# Antenna Selection Schemes in Quadrature Spatial Modulation Systems 

Sangchoon Kim

This paper presents antenna selection schemes for recently proposed quadrature spatial modulation (QSM) systems. The antenna selection strategy is based on Euclidean distance optimized antenna selection (EDAS). The symbol error rate (SER) performance of these schemes is compared with that of the corresponding algorithm associated with spatial modulation (SM) systems. It is shown through simulations that QSM systems using EDAS offer significant improvement in terms of SER performance over SM systems with EDAS. Their SER performance gains are seen to be about 2 dB 4 dB in $E_{5} / N_{0}$ values.

Keywords: Quadrature spatial modulation (QSM), spatial modulation (SM), antenna selection, Euclidean distance antenna selection (EDAS).

[^0]
## I. Introduction

A spatial modulation (SM) transmission technique is a recently proposed single-RF multiple input multiple output method [1]-[3]. Because only one transmit antenna from $N_{\mathrm{T}}$ transmit antennas is activated for the transmission of a symbol during a single symbol time, no synchronization between transmit antennas is necessary, and inter-channel interference at the receiver can be avoided. Transmit antennas are considered to have a spatial constellation dimension to convey additional information bits. The overall spectral efficiency is therefore proportional to $\log _{2} N_{\mathrm{T}}$. In [4], a new approach to improving the overall spectral efficiency of SM technology, called quadrature spatial modulation (QSM), was presented. In QSM, the spatial constellation symbols with in-phase and quadrature components are transmitted through two spatial dimensions. Thus, two spatial dimensions can carry $2 \log _{2} N_{\mathrm{T}}$ information bits.
Transmit antenna selection techniques have recently been developed to enhance the reliability of SM systems [5]-[9]. In [5] and [6], Euclidean distance optimized antenna selection schemes with low-complexity (SM-EDAS-LC) were presented. These schemes can achieve the same symbol error rate (SER) performance as EDAS with exhaustive search (SM-EDAS-ES) with low computational complexity. In [7], maximizing the instantaneous minimum squared Euclidean distance has been used based on singular value decomposition as a suboptimal antenna selection approach. This method has led to some performance degradation in comparison to an optimal SM-EDAS-ES algorithm. In [10], the achievable transmit diversity order of SM systems with EDAS-based antenna selection has been quantified. Although such researches on antenna selection for SM systems have been
conducted recently, to the best of our knowledge, there have been no investigations on antenna selection for QSM systems. The previous antenna selection algorithms for SM systems were unable to be directly applied to QSM systems because QSM systems exploit two spatial dimensions consisting of in-phase and quadrature parts for transmitting a signal constellation symbol. Thus, antenna selection schemes appropriate for QSM systems should be considered. The main contribution of this work is to present the first antenna selection schemes for QSM systems, which deals with transmit antenna selection methods for QSM systems not yet available in the literature to achieve transmit diversity gains. EDAS, which was proposed for SM systems, was extended to QSM systems. Furthermore, low-complexity QSM algorithms that trade performance for complexity are proposed. The SER performance of the proposed algorithms and their lowcomplexity counterparts are compared along with their computational complexities. Finally, we compare their performance and complexity with those of EDAS-ES in SM systems.

## II. System Model

A QSM system with $N_{\mathrm{T}}$ transmit antennas and $N_{\mathrm{R}}$ receive antennas is considered. An $N_{\mathrm{R}} \times 1$ received signal vector can then be expressed as

$$
\begin{equation*}
\mathbf{y}=\mathbf{h}_{l_{R}} s_{\mathrm{R}}+j \mathbf{h}_{l_{Q}} s_{\mathrm{Q}}+\mathbf{w}, l_{R}, l_{Q}=1,2, \ldots, N_{\mathrm{T}}, \tag{1}
\end{equation*}
$$

where $s_{\mathrm{R}}$ and $s_{\mathrm{Q}}$ are the real and imaginary parts of a QAM symbol, respectively, and $\mathbf{w}=\left[w_{1} w_{2}, \ldots, w_{N_{\mathrm{R}}}\right]^{\mathrm{T}}$ is an $N_{\mathrm{R}} \times 1$ complex additive white Gaussian noise vector with independent and identically distributed (i.i.d.) entries $w_{p} \sim C N$ $\left(0, N_{0}\right), p=1,2, \ldots, N_{\mathrm{R}}$, where $N_{0}$ is the noise variance. In addition, $\mathbf{h}_{l_{R}}$ and $\mathbf{h}_{l_{Q}}$ denote the $l_{R}$ th and $l_{Q}$ th column vectors of an $N_{\mathrm{R}} \times N_{\mathrm{T}}$ channel gain matrix $\mathbf{H}$ given by $\mathbf{H}=\left[\mathbf{h}_{1}, \mathbf{h}_{2}, \ldots, \mathbf{h}_{N_{\mathrm{T}}}\right]$, respectively. The elements of $\mathbf{H}$ are i.i.d. random variables with a circularly symmetric complexvalued Gaussian distribution $C N(0,1)$.

## III. Euclidean Distance-Based Antenna Selection for QSM Systems

## 1. EDAS for QSM Systems

To select $N_{\mathrm{S}}\left(<N_{\mathrm{T}}\right)$ antennas out of $N_{\mathrm{T}}$ transmit antennas, EDAS with an exhaustive search finds the specific antenna subset that maximizes the minimum Euclidean distance among all possible transmit symbol vectors, and can be defined as
$I_{\mathrm{ED}}$
$=\underset{I \in \Gamma}{\arg \max }\left\{\min _{\mathbf{x}_{1} \neq \mathbf{x}_{2} \in X}\left\|\mathbf{H}_{I}\left(\mathbf{x}_{1 R}-\mathbf{x}_{2 R}\right)+j \mathbf{H}_{I}\left(\mathbf{x}_{1 Q}-\mathbf{x}_{2 Q}\right)\right\|_{F}^{2}\right\}$,
where $\Gamma$ represents the set of enumerations of all possible $N_{\text {search }}=C_{N_{\mathrm{S}}}^{N_{\mathrm{T}}}$ combinations, in which $C_{N_{\mathrm{S}}}^{N_{\mathrm{T}}}$ denotes the number of $N_{\mathrm{S}}$ combinations from $N_{\mathrm{T}}$ elements. In addition, $\mathbf{H}_{I}$ is the $N_{\mathrm{R}} \times N_{\mathrm{S}}$ channel gain matrix based on the Ith enumeration of the set $\Gamma$, and $X$ is the set of all possible QSM transmit symbol vectors denoted by $\mathbf{e}_{k} s$, in which $\mathbf{e}_{k}$, for $k=1,2, \cdots, N_{\mathrm{S}}$, is an $N_{\mathrm{S}} \times 1$ vector with 1 as a single non-zero element at the $k$ th location, and $s$ is a transmit symbol from symbol set $S$.
The EDAS of (2) can be written as

$$
\begin{equation*}
I_{\mathrm{ED}}=\underset{I \in \Gamma}{\arg \max }\{\min \mathbf{D}(I)\}, \tag{3}
\end{equation*}
$$

where $\mathbf{D}(I)$ is an $N_{\mathrm{S}} \times N_{\mathrm{S}}$ matrix computed by an $N_{\mathrm{R}} \times N_{\mathrm{S}}$ matrix $\quad \mathbf{H}_{I}=\left[\mathbf{h}_{(1)}, \mathbf{h}_{(2)}, \ldots, \mathbf{h}_{\left(N_{\mathrm{s}}\right)}\right]$ obtained by eliminating the columns that are not present in $I$. Here, $\mathbf{h}_{(t)}$, for $t=1,2, \ldots, N_{\mathrm{S}}$, is the $t$ th column vector of $\mathbf{H}_{I}$ associated with the $I$ th enumeration of the set $\Gamma$. For $m=n$, the $(m, n)$ th element of $\mathbf{D}(I)$ can be obtained as

$$
\begin{align*}
& D_{m, n}(I)= \\
& \quad \min _{\substack{s_{1 \mathrm{R}} \neq s_{2 \mathrm{R}} \in \operatorname{Re}(S) \\
s_{1 \mathrm{Q}}, s_{2 Q} \in \operatorname{Im}(S) \\
m_{R}(=m)=n_{R}(=n) \\
m_{Q}=1,2, \ldots, N_{\mathrm{s}} \\
n_{Q}=m_{Q}, m_{Q}+1, \ldots, N_{\mathrm{S}}}}\left\|\mathbf{h}_{\left(m_{R}\right)} S_{1 \mathrm{R}}+j \mathbf{h}_{\left(m_{Q}\right)} S_{1 \mathrm{Q}}-\mathbf{h}_{\left(n_{R}\right)} S_{2 \mathrm{R}}-j \mathbf{h}_{\left(n_{Q}\right)} S_{2 \mathrm{Q}}\right\|_{F}^{2},
\end{align*}
$$

where $m=m_{R}=1,2, \ldots, N_{\mathrm{S}}$, and $s_{\mathrm{R}}$ and $s_{i \mathrm{Q}}$ denote real and imaginary parts of a symbol, $s_{i}$, respectively, in which $i=1,2$, from the symbol set $S$. For $m<n, D_{m, n}(I)$ can be written as

$$
\begin{align*}
& D_{m, n}(I)= \\
& \quad \min _{\substack{s_{1 \mathrm{R}}, s_{2 \mathrm{R}} \in \operatorname{Re}(S) \\
s_{1 \mathrm{Q}}, s_{2 Q} \in \operatorname{Im}(S) \\
m_{R}(=m) \neq n_{R}(=n) \\
m_{Q}=1,2, \ldots, N_{\mathrm{S}} \\
n_{Q}=m_{Q}, m_{Q}+1, \ldots, N_{\mathrm{S}}}}\left\|\mathbf{h}_{\left(m_{R}\right)} s_{1 \mathrm{R}}+j \mathbf{h}_{\left(m_{Q}\right)} s_{1 \mathrm{Q}}-\mathbf{h}_{\left(n_{R}\right)} s_{2 \mathrm{R}}-j \mathbf{h}_{\left(n_{Q}\right)} s_{2 \mathrm{Q}}\right\|_{F}^{2},
\end{align*}
$$

where $\quad m=m_{R}=1,2, \ldots, N_{\mathrm{S}}, \quad m_{Q}=1,2, \ldots, N_{\mathrm{S}}, \quad n=n_{R}=$ $m+1, m+2, \ldots, N_{\mathrm{s}}$, and $n_{Q}=m_{Q}, m_{Q}+1, \ldots, N_{\mathrm{s}}$.
Note that $\mathbf{D}(I)$ in QSM is computed in each enumeration of all possible combinations. For a comparison of the computational complexity, this work follows a complexity analysis similar to that performed in floating point operations [7]. Denoting the signal constellation size as $M$, an exhaustive search approach for EDAS in QSM systems (QSM-EDASES) requires the following approximate computational complexity in all of the flops.

$$
\begin{equation*}
C_{\mathrm{QSM}-\mathrm{EDAS}-\mathrm{ES}}=C_{N_{\mathrm{s}}}^{N_{\mathrm{T}}}\left(C_{2}^{N_{\mathrm{S}}}+N_{\mathrm{S}}\right)^{2}\left(12 N_{\mathrm{R}}+2\right) M^{2} . \tag{6}
\end{equation*}
$$

Here, instead of complex multiplications, real multiplications are reflected in the complexity.

## 2. EDAS with Reduced Complexity

EDAS-ES for QSM systems can be modified to reduce the computational complexity, which is called QSM-EDAS-LC in this work. The computation of $D_{m, n}(I)$ in (4) and (5) can be rewritten as
where $\mathbf{s}=\left[\begin{array}{lll}s_{1 \mathrm{R}} & s_{1 \mathrm{Q}} & -s_{2 \mathrm{R}}\end{array}-s_{2 \mathrm{Q}}\right]^{\mathrm{T}}$ and $\tilde{\boldsymbol{\Xi}}_{(m, n)}=\boldsymbol{\Xi}_{(m, n)}^{H} \boldsymbol{\Xi}_{(m, n)} \quad$ is a $4 \times 4$ matrix whose $(u, v)$ th element is denoted by $\tilde{h}_{u v}^{(m, n)}, \quad u$, $v=1,2,3,4$ with $\tilde{h}_{u v}^{(m, n)}=\tilde{h}_{v u}^{(m, n)}, u \neq v$. That is,

$$
\tilde{\boldsymbol{\Xi}}_{(m, n)}=\left[\begin{array}{llll}
\tilde{h}_{11}^{(m, n)} & \tilde{h}_{12}^{(m, n)} & \tilde{h}_{13}^{(m, n)} & \tilde{h}_{14}^{(m, n)}  \tag{9}\\
\tilde{h}_{21}^{(m, n)} & \tilde{h}_{22}^{(m, n)} & \tilde{h}_{23}^{(m, n)} & \tilde{h}_{24}^{(m, n)} \\
\tilde{h}_{31}^{(m, n)} & \tilde{h}_{32}^{(m, n)} & \tilde{h}_{33}^{(m, n)} & \tilde{h}_{34}^{(m, n)} \\
\tilde{h}_{41}^{(m, n)} & \tilde{h}_{42}^{(m, n)} & \tilde{h}_{43}^{(m, n)} & \tilde{h}_{44}^{(m, n)}
\end{array}\right] .
$$

Here, the modified channel matrix $\boldsymbol{\Xi}_{(m, n)}$ is defined as

$$
\boldsymbol{\Xi}_{(m, n)}=\left[\begin{array}{cccc}
\mathbf{h}_{\left(m_{R}\right) R} & -\mathbf{h}_{\left(m_{Q}\right) Q} & \mathbf{h}_{\left(n_{R}\right) R} & -\mathbf{h}_{\left(n_{Q}\right) Q}  \tag{10}\\
\mathbf{h}_{\left(m_{R}\right) Q} & \mathbf{h}_{\left(m_{Q}\right) R} & \mathbf{h}_{\left(n_{R}\right) Q} & \mathbf{h}_{\left(n_{Q}\right) R}
\end{array}\right],
$$

where $\mathbf{h}_{\left(m_{R}\right) R}=\operatorname{Re}\left(\mathbf{h}_{\left(m_{R}\right)}\right), \quad \mathbf{h}_{\left(m_{R}\right) Q}=\operatorname{Im}\left(\mathbf{h}_{\left(m_{R}\right)}\right), \quad \mathbf{h}_{\left(n_{R}\right) R}=$ $\operatorname{Re}\left(\mathbf{h}_{\left(n_{R}\right)}\right), \quad \mathbf{h}_{\left(n_{R}\right) Q}=\operatorname{Im}\left(\mathbf{h}_{\left(n_{R}\right)}\right), \quad \mathbf{h}_{\left(m_{Q}\right) R}=\operatorname{Re}\left(\mathbf{h}_{\left(m_{Q}\right)}\right)$, $\mathbf{h}_{\left(m_{Q}\right) Q}=\operatorname{Im}\left(\mathbf{h}_{\left(m_{Q}\right)}\right), \quad \mathbf{h}_{\left(n_{Q}\right) R}=\operatorname{Re}\left(\mathbf{h}_{\left(n_{Q}\right)}\right)$, and $\quad \mathbf{h}_{\left(n_{Q}\right) Q}=$ $\operatorname{Im}\left(\mathbf{h}_{\left(n_{Q}\right)}\right)$ for $m_{R}, m_{Q}=1,2, \ldots, N_{\mathrm{S}}, n_{R}=m_{R}, m_{R}+1, \ldots, N_{\mathrm{S}}$, and $n_{Q}=m_{Q}, m_{Q}+1, \cdots, N_{\mathrm{S}}$.

Thus, (7) and (8) can be computed as

$$
\begin{align*}
D_{m, n}(I)= & \min _{\substack{s_{12}, s_{2} \in \operatorname{Re}(S) \\
s_{1 Q}, s_{2 \Omega} \in \operatorname{Im}(S)}} \tilde{h}_{11}^{(m, n)}\left(s_{1 R}\right)^{2} \\
& +2 s_{1 R}\left(\tilde{h}_{12}^{(m, n)} s_{1 Q}-\tilde{h}_{13}^{(m, n)} s_{2 R}-\tilde{h}_{14}^{(m, n)} s_{2 Q}\right)  \tag{11}\\
& +\tilde{h}_{22}^{(m, n)}\left(s_{1 Q}\right)^{2}-2 s_{1 Q}\left(\tilde{h}_{23}^{(m, n)} s_{2 R}+\tilde{h}_{24}^{(m, n)} s_{2 Q}\right) \\
& +\tilde{h}_{33}^{(m, n)}\left(s_{2 R}\right)^{2}+2 \tilde{h}_{34}^{(m, n)} s_{2 R} s_{2 Q}+\tilde{h}_{44}^{(m, n)}\left(s_{2 Q}\right)^{2} .
\end{align*}
$$

Upon conditioning $s_{2 R}$ and $s_{2 \mathrm{Q}}$, (11) can be expressed as

$$
\begin{align*}
D_{m, n}(I)= & \tilde{h}_{33}^{(m, n)}\left(s_{2 \mathrm{R}}\right)^{2}+2 \tilde{h}_{34}^{(m, n)} s_{2 \mathrm{R}} s_{2 \mathrm{Q}}+\tilde{h}_{44}^{(m, n)}\left(s_{2 \mathrm{Q}}\right)^{2} \\
& -\frac{\left(\tilde{h}_{23}^{(m, n)} s_{2 R}+\tilde{h}_{24}^{(m, n)} s_{2 \mathrm{Q}}\right)^{2}}{\tilde{h}_{22}^{(m, n)}} \\
& -\frac{\left(-\tilde{h}_{12}^{(m, n)} s_{1 \mathrm{Q}}+\tilde{h}_{13}^{(m, n)} s_{2 \mathrm{R}}+\tilde{h}_{14}^{(m, n)} s_{2 \mathrm{Q}}\right)^{2}}{\tilde{h}_{11}^{(m, n)}} \\
& +\min _{\substack{s_{1 \mathrm{R}} \mathrm{Re}(S) \\
s_{1} \in \operatorname{Im}(S)}}\left\{\tilde{h}_{11}^{(m, n)}\left(s_{1 \mathrm{R}}-g_{1}\right)^{2}+\tilde{h}_{22}^{(m, n)}\left(s_{1 \mathrm{Q}}-g_{2}\right)^{2}\right\}, \tag{12}
\end{align*}
$$

where

$$
\begin{gather*}
g_{1}=\frac{-\tilde{h}_{12}^{(m, n)} s_{1 \mathrm{Q}}+\tilde{h}_{13}^{(m, n)} s_{2 \mathrm{R}}+\tilde{h}_{14}^{(m, n)} s_{2 \mathrm{Q}}}{\tilde{h}_{11}^{(m, n)}} \text { and }  \tag{13}\\
g_{2}=\frac{\tilde{h}_{23}^{(m, n)} s_{2 \mathrm{R}}+\tilde{h}_{24}^{(m, n)} s_{2 \mathrm{Q}}}{\tilde{h}_{22}^{(m, n)}} . \tag{14}
\end{gather*}
$$

Thus, the estimates of $s_{1 \mathrm{Q}}$ and $s_{1 \mathrm{R}}$ are obtained by

$$
\begin{align*}
\hat{s}_{1 \mathrm{Q}} & =F_{\mathrm{Q}}\left(g_{2}\right)=F_{\mathrm{Q}}\left(\frac{\tilde{h}_{23}^{(m, n)} s_{2 \mathrm{R}}+\tilde{h}_{24}^{(m, n)} s_{2 \mathrm{Q}}}{\tilde{h}_{22}^{(m, n)}}\right) \text { and }  \tag{15}\\
\hat{s}_{1 \mathrm{R}} & =F_{\mathrm{R}}\left(g_{1}\right) \\
& =F_{\mathrm{R}}\left(\frac{-\tilde{h}_{12}^{(m, n)} F_{\mathrm{Q}}\left(g_{2}\right)+\tilde{h}_{13}^{(m, n)} s_{2 \mathrm{R}}+\tilde{h}_{14}^{(m, n)} s_{2 \mathrm{Q}}}{\tilde{h}_{11}^{(m, n)}}\right), \tag{16}
\end{align*}
$$

where $F_{\mathrm{Q}}\left(g_{2}\right)$ and $F_{\mathrm{R}}\left(g_{1}\right)$ denote the function demodulating $g_{2}$ and $g_{1}$ to the nearest point of $\operatorname{Im}(S)$ and $\operatorname{Re}(S)$, respectively. Symbol $s_{1}$ is then obtained by $s_{1}=F_{\mathrm{R}}\left(g_{1}\right)+j F_{\mathrm{Q}}\left(g_{2}\right)$. One of the differences between the proposed algorithm and the method in [6] is that QSM-EDAS-LC does not require $Q R$ decomposition of the modified channel matrix and requires two spatial dimensional searches. Another difference is that the estimate of $s_{1 \mathrm{R}}$ in QSM-EDAS-LC should be calculated after the estimate of $s_{1 \mathrm{Q}}$ is obtained.

Then, using $\hat{s}_{1 \mathrm{R}}$ and $\hat{s}_{1 \mathrm{Q}}$, the computation of $D_{m, n}(I)$ in (7) and (8) can be equivalently defined respectively as
where

$$
\begin{align*}
\hat{\mathbf{s}}^{T} \tilde{\boldsymbol{\Xi}}_{(m, n)} \hat{\mathbf{s}} & =\left\|\mathbf{h}_{\left(m_{R}\right)} \hat{s}_{1 \mathrm{R}}+j \mathbf{h}_{\left(m_{Q}\right)} \hat{s}_{1 \mathrm{Q}}-\mathbf{h}_{\left(n_{R}\right)} s_{2 \mathrm{R}}-j \mathbf{h}_{\left(n_{Q}\right)} s_{2 \mathrm{Q}}\right\|_{F}^{2},  \tag{19}\\
\hat{\mathbf{s}} & =\left[\begin{array}{llll}
\hat{s}_{1 \mathrm{R}} & \hat{s}_{1 \mathrm{Q}} & -s_{2 \mathrm{R}} & -s_{2 \mathrm{Q}}
\end{array}\right]^{\mathrm{T}} . \tag{20}
\end{align*}
$$

Thus, the complexity of QSM-EDAS-LC is approximated by

$$
\begin{equation*}
C_{\mathrm{QSM}-\mathrm{EDAS}-\mathrm{CC}}=C_{N_{\mathrm{S}}}^{N_{\mathrm{T}}}\left(C_{2}^{N_{\mathrm{S}}}+N_{\mathrm{S}}\right)^{2}\left(12 N_{\mathrm{R}}+2\right) M \tag{21}
\end{equation*}
$$

To further reduce the computational complexity of (17) and (18), we can exploit the rotational symmetry of angle $\theta_{0}=\pi$ [8]. By using a polar coordinate representation, a given symbol $s_{2}$ can be given by $s_{2}=r e^{j \theta}$. Now, consider the symbol $s_{2}$ rotated by angle $\theta_{0}=\pi$, which is given by $s_{2} e^{j \pi}=-s_{2}$. From (14) and (13), we then have

$$
\begin{align*}
g_{2}^{\prime} & =\frac{\tilde{h}_{23}^{(m, n)} r \cos (\theta+\pi)+\tilde{h}_{24}^{(m, n)} r \sin (\theta+\pi)}{\tilde{h}_{22}^{(m, n)}} \text { and }  \tag{22}\\
& =-g_{2}, \\
g_{1}^{\prime} & =\frac{-\tilde{h}_{12}^{(m, n)} F_{\mathrm{Q}}\left(g_{2}^{\prime}\right)+\tilde{h}_{13}^{(m, n)} r \cos (\theta+\pi)+\tilde{h}_{14}^{(m, n)} r \sin (\theta+\pi)}{\tilde{h}_{11}^{(m, n)}} \\
& =-g_{1} . \tag{23}
\end{align*}
$$

Thus, this result can be re-expressed as $g_{1}^{\prime}+j g_{2}^{\prime}=$ $\left(g_{1}+j g_{2}\right) e^{j \pi}$. Therefore, assuming that $s_{1}$ is the constellation point to achieve the minimum $D_{m, n}(I)$ for a given $s_{2}$, the constellation point that makes $D_{m, n}(I)$ minimum is equal to $F_{\mathrm{R}}\left(g_{1}^{\prime}\right)+j F_{\mathrm{Q}}\left(g_{2}^{\prime}\right)=s_{1} e^{j \pi}$ for a given $s_{2} e^{j \pi}$. Hence, the complexity of QSM-EDAS-LC can be further reduced as

$$
\begin{equation*}
C_{\mathrm{QSM}-\mathrm{EDAS}-\mathrm{LC}}=C_{N_{\mathrm{S}}}^{N_{\mathrm{T}}}\left(C_{2}^{N_{\mathrm{S}}}+N_{\mathrm{S}}\right)^{2}\left(12 N_{\mathrm{R}}+2\right)(M / 2) \tag{24}
\end{equation*}
$$

## 3. EDAS with Further Complexity Reduction

In Sections I and II, $\mathbf{D}(I)$ was computed in each enumeration using the column vectors of the channel matrix associated with the Ith enumeration, and thus its computation with an exhaustive search results in a huge complexity. To significantly reduce this high complexity, we compute an upper triangular $N_{\mathrm{T}} \times N_{\mathrm{T}}$ matrix $\boldsymbol{\Sigma}$ only once before searching for $I_{\mathrm{ED}}$, as in SM [5], [6]. The $\left(m^{\prime}, n^{\prime}\right)$ th element of the matrix $\boldsymbol{\Sigma}$ can be calculated as follows.

```
\Sigma 珤,n}{\prime}{\prime}
```

$$
\begin{align*}
& \min _{\substack{s_{1 \mathrm{R}} \neq s_{2 \mathrm{~L}} \in \operatorname{Re}(S) \\
s_{1 \mathrm{Q}}, S_{2 Q} \in \operatorname{Im}(S)}}\left\|\mathbf{h}_{m_{R}^{\prime}} s_{1 \mathrm{R}}+j \mathbf{h}_{m_{Q}^{\prime}} s_{1 \mathrm{Q}}-\mathbf{h}_{n_{R}^{\prime}} s_{2 \mathrm{R}}-j \mathbf{h}_{n_{Q}^{\prime}} s_{2 \mathrm{Q}}\right\|_{F}^{2}, m^{\prime}=n^{\prime}, \\
& m_{R}^{\prime}\left(=m^{\prime}\right)=n_{R}^{\prime}\left(=n^{\prime}\right) \\
& n_{Q}^{\prime}=m_{Q}^{\prime}, m_{Q}^{\prime}+1, \ldots, N_{\mathrm{T}} \\
& \Sigma_{m^{\prime}, n^{\prime}}= \tag{25}
\end{align*}
$$

where $m^{\prime}=m_{R}^{\prime}=1,2, \ldots, N_{\mathrm{T}}, \quad m_{Q}^{\prime}=1,2, \ldots, N_{\mathrm{T}}, \quad n^{\prime}=n_{R}^{\prime}$
$=m^{\prime}+1, m^{\prime}+2, \ldots, N_{\mathrm{T}}$, and $n_{Q}^{\prime}=m_{Q}^{\prime}, m_{Q}^{\prime}+1, \ldots, N_{\mathrm{T}}$. Here, $\mathbf{h}_{t^{\prime}}, t^{\prime}=1,2, \ldots, N_{\mathrm{T}}$, is the $t^{\prime}$ th column vector of an $N_{\mathrm{R}} \times N_{\mathrm{T}}$ matrix $\mathbf{H}$.
After computing the $N_{\mathrm{T}} \times N_{\mathrm{T}}$ upper triangular matrix $\boldsymbol{\Sigma}$, its $N_{\mathrm{S}} \times N_{\mathrm{S}}$ sub-matrix $\boldsymbol{\Sigma}(I)$ can be obtained by eliminating the rows and columns that are not present in $I$. The decision metric whose approach is based on the full dimension of $\boldsymbol{\Sigma}$, which is called QSM-EDAS-F, can then be given as

$$
\begin{equation*}
I_{\mathrm{ED}}=\underset{I \in \Gamma}{\arg \max }\{\min \boldsymbol{\Sigma}(I)\} \tag{27}
\end{equation*}
$$

Furthermore, we can reduce the complexity by applying the methods used in Section II to this approach by assuming that $s_{2 \mathrm{R}}$ and $s_{2 \mathrm{Q}}$ are the optimal solution of $\Sigma_{m^{\prime}, n^{\prime}}$. This reduced algorithm is called QSM-EDAS-R, and (25) and (26) can be rewritten as
where $\tilde{\boldsymbol{\Xi}}_{\left(m^{\prime}, n^{\prime}\right)}=\boldsymbol{\Xi}_{\left(m^{\prime}, n^{\prime}\right)}^{H} \mathbf{\Xi}_{\left(m^{\prime}, n^{\prime}\right)}$ and

$$
\boldsymbol{\Xi}_{\left(m^{\prime}, n^{\prime}\right)}=\left[\begin{array}{cccc}
\mathbf{h}_{m_{R}^{\prime} R} & -\mathbf{h}_{m_{Q}^{\prime} Q} & \mathbf{h}_{n_{R}^{\prime} R} & -\mathbf{h}_{n_{Q}^{\prime} Q}  \tag{30}\\
\mathbf{h}_{m_{R}^{\prime} Q} & \mathbf{h}_{m_{Q}^{\prime} R} & \mathbf{h}_{n_{R}^{\prime} Q} & \mathbf{h}_{n_{Q}^{\prime} R}
\end{array}\right],
$$

where $\mathbf{h}_{m_{R}^{\prime} R}=\operatorname{Re}\left(\mathbf{h}_{m_{R}^{\prime}}\right), \mathbf{h}_{m_{R}^{\prime} Q}=\operatorname{Im}\left(\mathbf{h}_{m_{R}^{\prime}}\right), \mathbf{h}_{n_{R}^{\prime} R}=\operatorname{Re}\left(\mathbf{h}_{n_{R}^{\prime} R}\right)$, $\mathbf{h}_{n_{R}^{\prime} Q}=\operatorname{Im}\left(\mathbf{h}_{n_{R}^{\prime}}\right), \quad \mathbf{h}_{m_{Q}^{\prime} R}=\operatorname{Re}\left(\mathbf{h}_{m_{Q}^{\prime}}\right), \quad \mathbf{h}_{m_{Q}^{\prime} Q}=\operatorname{Im}\left(\mathbf{h}_{m_{Q}^{\prime}}\right)$, $\mathbf{h}_{n_{Q}^{\prime} R}=\operatorname{Re}\left(\mathbf{h}_{n_{Q}^{\prime}}\right)$, and $\quad \mathbf{h}_{n_{Q}^{\prime} Q}=\operatorname{Im}\left(\mathbf{h}_{n_{Q}^{\prime}}\right) \quad$ for $\quad m_{R}^{\prime}, m_{Q}^{\prime}$ $=1,2, \ldots, N_{\mathrm{T}}, \quad n_{R}^{\prime}=m_{R}^{\prime}, m_{R}^{\prime}+1, \ldots, N_{\mathrm{T}}, \quad$ and $\quad n_{Q}^{\prime}=m_{Q}^{\prime}$, $m_{Q}^{\prime}+1, \ldots, N_{\mathrm{T}}$. The complexity of QSM-EDAS-R can then be approximated as

$$
\begin{equation*}
C_{\mathrm{QSM}-\mathrm{EDAS}-\mathrm{R}}=3 N_{\mathrm{T}}^{2}\left(2 N_{\mathrm{R}}-1\right)+\left(C_{2}^{N_{\mathrm{T}}}+N_{\mathrm{T}}\right)^{2} 51(M / 2) . \tag{31}
\end{equation*}
$$

Here, let us compute $\tilde{\boldsymbol{\Xi}}_{\left(m^{\prime}, n^{\prime}\right)}=\boldsymbol{\Xi}_{\left(m^{\prime}, n^{\prime}\right)}^{H} \boldsymbol{\Xi}_{\left(m^{\prime}, n^{\prime}\right)}$ first. To do so, we calculate $\mathbf{h}_{t_{m}^{\prime} R}^{T} \mathbf{h}_{t_{n}^{\prime} R}, \mathbf{h}_{t_{m}^{\prime} Q}^{T} \mathbf{h}_{t_{n}^{\prime} Q}$, and $\mathbf{h}_{t_{m}^{\prime} R}^{T} \mathbf{h}_{t_{n}^{\prime} Q}, t_{m}^{\prime}$, $t_{n}^{\prime}=1,2, \ldots, N_{\mathrm{T}}$, which requires $3 N_{\mathrm{T}}^{2}\left(2 N_{\mathrm{R}}-1\right)$ flops. Then, (28) and (29) require the computation of $\left(C_{2}^{N_{\mathrm{T}}}+N_{\mathrm{T}}\right)^{2} 51 M^{2}$. Further, if the approaches presented in Section II are applied to QSM-EDAS-F, its complexity can be further reduced, and we thus have the complexity of QSM-EDAS-R.

## IV. Simulation Results

The receiver for QSM systems employs a maximum-
likelihood (ML) detector, which jointly estimates the indices of the activated antennas and the symbol transmitted from them. For comparison purposes, we consider the SM system based on EDAS-ES in [5] and [6]. Now, the SER performance of the QSM systems based on EDAS is evaluated and compared with that of a conventional SM-EDAS-ES. The spectral efficiency considered in the simulations is a rate of 6 bits per channel used. Figure 1 shows the simulation results of the SER performance versus $E_{s} / N_{0}$ in decibels for QSM-EDAS-ES, QSM-EDAS-LC, QSM-EDAS-F, QSM-EDAS-R, and SM-EDAS-ES. Here, $E_{\mathrm{s}}$ represents the QAM signal symbol energy. We employ $N_{\mathrm{T}}=6$, $N_{\mathrm{T}}=4$, and $N_{\mathrm{R}}=2$ with 4-QAM for QSM. Meanwhile, 16QAM is assumed for SM systems with $N_{\mathrm{T}}=6, N_{\mathrm{S}}=4$, and $N_{\mathrm{R}}$ $=2$. It was found that the QSM system using EDAS-ES offers a significantly better SER performance than with no antenna selection. Here, we observed a gain in $E_{\mathrm{s}} / N_{0}$ of about 3 dB to 8 dB . In addition, it outperforms the SM system with EDASES by about 2 dB to 3 dB in $E_{\mathrm{s}} / N_{0}$ values. Note that the QSM system uses two transmit RF chains, unlike the SM system, which employs only one. QSM-EDAS-LC achieves the same SER performance as QSM-EDAS-ES. It was also shown that QSM-EDAS-F and QSM-EDAS-R experience a performance loss compared with QSM-EDAS-ES. This is due to the single computation of an upper triangular $N_{\mathrm{T}} \times N_{\mathrm{T}}$ matrix $\boldsymbol{\Sigma}$ prior to the searching mode, which results in the inclusion of extra unnecessary channel components. However, they provide a slightly better performance than SM-EDAS-ES for the given $E_{8} / N_{0}$ ranges. In Fig. 2, four receive antennas are assumed with the same parameters shown in Fig. 1. It was shown that the QSM system employing EDAS-ES outperforms the SM system with EDAS-ES. Here, a gain of about 3 dB to 4 dB is seen. The performance degradation of QSM-EDAS-F and QSM-EDAS-R is relatively small compared to that of QSM-EDAS-ES. Thus, they still provide about a 1.5 dB to 3 dB better performance than no selection in QSM, and about a 2.3dB better performance than SM-EDAS-ES. Therefore, QSM-EDAS-R is more beneficial in terms of transmit diversity gains when there are four receive antennas than when there are two receive antennas.
Next, we compare the complexity of the EDAS algorithms used in the QSM and SM systems. The complexity of SM-EDMS-ES [5], [6] can be given as

$$
\begin{equation*}
C_{\text {SM-EDAS-ES }}=C_{2}^{N_{\mathrm{T}}}\left(10 N_{\mathrm{R}}-1\right) M^{2} . \tag{32}
\end{equation*}
$$

Note that the complexity includes the number of real multiplications. The complexities of QSM-EDAS-ES, QSM-EDAS-LC, QSM-EDAS-R, and SM-EDAS-ES are given in Tables 1 and 2 . Table 1 shows the number of flops for $N_{\mathrm{T}}=6$, $N_{\mathrm{S}}=4$, and $N_{\mathrm{R}}=2$ with 4-QAM and 16-QAM. Other than $N_{\mathrm{R}}=4$, the parameters in Table 2 are the same as those in


Fig. 1. SER performance comparison of SM-EDAS-ES, QSM-EDAS-ES, QSM-EDAS-LC, QSM-EDAS-F, and QSM-EDAS-R algorithms for $N_{\mathrm{T}}=6, N_{\mathrm{S}}=4$, and $N_{\mathrm{R}}=2$.


Fig. 2. SER performance comparison of SM-EDAS-ES, QSM-EDAS-ES, QSM-EDAS-LC, QSM-EDAS-F, and QSM-EDAS-R algorithms for $N_{\mathrm{T}}=6, N_{\mathrm{S}}=4$, and $N_{\mathrm{R}}=4$.

Table 1. It can be seen that the EDAS algorithm for QSM systems requires an extremely high complexity compared to the EDAS for SM systems because of two spatial dimensional searches. Recall that $\mathbf{D}(I)$ in QSM-EDAS-ES and QSM-EDAS-LC was computed in each enumeration of all possible combinations. On the other hand, QSM-EDAS-R and SM calculate an upper triangular $N_{\mathrm{T}} \times N_{\mathrm{T}}$ matrix $\boldsymbol{\Sigma}$ a single time. An $N_{\mathrm{S}} \times N_{\mathrm{S}}$ sub-matrix $\boldsymbol{\Sigma}(I)$ of the matrix $\boldsymbol{\Sigma}$ is then obtained by removing the rows and columns that are not present in $I$, which is why the complexity of QSM-EDAS-ES is much larger than that of SM-EDAS-ES. However, for a fair comparison, we have to assume the same data rate per channel used. Here, 6 bits per channel used are employed. The complexities in the highlighted cells of the tables should be

Table 1. Computational complexity for $N_{\mathrm{T}}=6, N_{\mathrm{S}}=4, N_{\mathrm{R}}=2$.

|  | SM- <br> EDAS-ES | QSM- <br> EDAS-ES | QSM- <br> EDAS-LC | QSM- <br> EDAS-R |
| :---: | :---: | :---: | :---: | :---: |
| 4-QAM | 4,560 | $\mathbf{6 2 4 , 0 0 0}$ | $\mathbf{7 8 , 0 0 0}$ | $\mathbf{4 5 , 3 0 6}$ |
| 16-QAM | $\mathbf{7 2 , 9 6 0}$ | $9,984,000$ | 312,000 | 180,252 |

Table 2. Computational complexity for $N_{\mathrm{T}}=6, N_{\mathrm{S}}=4, N_{\mathrm{R}}=4$.

|  | SM- <br> EDAS-ES | QSM- <br> EDAS-ES | QSM- <br> EDAS-LC | QSM- <br> EDAS-R |
| :---: | :---: | :---: | :---: | :---: |
| 4-QAM | 9,360 | $\mathbf{1 , 2 0 0 , 0 0 0}$ | $\mathbf{1 5 0 , 0 0 0}$ | $\mathbf{4 5 , 7 3 8}$ |
| 16-QAM | $\mathbf{1 4 9 , 7 6 0}$ | $19,200,000$ | 600,000 | 180,684 |

compared. In other words, SM systems with 16-QAM should be compared with QSM systems with 4-QAM. As can be seen, the antenna selection schemes for QSM systems offer a lower complexity than SM-EDAS-ES. In Tables 1 and 2, QSM-EDAS-R achieves approximately a 1.6 and three-times smaller complexity than SM-EDAS-ES, respectively. In addition, it was found that the increase in the complexity of QSM-EDASR occurring from $N_{\mathrm{R}}=2$ to $N_{\mathrm{R}}=4$ is minor, but the other values incur about a two-fold larger complexity. Thus, QSM-EDAS-R is advantageous for a larger number of receive antennas. To achieve transmit diversity gains through antenna selection in a QSM system, more efficient antenna selection techniques need to be developed for practical use. To the best of our knowledge, this algorithm for QSM systems is not yet available in the literature.

## V. Conclusion

Antenna selection schemes based on EDAS for QSM systems were introduced in this paper. We studied and compared their SER performance when EDAS-based antenna selection techniques are applied in QSM and SM systems. We found that the QSM systems employing EDAS-based antenna selection have a much better SER performance compared with SM systems utilizing EDAS-ES. The complexity of the EDAS-ES algorithm for QSM systems can be reduced through modifications. Thus, QSM-EDAS-R has a lower complexity compared to the EDAS-ES approach for SM systems with the same data rate per channel used.

## References

[1] R.Y. Mesleh et al., "Spatial Modulation," IEEE Trans. Veh.

Technol., vol. 57, no. 4, July 2008, pp. 2228-2241.
[2] M. Di Renzo, H. Haas, and P.M. Grant, "Spatial Modulation for Multiple-Antenna Wireless Systems: A Survey," IEEE Commun. Mag., vol. 49, no. 12, Dec. 2011, pp. 182-191.
[3] J. Jeganatha, A. Ghrayeb, and L. Szczecinski, "Spatial Modulation: Optimal Detection and Performance Analysis," IEEE Commun. Lett., vol. 12, no. 8, Aug. 2008, pp. 545-547.
[4] R. Mesleh, S.S. Ikki, and H.M. Aggoune, "Quadrature Spatial Modulation," IEEE Trans. Veh. Technol., vol. 64, no. 6, July 2015, pp. 2738-2742.
[5] R. Rajashekar, K.V.S. Hari, and L. Hanzo, "Antenna Selection in Spatial Modulation Systems," IEEE Comтии. Lett., vol. 17, no. 3, Mar. 2013, pp. 521-524.
[6] N. Pillay and H. Xu, "Comments on Antenna Selection in Spatial Modulation Systems," IEEE Commun. Lett., vol. 17, no. 9, Sept. 2013, pp. 1681-1683.
[7] K. Ntontin et al., "A Low-Complexity Method for Antenna Selection in Spatial Modulation Systems," IEEE Commun. Lett., vol. 17, no. 12, Dec. 2013, pp. 2312-2315.
[8] N. Wang et al., "Further Complexity Reduction Using Rotational Symmetry for EDAS in Spatial Modulation," IEEE Commun. Lett., vol. 18, no. 10, Oct. 2014, pp. 1835-1838.
[9] N. Pillay and H. Xu, "Low-Complexity Transmit Antenna Selection Schemes for Spatial Modulation," IET Commun., vol. 9, no. 2, Jan. 2015, pp. 239-248.
[10] R. Rajashekar, K.V.S. Hari, and L. Hanzo, "Quantifying the Transmit Diversity Order of Euclidean Distance Based Antenna Selection in Spatial Modulation," IEEE Signal Process. Lett., vol. 22, no. 9, Sept. 2015, pp. 1434-1437.


Sangchoon Kim received his BS degree from Yonsei University, Seoul, Rep. of Korea, in 1991, and his ME and PhD from the University of Florida, Gainesville, degree USA, in 1995 and 1999, respectively, all in electrical and computer engineering. From 2000 to 2005, he was a senior research engineer at LG Corporate Institute of Technology, Seoul, Rep. of Korea, and a chief research engineer with LG Electronics, Anyang, Rep. of Korea, working on a range of research projects in the field of wireless/mobile communications. In 2005, he joined Dong-A University, Busan, Rep. of Korea, where he is currently a full professor in the Department of Electronics Engineering. His research interests cover a range of areas in wireless/mobile communications, signal processing, and antenna design.


[^0]:    Manuscript received Nov. 16, 2015; revised Feb. 4, 2016; accepted Mar. 15, 2016.
    This present study was supported by research funds from Dong-A University.
    Sangchoon Kim (sckim@dau.ac.kr) is with the Department of Electronics Engineering, Dong-A University, Busan, Rep. of Korea.

