# Novel Peak-to-Average Power Ratio Reduction Methods for OFDM/OQAM Systems

Vangala Sandeep and Sundru Anuradha

**The tone reservation method is one of the most effective pre-distortion methods for peak-to-average power ratio reduction in orthogonal frequency division multiplexing (OFDM) systems. Its direct application to OFDM systems with offset quadrature amplitude modulation (OQAM) is, however, not effective. In this paper, two novel TR-based methods are proposed, specifically designed for OFDM/OQAM systems by taking into consideration the overlapping nature of OQAM signals. These two methods have different approaches to the generation of the peak-cancelling signal. The first one (*overlapped scaling tone reservation*) generates the peak-cancelling signal using a least squares approximation algorithm with possible adjacent symbol overlap; the second one (*multi-kernel tone reservation*) generates the peak-cancelling signal by using multiple impulse-like time domain kernels. It is shown by simulation that, when used in OFDM/OQAM systems, the proposed methods can provide better performance than the direct application of the existing *controlled clipping tone reservation* method, and even outperform the *multi-block tone reservation* method.**

**Keywords: OFDM/OQAM, PAPR, OS-TR, MK-TR, CC-TR, MB-TR.**

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## I. Introduction

Orthogonal frequency division multiplexing (OFDM) is currently the most commonly used multicarrier method. To prevent inter-block interference in OFDM implementations, a cyclic prefix (CP) code is appended before each data block, with a duration longer than the delay spread of the channel [1], [2]. Although the use of OFDM carries many advantages to communication systems, the performance of this method degrades dramatically in doubly dispersive channels with large delays and Doppler spreads, because of the usage of rectangular windows for symbol transmission [3], [4]. Rectangular windows have a sinc shape in the frequency domain, with large side-lobes that generate considerable inter-carrier interference, even in channels with small Doppler shifts. Moreover, when the delay spread of the channel is large, the bandwidth efficiency of OFDM will be very low, because of the need to increase the CP duration. To overcome the shortcomings of OFDM systems, OFDM with offset quadrature amplitude modulation (OFDM/OQAM) has been proposed, and has drawn significant interest in recent years [5], [6]. Compared with traditional OFDM, OFDM/OQAM exhibits lower side lobes, because of the use of pulse shaping filters [7], [8].

Even though OFDM/OQAM has many advantages over OFDM, the transmitted OFDM/OQAM signals still suffer from a high peak-to-average power ratio (PAPR), leading to the saturation of the high power amplifiers [9]. Moreover, the high PAPR generates out-of-band radiation, which causes serious adjacent channel interference [10], [11]. Many methods have been introduced to deal with the PAPR problem in OFDM, including tone reservation (TR) [12], companding [13], partial transmit sequence (PTS) [14], and selective mapping [15]. An overview of the above methods can be found in [16].

  
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The TR method has significant advantages when compared with the other PAPR reduction methods, without introducing interference or transmitting additional side information. Because of this, it has been considered for the use in future technologies such as DVB-T2 [17]. Given that OFDM/ OQAM and OFDM have similar characteristics, most research has been focused on the use of the conventional OFDM PAPR reduction methods in the newer OFDM/OQAM systems. However, a direct application of these methods is not efficient, because OFDM/OQAM signals overlap with adjacent data blocks.

In recent years, only a limited number of PAPR reduction methods have been proposed for the specific case of OFDM/OQAM systems. In [18], the multi-block joint optimization (MBJO) method was proposed for the reduction of PAPR in OFDM/OQAM signals. The overlapping structure of OFDM/OQAM is used in MBJO, and multiple data blocks are jointly considered and optimized. In [17], a novel segmental PTS method was proposed. In this method, OFDM/OQAM signals are divided into a number of segments, and each segment is then partitioned into individual sub-blocks and is multiplied by different phase rotation factors. These two methods provide effective solutions for the high PAPR problem by taking into account the overlapping nature of OFDM/OQAM signals; their common approach can be summarized as follows: deal with multiple adjacent data blocks simultaneously, instead of optimizing each data block independently.

In this article, two new methods are proposed to reduce PAPR in OFDM/OQAM systems: the overlapped scaling TR (OS-TR) method and the multi-kernel TR (MK-TR) method. These methods exploit the overlapping nature of OFDM/OQAM signals, and thereby exhibit better performances than the controlled clipping TR (CC-TR) and multi-block TR (MB-TR) methods in [19]. Given that adjacent data blocks in the OFDM/OQAM signals overlap with each other—because of the introduction of the filter bank—the OS-TR method scales the filtered clipping noise to produce the peak-cancelling signal, and all the influences of the associated data block overlap are taken into consideration when computing the scaling factors. The OS-TR method can achieve a good trade-off between PAPR reduction and computational complexity, but it often requires several iterations when reducing large PAPR values. The MK-TR method is therefore proposed, to overcome this problem of high complexity. The MK-TR method uses multiple impulse-like time domain kernels—subjected to TR constraints—to eliminate the high peaks of the OFDM/OQAM signal. The time domain kernels of OFDM/OQAM systems are different from those of conventional OFDM systems, because OFDM/OQAM systems are more complex than conventional OFDM systems. Compared with the OS-TR method, the MK-TR method is more focused on peak elimination, resulting in better PAPR reduction.

This paper is organized as follows. Section II presents a brief review of the TR method as a means to reduce PAPR in OFDM/OQAM systems. Section III discusses the two novel TR methods being proposed. In Section IV, the proposed methods are simulated. Finally, Section V presents some concluding remarks.

## II. Tone Reservation in OFDM/OQAM Systems

### 1. PAPR of OFDM/OQAM Signals

As shown in Fig. 1, the baseband modulated OFDM/ OQAM system consists of $N$ subcarriers.

Real valued symbols modulated by offset-QAM are transmitted on each subcarrier; the transmitted signal can be written as follows [19]:

$$s(n)=\sum_{m=0}^{M-1}\underbrace{\sum_{k=0}^{N-1}X_m^k\,\underbrace{h\left(n-m\frac{N}{2}\right)e^{jk\left(\frac{2\pi}{N}n+\frac{\pi}{2}\right)}}_{h_m^k(n)}}_{s_m(n)}, \qquad (1)$$

where $M$ represents the number of input data blocks, $X_m^k$ is a real valued symbol with time-frequency index ($m$, $k$), and $N$/2 is the symbol interval, as shown in Fig. 2. $X_{2\delta}^k$ and $X_{2\delta+1}^k(\delta\in Z)$ are the real and imaginary parts of the staggered complex QAM symbol, $h(n)$ is the prototype filter impulse response, with length $LN(L\in Z)$ [19].

Given that the time domain signal of each data block is overlapped with the time domain signals of the adjacent OFDM/OQAM data blocks (as shown in Fig. 2), the conventional definition of PAPR for OFDM systems no longer matches for OFDM/OQAM systems. If we divide the time domain OFDM/OQAM signals into ($M$ + $L$) intervals with equal time duration $N$, the PAPR of each interval is given by:

$$PAPR_s=\frac{\max\limits_{sN\le n\le(s+1)N}|s(n)|^2}{\mathrm{E}\left[|s(n)|^2\right]} \qquad s=0,1,\ldots,(M+L-1), \qquad (2)$$

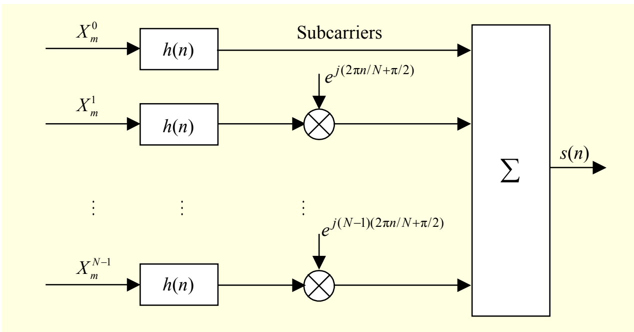


Fig. 1. Baseband OFDM/OQAM transmitter.

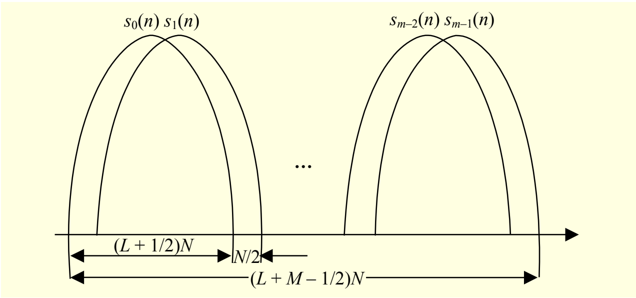


Fig. 2. OFDM/OQAM signal structure.

where $\mathrm{E}[\cdot]$ represents the expectation operator.

### 2. Tone Reservation Techniques for OFDM/OQAM Systems

In the TR method, a small percentage of subcarriers $N_r$ are preserved as peak reduction tones (PRTs), whereas the remaining subcarriers ($N - N_r$) are used for data transmission. The ordered set of PRT indices is denoted by $R = \{k_0, k_1, \dots, k_{N_r - 1}\}$, where $N_r$ is the size of the PRT set. The *mth* symbol on the *kth* tone of $x_m^k(n)$ consists of two parts: the peak reduction signal (on the PRTs) and the data signal (on the unreserved tones), as:

$$S_m^k = D_m^k + C_m^k = \begin{cases} D_m^k, & k \in R^c, \\ C_m^k, & k \in R \ , \end{cases} \tag{3}$$

where $R^c$ is the complement set of $R$ in $\delta = \{0, 1, \dots, N-1\}$, $D_m^k$ and $C_m^k$ represent the data and peak reduction signals on the $k$th tone of the $m$th data block, respectively, and

$$D_m^k = 0, \quad \text{for } \ k \in R, \quad C_m^k = 0, \quad \text{for } \ k \in R^c . \tag{4}$$

At the receiving end, the peak reduction subcarriers are discarded, and signal demodulation is performed on the data subcarriers. If a cancelling signal is superimposed on the transmitted data subcarriers, the peak reduction signal is given as follows:

$$\begin{aligned} \hat{s}(n) &= \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} \left(D_m^k + C_m^k\right) h\left(n - mN/2\right) e^{jk\left(\frac{2\pi n}{N} + \frac{\pi}{2}\right)}, \\ &= \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} D_m^k h_m^k(n) + \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} C_m^k h_m^k(n) = s(n) + c(n), \end{aligned} \tag{5}$$

where $c(n)$ is the peak cancellation signal; that is,

$$c(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} C_m^k h\left(n - m\frac{N}{2}\right) e^{j\frac{2\pi k}{N} n} e^{j\phi_m^k} . \tag{6}$$

In this case, the PAPR is defined as

$$PAPR_s = \frac{\max\limits_{sT \le n \le (s+1)T} \left|s(n) + c(n)\right|^2}{\mathrm{E}\left[\left|s(n)\right|^2\right]}, \quad s = 0, 1, \dots, (M + L - 1). \tag{7}$$

From (6), the peak-cancelling signal $c(n)$ determines the PAPR reduction effect. Therefore, $C$ must be chosen to minimize the peak of the time domain signal and achieve the optimal peak-cancelling symbol $C^{\mathrm{opt}}$; that is:

$$C^{\mathrm{opt}} = \arg\min_{C} \max_{qN \le n \le (q+1)N} |s(n) + c(n)|^2, \quad q = 0, 1, \dots, M + L - 1. \tag{8}$$

A simple algorithm was therefore required to effectively obtain the optimum peak-cancelling signal $C^{\mathrm{opt}}$ with low computational complexity.

## III. Proposed PAPR Reduction Methods for OFDM/OQAM Systems

In this section, two novel TR methods are proposed; both methods consider the adjacent data blocks when calculating the optimal peak-cancelling signal for time domain signals exceeding a given threshold $A$. In addition, the influence of the associated data block overlaps is taken into consideration to overcome the problem of peak regrowth created by OFDM/OQAM signals, thus improving the convergence rate and time efficiency.

### 1. Overlapped Scaling Tone Reservation

The proposed OS-TR method is executed at a multi-block level to generate the optimal peak-cancelling signal, thereby maintaining the continuous nature of OFDM/OQAM systems. The OS-TR method approximates the peak-cancelling signal to the clipping noise waveform using the least squares algorithm and considering the influence of the data block overlaps associated with OFDM/OQAM signals.

The time domain signal $s(n)$ is clipped using a soft limiter [20], resulting in the signal $\bar{s}(n)$, as follows:

$$\bar{s}(n) = \begin{cases} s(n), \text{if } |s(n)| \le A, \\ Ae^{j\theta_n}, \text{if } |s(n)| > A, \end{cases} \quad 0 \le n \le (M + L - 1/2)N, \tag{9}$$

where $A$ is the predefined threshold and $\theta_n$ is the phase of $s(n)$. The clipping noise is defined as

$$f(n) = \bar{s}(n) - s(n), \ \ 0 \le n \le (M + L - 1/2)N. \tag{10}$$

The clipping noise $f(n)$ is demodulated and converted to the corresponding frequency domain signal, as

$$F_m^k = \Re\left\{ \sum_{n=0}^{(M+L-1/2)N} f(n) h\left(n - m\frac{N}{2}\right) e^{-jk\left(\frac{2\pi}{N}n + \frac{\pi}{2}\right)} \right\}. \tag{11}$$

Given that the input symbols in OFDM/OQAM are real valued, the symbols on the peak reduction tones $F_m^k$ must also be real valued, to satisfy the OFDM/OQAM symbol structure. All




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tones other than the peak reduction tones $F_m^k$ are then set to zero; that is,

$$F_m^k = \begin{cases} F_m^k, & k \in R, \\ 0, & k \in R^c, \end{cases} \quad m = 0,1, \dots, M-1. \tag{12}$$

Therefore, the frequency domain peak-cancelling signal can be written as $F = [F_0, F_1, \dots, F_{M-1}]^{\mathrm{T}}$, where $F_m = [F_m^0, F_m^1, \dots, F_m^{N-1}]^{\mathrm{T}}$, and $m = 0,1, \dots, M-1$. The tones $F_m^k$ are modulated to obtain the time domain clipped signal $\hat{f}_m(n)$. To approximate $c(n)$ as much as possible to $f(n)$, $\hat{f}_m(n)$ is scaled by a constant $p_m$, resulting in

$$c(n) = \sum_{m=0}^{M-1} p_m \hat{f}_m(n), \tag{13}$$

where

$$\hat{f}_m(n) = \sum_{k=0}^{N-1} F_m^k(n) h\left(n - m\frac{N}{2}\right) e^{jk\left(\frac{2\pi}{N}n + \frac{\pi}{2}\right)}. \tag{14}$$

The objective function of the OS-TR method leads to a decrease in the Euclidean distance between $f(n)$ and the peak-cancelling signal $c(n)$ through a least squares approximation, considering the possible symbol overlap in OFDM/OQAM symbol structure. It is given by

$$\min_{p_0, p_1, \dots, p_{M-1}} \left\{ \sum_{n=0}^{(M+L-1/2)N} \left[ \sum_{m=0}^{M-1} p_m \hat{f}_m(n) - f(n) \right]^2 \right\}. \tag{15}$$

As previously mentioned, $\hat{f}_m(n)$ is the time domain signal ranging from $mN/2$ to $(L + m/2)/N$, and $L$ signals of $\hat{f}_m(n)$ overlap with the previous section; that is, $\hat{f}_{m-1}(n)$, $\hat{f}_{m-2}(n), \dots, \hat{f}_{m-(L+1)}(n)$. Therefore, when calculating $p_m$, the possible signal overlaps are considered over the full interval $mN/2 \le n \le (L + m/2)N$, for $m = 0, 1, \dots, M-1$. The scaling factor calculation considering the possible symbol overlaps proceeds as follows:

$$p_m = \begin{cases} \dfrac{\sum_{n \in S_0} |\hat{f}_0(n)||f(n)|}{\sum_{n \in S_0} |\hat{f}_0(n)|^2}, & m = 0, \\ \dfrac{\sum_{n \in S_m} |\hat{f}_m(n)| \left| f(n) - \sum_{k=0}^{m-1} p_k \hat{f}_k(n) \right|}{\sum_{n \in S_m} |\hat{f}_m(n)|^2}, & 1 \le m \le M-1, \end{cases} \tag{16}$$

where

$$S_m = \left\{ n / |f(n)| > 0, \frac{mN}{2} \le n \le \left(L + \frac{m}{2}\right)N \right\}, m = 0,1, \dots, M-1,$$

and $p_m$ is the optimized scaling factor. Moreover, from (15), as the distance between (13) and (10) is very small, the new peak cancelling signal amplitude approximates the original clipping noise within only four iterations.

The OS-TR algorithm can be illustrated as follows:

*Algorithm (overlapped scaling algorithm)*

**Algorithm 1. OS-TR**

*Initialization:*

1. Set the threshold value to A, the maximum number of iterations to $I$, and randomly generate a set of reserved subcarriers.
2. Encode, interleave and modulate the input bit streams.

*Runtime:*

1. Using (3), place $N - N_r$ frequency domain data on data subcarriers $R^c$, and calculate the corresponding time domain signal $s(n)$.
2. If $\max(|s(n)|) > A$, set the number of iterations to $i = 1$, and proceed to Step 3. Otherwise, output the signal $s(n)$ and terminate the algorithm.
3. Generate a peak-*cancelling* signal $c(n)$:
   a. Using (9) and (10) process the signal $s(n)$ and generate the clipping noise, denoted by $f(n)$.
   b. Transform $f(n)$ to the frequency domain to produce $F_m^k$, and set $F_m^k = 0$, for all $k \notin R$.
   c. Using (14), transform $F_m^k$ to $\hat{f}_m(n)$.
   d. Accordingly, calculate the $p_m$ as given by (16).
   e. Calculate $c(n)$ using (13).
4. Superimpose the peak-cancelling signal $c(n)$ on the original time domain signal $s(n)$, to obtain the new peak-reduced signal:
$$\hat{s}(n) = s(n) + c(n). \tag{17}$$
5. If $i < I$ and $\max(|\hat{s}(n)|) > A$, calculate the current time domain signal peak $\max(|\hat{s}(n)|)$, calculate $s(n) = \hat{s}(n)$, $i = i + 1$, and jump to Step 3; otherwise, output the current time domain signal $\hat{s}(n)$.

*Computational Complexity Analysis:*

The computational complexity is measured here by the counts of real multiplications and real divisions. The two initialization steps and runtime Step 5 are not considered, because these steps are regular operations required in general by TR-based methods. The complete computational complexity for modulation and demodulation of an OFDM/OQAM system is given in [21].

Step 3 requires the calculation of the optimal scaling factor. In (16), the number of possible samples in $S$ is taken as $K$, where $K$ is a random variable and varies with the iteration; here we denote the average value of $K$ by $\bar{K}$. The number of real multiplications and real divisions required to calculate $p_m$ can then be estimated as $(2\bar{K} + 1)$ and $(\bar{K} + 1)$, respectively. Step 4 requires no real multiplications or divisions. The complexity is given by

$$
\begin{aligned}
MUL_{\text{Total, OS-TR}} &= I\Big(MUL_{\text{Mod, OS-TR}} + \\
&\quad (MUL_{\text{Mod, OS-TR}} + MUL_{\text{DeMod, OS-TR}}) + (2\bar{K}+1)\Big), \\
&= I\Big(2\big(N(\log_2(N/2)-3)+8+4(NL+1)\big)+ \\
&\quad \big(2N(\log_2(N)-3)+8+4(NL+1)\big)+(2\bar{K}+1)\Big), \\
DIV_{\text{Total, OS-TR}} &= I(\bar{K}+1).
\end{aligned} \tag{18}
$$

## 2. Multi-kernel Tone Reservation Method

The OS-TR method is simple and effective, but it requires several iterations to reduce large PAPR. To overcome this problem, the MK-TR method is proposed, which uses the reserved subcarriers to generate pulse-like signals to eliminate the peak values of OFDM/OQAM signals.

In conventional OFDM systems, the signal obtained after the IFFT operation can be directly used as a time domain signal to synthesize a new peak cancellation signal. However, the time domain peak-cancelling pulse waveform of OFDM/OQAM is completely different from that of traditional OFDM systems. In OFDM/OQAM systems, the real input signal needs to go through a pre-modulator and a filter before and after the IFFT operation. If the IFFT operation is carried out on a frequency domain data block containing only ones and zeros, the IFFT output signal is an impulse-like signal waveform [22], and reaches its peak at $n = 0$.

The time domain kernel is thus obtained as

$$
q(n) = \frac{1}{N}\sum_{k=0}^{N-1} Q_k e^{j\frac{2\pi kn}{N}}, \quad n = 0, 1, \ldots, N-1, \tag{19}
$$

where $Q_k$ is the frequency domain kernel. $Q_k$ is calculated by labeling with 1 the set $R$ of $N_r$ reserved tones, and is defined as

$$
Q_k = \begin{cases} 1, & k \in R, \\ 0, & k \in R^c. \end{cases} \tag{20}
$$

Based on the Fourier transform cyclic shift characteristics, the signal $q(n)$ can be cyclic-shifted using the $Q_k$ frequency kernel, as follows:

$$
q(n+v)_N = \frac{1}{N}\sum_{k=0}^{N-1} Q_k e^{j\frac{2\pi k(n+v)}{N}}, \tag{21}
$$

where $q(n+v)_N$ represents a left cyclic shift of the signal $q(n)$ by $v$, an arbitrary positive integer.

If the $Q_k$ frequency domain data blocks are input into the OFDM/OQAM system, the output signal $q_m^0(n)$, $m = 0, 1, \ldots, M-1$, can be expressed as

$$
\begin{aligned}
q_m^0(n) &= \sum_{k=0}^{N-1} Q_k h\left(n - \frac{mN}{2}\right) e^{jk\left(\frac{2\pi n}{N}+\frac{\pi}{2}\right)}, \quad \frac{mN}{2} \le n \le \left(L+\frac{m}{2}\right)N, \\
&= Nq\left(n+\frac{N}{4}\right)_N h\left(n-\frac{mN}{2}\right),
\end{aligned} \tag{22}
$$

where $q(n+N/4)$ is an impulse-like signal with its peak position at $n = 3N/4$, and $h(n - mN/2)$ is a symmetrical waveform having well localized time-frequency characteristics, with a maximum value much larger than the second largest value, and reaching its peak value at $n = ((L+m)N)/2$. Signal $q_m^0(n)$ is obtained by multiplying $q(n+(N/4))_N$ and $h(n-(mN/2))$, and is still a similarly impulse-like signal. Therefore, when $m$ is even, the maximum value of $q_m^0(n)$ is obtained for $n = ((L+m-1/2)N)/2$; when $m$ is odd, the maximum value of $q_m^0(n)$ is obtained for $n = ((L+m+1/2)N)/2$.

Similarly, if the input signal to the OFDM/OQAM system is $Q^v = \left[Q_0 e^{jk\frac{2\pi v}{N}}, \ldots, Q_{N-1} e^{jk\frac{2\pi v}{N}}\right]$, the output signal $q_m^v(n)$ will be:

$$
q_m^v(n) = Nq\left(n+v+\frac{N}{4}\right)_N h\left(n-\frac{mN}{2}\right), \quad \frac{mN}{2} \le n \le \left(L+\frac{m}{2}\right)N. \tag{23}
$$

When $m$ is even, the maximum value of $q_m^v(n)$ is obtained for $n = ((L+m-1/2)N)/2 - v$; when $m$ is odd, the maximum value of $q_m^v(n)$ is obtained for $n = ((L+m+1/2)N)/2 - v$. Therefore, when $v$ assumes an arbitrary value, the signal peak of $q_m^v(n)$ will always be between $n = ((L+m-1)N)/2$ and $n = ((L+m+1)N)/2$.

Figure 3 shows how the peak of $q_m^v(n)$ is determined by $v$. The $q_m^v(n)$ signal peak position can be controlled by imposing cyclic shifts on $q(n)$. It is worth noting that the cyclic-shifted

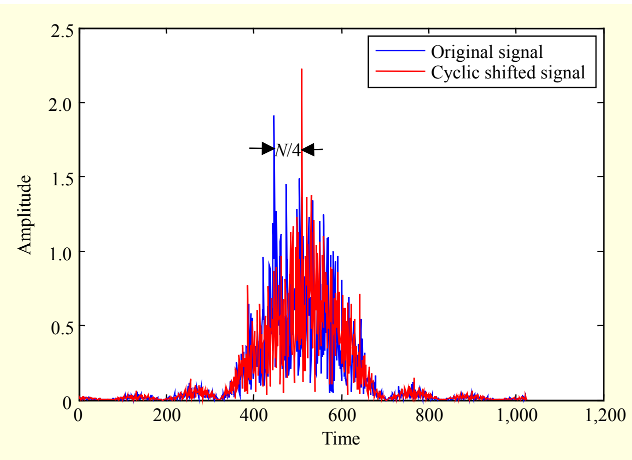


Fig. 3. Waveforms of signals $q_m^m(n)$ and $q_m^{\frac{N}{4}}(n)$.

signal $q(n+v)$ does not destroy the OFDM/OQAM system orthogonality, because of the cyclic shift characteristics of the Fourier transform. Therefore, the $q_m^v(n)$ signal is the time domain peak cancellation signal in OFDM/OQAM systems.

In the following analysis, multiple signals exceeding the threshold $A$ are simultaneously suppressed, by applying multiple impulse-like time domain kernels in a single iteration. This improves the time efficiency and accelerates the convergence rate.

For the MK-TR method, multiple adjacent data blocks need to be jointly considered, to obtain a series of pulses $f = \left[f[0], f[1], \dots, f\left[\left(M+L-1\right)N\right]\right]^{\mathrm{T}}$ when $|s(n)| > A$, given by

$$f(n) = \begin{cases} 0, & |s(n)| \le A, \\ s(n) - Ae^{j\theta(n)}, & |s(n)| > A, \end{cases} \tag{24}$$

where $\theta(n)$ is the phase of $s(n)$.

The time domain kernel $q_m^v(n)$ is defined to construct the peak-cancelling signal $c(n)$. Given that the length of $s(n)$ is much greater than the length of $q_m^v(n)$, the $s(n)$ is segmented into $(M + L)$ intervals, to obtain the individual peak positions of the clipping noise $f(n)$ for each segment. Therefore, the location of each segment (segment $v_m$ peaks at positions $n_m^0$, $n_m^1, \dots, n_m^{v_m-1}$) is given as

$$S_m = \left\{ n \backslash f(n) > f(n-1), f(n) > f(n+1), \frac{(L+m-1)N}{2} \le n \le \frac{(L+m+1)N}{2} \right\}, m = 0, 1, \dots, M-1. \tag{25}$$

The time domain kernel $q_m^{u_m^v}(n)$ is then constructed by cyclic-shifting $q_m^v(n)$ to the right by the value of median $u_m^v$. The median $u_m^v$ of the time domain kernel at the peak positions is given by

$$u_m^v = \begin{cases} n_m^v - \dfrac{(L+m-1/2)N}{2}, & m \text{ is even}, \\ n_m^v - \dfrac{(L+m+1/2)N}{2}, & m \text{ is odd}. \end{cases} \tag{26}$$

After finding these peak positions, the peak cancellation signal $c(n)$ is generated using the time domain kernel $q_m^{u_m^v}(n)$. It is clear that the performance improves if the peak-cancelling signal amplitude approaches the original clipping noise $f(n)$ within a single iteration. Therefore, the time domain kernel of each segment $q_m^{u_m^v}$ is scaled, to approximate the corresponding pulse in $f(n)$ to location $S_m$. Defining $a_m^v$ as the scaling factor of the $v$th peak of the $m$th symbol, the objective function of the optimization problem of the MK-TR method can be formulated as

$$\min \left\| \sum_{m=0}^{M-1} \sum_{v=0}^{v_m-1} q_m^{u_m^v}(n) a_m^v - f(n) \right\|_2^2. \tag{27}$$

The optimum $a_m^v$ in (27) is the minimum norm least squares approximation of the linear equation at the median position $n_m^v$; it is given by

$$q_m^{u_m^v}\left(n_m^v\right) a_m^v - f\left(n_m^v\right) = 0; \; a_m^v = f\left(n_m^v\right) / q_m^{u_m^v}\left(n_m^v\right). \tag{28}$$

Therefore, the time domain peak reduction signal is

$$\begin{aligned} c(n) &= \sum_{m=0}^{M-1} \sum_{v=0}^{v_m-1} a_m^v q_m^{u_m^v}(n) \\ &= \sum_{m=0}^{M-1} \sum_{v=0}^{v_m-1} a_m^v q\left(n + \frac{N}{4} - u_m^v\right) h\left(n - \frac{mN}{2}\right). \end{aligned} \tag{29}$$

*Algorithm (Multi-kernel tone reservation)*

As shown in Fig. 4, the MK-TR algorithm can be summarized as follows:

**Algorithm 2. MK-TR**

*Initialization:*

1. Set the threshold value to $A$, the maximum number of iterations to $I$, and randomly generate a set of reserved subcarriers.
2. Encode, interleave and modulate the input bit streams.
3. Calculate the time domain kernel $q(n)$ using (19).

*Runtime:*

1. Using (3), place $N - N_r$ frequency domain data on data subcarriers $R^c$ and calculate the corresponding time domain signal $s(n)$.
2. If $\max(|s(n)|) > A$, set the number of iterations to $i = 1$ and go to Step 3. Otherwise, output the signal $s(n)$ and terminate the algorithm.
3. Using (24), process the time-domain signal $s(n)$, to obtain the time domain clipping noise $f(n)$ and its peak position using (25).
4. Using (28), calculate the complex scaling factor $a_m^v$ and shift the median $u_m^v$; then, using (29), calculate the peak cancellation signal $c(n)$, and superimpose $c(n)$ on the original time domain signal $s(n)$, calculating the reduced signal peak as
$$\hat{s}(n) = s(n) + c(n). \tag{30}$$
5. If $i < I$ and $\max\left(|\hat{s}(n)|\right) > A$, calculate the current time domain signal peak $\max\left(|\hat{s}(n)|\right)$, $\hat{s}(n) = s(n)$, increment $i$ ($i = i + 1$) and jump back to Step 3; otherwise, output the current time domain signal $s(n)$.

*Computational complexity analysis*

The computational complexity is measured here again by the counts of real multiplications and real divisions. The three initialization steps and runtime Step 5 are not considered, because these steps are regular operations required in general by TR-based methods.

In Step 3 of the proposed MK-TR method, $f(n)$ can be rewritten using (24). The size of set $S$ using (25) is taken as $K$, where $K$ is a random variable and varies with the iteration; here we denote the average value of $K$ by $\bar{K}$. The complexity of

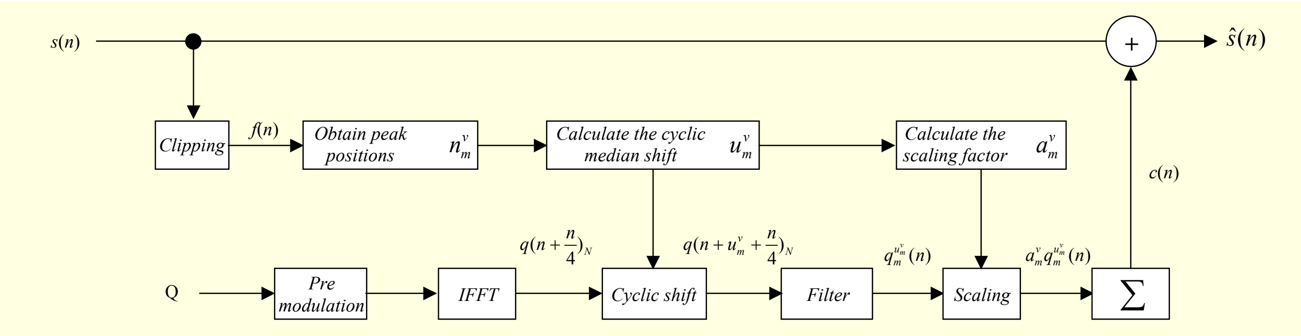


Fig. 4. Multi-kernel tone reservation method.

Step 3 can then be found to be $2\bar{K}$ real multiplications and $\bar{K}$ real divisions.

The time domain kernel is generated by circularly shifting $q$ in Step 4, and requires no multiplications or divisions. However, calculating $a$ using (28) requires $\bar{K}$ real divisions. In Step 5, calculating the peak-reduced signal requires $2LN\bar{K}$ real multiplications. The total complexity in terms of real multiplications and divisions with $I$ iterations can therefore be expressed as:

$$\begin{aligned} MUL_{\text{MK-TR}} &= I(3\bar{K}+2LN\bar{K}), \\ DIV_{\text{MK-TR}} &= I(2\bar{K}). \end{aligned} \tag{31}$$

The complexity of the MB-TR method—which is estimated in [19]—is provided here for comparison:

$$\begin{aligned} MUL_{\text{total, MB-TR}} &= MUL_{\text{Mod, MB-TR}} + (MUL_{\text{Mod, MB-TR}} + MUL_{\text{DeMod, MB-TR}}) \\ &\quad + (6LN), \\ &= 2\left(N\left(\log_2(N/2)-3\right)+8+4(NL+1)\right)+ \\ &\quad \left(2N\left(\log_2(N)-3\right)+8+4(NL+1)\right)+(6LN), \\ DIV_{\text{MB-TR}} &= N. \end{aligned} \tag{32}$$

Given that the analytic expression of $\bar{K}$ cannot be derived, we carried out some numerical calculations to obtain the value of $\bar{K}$. In an OFDM/OQAM system with $N = 256$, $L = 4$, 32 reserved tones, and a clipping ratio of 7 dB, we obtained value of $\bar{K} = 4.57$ per symbol and iteration. The computational complexity in these conditions will be discussed in the next section.

## IV. Simulation Results

In this section, the PAPR reduction and BER performance of the proposed OS-TR and MK-TR methods are compared with those of existing methods. In the simulations, the OFDM/OQAM system uses 256 subcarriers with $10^4$ randomly generated data blocks, convolution coded ($K = 7$ with $g_1 = (133)$ and $g_2 = (171)$) and then QPSK modulated. The

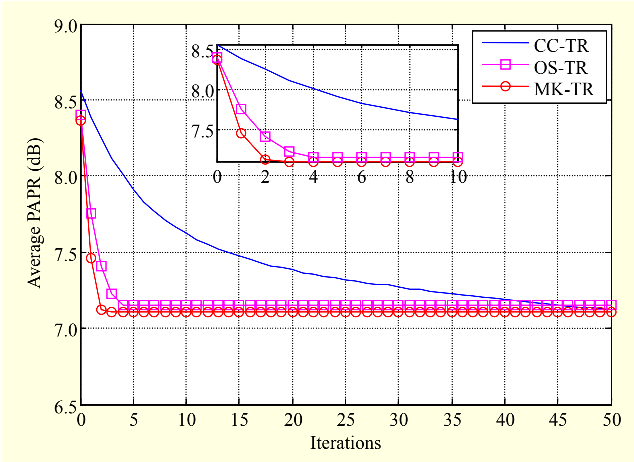


Fig. 5. Average PAPR in successive iterations, for different methods.

number of randomly generated PRTs is $N_r = 32$. The square-root raised cosine filter employed in this paper has a unit roll-off factor, and a length of $4N$.

The convergence rate of the MK-TR and OS-TR methods is compared with that of CC-TR; the average PAPR is computed with 50 fixed iterations. Figure 5 illustrates the average power descent along the number of iterations for the proposed MK-TR, OS-TR, and CC-TR methods. As shown, all three methods converge to the same value, even though they require different numbers of iterations. In terms of convergence rate, the MK-TR attains the minimal value of the averaged PAPR with only two iterations, OS-TR reaches this value after four iterations, and CC-TR approximates the minimal value of averaged PAPR much more slowly, requiring approximately 50 iterations to converge.

Based on the analysis of convergence rate considered above, a parameter for the typically required number of iterations of each method is considered in the following simulation: 2, 4, and 10 for the MK-TR, OS-TR, and CC-TR methods, respectively. The proposed methods are also compared with the MB-TR, which is a non-iterative method. The complementary cumulative distribution function (CCDF) is considered in the


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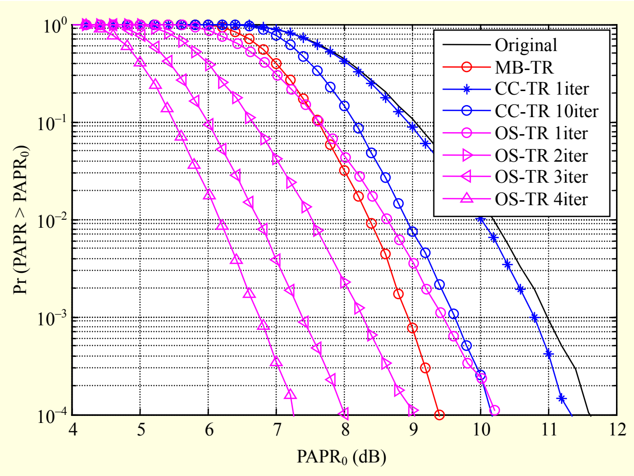


Fig. 6. PAPR reduction performance of the OS-TR, CC-TR, and MB-TR methods when $N = 256$ and $R = 32$.

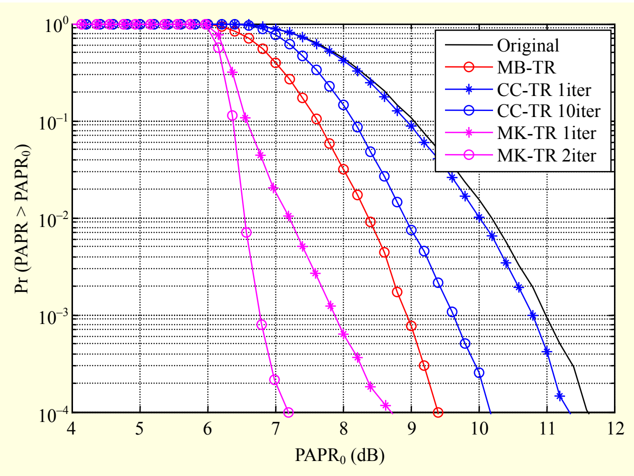


Fig. 7. PAPR reduction performance of the MK-TR, CC-TR, and MB-TR methods when $N = 256$ and $R = 32$.

PAPR calculations.

Figures 6 and 7 compare the proposed methods for 7.0 dB clipping, 12.5% TR ratio, and a randomly selected set of reserved tones. As shown in Fig. 6, after ten iterations a PAPR reduction of 1.3 dB is observed in the case of CC-TR at a $10^{-4}$ clip probability. At each iteration, a small reduction in PAPR is achieved by the CC-TR method; to obtain the maximum possible PAPR reduction, more iterations are therefore required.

To be effective in PAPR reduction, at least ten iterations have to be carried out for the CC-TR method, because of its slow convergence rate. The MB-TR method achieves a 2.3 dB reduction in PAPR at a $10^{-4}$ clip probability, without any type of iteration.

As shown in the same figure, the OS-TR method achieves 1.2 dB, 2.5 dB, 3.6 dB, and 4.2 dB reductions after one, two, three, and four iterations, respectively. However, after the first three iterations, a smaller PAPR reduction is observed (approximately 0.6 dB), which indicates that the OS-TR convergence rate is faster than the one of the CC-TR method. It should be noted that the proposed method requires additional modulation and demodulation operations of OFDM/OQAM signals to calculate the optimal scaling factor at each iteration.

The performance of the MK-TR method is shown in Fig. 7, for one and two iterations. As shown, when CCDF = $10^{-4}$, the PAPR reductions of this method are 3.0 dB and 4.4 dB for one and two iterations, respectively. Operating the MK-TR algorithm for more than two iterations could result in an additional (albeit negligible) performance improvement, but at the expense of computational complexity. Therefore, the number of iterations is limited here to two. It is noteworthy that the MB-TR method can attain a 2.3 dB PAPR reduction without any iteration, whereas the CC-TR method takes almost ten iterations to attain a 1.3 dB reduction in PAPR. The reason is that in the MK-TR method the overlapping nature of the OFDM/OQAM signals is adequately exploited, by jointly considering the adjacent data blocks in the process of obtaining the optimal time domain peak-cancelling signal.

As shown in Figs. 6 and 7, at $10^{-4}$ clip probability, the OS-TR and MK-TR methods achieve (respectively) a 1.2 dB and 3.0 dB reduction in PAPR with a single iteration. During each iteration, the optimal scaling factor is calculated. Taking into consideration the complexity analysis presented in the previous section, to achieve the desired PAPR reduction performance, MK-TR needs two iterations, OS-TR needs four iterations, and CC-TR needs 50 iterations. However, MB-TR is a non iterative process. Therefore, the computational complexity of the proposed methods is evaluated using the computational complexity reduction ratio (CCRR), defined as follows:

$$CCRR = \left(1 - \frac{Complexity\ \ of\ \ proposed\ \ method}{Complexity\ \ of\ \ previous\ \ method}\right) \times 100\%. \quad (33)$$

The computational complexity comparison of the proposed MK-TR and OS-TR methods using equations (18) and (31) is listed in Table 1. As shown, the real multiplications computational complexity of the MK-TR method is reduced by 71.47 %, 18.76 %, and 88.5 % when compared with the OS-TR, MB-TR and CC-TR methods, respectively. In other words, the MK-TR and OS-TR methods obtain a better PAPR reduction, but the OS-TR method is computationally more complex. Therefore, the proposed MK-TR method simultaneously achieves the more significant PAPR reduction and exhibits the lower computational complexity when compared with the OS-TR and MB-TR methods, thus allowing both computational and time savings.

Table 2 illustrates the average power variations of the

Table 1. Computational complexity comparison of the MK-TR, OS-TR, MB-TR, and CC-TR methods.

| Pulse | Real multiplications | CCRR of real multiplications |
|---|---|---|
| MK-TR | 18,746 | |
| OS-TR | 65,721 | 71.47% |
| MB-TR | 23,076 | 18.76% |
| CC-TR | 164,200 | 88.5% |

Table 2. Comparison of average power (in decibels) and PAPR when $CCDF = 10^{-4}$, for CC-TR, MB-TR, OS-TR, and MK-TR, with randomly selected reserved tones and a 7 dB clipping ratio.

| Pulse | Average power (dB) | PAPR at $10^{-4}$ (dB) |
|---|---|---|
| CC-TR | 0.2216 | 10.2 |
| MB-TR | 0.2299 | 9.4 |
| OS-TR | 0.1715 | 7.4 |
| MK-TR | 0.1862 | 7.2 |

proposed methods. The larger the PAPR reduction is, the larger the average power increases. Given that 0.2299 dB is the largest average power increase, this effect should not significantly increase the BER performance. However, a negligible average power increase is observed for the OS-TR and MK-TR methods, because the length of the set $S(\bar{K})$ is usually small when calculating the peak cancellation signal $c(n)$ with equations (16) and (29).

It can be observed from Figs. 6, 7, and Table 2, that when using the MK-TR method, the PAPR reductions at the first and second iterations are 3.0 dB and 4.4 dB, respectively, values that are almost equivalent to the ones of the second and fourth iterations of the OS-TR method (2.5 dB and 4.2 dB) and to the reduction achieved by MB-TR with no iterations. The PAPR reductions of the MK-TR, OS-TR, and MB-TR methods are better than that of the CC-TR method, because in the CC-TR method the adjacent data blocks are independently considered, thus introducing a serious peak regrowth effect. The neighboring data blocks are effectively processed in the OS-TR and MB-TR methods. However, calculating the optimal scaling factor in these methods requires additional demodulation and modulation operations, thereby increasing computational complexity. Therefore, the proposed MK-TR method achieves faster convergence by generating optimal peak-cancelling signals using multiple impulse-like time domain kernels. As indicated in Section III-2, the proposed MK-TR method is focused on eliminating signal peaks without any additional demodulation and modulation per iteration,

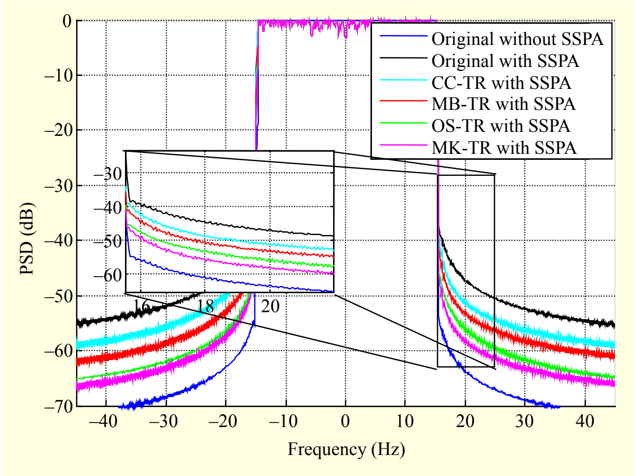


Fig. 8. Power spectrum comparison of the MK-TR, OS-TR, CC-TR, and MB-TR methods.

thereby improving computational complexity.

Taking into consideration the average power increase, PAPR reduction, and computational complexity, OS-TR with four iterations is a good choice if a larger PAPR can be accommodated, and less average transmit power is desired. If a large PAPR reduction, less average power, and low complexity are all desired features, then MK-TR with two iterations is a good choice for 12.5% TR ratio.

The peak reduced OFDM/OQAM signal is passed through a solid-state power amplifier (SSPA). The SSPA smoothing parameter $r$ is set to two, and the input back-off (IBO) is set to 1.6 dB. The IBO is defined as $IBO = 10\log_{10}\left(C^2/\sigma^2\right)$, where C is the clipping threshold, and $\sigma^2$ is the average power of the input signal.

In Fig. 8, the out-of-band radiation of the proposed methods is compared with that of the existing methods. The simulation parameters are the same as before. As shown, if no PAPR reduction is used, the out-of-band radiation is −36.5 dB; with the CC-TR method (ten iterations) and MB-TR method (zero iterations), the out-of-band radiation is reduced to −38.5 dB and −42dB. However, by using the OS-TR method (four iterations) and the MK-TR method (two iterations), the out-of band radiation is further reduced to −45.5 dB and −47dB, respectively. This is an acceptable trade-off, when the large decrease in out-of-band leakage that is achieved using the proposed OFDM/OQAM methods is considered.

Let us now evaluate the BER performance of the MK-TR, OS-TR, CC-TR, and MB-TR methods over Rayleigh fading and additive white Gaussian noise (AWGN) channels, considering the existence of the SSPA. (In practice, to obtain a sufficient transmit average power SSPAs are often employed on the transmitter side.) For the simulation, the smoothing parameter of the SSPA model is set to two, and the input back

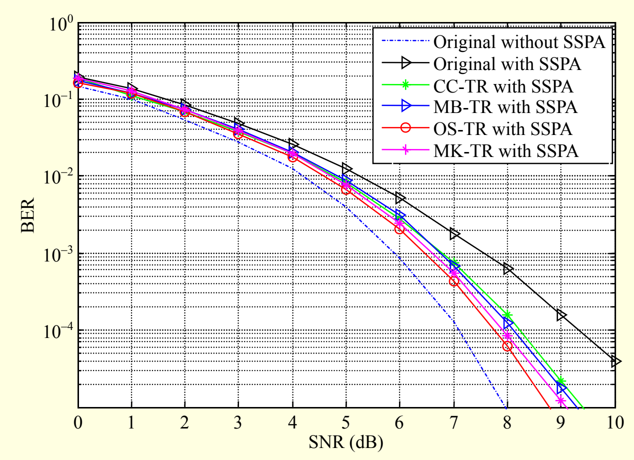


Fig. 9. BER performance of MK-TR, OS-TR, CC-TR, and MB-TR over AWGN channels, with QPSK modulation.

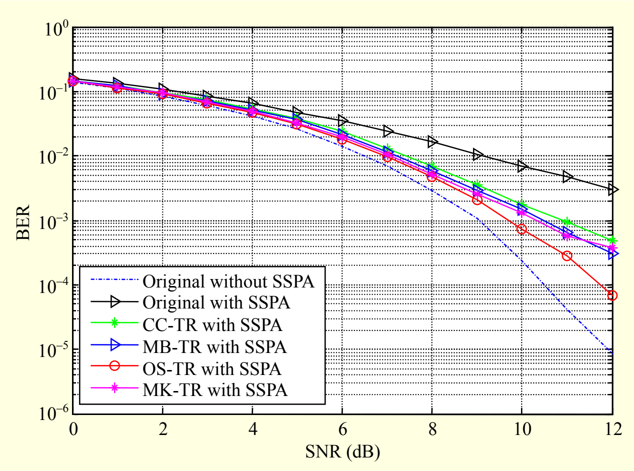


Fig. 10. BER performance of MK-TR, OS-TR, CC-TR, and MB-TR over Rayleigh channels, with QPSK modulation.

off (IBO) to 1.6 dB, to avoid phase distortion. The channel between receive and transmit antennas is patterned as a frequency selective Rayleigh fading channel with impulse response $h_i(n), \left(i=1,2; n=0,1,\ldots,L_{h_i}-1\right)$. The considered maximum multipath delay length is $L_{h_i}=N/32$, where $N$ denotes the length of the OFDM/OQAM symbols; the multipath fading factor is taken as 0.8. The BER performances of the MK-TR, OS-TR, and CC-TR methods are shown in Figs. 9 and 10 for $N=256$ QPSK modulated subcarriers. The number of randomly generated PRTs is 32. The number of iterations of each method was chosen as 2, 4, and 10, for the MK-TR, OS-TR, and CC-TR methods respectively. The best BER performance is obtained with the original signal by ignoring the effect of SSPA and its nonlinear characteristics. However, it shows a more pronounced PAPR, when compared with MK-TR, OS-TR, CC-TR, and MB-TR, which makes it ineffective in practice. The proposed MK-TR, OS-TR, CC-TR, and MB-TR methods are effective in improving the BER performance, when compared with the original signal with SSPA. When BER = $10^{-3}$, the SNRs are respectively 6.5 dB, 6.4 dB, 6.75 dB, and 6.7 dB for MK-TR, OS-TR, CC-TR, and MB-TR over AWGN channel, and 10.3 dB, 9.6 dB, 11 dB, and 10.5 dB for MK-TR, OS-TR, CC-TR, and MB-TR over Rayleigh channels, respectively. It can therefore be concluded that the MK-TR and OS-TR methods achieve better performance when compared to the MB-TR and CC-TR methods. The BER performance of OS-TR is better than that of MK-TR. It should be noted that the MK-TR method BER performance degradation is completely within the acceptable range.

## V. Conclusion

In this paper, two novel methods (OS-TR and MK-TR) were proposed. The OS-TR method is simple and effective; it is generated by jointly considering adjacent data blocks. The peak-cancelling signal in this method is obtained with the least squares algorithm, by fitting the peak-cancelling signal waveform to the clipping noise. In addition, a fast convergence method (MK-TR) was proposed, which uses multiple impulse-like time domain kernels to generate the optimal peak-cancelling signal. In particular, the newly generated peak-cancelling signal amplitude is approximated to that of the original clipping noise by scaling the multiple time domain kernels. When compared with the OS-TR, MB-TR, and CC-TR methods, the MK-TR method achieves a better PAPR reduction, and does so with a very fast convergence rate Finally, from the simulation results we conclude that among the proposed OS-TR and MK-TR methods, MK-TR is the method that can provide a more effective reduction in PAPR and the greater improvement in BER performance, while simultaneously presenting a much lower computational complexity.

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