High Diversity Transceiver for Low Power Differentially Encoded OFDM System

Faisal Nadeem, Muhammad Zia, Hasan Mahmood, Naeem Bhatti, and Ihsan Haque

In this work, we investigate differentially encoded blind transceiver design in low signal-to-noise ratio (SNR) regimes for orthogonal frequency-division multiplexing (OFDM) signaling. Owing to the fact that acquisition of channel state information is not viable for short coherence times or in low SNR regimes, we propose a time-spread frequency-encoded method under OFDM modulation. The repetition (spreading) of differentially encoded symbols allows us to achieve a target energy per bit to noise ratio and higher diversity. Based on the channel order, we optimize subcarrier assignment for spreading (along time) to achieve frequency diversity of an OFDM modulated signal. We present the performance of our proposed transceiver design and investigate the impact of Doppler frequency on the performance of the proposed differentially encoded transceiver design. To further improve reliability of the decoded data, we employ capacity-achieving low-density parity-check forward error correction encoding to the information bits.

Keywords: DQPSK, diversity, low SNR, SISO-OFDM, FEC, LDPC.

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Faisal Nadeem (faisal@ele.qau.edu.pk), Muhammad Zia (corresponding author, mzia@qau.edu.pk), Hasan Mahmood (hasan@qau.edu.pk), and Naeem Bhatti (nbhatti@qau.edu.pk) are with the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan.

Ihsan Haque (ihsanulhaq@iiu.edu.pk) is with the Department of Electronics Engineering, International Islamic University, Islamabad, Pakistan.

I. Introduction

Over the years, blind detection for orthogonal frequency-division multiplexing (OFDM) signals for frequency selective channels has received considerable attention. OFDM modulation combats the multipath fading effect by converting frequency selective channels into parallel flat-fading channels. Owing to the fact that consecutive OFDM subcarriers are highly correlated, differential encoding along subcarriers of the same OFDM symbol is viable. For a moderate coherence time, subcarriers of consecutive OFDM symbols have a strong correlation, thus making them suitable for differential encoding along time. Therefore, OFDM modulation allows differential encoding of information bits along both frequency and time [1], [2]. Differential encoding provides a viable alternative when pilot-assisted channel estimation is not possible due to fast-fading [3]. In addition, in low signal-to-noise ratio (SNR) regimes, acquisition of channel state information (CSI) has an adverse impact on bandwidth efficiency. In such a scenario, differential encoding with repetition coding using blind detection provides for an alternative solution. Note that non-coherent differential detection suffers from 3 dB SNR loss as compared to coherent detection [4]. In [5] and [6], high-complexity joint differential detection of multiple information symbols is presented to improve performance.

In wireless communication, capacity-achieving low-density parity-check (LDPC) codes and Turbo codes are an effective means of lowering bit error rates (BERs) [7], [8]. The performances of LDPC codes and Turbo codes over Rayleigh fading channels is analyzed in [9]–[11]. Differential decoding along with capacity-achieving forward error correction (FEC) encoding, such as LDPC codes and Turbo codes, is employed to further enhance BER performance [11]–[14].

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OFDM modulation combats the fading of a multipath channel and allows low-complexity receiver design. Differential encoding over frequency selective channels using OFDM signaling is proposed in [1], [15], and [16]. OFDM modulation permits differential encoding along time and frequency dimensions under low and moderate mobility offering both time and frequency diversities.

In [1], differential encoding along frequency and time directions for OFDM signaling, denoted by frequency-domain differential modulation and time-domain differential modulation, respectively, is investigated. The conventional differential detection suffers from a BER error-floor under various channel conditions. The issue of poor performance is addressed in [16] using semi-blind detection at the expense of pilot symbols. Maximum-likelihood multiple-symbol differential detection (ML-MSDD) is proposed in [5] and references therein claim to have mitigated any BER error-floor, which has high computational cost. The multiple-symbol differential sphere decoding (MSDSD) in [17] reduces the complexity of the ML-MSDD method at the expense of moderate performance loss. We propose a time-spread frequency-encoded (TSFE) method for differential encoding, which addresses the BER error-floor bottleneck and achieves higher diversity over multipath Rayleigh fading channels at low computational cost.

In [1], [5], and [6], performance of uncoded differential encoding along time and frequency dimensions is compared in the presence of Doppler frequency. High-complexity joint differential decoding of multiple information symbols is considered in [5] and [6]. For a multiple-input and multiple-output OFDM (MIMO-OFDM) system, differential space–time block coding that achieves higher diversity with a low complexity receiver is proposed in [18] and [19] and references therein.

In this work, we propose a differentially encoded blind transceiver design that achieves low complexity, low power, and high diversity for single-input and single-output OFDM (SISO-OFDM) signaling. The proposed transceiver operates in low SNR regimes in a non-coherent fashion under multipath channels without CSI. We achieve full channel diversity by repeating low-power differentially-encoded information symbols along subcarriers and spreading along time. In our TSFE method, we exploit the fact that consecutive subcarriers have a strong tendency to employ differential encoding along subcarriers. We achieve full channel diversity by spreading differentially-encoded information along time and assigning groups of uncorrelated subcarriers to differentially-encoded symbols denoted by optimal interleaving, I_o. The aforementioned uncorrelated assignment of subcarriers for spreading assumes knowledge of the channel order, P. The proposed method is robust to channel order mismatch. We also investigate the BER performance of the proposed method under Doppler frequency. We also present performance comparisons between optimal and random subcarrier assignment under relative mobility of the nodes. To further improve the reliability of the proposed modulation schemes, we integrate a capacity-achieving LDPC FEC code with the proposed transceiver design.

Simulation analyses demonstrate that differential encoding along correlated subcarriers with spreading along time (TSFE) is resilient to Doppler frequency and achieves higher diversity. We investigate the impact of both a spreading factor and Doppler frequency on the BER performance of the proposed TSFE method. The BER performance of the proposed method outperforms that of the MSDSD method presented in [17].

We organize this manuscript as follows. First, we present the system model and problem formulation of differential quadrature phase shift keying (DQPSK) as blind detection for SISO-OFDM signaling in Section II and Section III, respectively. The receiver design for the proposed differentially encoded scheme is presented in Section IV. The log-likelihood ratio (LLR) of DQPSK modulation for an LDPC decoder is evaluated in Section V. The BER performances of the proposed methods are presented in Section VI. Finally, we conclude our work in Section VII.

II. System Model

In this work, we consider SISO-OFDM signaling in low SNR regimes over a Rayleigh fading channel of P significant paths as shown in Fig. 1. The reason for considering a low SNR regime arises from the fact that under deep fading, a transmitted signal undergoes severe channel attenuation, resulting in low SNR. In such scenarios, acquisition of CSI is not viable. Blind detection of a differentially encoded signal provides an alternative solution to coherent detection. Spreading (repetition) of differentially-encoded information symbols along frequency or time dimensions for an OFDM

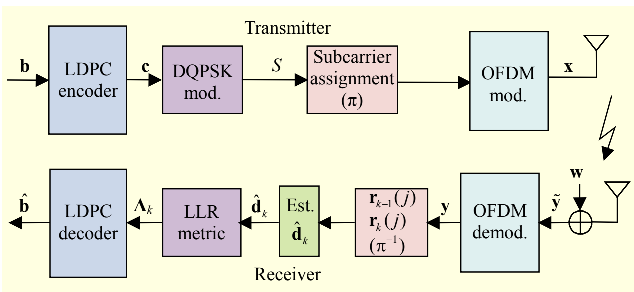

Fig. 1. System model of differential encoded blind transceiver design with LDPC code under SISO-OFDM signaling.

system combats severe channel attenuation to achieve a target energy per bit to noise ratio. Furthermore, even in moderate and low channel attenuation, a transmitter transmits at low power to achieve a low probability of detection. In such a scenario, the target energy per bit to noise ratio is also achieved by considering information spreading (repetition) along time–frequency dimensions of an OFDM modulation. As depicted in the system model in Fig. 1, the LDPC encoder with code rate R = 1/2 encodes an information bit vector $\mathbf{b}$ of q bits into a codeword $\mathbf{c}$ of n bits. A differential encoder under a Gray mapping maps codeword $\mathbf{c}$ to a complex constellation set of cardinality M. The differential encoder maps information bits to phase shift between the complex constellation points transmitted over the $(\ell - 1)$th and ℓth subcarriers of the same OFDM symbol. Conventional differential encoding encodes information along time [20]. Note that in the case of differential encoding along OFDM subcarriers, information bits are encoded in phase difference between the information symbols transmitted over the consecutive subcarriers of the same OFDM symbol. The system model in Fig. 1 also considers the relative mobility of the communicating nodes.

To achieve a target energy per bit to noise ratio, our proposed method spreads (repeats) differentially-encoded information symbols along a time dimension. Let β be the spreading factor of a differentially encoded symbol of an OFDM modulation of N_s subcarriers. In addition, let $\mathbf{h}_k \in C^{N_s \times 1}$ be an unknown gain vector of N_s subcarriers of an OFDM system, k is time index. The unknown gain vector $\mathbf{h}_k$ is in fact a fast Fourier transformation (FFT) of the random channel vector $\tilde{\mathbf{h}}_k$ between the transmitter and the receiver pair with zero-mean and variance $\sigma_{\mathbf{h}}^2 = 1/P$. That is, $\mathbf{h}_k = \mathbf{F}\tilde{\mathbf{h}}_k$, where $\mathbf{F}$ is an FFT matrix. Due to the dynamic nature of the wireless channels, spreading of differentially-encoded information symbols along time or frequency dimensions provides diversity.

To differentially encode information along subcarriers of an OFDM system, we exploit the fact that a pair of consecutive subcarriers of an OFDM system have strong correlation. The correlation between consecutive subcarriers increases by increasing the number of OFDM subcarriers (N_s). We achieve a target energy per bit to noise ratio by spreading differentially encoded information symbols along time. To achieve maximum diversity, subcarriers assigned to encoded symbols should have minimum correlation; that is, equal to the number of independent channel paths, P, of a frequency selective channel. With repetition (we also call it spreading) of factor β along time of an OFDM symbol, $(N_s - 1)$ information symbols are transmitted over β OFDM symbols using β interleavers, $\pi_1, \pi_2, \ldots, \pi_\beta$. We observe that differential encoding along subcarriers is robust to the relative mobility of the communicating nodes. This is due to the fact that the TSFE method exploits both time and frequency diversity. Next, we discuss the proposed TSFE method.

III. Problem Formulation

Now, we present our proposed blind TSFE approach, which is effective in a low SNR regime and high relative mobility of the communicating nodes.

1. TSFE Method

In a TSFE method, information is differentially encoded along subcarriers and spread along time. To differentially encode information along subcarriers, we rely on the fact that neighboring subcarriers of an OFDM system have high correlation, as shown in Fig. 2. The correlation coefficient in (3), $r_k(\upsilon)$, between two consecutive subcarriers (υ = 1), $h_k(\ell)$ and $h_k(\ell + 1)$, increases by increasing the number of subcarriers (N_s) for a given channel; k is the time index. Note that the subcarriers corresponding to low gain severely distort encoded information. We combat the nulling effect of OFDM subcarriers by interleaved spreading (repetition) of differentially encoded information symbols along time. An OFDM modulation with N_s subcarriers differentially encodes $(N_s - 1)$ information symbols using the first subcarrier as a reference symbol. For a large number of subcarriers, the overhead due to reference subcarriers is negligible. The repetition of encoded information symbols over independent subcarriers achieves a target energy per bit to noise ratio and higher diversity at the same time. Note

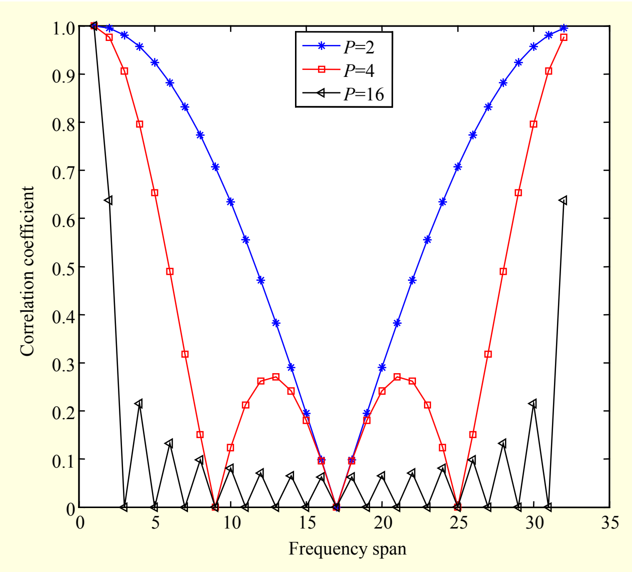

Fig. 2. Auto-correlation of OFDM subcarriers for P = 2, 4, and 16 (for 32 subcarriers).

that the TSFE method exploits frequency and time diversity by employing β interleavers — one for each repetition of the frequency encoded OFDM symbol. The interleaver $\boldsymbol{\pi}_m$, where $m = 1, \ldots, \beta$ assigns subcarriers for differential encoding to each phase of the mth OFDM symbol.

Let $\Delta\boldsymbol{\phi} = \left[\Delta\phi_1 \ \Delta\phi_2 \ \Delta\phi_3 \ldots \Delta\phi_{N_s-2} \ \Delta\phi_{N_s-1} \right]^{\mathrm{T}}$ denote the phase differences between symbols transmitted over N_s consecutive subcarriers, resulting from the mapping of information bits to the phase differences between symbols transmitted over consecutive subcarriers of an OFDM system. We construct β differentially encoded symbol vectors of length N_s, each by interleaving the phase difference vector $\Delta\boldsymbol{\phi}$. The phase difference vector $\Delta\boldsymbol{\phi}_m = \boldsymbol{\pi}_m(\Delta\boldsymbol{\phi})$, where $m = 1, \ldots, \beta$. Note that $\Delta\boldsymbol{\phi}_1 = \boldsymbol{\pi}_1(\Delta\boldsymbol{\phi}) = \Delta\boldsymbol{\phi}$. The differentially encoded symbol matrix of the TSFE method corresponding to β phase difference vectors transmitted over β OFDM symbols is

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_1^{\mathrm{T}} \\ \mathbf{s}_2^{\mathrm{T}} \\ \vdots \\ \mathbf{s}_\beta^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} s_1(1) & s_1(2) & \ldots & s_1(N_s) \\ s_2(1) & s_2(2) & \ldots & s_2(N_s) \\ \vdots & \vdots & \ddots & \vdots \\ s_\beta(1) & s_\beta(2) & \cdots & s_\beta(N_s) \end{bmatrix}. \quad (1)$$

In (1), $s_m(j)$ is an information symbol transmitted over the jth subcarrier of the mth OFDM symbol. Let $\mathbf{I}(j) = [I_1(j), I_2(j), \ldots, I_\beta(j)]^{\mathrm{T}}$ be the edge indices of subcarriers assigned to phase difference $\Delta\phi_j$ in vector $\Delta\boldsymbol{\phi}$, where $j = 1, \ldots, N_s-1$. Then, we have

$$\Delta\phi_j = \angle\left(s_m^*\left(I_m(j)+1\right) s_m\left(I_m(j)\right)\right). \quad (2)$$

Each symbol $s_m\left(I_m(j)\right)$ is scaled by an OFDM subcarrier gain, $h_m\left(I_m(j)\right)$. Note that the first symbol of each OFDM symbol, $s_m(1)$, serves as a reference signal for differential encoding.

If the receiver has knowledge of the channel order, P, then we can assign uncorrelated subcarriers to a differentially encoded information symbol sequence using an optimal interleaver to achieve full channel diversity. The assignment of subcarriers does not change as long as the channel order stays the same. In Section VI, we present the BER performances of optimal and random subcarrier assignments for spreading.

2. Optimal Interleaving

For known channel order P, the correlation function between subcarriers of an OFDM modulation is [21], [22]

$$r_k(\upsilon) = \mathrm{E}\left(H_k(\ell) H_k^*(\ell+\upsilon) \right) = \sum_{n=0}^{P-1} \sigma_{\mathbf{h}}^2 e^{-\frac{\mathrm{i}2\pi\upsilon n}{N_s}}, \quad (3)$$

where υ is the index of the correlation between subcarrier gains $h_k(\ell)$ and $h_k(\ell+\upsilon)$.

The diversity order P, of a multipath channel, can be achieved by grouping OFDM subcarriers into groups of uncorrelated subcarriers. Figure 2 provides correlation coefficients of OFDM subcarriers for $N_s = 32$ for channel orders $P = 2$, 4, and 16. The correlation coefficients are zero for $\upsilon = D, 2D, \ldots, (P-1)D$, where $D = N_s/P$. There are at most $\beta = P$ elements of zero correlation that can achieve full diversity P. Thus, to achieve full diversity, the spreading factor, β, should be greater than or equal to P. We assign one group of uncorrelated subcarriers to a differentially encoded information symbol to achieve full diversity. When $\beta > P$, there are at least P uncorrelated subcarriers. In the TSFE approach, one differentially encoded information symbol is transmitted in β consecutive OFDM symbols over uncorrelated subcarriers. For example, for $P = 4$ and $N_s = 32$ subcarriers, there are four uncorrelated channel gains in a group; $\beta = P = 4$. Thus, one phase difference, $\Delta\phi_j$, can be transmitted over the edges of four uncorrelated subcarriers in $\beta = 4$ epochs. To further elaborate on optimal interleaving, let us consider the example where $\beta = 4$, $P = 4$, and $N_s = 16$. Here, there are four optimal interleavers that spread a phase difference vector of size 15 corresponding to differential encoding in four consecutive OFDM symbols. Thus, the required four optimal interleaves are as follows:

$$\boldsymbol{\pi}_1 = \left[1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15\right]^{\mathrm{T}},$$

$$\boldsymbol{\pi}_2 = \left[12 \ 13 \ 14 \ 15 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11\right]^{\mathrm{T}},$$

$$\boldsymbol{\pi}_3 = \left[8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\right]^{\mathrm{T}},$$

$$\boldsymbol{\pi}_4 = \left[4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 1 \ 2 \ 3\right]^{\mathrm{T}}.$$

For the aforementioned example, we require a phase difference of $\Delta\phi_1 = \angle\left(s_1^*(2)s_1(1)\right)$ for an OFDM symbol, based on interleaver $\boldsymbol{\pi}_1$. The phase difference $\Delta\phi_1$ is spread over four consecutive OFDM symbols. Similarly, for $\boldsymbol{\pi}_2$, $\boldsymbol{\pi}_3$, and $\boldsymbol{\pi}_4$, $\Delta\phi_1$ equals $\angle\left(s_2^*(6)s_2(5)\right)$, $\angle\left(s_3^*(10)s_3(9)\right)$, and $\angle\left(s_4^*(14)s_4(13)\right)$, respectively. Note that we obtain optimal interleaver $\boldsymbol{\pi}_m$ for the mth OFDM symbol by circularly shifting interleaver $\boldsymbol{\pi}_{m-1}$ by $D = N_s/\beta$, where $m = 1, \ldots, \beta$. Note that spreading factor β achieves diversity P by spreading $\Delta\boldsymbol{\phi}$ over $\beta \geq P$ OFDM symbols and by assigning uncorrelated subcarriers to encoded information symbols.

The channel order can be estimated without pilot assistance using existing methods [23]. When the channel order is not known or the channel coherence time is short, optimal assignment of subcarriers is not possible and random subcarrier assignment is a viable solution. In Section VI, we provide BER performance comparisons of optimal and random assignments of subcarriers for the proposed TSFE method. We also provide a study on the impact of channel-order mismatch on BER in Section VI.

IV. Receiver Design

Now, we provide the receiver design of the proposed TSFE method for the OFDM system presented in Section II. The proposed differential encoding method combats channel attenuation and spectrum nulls of OFDM subcarriers by spreading encoded information along time in β OFDM symbols.

Let $\tilde{\mathbf{y}}_m$ be the observation vector corresponding to the kth OFDM symbol. After removing the cyclic prefix (CP) from $\tilde{\mathbf{y}}_m \in C^{(N_s+P)\times 1}$, we have $y_m(\ell)$ as the demodulated observation corresponding to the ℓth subcarrier, where $\ell = 1,\ldots,N_s$ and $\mathbf{y}_m = \mathrm{FFT}(\tilde{\mathbf{y}}_m) \in C^{N_s\times 1}$. Note that the phase difference $\Delta\boldsymbol{\phi}$ between two symbols along subcarriers, which represents bit information, is spread along time for the TSFE method in β epochs.

The vector $\mathbf{y}_m$ represents observations corresponding to $(N_s - 1)$ phase differences encoded along subcarriers, and m represents the time index. The time index is reset to 1 after β OFDM symbols. That is, $m = 1,\ldots,\beta,$ where m is the index of the OFDM symbol corresponding to the spreading of frequency encoded information symbol vector $\Delta\boldsymbol{\phi}$. The phase difference vector transmitted over the mth OFDM symbol is $\Delta\boldsymbol{\phi}_m = \boldsymbol{\pi}_m(\Delta\boldsymbol{\phi})$. We know that $I_1(j), I_2(j),\ldots,I_\beta(j)$ are the edge indices of subcarriers assigned to phase difference $\Delta\phi_j$ in vector $\Delta\boldsymbol{\phi}$, where $j = 1,\ldots,N_s-1$. Then,

$$\mathbf{r}_1(j) = \begin{bmatrix} y_1(I_j(1)) \\ \vdots \\ y_\beta(I_j(\beta)) \end{bmatrix} = \begin{bmatrix} h_1(I_j(1))s(I_j(1)) \\ \vdots \\ h_\beta(I_j(\beta))s(I_j(\beta)) \end{bmatrix} + \begin{bmatrix} w_1(I_j(1)) \\ \vdots \\ w_\beta(I_j(\beta)) \end{bmatrix}, \tag{4}$$

and

$$\mathbf{r}_2(j) = \begin{bmatrix} y_1(I_j(1)+1) \\ \vdots \\ y_\beta(I_j(\beta)+1) \end{bmatrix} = \begin{bmatrix} h_1(I_j(1))s(I_j(1)+1) \\ \vdots \\ h_\beta(I_j(\beta))s(I_j(\beta)+1) \end{bmatrix} + \begin{bmatrix} w_1(I_j(1)+1) \\ \vdots \\ w_\beta(I_j(\beta)+1) \end{bmatrix}. \tag{5}$$

Note that $\Delta\phi_j = \angle\left(s^*(I_j(1)+1)s(I_j(1))\right) = \angle e^{-\mathrm{i}\Delta\phi_j}$ and despreading is represented as follows:

$$\hat{d}(j) = \mathbf{r}_1^{\mathrm{H}}(j)\mathbf{r}_2(j) \text{ and } \Delta\hat{\phi}_j = \angle\hat{d}(j), \tag{6}$$

where H denotes the Hermitian conjugate of a vector. Thus, estimates of $(N_s - 1)$ elements of vector $\Delta\boldsymbol{\phi}$ are evaluated by constructing $(N_s - 1)$ pairs of $\mathbf{r}_1(j)$ and $\mathbf{r}_2(j)$ vectors and by using (6). We use $\hat{d}(j)$ to estimate $\Delta\phi_j$ and LLR; thus, we have

$$\begin{aligned} \hat{d}(j) &= \mathbf{r}_1^{\mathrm{H}}(j)\mathbf{r}_2(j) \\ &= \sum_{m=1}^{\beta} \left|h_m(I_j(m))\right|^2 \left|s(I_j(m))\right|^2 e^{\mathrm{i}\Delta\phi_j} \\ &= d(j) + w_e(j), \end{aligned} \tag{7}$$

where

$$\begin{aligned} w_e(j) &= h_1^*(I_j(1))s^*(I_j(1))w_1(I_j(1)+1) \\ &+ h_1(I_j(1))s(I_j(1)+1)w_1^*(I_j(1)) \\ &+ w_1^*(I_j(1))w_1(I_j(1)+1) \\ &+ h_2^*(I_j(2))s^*(I_j(2))w_1(I_j(2)+1) \\ &+ h_2(I_j(2))s(I_j(2)+1)w_2^*(I_j(2)) \\ &+ w_2^*(I_j(2))w_2(I_j(2)+1) + \ldots \\ &+ h_\beta^*(I_j(\beta))s^*(I_j(\beta))w_\beta(I_j(\beta)+1) \\ &+ h_\beta(I_j(\beta))s(I_j(\beta)+1)w_\beta^*(I_j(\beta)) \\ &+ w_\beta^*(I_j(\beta))w_\beta(I_j(\beta)+1), \end{aligned} \tag{8}$$

$$\begin{aligned} w_e(j) = \sum_{m=1}^{\beta} \Big[& h_m^*(I_j(m))s^*(I_j(m))w_\beta(I_j(m)+1) \\ & + h_m(I_j(m))s(I_j(m)+1)w_m^*(I_j(m)) \\ & + w_m^*(I_j(m))w_m(I_j(m)+1)\Big]. \end{aligned} \tag{9}$$

Note that subcarrier channel realizations $h_k(j)$, noise realizations $w_k(j)$, and encoded symbols $s_k(j)$ are independent with zero-mean. Therefore, variance σ_e^2 of effective noise $w_e(j)$ is $\sigma_e^2 = 2\beta N_0 + \beta N_0^2$, where N_0 is the variance of additive white Gaussian noise. The estimate of phase difference vector $\Delta\boldsymbol{\phi}$ of $(N_s - 1)$ phase differences of the TSFE method is

$$\Delta\hat{\boldsymbol{\phi}}_k = \left[\Delta\hat{\phi}_1\ \Delta\hat{\phi}_2\ \Delta\hat{\phi}_3 \ldots \Delta\hat{\phi}_{N_s-2}\ \Delta\hat{\phi}_{N_s-1}\right]^{\mathrm{T}}. \tag{10}$$

V. LLR for Differential Detection

FEC codes are commonly used to improve the reliability of a receiver. We use a capacity-achieving half-rate LDPC code in conjunction with the proposed method. Now, we discuss our LLR evaluation of the differentially encoded observations. Optimal and suboptimal methods for computing LLR for differential encoding over a Rayleigh fading channel are presented in [12], [13], and [24]. Optimal LLR estimation requires partial channel knowledge. We consider a suboptimal estimate of the LLR metric for FEC decoding [24]. For simplicity, we omit subcarrier index j in this section. To estimate LLR for decoding, we use $\hat{d}$ in (7) for the TSFE method after omitting index j.

Under the assumption of a Gaussian approximation [13], the probability of $\hat{d}$ given that d is encoded is given as

$$P_{\rm r}\left(\hat{d} \mid d\right) = \frac{1}{\sqrt{2\pi\sigma_{\rm e}^2}} e^{\left(-\frac{\left|\hat{d}-d\right|}{2\sigma_{\rm e}^2}\right)}. \quad (11)$$

We assume that any noise variance is known at the receiver.

Note that we use a gray mapping and the fact that $\angle d_k \in \{\pm\pi/4, \pm 3\pi/4\}$. The soft decision corresponding to the most-significant (MS) and least-significant (LS) bits is given as [13]

$$\lambda_{\rm MS} = \ln\left(\frac{P_{\rm r}\left(\angle d = \frac{\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right) + P_{\rm r}\left(\angle d = \frac{3\pi}{4} \mid \mathbf{r}_1,\mathbf{r}_2\right)}{P_{\rm r}\left(\angle d = -\frac{\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right) + P_{\rm r}\left(\angle d = -\frac{3\pi}{4} \mid \mathbf{r}_1,\mathbf{r}_2\right)}\right), \quad (12)$$

$$\lambda_{\rm LS} = \ln\left(\frac{P_{\rm r}\left(\angle d = \frac{\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right) + P_{\rm r}\left(\angle d = -\frac{\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right)}{P_{\rm r}\left(\angle d = \frac{3\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right) + P_{\rm r}\left(\angle d = -\frac{3\pi}{4}|\mathbf{r}_1,\mathbf{r}_2\right)}\right), \quad (13)$$

where $\lambda_{\rm MS}$ and $\lambda_{\rm LS}$ denote the LLRs corresponding to MS and LS bits, respectively. Considering that the channel remains constant during consecutive encoded symbols, LLR is given as [13].

$$\lambda_{\rm MS} = \frac{2{\rm Re}\{\mathbf{r}_1^{\rm H}\mathbf{r}_2\}}{\sigma_{\rm e}^2}, \quad (14)$$

$$\lambda_{\rm LS} = \frac{2{\rm Im}\{\mathbf{r}_1^{\rm H}\mathbf{r}_2\}}{\sigma_{\rm e}^2}. \quad (15)$$

There are $(N_{\rm s} - 1)$ LLR pairs — one for each $\Delta\phi_j$. We use the LLR vector in the LDPC decoder to estimate transmitted binary message vector $\hat{\mathbf{b}}$, as shown in Fig 1. Note that we use $\hat{d}$ from (7) to compute the LLR vector in (14) and (15) for the TSFE method.

VI. Simulations

Now, we present the simulation results of the proposed TSFE method over a Rayleigh fading frequency selective channel for a SISO-OFDM system. We present the effects of Doppler frequency $f_{\rm d}$ (resulting from the relative mobility of nodes), the spreading factor, and the number of paths on the BER performance of the proposed TSFE method. We also provide a simulation analysis of the TSFE method with capacity-achieving half-rate LDPC (648, 324) FEC code. We investigate the effect of spreading factor β on the diversity of the proposed scheme with different channel orders and Doppler frequencies. We also compare the BER performance of the proposed TSFE method with the MSDSD approach of [17]. In our simulation setup, we consider a 4-point DQPSK modulation over a Rayleigh fading channel with bandwidth $B = 10^6$ Hz, which corresponds to the symbol interval $T_{\rm s} = 10^{-6}$ s. Note that we use a uniform power delay profile of P independent and identically distributed paths of mean zero and variance $\sigma_{\rm h}^2 = 1/P$. The delay of the kth path is $kT_{\rm s}$, where $k = 0, 1, \ldots, P-1$.

1. Impact of Interleaving

Now, we present the impact of optimal assignment of subcarriers (interleaving), $I_{\rm o}$, and random assignment (interleaving), $I_{\rm r}$, to the differentially encoded information symbols in Fig. 3. We consider a SISO-OFDM modulated signal with $N_{\rm s}$ = 1,024 and β = 4, 8; P = 4 for Doppler frequencies of 0 Hz and 80 Hz. For optimal interleaving, $I_{\rm o}$, we assume that channel order P is known. As Fig. 3 reveals, optimal interleaving using (3) achieves full diversity P with $f_{\rm d}$ = 0 Hz, whereas random interleaving suffers from diversity loss. In the presence of Doppler frequency $f_{\rm d}$ = 80 Hz, random and optimal interleaving provide a similar performance. It is important to note that the TSFE method is resilient to the

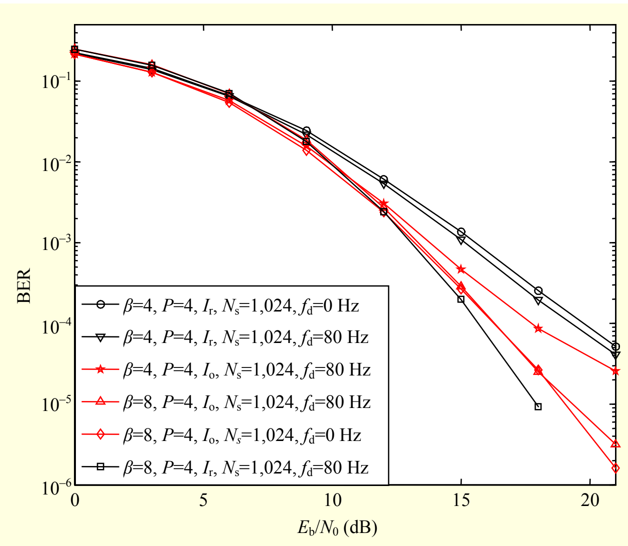

Fig. 3. BER performance of optimal and random interleaving for 1,024 subcarriers: P = 4; spreading factors of β = 4 and β = 8 at Doppler frequencies of 0 Hz and 80 Hz.

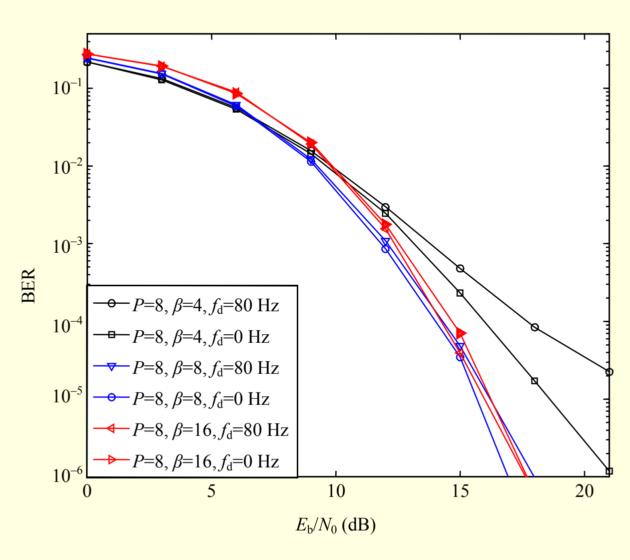

Fig. 4. Effect of spreading factor β on diversity: $P = 4$, with 1,240 OFDM subcarriers and Doppler shifts of $f_d = 0$ Hz and 80 Hz.

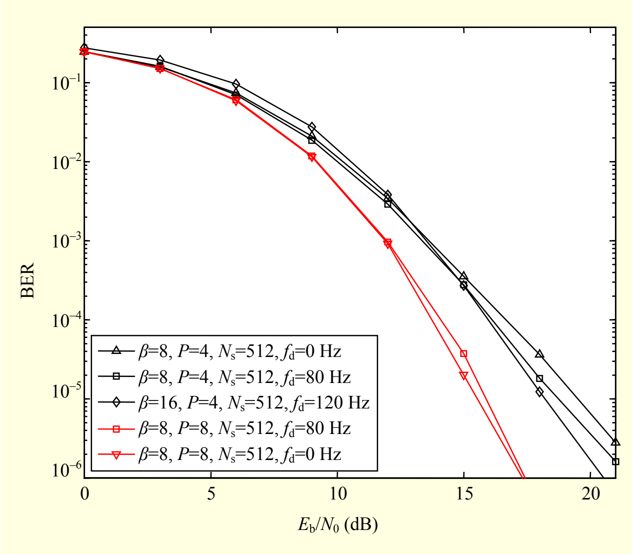

Fig. 5. Impact of number of paths P of SISO-OFDM system on BER performance of TSFE method.

Doppler frequency, as shown in Fig. 3. Since, in most of the channel conditions, optimal interleaving performs better than random interleaving, we consider optimal interleaving in the following discussion.

2. Impact of Spreading

The impact of spreading factor β on the BER performance of the TSFE method is twofold. That is, we achieve higher diversity and target energy per bit to noise ratio. Figure 4 depicts the impact of spreading on the BER performance under optimal interleaving. We consider the case where N_s = 1,024 and $f_d = 0$ Hz and 80 Hz. We select $P = 8$ and evaluate the BER performance for $\beta = 4$, 8, and 16. It is clear from Fig. 4 that the TSFE method suffers from diversity loss when $\beta = 4$ ($\beta < P$) at both $f_d = 0$ Hz and 80 Hz. The spreading factor of $\beta = 4$ does not achieve full diversity when $P = 8$ at $f_d = 0$ Hz. Figure 4 also shows that the TSFE method achieves a maximum frequency diversity of $P = 8$ when $\beta = 8$ and 16 at $f_d = 0$ Hz and 80 Hz, respectively.

3. Impact of Paths

Figure 5 presents the impact of P on the BER performance of the proposed method. An OFDM receiver can achieve a maximum frequency diversity of P [21]. In Fig. 5, we study the impact of the number of paths on the BER performance, for $P = 4$ and $P = 8$ with $N_s = 512$ and $\beta = 8$ under optimal interleaving. As Fig. 5 reveals, the proposed TSFE method achieves full frequency diversity when $\beta = P$ for $f_d = 0$ Hz and $f_d = 80$ Hz. The TSFE method achieves diversity order 4 with $P = 4$ for both $\beta = 4$ and $\beta = 8$ under $f_d = 0$. Thus, $\beta \geq P$ does increase diversity at low relative mobility. Note that OFDM modulation can achieve time diversity due to relative mobility. The TSFE method exploits time diversity along with frequency diversity under high relative mobility and $\beta \geq P$.

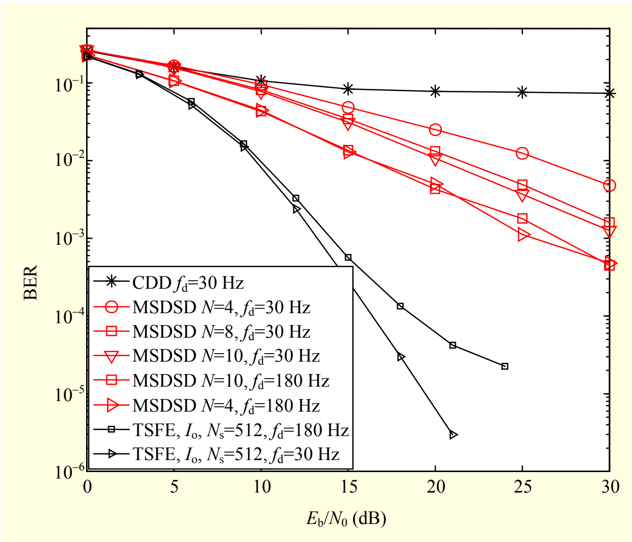

Fig. 6. BER performance of TSFE method vs. MSDSD method [17] using $P = 4$ and $\beta = 4$.

4. TSFE vs. MSDSD

Figure 6 presents the BER performance of the TSFE method in comparison with the MSDSD method presented in [17]. In this comparison, we consider the relative mobility of $f_d = 30$ Hz

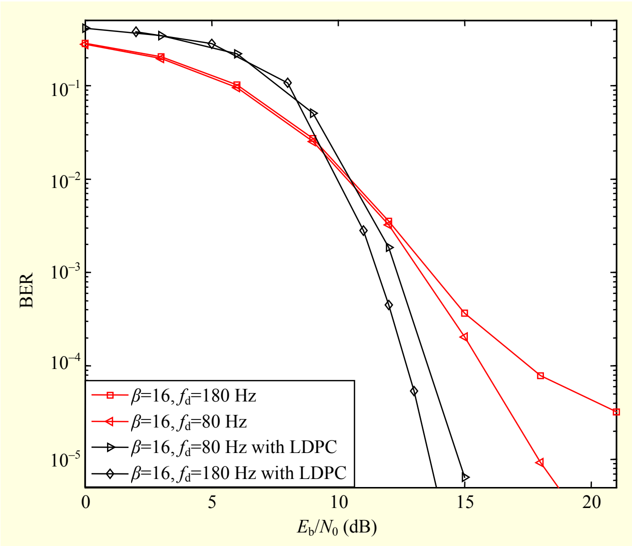

Fig. 7. BER performance of TSFE method with LDPC (648, 324) and without LDPC for different spreading factors.

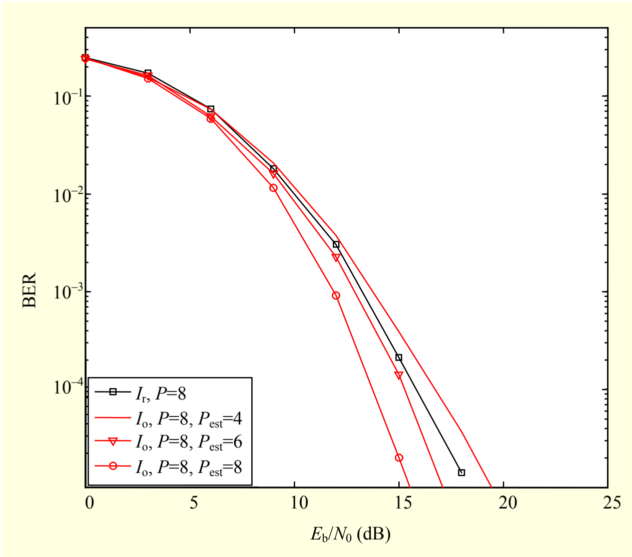

Fig. 8. BER performance of TSFE method with channel-order mismatch and optimal interleaving using 512 OFDM subcarriers and Doppler frequency of 0 Hz.

and f_d = 180 Hz with P=4 and β=4. Note that the MSDSD method jointly decodes information without channel knowledge and has high complexity [17]. We use window sizes of 4, 8, and 10 for joint detection to evaluate the BER performance of the MSDSD method. It is obvious from Fig. 6 that the TSFE method outperforms the MSDSD method and achieves higher diversity due to spreading. The performance of the MSDSD method improves by increasing the window size. However, for large window sizes, there are only subtle improvements in performance.

5. Performance with LDPC

Now, we present the BER performance versus E_b/N_0 of the TSFE method with a half-rate LDPC (648, 324) encoder over a Rayleigh fading channel of P = 4. We use a spreading factor of β = 16 with optimal interleaving for the spreading of an OFDM system with N_s=1,296. We compare the BER performances of the uncoded and coded (LDPC) at f_d=80 Hz and 180 Hz to study the impact of channel coding. It is clear from Fig. 7 that the LDPC encoder provides significant performance gain as compared to uncoded differential encoding for the TSFE method.

Figure 7 reveals that the TSFE method with large spreading factor β is more effective in a low SNR regime. Furthermore, our proposed TSFE method is more effective under low transmit power constraints and higher relative mobility.

6. Impact of Channel-Order Mismatch

Figure 8 presents the impact of mismatch of channel order P on the BER performance of the TSFE method using optimal interleaving. We consider the situation where N_s=512; P=8 without relative mobility (f_d = 0 Hz). We compare the performance of random interleaving with that of optimal interleaving under channel mismatch. The simulation results suggest that optimal interleaving I_o outperforms random interleaving I_r when P_{est} = 6 and 8. The performance gap between random interleaving and optimal interleaving is marginal when P_{est}=4.

VII. Conclusion

In this work, we investigated a non-coherent SISO-OFDM communication system that works in a low SNR regime and high relative mobility. The proposed TSFE method employs differential encoding along frequency dimensions and spreading along time to exploit frequency and time diversity for dynamic nodes. Our simulation analysis reveals that differential encoding along subcarriers achieves higher diversity and is resilient to the relative mobility of the nodes. The proposed method can achieve higher diversity and target energy per bit to noise ratio by employing repetition of information symbols along the time dimension. To further improve error-rate, we embedded an LDPC FEC code. The proposed method is effective for application with high mobility and low power constraints.

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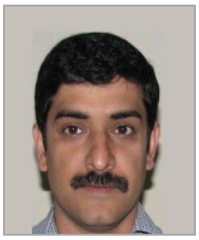
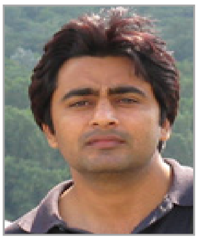
Faisal Nadeem received his BS degree in electronics from COMSATS Institute of Information Technology, Islamabad, Pakistan, in 2011 and his MPhil degree in electronics from the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan, in 2014. His research interests include wireless communication and signal processing, with an emphasis on wireless security and efficient blind transceiver design.

Muhammad Zia received his MS degree in electronics in 1991 and his MPhil degree in 1999, both from the Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan. He received his PhD degree in electrical engineering from the Department of Electrical and Computer Engineering, University of California, Davis, USA, in 2010. He is currently with the Department of Electronics, Quaid-i-Azam University, as an assistant professor.



Hasan Mahmood received his MS degree in electronics from Quaid-i-Azam University, Islamabad, Pakistan, in 1991 and his MS degree in electrical engineering from the University of Ulm, Germany, in 2002. From 1994 to 2000, he was with the Department of Electronics, Quaid-i-Azam University, as a faculty member. He received his PhD degree in electrical engineering from the Stevens Institute of Technology, Hoboken, NJ, USA, in 2007. He is currently with the Department of Electronics, Quaid-i-Azam University, as an assistant professor.



Naeem Bhatti received his MS and MPhil degrees in electronics from Quaid-i-Azam University, Islamabad, Pakistan, in 2002 and 2004, respectively. He received his PhD degree in Informatics from the Vienna University of Technology, Austria, in 2012. He is currently with the Department of Electronics, Quaid-i-Azam University, as an assistant professor.

Ihsan Haque received his MS degree in physics with specialization in electronics from the Bahauddin Zakariya University, Bosan Road, Pakistan, in 1998 and his MPhil degree in electronics from Quaid-i-Azam University, Islamabad, Pakistan, in 2004. He received his PhD degree in information and electronic engineering from BeiHang University, Beijing, China, in 2009. He is currently with the Department of Electronic Engineering, International Islamic University, Islamabad, Pakistan, as an assistant professor.