the normalized beamforming vector through

$$\tilde{\mathbf{w}}_1 = \frac{\hat{\mathbf{w}}_1}{\|\hat{\mathbf{w}}_1\|}, \hat{\mathbf{w}}_1 = (\mathbf{I} + \lambda_2 z_{s,th} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{\mathsf{H}})^{-1} \mathbf{h}_{s,p} \quad \text{and} \quad (27)$$

$$\tilde{\mathbf{w}}_2 = \frac{\hat{\mathbf{w}}_2}{\|\hat{\mathbf{w}}_2\|}, \hat{\mathbf{w}}_2 = (\mathbf{I} + \lambda_1 \tilde{z}_{p,th} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{\mathrm{H}})^{-1} \mathbf{h}_{s,s} .$$
(28)

Let $\mathbf{w}_1 = \sqrt{\chi_1} \tilde{\mathbf{w}}_1$, $\mathbf{w}_2 = \sqrt{\chi_2} \tilde{\mathbf{w}}_2$, and the power coefficients be calculated as follows.

$$\left[\chi_1,\chi_2\right]^{\mathrm{T}} = \mathbf{X}^{-1}\mathbf{Y},\tag{29}$$

$$\mathbf{X} = -\begin{bmatrix} -\left|\mathbf{h}_{s,p}^{\mathrm{H}}\tilde{\mathbf{w}}_{1}\right|^{2} & \tilde{z}_{p,\mathrm{th}}\left|\mathbf{h}_{s,p}^{\mathrm{H}}\tilde{\mathbf{w}}_{2}\right|^{2} \\ z_{s,\mathrm{th}}\left|\mathbf{h}_{s,s}^{\mathrm{H}}\tilde{\mathbf{w}}_{1}\right|^{2} & -\left|\mathbf{h}_{s,s}^{\mathrm{H}}\tilde{\mathbf{w}}_{2}\right|^{2} \end{bmatrix}, \mathbf{Y} = \begin{pmatrix} \tilde{z}_{p,\mathrm{th}}\sigma_{p}^{2} \\ z_{s,\mathrm{th}}\sigma_{r}^{2} \end{pmatrix}, \quad (30)$$

According to above derivation, a joint power splitting and beamforming optimization algorithm for DF mode (referred to as JPSBO-DF) is proposed, as shown in Table 1.

2. Problem Solving for AF

For the case of AF, the following contrapositive problem is

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Table 1. JPSBO-DF.

1: Input $\mathbf{h}_{p,s}$, $\mathbf{h}_{s,p}$, $\mathbf{h}_{s,s}$, $h_{p,p}$, P_1 , P_2 , η , N , Δf , σ_s^2 , σ_I^2 , σ_p^2 , σ_r^2 , $R_{p,th}$, and
tolerable error ε
2: Compute β^* as (23)
3: Initialize $z_{\text{s,th}}^{\text{up}}$, $z_{\text{s,th}}^{\text{down}}$ with $f(z_{\text{s,th}}^{\text{up}}) > P_2 + Q(\beta^*)$, $f(z_{\text{s,th}}^{\text{down}}) < P_2 + Q(\beta^*)$.
Iteration: (Obtain $z_{s,th}$ using bisection methods)
4: Let $z_{s,th} = (z_{s,th}^{up} + z_{s,th}^{down})/2$, and solve problem $\mathcal{OP}_{3 DF}$ to gain $f(z_{s,th})$.
5: If $f(z_{s,th}) > P_2 + Q(\beta^*)$, let $z_{s,th}^{up} = z_{s,th}$; Else, let $z_{s,th}^{down} = z_{s,th}$.
6: Compute $\Pi = \left f(z_{s,th}) - (P_2 + Q(\beta^*)) \right $, and repeat steps 4 and 5 until
$\Pi < \varepsilon.$
7: Calculate λ_1 and λ_2 with (25) and (26).
8: Determine $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$ following (27) and (28), and the power
coefficient χ_1 and χ_2 according to (29) and (30).
Output:
9: Compute $\mathbf{w}_1 = \sqrt{\chi_1} \tilde{\mathbf{w}}_1$ and $\mathbf{w}_2 = \sqrt{\chi_2} \tilde{\mathbf{w}}_2$.

established, in which we optimize the beamforming vectors to minimize the total power consumption subject to the minimum transmission SINR constraints for both the PR and SR.

$$\begin{aligned} \mathcal{OP}_{2|AF} &: \min_{\{\mathbf{w}_{1},\mathbf{w}_{2}\}} \beta P_{1} \| \mathbf{B}_{2} \mathbf{w}_{1} \|^{2} + (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) \| \mathbf{w}_{1} \|^{2} + \| \mathbf{w}_{2} \|^{2} \\ &- \eta (1 - \beta) (P_{1} | \mathbf{h}_{p,s} |^{2} + N \sigma_{s}^{2}) \\ \mathcal{C}_{2}^{\prime} &: \beta P_{1} | \mathbf{h}_{s,p}^{H} \mathbf{B}_{2} \mathbf{w}_{1} |^{2} \geq \\ &\tilde{z}_{p,th} \left[(\beta \sigma_{s}^{2} + \sigma_{I}^{2}) | \mathbf{h}_{s,p}^{H} \mathbf{w}_{1} |^{2} + | \mathbf{h}_{s,p}^{H} \mathbf{w}_{2} |^{2} + \sigma_{p}^{2} \right] \\ \mathcal{C}_{3}^{\prime} &: | \mathbf{h}_{s,s}^{H} \mathbf{w}_{2} |^{2} \geq \\ &z_{s,th} \left[(\beta \sigma_{s}^{2} + \sigma_{I}^{2}) | \mathbf{h}_{s,s}^{H} \mathbf{w}_{1} |^{2} + \beta P_{1} | \mathbf{h}_{s,s}^{H} \mathbf{B}_{2} \mathbf{w}_{1} |^{2} + \sigma_{r}^{2} \right]. \end{aligned}$$

Define the optimal objective value, that is, the minimum power consumption, as $f_{AF}(\beta, z_{s,th})$. The following theorems are then presented to facilitate the algorithm design.

Theorem 1: $f_{AF}(\beta, z_{s,th})$ is monotonically increasing with $z_{s,th}$ and is quasi-convex about β .

Proof: See Appendix A.

Theorem 2: Denote the optimal value of problem $\mathcal{OP}_{I|AF}$ as $F(\beta)$. Then, $F(\beta)$ is quasi-concave about β .

Proof: See Appendix B.

With the help of Theorem 2, we can first optimize the problem $\mathcal{OP}_{I|AF}$ for the given β , and then search for the optimal β using the Golden searching method [21]. To solve problem $\mathcal{OP}_{I|AF}$ for the given β , the following bisection algorithm is designed in detail.

First, Lagrangian multipliers λ_1 and λ_2 are introduced. The

Lagrangian function is as follows.

$$\begin{aligned} \mathbf{L} &= \beta P_1 \| \mathbf{B}_2 \mathbf{w}_1 \|^2 + (\beta \sigma_s^2 + \sigma_l^2) \| \mathbf{w}_1 \|^2 + \| \mathbf{w}_2 \|^2 \\ &- \eta (1 - \beta) (P_1 | \mathbf{h}_{p,s} |^2 + N \sigma_s^2) - \lambda_1 t_1 - \lambda_2 t_2 \\ &= \mathbf{w}_1^{\mathrm{H}} \mathbf{C}_1 \mathbf{w}_1 + \mathbf{w}_2^{\mathrm{H}} \mathbf{C}_2 \mathbf{w}_2 + \lambda_1 \tilde{z}_{p,\mathrm{th}} \sigma_p^2 + \lambda_2 z_{s,\mathrm{th}} \sigma_r^2 \\ &- \eta (1 - \beta) (P_1 | \mathbf{h}_{p,s} |^2 + N \sigma_s^2), \end{aligned}$$
(31)

where

$$t_{1} = \beta P_{1} |\mathbf{h}_{s,p}^{H} \mathbf{B}_{2} \mathbf{w}_{1}|^{2} - \tilde{z}_{p,th} (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) |\mathbf{h}_{s,p}^{H} \mathbf{w}_{1}|^{2}
- \tilde{z}_{p,th} |\mathbf{h}_{s,p}^{H} \mathbf{w}_{2}|^{2} - \tilde{z}_{p,th} \sigma_{p}^{2},
t_{2} = |\mathbf{h}_{s,s}^{H} \mathbf{w}_{2}|^{2} - z_{s,th} (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) |\mathbf{h}_{s,s}^{H} \mathbf{w}_{1}|^{2}
- z_{s,th} \beta P_{1} |\mathbf{h}_{s,s}^{H} \mathbf{B}_{2} \mathbf{w}_{1}|^{2} - z_{s,th} \sigma_{r}^{2},
\mathbf{C}_{1} = \beta P_{1} \mathbf{B}_{2}^{H} \mathbf{B}_{2} + (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) \mathbf{I} - \lambda_{1} \beta P_{1} \mathbf{B}_{2}^{H} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} \mathbf{B}_{2}
+ \lambda_{1} \tilde{z}_{p,th} (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} + \lambda_{2} z_{s,th} (\beta \sigma_{s}^{2} + \sigma_{I}^{2}) \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H}$$
(32)
+ $\lambda_{2} z_{s,th} \beta P_{1} \mathbf{B}_{2}^{H} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H} \mathbf{B}_{2}, \text{ and}
\mathbf{C}_{2} = \mathbf{I} + \lambda_{1} \tilde{z}_{p,th} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} - \lambda_{2} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H}.$ (33)

The dual problem is presented as follows.

$$\begin{split} \mathcal{F}_{AF} &: \max_{\{\lambda_{1},\lambda_{2}\}} \lambda_{1} \tilde{z}_{p,th} \sigma_{p}^{2} + \lambda_{2} z_{s,th} \sigma_{r}^{2} - \eta (1 - \beta) (P_{1} \mid \mathbf{h}_{p,s} \mid^{2} + N \sigma_{s}^{2}), \\ \text{s.t.} \quad \mathbf{G}(\lambda_{1},\lambda_{2}) \succeq \lambda_{1} \mathbf{B}_{2}^{H} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} \mathbf{B}_{2} \\ \mathbf{I} + \lambda_{1} \tilde{z}_{p,th} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{H} \succeq \lambda_{2} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{H}, \end{split}$$

where $\mathbf{G}(\lambda_1, \lambda_2) = \tilde{\mathbf{G}}(\lambda_1, \lambda_2) / \beta P_1$, $\tilde{\mathbf{G}}(\lambda_1, \lambda_2) = \beta P_1 \mathbf{B}_2^{\mathrm{H}} \mathbf{B}_2 + \lambda_1 \tilde{z}_{\text{p,th}} (\beta \sigma_s^2 + \sigma_l^2) \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{\mathrm{H}} + \lambda_2 z_{s,\text{th}} [(\beta \sigma_s^2 + \sigma_l^2) \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{\mathrm{H}} + \beta P_1 \mathbf{B}_2^{\mathrm{H}} \mathbf{h}_{s,s} \mathbf{h}_{s,s}^{\mathrm{H}} \mathbf{B}_2] + (\beta \sigma_s^2 + \sigma_l^2) \mathbf{I}$.

The two linear matrix inequalities uniquely determine λ_1 and λ_2 as follows.

$$\lambda_1 = \frac{1}{\mathbf{h}_{s,p}^{\mathrm{H}} \mathbf{B}_2 \mathbf{G}(\lambda_1, \lambda_2)^{-1} \mathbf{B}_2^{\mathrm{H}} \mathbf{h}_{s,p}} \quad \text{and} \tag{34}$$

$$\lambda_2 = \frac{1}{\mathbf{h}_{s,s}^{H} (\mathbf{I} + \lambda_1 \tilde{z}_{p,th} \mathbf{g}_{s,p} \mathbf{g}_{s,p}^{H})^{-1} \mathbf{h}_{s,s}}.$$
 (35)

From (34) and (35), the optimal λ_1 and λ_2 can be obtained using the fixed point iteration algorithm [22]. Then, compare $f_{\rm AF}(\beta, z_{\rm s,th})$ with P_2 , and search the maximum $z_{\rm s,th}$ to satisfy $f_{\rm AF}(\beta, z_{\rm s,th}) \leq P_2$.

Finally, according to the two constraints of $\mathcal{OP}_{\!2|AF}$, we compute the normalized beamforming vectors as

$$\tilde{\mathbf{w}}_1 = \hat{\mathbf{w}}_1 / \|\hat{\mathbf{w}}_1\|, \hat{\mathbf{w}}_1 = \mathbf{G}(\lambda_1, \lambda_2)^{-1} \mathbf{B}_2^{\mathrm{H}} \mathbf{h}_{\mathrm{s},\mathrm{p}} \quad \text{and} \qquad (36)$$

$$\tilde{\mathbf{w}}_{2} = \hat{\mathbf{w}}_{2} / \|\hat{\mathbf{w}}_{2}\|, \hat{\mathbf{w}}_{2} = (\mathbf{I} + \lambda_{1} \tilde{z}_{p,\text{th}} \mathbf{h}_{s,p} \mathbf{h}_{s,p}^{\text{H}})^{-1} \mathbf{h}_{s,s} .$$
(37)

Letting $\mathbf{w}_1 = \sqrt{\chi_1} \tilde{\mathbf{w}}_1$ and $\mathbf{w}_2 = \sqrt{\chi_2} \tilde{\mathbf{w}}_2$, the power coefficients can be calculated as follows.

Table 2. JPSBO-AF.

1: Input $\mathbf{h}_{p,s}$, $\mathbf{h}_{s,p}$, $\mathbf{h}_{s,s}$, $h_{p,p}$, P_1 , P_2 , η , N , Δf , σ_s^2 , σ_I^2 , σ_p^2 , σ_r^2 , R_{p,th_s} and
tolerable error $\varepsilon_1, \varepsilon_2$.
2: Initialize $a = 0$, and $b = 1$ (obtain β using Golden section methods).
3: Initialize $\beta^{\text{down}} = a + 0.382(b-a)$ and $\beta^{\text{up}} = a + 0.618(b-a)$.
Iteration: (Obtain $F(\beta^{\text{down}})$ and $F(\beta^{\text{up}})$ using bisection method)
4: Initialize $z_{s,th}^{up}$, $z_{s,th}^{down}$ with $f_{AF}(\beta^{up}, z_{s,th}^{up}) > P_2$, $f_{AF}(\beta^{up}, z_{s,th}^{down}) < P_2$;
5: Let $z_{s,th} = (z_{s,th}^{up} + z_{s,th}^{down})/2$;
6: Update λ_1 and λ_2 according to (34) and (35) until λ_1 and λ_2 converge;
7: If $f_{AF}(\beta^{up}, z_{s,th}) > P_2$, let $z_{s,th}^{up} = z_{s,th}$; Else, let $z_{s,th}^{down} = z_{s,th}$;
8: Compute $\Pi = z_{s,th}^{up} - z_{s,th}^{down} $, and repeat steps 5 through 7 until
$\Pi < \varepsilon_2 ;$
9: $F(\beta^{\rm up}) = z_{\rm s,th};$
10: Repeat steps 3 through 8 with β^{up} replaced by β^{down} ;
11: $F(\beta^{\text{down}}) = z_{s,\text{th}};$
12: If $F(\beta^{up}) > F(\beta^{down})$, let $a = \beta^{down}$; Else, $b = \beta^{up}$.
13: Repeat steps 3 through 12 until $ b-a < \varepsilon_1$
Output:
20: Set $z_{s,th} = [F(\beta^{up}) + F(\beta^{down})]/2$, $\beta = (\beta^{up} + \beta^{down})/2$.
21: Determine $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_1$ following (36) and (37), and the power
coefficients χ_1 and χ_2 according to (38) through (40).
22: Compute $\mathbf{w}_1 = \sqrt{\chi_1} \tilde{\mathbf{w}}_1$ and $\mathbf{w}_2 = \sqrt{\chi_2} \tilde{\mathbf{w}}_2$.

$$\left[\boldsymbol{\chi}_1, \boldsymbol{\chi}_2\right]^{\mathrm{T}} = \mathbf{X}^{-1} \mathbf{Y}, \qquad (38)$$

$$\mathbf{X}(:,1) = \begin{pmatrix} \beta P_1 \left| \mathbf{h}_{s,p}^{\mathrm{H}} \mathbf{B}_2 \tilde{\mathbf{w}}_1 \right|^2 - \tilde{z}_{p,\mathrm{th}} \left(\beta \sigma_s^2 + \sigma_l^2 \right) \left| \mathbf{h}_{s,p}^{\mathrm{H}} \tilde{\mathbf{w}}_1 \right|^2 \\ -z_{s,\mathrm{th}} \left(\beta \sigma_s^2 + \sigma_l^2 \right) \left| \mathbf{h}_{s,s}^{\mathrm{H}} \tilde{\mathbf{w}}_1 \right|^2 - z_{s,\mathrm{th}} \beta P_1 \left| \mathbf{h}_{s,s}^{\mathrm{H}} \mathbf{B}_2 \tilde{\mathbf{w}}_1 \right|^2 \end{pmatrix}, \quad (39)$$

$$\mathbf{X}(:,2) = \begin{pmatrix} -\tilde{z}_{p,\text{th}} | \mathbf{h}_{s,p}^{\text{H}} \tilde{\mathbf{w}}_{2} |^{2} \\ | \mathbf{h}_{s,s}^{\text{H}} \tilde{\mathbf{w}}_{2} |^{2} \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} \tilde{z}_{p,\text{th}} \sigma_{p}^{2} \\ z_{s,\text{th}} \sigma_{r}^{2} \end{pmatrix}$$
(40)

According to the above derivation, a joint power splitting and beaming optimization algorithm for AF mode (referred to as JPSBO-AF) is proposed, as elaborated upon in Table 2.

V. Performance Analysis

In this section, simulation results are presented to demonstrate the performance of the JPSBO-DF/AF algorithm. Unless stated otherwise, the following parameter settings are used throughout the simulation. The transmit power at the PT and ST is set to 1.5 W and 0.1 W, respectively, and the receiving noise power is set to $\sigma_p^2 = \sigma_r^2 = N_0 = -36$ dBW, and $\sigma_s^2 = \sigma_I^2 = N_0 - 3$. The target transmission rate of PU $R_{\text{p,th}}$ is



Fig. 2. Average successful transmission probability of PU vs. noise power with $R_{p,th} = 6$ Mbps.

set to 6 Mbps, and the number of antennas N is 8. The bandwidth Δf and energy conversion efficiency η are 2 MHz and 0.8, respectively.

The transmission channel vectors/variables are generated as $\mathbf{h}_{p,s} = d_{p,s}^{-\alpha/2} \tilde{\mathbf{h}}_{p,s}$, $\mathbf{h}_{s,s} = d_{s,s}^{-\alpha/2} \tilde{\mathbf{h}}_{s,s}$, $\mathbf{h}_{s,p} = d_{s,p}^{-\alpha/2} \tilde{\mathbf{h}}_{s,p}$, and $h_{p,p} = d_{p,p}^{-\alpha/2} \tilde{h}_{p,p}$, where $\tilde{\mathbf{h}}_{p,s}$, $\tilde{\mathbf{h}}_{s,s}$, $\tilde{\mathbf{h}}_{s,p}$, and $\tilde{h}_{p,p}$ are independent identically distributed Rayleigh fading channel vectors/variables; $d_{p,s}$, $d_{s,s}$, $d_{s,p}$, and $d_{p,p}$ are the normalized distances from the PT to ST, from the ST to SR, from the ST to PR, and from the PT to PR, which are set to 5, 5, 6, and 10, respectively; and α is the path loss exponent, which is chosen as 2 [23], [24].

For brevity, the proposed JPSBO-DF/AF algorithms are referred to as "Proposed Scheme for DF" and "Proposed Scheme for AF," and are compared with the following schemes. 1) SWIPT-PSC with CB in [7]: The proposed SWIPT-enabled

- multi-antenna PSC scheme with centralized beamforming.This scheme is limited to AF mode, and requires a large information exchange and more complex signal processing.
- 2) Non-SWIPT-PSC for DF and Non-SWIPT-PSC for AF: ST assists the transmission of the PT using the DF or AF protocol, and transmits its own signal simultaneously through *N* antennas without energy harvesting from the PT.

Figures 2 and 4 depict the performance of these five schemes when noise power N_0 changes. Specifically, Fig. 2 presents the successful transmission probability for the PU, and Fig. 3 elaborates the average achieved transmission rate for the SU. It is observed that, as the channel condition worsens, the successful transmission probability decreases, as does the achievable transmission rate of the SU accordingly. Meanwhile, compared with traditional non-SWIPT-enabled PSC schemes, that is, "Non-SWIPT-PSC for AF" and "Non-SWIPT-PSC for



Fig. 3. Average transmission rate of SU vs. noise power with $R_{\text{p,th}} = 6$ Mbps.

DF," allowing the ST to harvest energy provides an apparent improvement in performance. Taking the DF relay for example, the achieved successful transmission probability for the PU and the transmission rate for the SU increase by about 48% and 52%, respectively, in our proposed schemes when $N_0 = -42$ dBW.

In addition, it was found that the reference scheme, that is, the proposed SWIPT-enabled multi-antenna PSC scheme in [7], consistently achieves a better performance in terms of the successful transmission probability for the PU, whereas the proposed scheme outperforms this in terms of the average achievable transmission rate of the SU. The performance gain of the reference scheme in [7] results from the adopted centralized beamforming at the SU, which supports more flexible signal processing and hence better spatial gain. However, the performance gain is at the cost of a large resource overhead for information exchange. Meanwhile, because the authors only utilize a simple 1-D exhaustive searching method to determine the optimal PS factor, the corresponding computation complexity is prohibitively high. In contrast, the proposed scheme aims at designing a distributed beamforming scheme in which the received signal at each antenna is forwarded independently with only the exchange of channel state information and no data information, resulting in a great reduction in the signal overhead. Meanwhile, our proposed JPSBO algorithms are based on the bisection search and/or Golden search methods, whose complexity is much lower than the exhaustive searching method.

Figures 4 and 5 further plot the average successful transmission probability and the average achieved transmission rate of these five schemes when the target transmission rate of the PU varies. Not surprisingly, along with the increase in $R_{p,th}$, both the successful transmission probability for the PU and the



Fig. 4. Average successful transmission probability of PU vs. trget transmission rate.



Fig. 5. Average transmission rate of SU vs. target transmission rate.

achieved transmission rate for the SU decrease accordingly. In addition, the proposed schemes, which integrate energy harvesting capacity into the ST, provide a significant performance improvement over the traditional PSC scheme. For instance, when $R_{p,th} = 4.5$ Mbps, the average successful transmission probability for the PU and the average transmission rate for the SU of the proposed DF and AF schemes are 99% and 107%, and 34% and 42%, higher than that in the traditional schemes, respectively. Finally, similar to the observations in Figs. 2 and 3, the reference scheme in [7] behaves better in terms of the average successful transmission probability of the PU, and our proposed scheme is superior, providing a higher average transmission rate for the SU.

Moreover, because the PS factor is critical to balancing the information forwarding and energy harvesting, as analyzed in Section IV, we further illustrate the average optimal PS factor obtained in the proposed schemes for different transmit power



Fig. 6. Optimal power splitting ratio vs. target transmission rate.



Fig. 7. Average transmission rate of SU vs. noise power.

settings and different target transmission rates of the PU in Fig. 6. It is shown that, as $R_{p,th}$ increases, more signal energy should be reserved to support a better signal transmission for the PU, that is, a larger PS factor is adopted at the ST. Meanwhile, when the PT has more transmit power, the received RF signal at the ST is of high energy. Hence, the PS factor should be reduced, and the ST can harvest more energy without deteriorating the signal reception (DF) or signal forwarding (AF) of the PU. On the contrary, when the transmit power at the ST varies, the PS factor in DF mode remains unchanged because the same portion of signal energy is required to guarantee the successful signal reception in the first hop. In addition, for AF mode, a higher transmit power at the ST implies a smaller PS factor, indicating that more energy is harvested at the ST. This is because the PS factor is set as $\beta = 1$ in our simulation when the target transmission rate of the PU is unsatisfied. At the same time, $P_2 < P_1$, and the second hop is a bottleneck. As a result, when the ST has a small transmit power, the probability that the target transmission rate of the PU is

satisfied is reduced, leading to a larger probability of $\beta = 1$ and hence a larger average PS factor, β .

Finally, we demonstrate the performance of the proposed schemes when different antennas are deployed at the ST, as shown in Fig. 7. Owing to spatial limitations, only the average achievable transmission rate of the SU is depicted. Because an ST equipped with more antennas can receive more RF energy and provide a large spatial gain, the performance achieved should increase accordingly along with the increase in deployed antennas. This coincides perfectly with the simulation results in Fig. 7.

VI. Conclusions

This paper investigated the power splitting and beaming optimization for multi-antenna primary-secondary cooperation in a cognitive radio network. Concentrating on maximizing the data transmission rate of an SU and guaranteeing the target transmission rate of a PU, the optimization problems in DF and AF modes were formulated. Through transforming the formulated problems into contrapositive power minimization problems, the problems are solved using a dual method. Joint power splitting and beamforming algorithms were derived through monotonicity and quasi-convexity/quasi-concavity analyses. Simulations show that the scheme with an energy harvesting function is superior to the scheme with only a relay forwarding function.

Appendix

1. Appendix A: Proof of Theorem 1

First, it is easy to see that the feasible region of $\mathcal{OP}_{2|AF}$ decreases along with an increase in $z_{s,th}$. Hence, the minimum value of the objective function, that is, $f_{AF}(\beta, z_{s,th})$, is monotonically increasing with $z_{s,th}$.

Second, modify problem $\mathcal{OP}_{2|AF}$ as

$$\begin{aligned} \mathcal{OP}_{3|AF}: & \min_{\{\mathbf{w}_{3},\mathbf{w}_{1},\mathbf{w}_{2}\}} P_{1} \|\mathbf{B}_{2}\mathbf{w}_{3}\|^{2} + \sigma_{s}^{2} \|\mathbf{w}_{3}\|^{2} + \sigma_{I}^{2} \|\mathbf{w}_{1}\|^{2} + \|\mathbf{w}_{2}\|^{2} \\ & \mathcal{C}_{2}': P_{1}|\mathbf{h}_{s,p}^{H}\mathbf{B}_{2}\mathbf{w}_{3}|^{2} \geq \tilde{z}_{p,h} \left[\sigma_{s}^{2}|\mathbf{h}_{s,p}^{H}\mathbf{w}_{3}|^{2} + \sigma_{I}^{2}|\mathbf{h}_{s,p}^{H}\mathbf{w}_{1}|^{2} \\ & + |\mathbf{h}_{s,p}^{H}\mathbf{w}_{2}|^{2} + \sigma_{P}^{2}\right], \\ & \mathcal{C}_{3}': |\mathbf{h}_{s,s}^{H}\mathbf{w}_{2}|^{2} \geq z_{s,h} \left[\sigma_{s}^{2}|\mathbf{h}_{s,s}^{H}\mathbf{w}_{3}|^{2} + \sigma_{I}^{2}|\mathbf{h}_{s,p}^{H}\mathbf{w}_{1}|^{2} \\ & + P_{1}|\mathbf{h}_{s,s}^{H}\mathbf{B}_{2}\mathbf{w}_{3}|^{2} + \sigma_{I}^{2}\right], \\ & \mathcal{C}_{4}: \mathbf{w}_{2} = \sqrt{\beta}\mathbf{w}_{1}, \end{aligned}$$

and define the minimum value of problem $\mathcal{OP}_{3|AF}$ as $\tilde{f}_{AF}(\beta, z_{s,th})$. It is then found that

$$f_{\rm AF}(\beta, z_{\rm s,th}) = \tilde{f}_{\rm AF}(\beta, z_{\rm s,th}) - \eta (1 - \beta) (P_1 | \mathbf{h}_{\rm p,s} |^2 + N \sigma_s^2) .$$
(41)

Meanwhile, because $\mathcal{OP}_{3|AF}$ can be transformed into a cone programming problem, its dual gap is zero. Define the Lagrangian multipliers λ , μ , and a_n for constraints \mathcal{C}'_2 , \mathcal{C}'_3 , and \mathcal{C}_4 , respectively, and let $\mathbf{x} = [\lambda, \mu, a_1, a_2, \dots, a_N]$. It can then be derived that

$$\begin{split} \hat{f}_{AF}(\beta, z_{s,th}) &= \max_{\mathbf{x}} \min_{\mathbf{w}_{3}, \mathbf{w}_{1}, \mathbf{w}_{2}} g(\beta, z_{s,th}, \mathbf{x}, \mathbf{w}_{3}, \mathbf{w}_{1}, \mathbf{w}_{2}) \text{ and } (42) \\ g(\beta, z_{s,th}, \mathbf{x}, \mathbf{w}_{3}, \mathbf{w}_{1}, \mathbf{w}_{2}) \\ &= P_{1} ||\mathbf{B}_{2} \mathbf{w}_{3}||^{2} + \sigma_{s}^{2} ||\mathbf{w}_{3}||^{2} + \sigma_{l}^{2} ||\mathbf{w}_{1}||^{2} + ||\mathbf{w}_{2}||^{2} \\ &- \lambda \Big[P_{1} |\mathbf{h}_{s,p}^{H} \mathbf{B}_{2} \mathbf{w}_{3}|^{2} - \tilde{z}_{p,th} \sigma_{s}^{2} |\mathbf{h}_{s,p}^{H} \mathbf{w}_{3}|^{2} \\ &- \tilde{z}_{p,th} \sigma_{l}^{2} |\mathbf{h}_{s,p}^{H} \mathbf{w}_{1}|^{2} - \tilde{z}_{p,th} |\mathbf{h}_{s,p}^{H} \mathbf{w}_{2}|^{2} - \tilde{z}_{p,th} \sigma_{p}^{2} \Big], \quad (43) \\ &- \mu \Big[|\mathbf{h}_{s,s}^{H} \mathbf{w}_{2}|^{2} - z_{s,th} \sigma_{s}^{2} |\mathbf{h}_{s,s}^{H} \mathbf{w}_{3}|^{2} - z_{s,th} \sigma_{l}^{2} |\mathbf{h}_{s,p}^{H} \mathbf{w}_{1}|^{2} \\ &- z_{s,th} P_{1} |\mathbf{h}_{s,s}^{H} \mathbf{B}_{2} \mathbf{w}_{3}|^{2} - z_{s,th} \sigma_{r}^{2} \Big] \\ &+ \sum_{n=1}^{N} a_{n} \bigg(\frac{\mathbf{w}_{3}[n]}{\mathbf{w}_{1}[n]} - \sqrt{\beta} \bigg), \end{split}$$

where $\mathbf{w}_1[n]$ and $\mathbf{w}_3[n]$ denote the *n*-th elements of \mathbf{w}_1 and \mathbf{w}_3 , respectively.

Let $\mathbf{w}_3(\mathbf{x})$, $\mathbf{w}_1(\mathbf{x})$, and $\mathbf{w}_2(\mathbf{x})$ denote the optimal \mathbf{w}_3 , \mathbf{w}_1 , and \mathbf{w}_2 with dual variables $\mathbf{x} = [\lambda, \mu, a_1, a_2, ..., a_N]$. Then, (42) can be further written as

$$\hat{f}_{AF}(\boldsymbol{\beta}, z_{s,th}) = \max_{\mathbf{x}} g\left(\boldsymbol{\beta}, z_{s,th}, \mathbf{x}, \mathbf{w}_{3}(\mathbf{x}), \mathbf{w}_{1}(\mathbf{x}), \mathbf{w}_{2}(\mathbf{x})\right)$$

= max $\tilde{g}(\boldsymbol{\beta}, z_{s,th}, \mathbf{x}).$ (44)

For $\forall \beta_1 < \beta_2, \ \theta \in [0,1], \beta_3 = \theta \beta_1 + (1-\theta)\beta_2$, we have

$$f_{AF}(\beta_{3}, z_{s,th}) = \max_{\mathbf{x}} \tilde{g}(\beta_{3}, z_{s,th}, \mathbf{x})$$

$$\leq \max_{\mathbf{x}} \max\left\{ \tilde{g}(\beta_{1}, z_{s,th}, \mathbf{x}), \tilde{g}(\beta_{2}, z_{s,th}, \mathbf{x}) \right\}$$

$$= \max\left\{ \max_{\mathbf{x}} \tilde{g}(\beta_{1}, z_{s,th}, \mathbf{x}), \max_{\mathbf{x}} \tilde{g}(\beta_{2}, z_{s,th}, \mathbf{x}) \right\},$$

$$= \max\left\{ \tilde{f}_{AF}(\beta_{1}, z_{s,th}), \tilde{f}_{AF}(\beta_{2}, z_{s,th}) \right\}.$$
(45)

where the inequality is due to $\tilde{g}(\beta, z_{s,th}, x)$ being quasiconvex about β .

Therefore, $\tilde{f}_{AF}(\beta, z_{s,th})$ is also quasi-convex about β . Finally, according to (41), $f_{AF}(\beta, z_{s,th})$ is also a quasi-convex function about β .

2. Appendix B: Proof of Theorem 2

First, $F(\beta)$ can be written as

$$F(\beta) = \max\{x \mid f_{AF}(\beta, x) \le P_2\}.$$
 (46)

In addition, it can be seen that $f_{AF}(\beta, F(\beta)) \le P_2$. Then, for $\beta_i, F(\beta_i), i = 1, 2$, and $\theta \in [0, 1], \beta_3 = \theta \beta_1 + \theta \beta_1$

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 $(1-\theta)\beta_2$. Let $z_0 = \min_{i=1,2} F(\beta_i)$, and we have $f_{AF}(\beta_i, z_0) \le f_{AF}[\beta_i, F(\beta_i)] \le P_2$ for i = 1, 2 because $f_{AF}(\beta_i, z)$ monotonically increases with z. Thus, we have

$$\max_{i=1,2} \left\{ f_{\rm AF}(\beta_i, z_0) \right\} \le P_2 \,. \tag{47}$$

Moreover, because for a fixed $z_{s,th}$, $f_{AF}(\beta, z_{s,th})$ is quasiconvex about variable β , we have

$$f_{\rm AF}\left((\theta\beta_1 + (1-\theta)\beta_2), z_0\right) \le \max_{i=1,2} \left\{ f_{\rm AF}(\beta_i, z_0) \right\}.$$
 (48)

Combine (47) with (48), we can obtain $f_{AF}((\theta\beta_1 + (1-\theta)\beta_2), z_0) \le P_2$. Therefore, $F(\beta_3) \ge z_0$ and $F(\beta)$ is a quasi-concave function of β .

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References

- J. Mitola and GQ. Jr. Maguire, "Cognitive Radio: Making Software Radios More Personal," *IEEE Pers. Commun.*, vol. 6, no. 4, Aug. 1999, pp. 13–18.
- [2] J.A. Paradiso and T. Starner, "Energy Scavenging for Mobile and Wireless Electronics," *IEEE Pervasive Comput.*, vol. 4, no. 1, Jan. 2005, pp. 18–27.
- [3] A. Nosratinia, T.E. Hunter, and A. Hedayat, "Cooperative Communication in Wireless Networks," *IEEE Commun. Mag.*, vol. 42, no. 10, Oct. 2004, pp. 74–80.
- [4] Q. Shi et al., "Joint Transmit Beamforming and Receive Power Splitting for MISO SWIPT Systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 6, June 2014, pp. 3269–3280.
- [5] Y. Huang and B. Clerckx, "Joint Wireless Information and Power Transfer for an Autonomous Multiple Antenna Relay System," *IEEE Commun. Lett.*, vol. 19, no. 7, July 2015, pp. 1113–1116.
- [6] Z. Wang et al., "Wireless Energy Harvesting and Information Transfer in Cognitive Two-Way Relay Networks," *IEEE Global Commun. Conf.*, Austin, TX, USA, Dec. 2014, pp. 3465–3470.
- [7] G Zheng et al., "Information and Energy Cooperation in Cognitive Radio Networks," *IEEE Trans. Signal Process.*, vol. 62, no. 9, May 2014, pp. 2290–2303.
- [8] W. Lu and J. Wang, "Opportunistic Spectrum Sharing Based on Full-Duplex Cooperative OFDM Relaying," *IEEE Commun. Lett.*, vol. 18, no. 2, Feb. 2014, pp. 241–244.
- [9] Y. Zhou et al., "Two-Stage Cooperative Multicast Transmission with Optimized Power Consumption and Guaranteed Coverage,"

IEEE J. Sel. Area Commun., vol. 32, no. 2, 2013, pp. 274-284.

- [10] Q, Wu, J. Zou, and Z. Wu, "Joint Spectrum and Power Allocation in Coded Cooperative Cognitive Radio Networks," *IEEE Int. Conf. Commun.*, Sydney, Australia, June 2014, pp. 1596–1601.
- [11] J. Zou et al., "Joint Spectrum and Power Auction with Multiauctioneer and Multibidder in Coded Cooperative Cognitive Radio Networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, Oct. 2014, pp. 5768–5780.
- [12] F. Gabry et al., "On the Optimization of the Secondary Transmitter's Strategy in Cognitive Radio Channels with Secrecy," *IEEE J. Sel. Area Commun.*, vol. 32, no. 3, Mar. 2013, pp. 451–463.
- [13] Z. Liu et al., "Network-Coded Primary-Secondary Cooperation in OFDM-Based Cognitive Multicast Networks," *EURASIP J. Wireless Commun. Netw.*, vol. 2015, no. 1, 2015, pp. 144–156.
- [14] Z. Chu, M. Johnston, and S.L. Goff, "SWIPT for Wireless Cooperative Networks," *Electron. Lett.*, vol. 51, no. 6, Mar. 2015, pp. 536–538.
- [15] H. Chen et al., "Distributed Power Splitting for SWIPT in Relay Interference Channels Using Game Theory," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, Jan. 2014, pp. 410–420.
- [16] Z. Zhou et al., "Joint Power Splitting and Antenna Selection in Energy Harvesting Relay Channels," *IEEE Signal Process. Lett.*, vol. 22, no. 7, July 2015, pp. 823–827.
- [17] J. Xiao et al., "Robust Transceiver Design for Two-User MIMO Interference Channel with Simultaneous Wireless Information and Power Transfer," *IEEE Trans. Veh. Technol.*, vol. 65, no. 5, May 2016, pp. 3823–3828.
- [18] Y. Zeng and R. Zhang, "Full-Duplex Wireless-Powered Relay With Self-Energy Recycling," *IEEE Wireless Commun. Lett.*, vol. 4, no. 2, Apr. 2015, pp. 201–204.
- [19] K. Tutuncuoglu, B. Varan, and A. Yener, "Throughput Maximization for Two-way Relay Channels with Energy Harvesting Nodes: The Impact of Relaying Strategies," *IEEE Trans. Commun.*, vol. 63, no. 6, June 2015, pp. 2081–2093.
- [20] R.V. Driessche and D. Roose, "An Improved Spectral Bisection Algorithm and its Application to Dynamic Load Balancing," *Parallel Comput.*, vol. 21, no. 1, Jan. 1995, pp. 29–48.
- [21] C. He, Y.F. Zheng, and S.C. Ahalt, "Object Tracking Using the Gabor Wavelet Transform and the Golden Section Algorithm," *IEEE Trans. Multimedia*, vol. 4, no. 4, Dec. 2002, pp. 528–538.
- [22] G Zheng et al., "Cooperative Cognitive Networks: Optimal, Distributed and Low-Complexity Algorithms," *IEEE Trans. Signal Process.*, vol. 61, no. 11, June 2013, pp. 2778–2790.
- [23] K. Xiong et al., "Wireless Information and Energy Transfer for Two-Hop Non-regenerative MIMO-OFDM Relay Networks," *IEEE J. Sel. Area Commun.*, vol. 33, no. 8, 2015, pp. 1595–1611.
- [24] S. Li et al., "Independent Power Splitting for Interference-Corrupted SIMO SWIPT Systems," *IEEE Commun. Lett.*, vol. 20, no. 3, Mar. 2016, pp. 478–481.

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