

# The New Approach of the Impedance Calculation Method of Fault Current Analysis in the AT Feeding Method of the Electric Railway

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**Abstract** – The cause of failure in the AT feeding method of the electric railway is divided into ground, short of feeding circuit and faults on the inside of the substation. Because the fault current is high, the real-time current is detected and the failure promptly has to be removed.

In this paper, short-circuit current is analyzed by numerical analysis and simulation is made for the correct operation of the protective relay and the impedance calculation method has been proposed for simple and accurate calculation of the short-circuit current. Also, In order to confirm the validity of the proposed calculation method, the simulation error rate was analyzed compared with the conventional numerical methods.

**Keywords:** AC electric railway, Fault current, Short-circuit, Numerical analysis

## 1. Introduction

The electrification of the railroad is continuously accomplished with the opening of the demands of the domestic high speed railway and need for a quality-enhanced means of railroad transportation is increasing dependent on changing transportation demands. In addition, the reliance on electrical power is gradually rising in the railroad field of industry [1-2]. Therefore, the stable supply of electricity power applied to the electric railway vehicle and claims about improvement of power quality are suddenly increasing and the importance of the protective relay acting in the stability maintenance of the power system is increasing. The cause of the failure in the AC electric railway power system is divided into Ground, Short of feeding circuit and faults on the inside of the substation[3-4], because the fault current is high, the real-time current is detected and the failure promptly has to be removed. because of high speed of the railway vehicle powered by the electrical energy and the increase of power load and regenerative power available from the electric railway vehicle, there is not a big difference between fault current and load current. The high performance protection system is necessary in order to prevent malfunction to the protective relay due to the load current. In order to implement the high performance protection system, detailed modelling about the element of the power feeding system has to be preceded. The foreign product is mostly applied to the AC protective relay for the feed system that is the constituent of AC feeding system that is being applied domestically. High cost of the protective relay construction and maintenance is occurred with the independent technical

absence of the protective relay. Protection function of every foreign product is already equipped, standardized protection for the AC feeder system is difficult.

In this paper, the short-circuit current is analyzed by numerical analysis and simulation for the correct operation of the protective relay and the formula has been proposed for simple and accurate calculation of the short-circuit current.

Also, in order to confirm the validity of the proposed formula, the simulation error rate was analyzed compared with the conventional numerical methods. And the formula applying the AT leakage impedance was derived and compared with simulation.

## 2. Impedance Calculation and Simulation

The AT feeding method is widely used, and for power, has good characteristics for high power long-distance feeding and induced obstacle because of the recent surge in load current. AT feeding method is the system for connecting the neutral point of the transformer winding on a rail at about 10 [km] intervals along the track.

In this paper, a short-circuit current was calculated through a short-circuit impedance derived in accordance with the circuit configuration for fault current analysis.

### 2.1 AC AT feeder system

Typically, the feed circuit applied in Korea for supporting a train which uses electricity as the energy source is shown as in Fig. 1.

This circuit consists of substation, sectioning post which uses closing and opening for division and extension of feeder section, sub sectioning post for restriction from power failure or blackout when working and accident, auto

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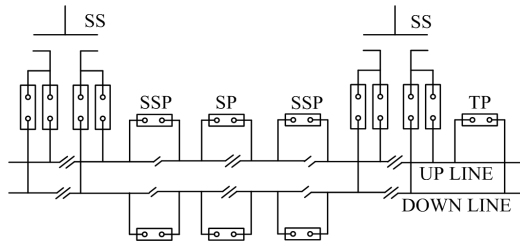


Fig. 1. Configuration diagram of AC feeder circuit

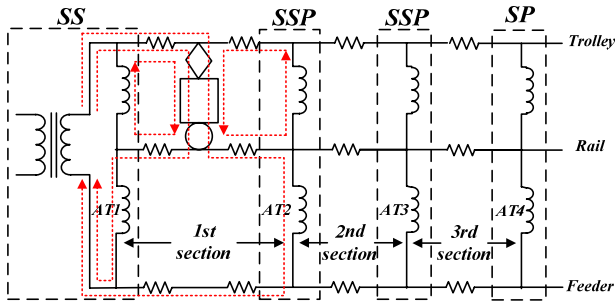


Fig. 2. AC Electric railway of AT feeder method

transformer post for voltage drop compensation of overhead trolley wire on catenary terminal and reduction of inductive disturbance [5-7].

Also, TP(Tie-Post) is installed and operated as arc measures for cross over road for short railroad terminal.

Fig. 2 is equivalent circuit of Fig. 1 and shows AT method of electric supply which most of Korea has been using. The AT is installed on the trolley and feeder, and the neutral point is installed on the rail. This method has both ends of ATs connected to trolley and feeder.

Thus, trolley has higher electric potential than rail and feeder has lower electric potential than rail and earth.

AT applies 55[kV] between trolley and feeder based on scott transformer secondary-side and 27.5[kV] between trolley and rail since neutral point is connected to the trolley and rail.

2.2 Conventional calculation method

Fig. 3 shows power supply circuit impedance characteristic between trolley, rail and feeder, as shown in Fig. 2. There are T-R short circuit, F-R short circuit, T-F short circuit in AC electric railway AT feeding method. Impedance increases with distance increase of fault location from traction substation (SS). However, impedance decreases if it gets closer to the specific location AT such as sectioning post (SP), then sub sectioning post (SSP) is installed [8].

Conversely, in the first section of Fig. 3, when AT interval distance is represented by  $D$ , the distance to the T-R short circuit point at the substation installed AT is represented by  $x$ , circuit excluding the load of the vehicle is the same as Fig. 4. Short circuit impedance as a short circuit occurs in first section impedance is represented by  $Z$  and Short circuit impedance, as a short circuit, occurs in a

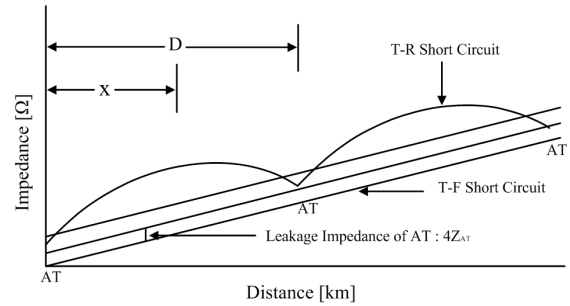


Fig. 3. Impedance characteristic of AT feeder method

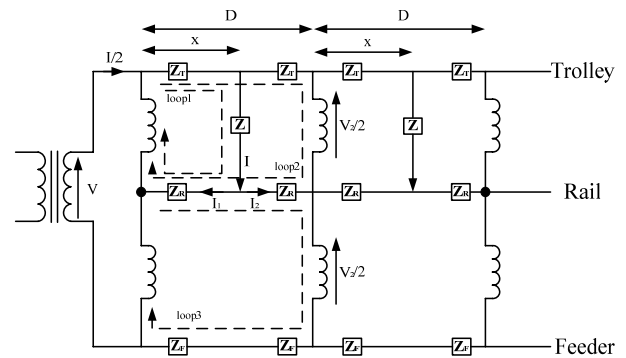


Fig. 4. Single-line simplified equivalent circuit

second section represented by  $Z'$  [9].

Eqs. (1)~(3) of equivalent circuit of Fig. 4 are satisfied by the Kirchoff's law in the loop 1~3 single-line simplified equivalent circuit of Fig. 4[10].

$$\frac{V}{2} = Z(I_1 + I_2) + Z_T x \left( \frac{2I_1 + I_2}{2} \right) + Z_R x I_1 \tag{1}$$

$$\frac{V}{2} - \frac{V_2}{2} = Z_T x \left( \frac{2I_1 + I_2}{2} \right) - Z_T (D - x) \frac{I_2}{2} + Z_R x I_1 - Z_R x I_1 - Z_R (D - x) I_2 \tag{2}$$

$$\frac{V}{2} - \frac{V_2}{2} = \frac{Z_F D I_2}{2} - Z_R x I_1 + Z_R (D - x) I_2 \tag{3}$$

$I_1, I_2$  can be derived from equations (1)~(3). And it is possible to determine the line impedance of the case of T-R short circuit at a distance  $x$  through following equation.

$$Z_{AL} = \frac{2V}{(I_1 + I_2)} \tag{4}$$

$$Z_{AL} = 4Z + \frac{4Dx(Z_F Z_T + Z_R Z_F + Z_T Z_R)}{4Z_R D + Z_F D + Z_T D} + \frac{4(D-x)x(Z_T + 2Z_R)^2}{4Z_R D + Z_F D + Z_T D} \tag{5}$$

In the second section, the addition of  $\frac{4DH}{G}$  in Eq. (5) is the same as Eq. (6).

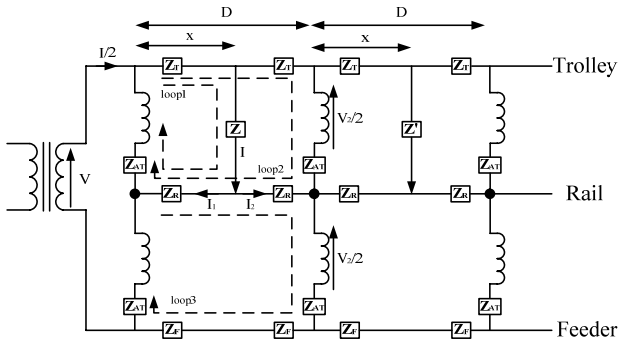


Fig. 5. Simple equivalent circuit in 2<sup>nd</sup> section

$$Z_{AL} = 4Z + \frac{4Dx(Z_F Z_T + Z_R Z_F + Z_T Z_R)}{4Z_R D + Z_F D + Z_T D} + \frac{4(D-x)x(Z_T + 2Z_R)^2}{4Z_R D + Z_F D + Z_T D} + \frac{4DH}{G} \quad (6)$$

where,  $G = Z_T + Z_F + 4Z_R$ ,  $H = Z_T Z_F + Z_T Z_R + Z_F Z_R$

### 2.3 Proposed calculation method

The conventional formula does not include the value of AT transformer. But the proposed method is calculated by the conventional method in addition to AT leakage reactance in section 1. but in this paper, a new calculation method is proposed for a simple and fast calculation in section 2~3

Eqs. (7)~(9) of equivalent circuit of Fig. 5 are satisfied by the Kirchhoff's law in the loop 1~3 single-line simplified equivalent circuit of Fig. 5.

$$\frac{V}{2} = Z(I_1 + I_2) + Z_T x \left( \frac{2I_1 + I_2}{2} \right) + Z_R x I_1 + \frac{Z_{AT} I_1}{2} \quad (7)$$

$$\frac{V}{2} - \frac{V_2}{2} = Z_T x \left( \frac{2I_1 + I_2}{2} \right) - Z_T (D-x) \frac{I_2}{2} + Z_R x I_1 - Z_R x I_1 - Z_R (D-x) I_2 - \frac{Z_{AT} (I_1 - I_2)}{2} \quad (8)$$

$$\frac{V}{2} - \frac{V_2}{2} = \frac{Z_F D I_2}{2} - Z_R x I_1 + Z_R (D-x) I_2 - \frac{Z_{AT} (I_1 - I_2)}{2} \quad (9)$$

$I_1, I_2$  can be derived from equations (7)~(9). And it is possible to determine the line impedance of the case of T-R short circuit at a distance  $x$  through following equation.

$$Z_{AL} = 4Z + \frac{4Dx(Z_F Z_T + Z_R Z_F + Z_T Z_R)}{4Z_R D + Z_F D + Z_T D} + \frac{4(D-x)x(Z_T + 2Z_R)^2}{4Z_R D + Z_F D + Z_T D} + \frac{(Z_F D + 4Z_R (D-x) - Z_T (2x-D))2Z_{AT}}{4Z_R D + Z_F D + Z_T D} \quad (10)$$

Table 1. Catenary Data for fault impedance calculation

Type		Data
Self impedance [Ω/km]	Trolley	0.1960+j0.7353
	Rail	0.1851+j0.6219
	Feeder	0.1782+j0.8672
Scott transformer secondary-side voltage		55[kV]
Auto-transformer secondary-side voltage		27.5[kV]
Auto-transformer leakage reactance	Ideal	0.033j[Ω]/10[MVA]
	Real	0.449j[Ω]/10[MVA]

In the second section, the addition of  $(Z_T + Z_F)D$  in Eq. (10) is the same as Eq. (11).

$$Z_{AL} = 4Z + \frac{4Dx(Z_F Z_T + Z_R Z_F + Z_T Z_R)}{4Z_R D + Z_F D + Z_T D} + \frac{4(D-x)x(Z_T + 2Z_R)^2}{4Z_R D + Z_F D + Z_T D} + \frac{(Z_F D + 4Z_R (D-x) - Z_T (2x-D))2Z_{AT}}{4Z_R D + Z_F D + Z_T D} + (Z_T + Z_F)D \quad (11)$$

where,  $Z_T$  is catenary impedance [Ω/km],  $Z_R$  is rail impedance [Ω/km],  $Z_F$  is feeder line impedance [Ω/km],  $Z$  is impedance of short point[Ω],  $Z_{AL}$  is line impedance of T-R short,  $D$  is distance between AT[km], Distance to the short circuit point at AT [km],  $Z_{AT}$  is leakage reactance of auto transformer[Ω]

Similarly, each time a section is increased,  $(Z_T + Z_F)D$  is added.

### 2.4 Impedance for the fault current numerical calculation

When an accident, such as a short circuit and ground fault occurs, EMTDC/PSACD was used to analyze the fault current. A short-circuit accident in a section of 1~20 [km], has been simulated at intervals of 1 [km].

In order to calculate the fault point impedance, applied catenary impedance is shown in Table 1 as the data of the real system between the Wonju ~ Gangneung.

Table 2 shows the calculated impedance values using the conventional numerical calculation of the impedance. The short-circuit impedance of 1[km] is 5.2499 [A], which was the largest current. And it was confirmed that if the distance increases and short-circuit impedance increases.

### 2.5. Simulation

Simulation was modelled with each one of the auto-transformer of SP, SSP from scott transformer secondary and was modelled to be powered up to 27.5[kV]. The catenary was modelled in detail.

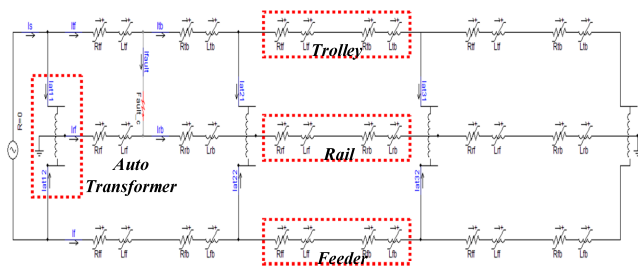
Fig. 6 is configured in the catenary modelling, using PSCAD / EMTDC to apply the resistance and reactance components of trolley, rail and feeder and it is also possible to short-circuit simulation.

**Table 2.** Data of short-circuit Impedance according to distance

Distance [km]	Short-circuit Impedance[Ω]		
	Conventional equation	proposed equation	
		0.03j	0.45j
1	5.2499	6.8262	7.0279
2	9.6893	11.0912	11.2062
3	13.3292	14.5604	14.6510
4	16.1698	17.2312	17.3556
5	18.211	19.1032	19.3186
6	19.453	20.1761	20.5454
7	19.896	20.4499	21.0325
8	19.5402	19.925	20.7782
9	18.3862	18.6016	19.7913
10	16.4352	16.4807	18.0683
11	21.6712	23.2817	22.7273
12	26.1094	27.547	27.0005
13	29.7491	31.0152	30.5216
14	32.5898	33.6852	33.2930
15	34.6316	35.5568	35.3243
16	35.8745	36.6298	36.5935
17	36.3185	36.904	37.1120
18	35.964	36.3797	36.8880
19	34.8113	35.0571	35.9008
20	32.8611	32.9368	34.1827
21	38.0964	39.7377	39.0903
22	42.534	44.003	43.3754
23	46.1732	47.4709	46.8883
24	49.0138	50.1406	49.6838
25	51.0557	52.012	51.6917
26	52.2988	53.0849	52.9865
27	52.7432	53.3593	53.5019
28	52.3892	52.8353	53.2430
29	51.237	51.513	52.2814
30	49.287	49.3929	50.5515

**Table 4.** Comparison of conventional calculation and simulation value for short-circuit current

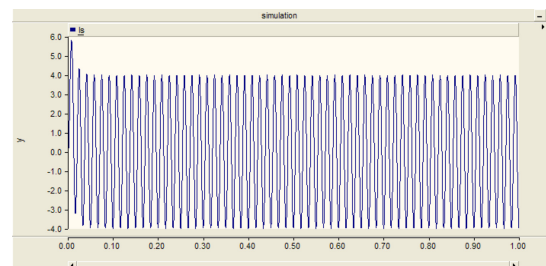
Distance [km]	Short-circuit current[A]		Error rate [%]
	Conventional calculation data	Simulation data	
1	10476	10196	2.676%
2	5676	5606	1.240%
3	4126	4094	0.782%
4	3401	3382	0.570%
5	3020	3006	0.469%
6	2827	2815	0.436%
7	2764	2752	0.448%
8	2815	2800	0.523%
9	2991	2972	0.648%
10	3346	3317	0.881%
11	2538	2525	0.509%
12	2107	2100	0.310%
13	1849	1845	0.205%
14	1688	1686	0.097%
15	1588	1587	0.072%
16	1533	1532	0.073%
17	1514	1513	0.091%
18	1529	1528	0.085%
19	1580	1578	0.123%
20	1674	1671	0.162%
21	1444	1441	0.187%
22	1293	1292	0.084%
23	1191	1191	0.014%
24	1122	1122	0.012%
25	1077	1078	0.069%
26	1052	1052	0.033%
27	1043	1043	0.020%
28	1050	1050	0.016%
29	1073	1074	0.052%
30	1116	1116	0.008%
Average error rate			0.363%



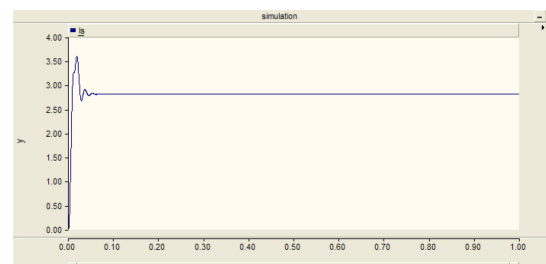
**Fig. 6.** AT feeder system modeling of AC electric railway (The 1st Section)

The scott transformer secondary voltage is 55 [kV] and modelling is made in the order of SS-SSP-SP. The primary winding is connected to twice times the number of turns of the secondary side in auto-transformer (AT). The voltage on the primary side is 55 [kV], Voltage on the secondary side is 27.5 [kV]. The point connecting both windings is connected to the rail.

The short circuit fault is simulated at each 1[km] interval in a section of 1~30 [km]. Fig. 7 shows an example of short circuit fault, and shows the (a) instantaneous value and the (b) RMS current value. Comparison of numerical analysis



(a) 6[km] point short circuit fault instantaneous current (X axis : [sec], Y axis : [kA])



(b) 6[km] point short circuit fault RMS current (X axis : [sec], Y axis : [kA])

**Fig. 7.** 6[km] point short circuit fault current waveform

and simulation result was done in a steady-state of the RMS values.

**2.5.1 Simulation of conventional calculation method ( $Z_{AT}=0.033j[\Omega]$ )**

The leakage reactance is applied to confirm the effect of leakage reactance in the simulation. First, the leakage reactance is set to a very small value in order to analyze the values of the condition without leakage reactance of auto-transformer in 2.5.2, and the simulation was carried out. Table 4 shows comparison of simulation results and short-circuit current value using conventional numerical calculation. And the maximum error rate in the section 1 is 2.676 [%] of 1[km] point, and the minimum error rate is 0.008 [%] of 30[km] points. The result is less than 1 [%] excepting at point 1[km] and 2[km]. Because the short-circuit current of 1, 2[km] is larger than the short-circuit current of the other point, the error rate is increased. The short circuit current value and the value of the numerical simulation calculations were almost similar as the average error rate was 0.363[%].

**Table 5.** Comparison of proposed calculation and simulation value for short-circuit current

Distance [km]	Short-circuit current[A]		Error rate [%]
	Proposed calculation data	Simulation data	
1	10363	10196	1.641%
2	5647	5606	0.724%
3	4112	4094	0.449%
4	3393	3382	0.333%
5	3015	3006	0.291%
6	2823	2815	0.301%
7	2762	2752	0.348%
8	2813	2800	0.453%
9	2990	2972	0.609%
10	3346	3317	0.879%
11	2528	2525	0.107%
12	2100	2100	0.000%
13	1844	1845	0.046%
14	1684	1686	0.113%
15	1585	1587	0.108%
16	1531	1532	0.083%
17	1512	1513	0.047%
18	1527	1528	0.037%
19	1578	1578	0.014%
20	1672	1671	0.065%
21	1439	1441	0.121%
22	1290	1292	0.178%
23	1188	1191	0.213%
24	1120	1122	0.189%
25	1075	1078	0.250%
26	1050	1052	0.198%
27	1041	1043	0.172%
28	1048	1050	0.157%
29	1072	1074	0.184%
30	1115	1116	0.133%
Average error rate			0.281%

**2.5.2 Simulation of proposed calculation method ( $Z_{AT}=0.033j[\Omega]$ )**

Table 5 shows comparison of the simulation results and short-circuit current value using the proposed numerical calculation and the maximum error rate in section 1 is 1.641 [%] of 1[km] point like the conventional calculation method, and minimum error rate is 0[%] over 12[km] point and the result is less than 1 [%] except points 1[km].

The proposed formula is simpler but the results were more accurate as the average error rate is 0.281 [%].

**2.5.3 Simulation of conventional calculation method ( $Z_{AT}=0.45j[\Omega]$ )**

The leakage impedance of the actual auto-transformer is applied in the simulation. Table 6 is tables comparing the simulation results with short-circuit current value using conventional numerical calculation and the maximum error rate in the section 1 is 25.299 [%] of 1[km] point, and the minimum error rate is 1.23 [%] of 25[km] points. The average error rate was 4.624[%].

**Table 6.** Comparison of conventional calculation and simulation value for short-circuit current

Distance [km]	Short-circuit current[A]		Error rate [%]
	Conventional calculation data	Simulation data	
1	10476	7826	25.299%
2	5676	4908	13.536%
3	4126	3754	9.022%
4	3401	3169	6.833%
5	3020	2847	5.733%
6	2827	2677	5.317%
7	2764	2615	5.404%
8	2815	2647	5.958%
9	2991	2779	7.100%
10	3346	3044	9.039%
11	2538	2420	4.647%
12	2107	2037	3.300%
13	1849	1802	2.531%
14	1688	1652	2.112%
15	1588	1557	1.961%
16	1533	1503	1.965%
17	1514	1482	2.138%
18	1529	1491	2.505%
19	1580	1532	3.035%
20	1674	1609	3.866%
21	1444	1407	2.542%
22	1293	1268	1.940%
23	1191	1173	1.525%
24	1122	1107	1.349%
25	1077	1064	1.230%
26	1052	1038	1.298%
27	1043	1028	1.418%
28	1050	1033	1.604%
29	1073	1052	1.998%
30	1116	1088	2.501%
Average error rate			4.624%

**Table 7.** Comparison of proposed calculation and simulation value for short-circuit current

Distance [km]	Short-circuit current[A]		Error rate [%]
	Proposed calculation data	Simulation data	
1	8057	7826	2.869%
2	4959	4908	1.026%
3	3777	3754	0.619%
4	3192	3169	0.717%
5	2879	2847	1.115%
6	2726	2677	1.797%
7	2689	2615	2.770%
8	2760	2647	4.106%
9	2957	2779	6.011%
10	3337	3044	8.787%
11	2362	2420	2.439%
12	1997	2037	2.024%
13	1773	1802	1.617%
14	1633	1652	1.178%
15	1547	1557	0.658%
16	1502	1503	0.099%
17	1490	1482	0.560%
18	1512	1491	1.378%
19	1569	1532	2.350%
20	1670	1609	3.645%
21	1384	1407	1.656%
22	1250	1268	1.447%
23	1159	1173	1.242%
24	1097	1107	0.919%
25	1057	1064	0.620%
26	1036	1038	0.186%
27	1031	1028	0.267%
28	1041	1033	0.766%
29	1068	1052	1.470%
30	1114	1088	2.292%
Average error rate			1.888%

**2.5.4 Simulation of proposed calculation method ( $Z_{AT} = 0.45j[\Omega]$ )**

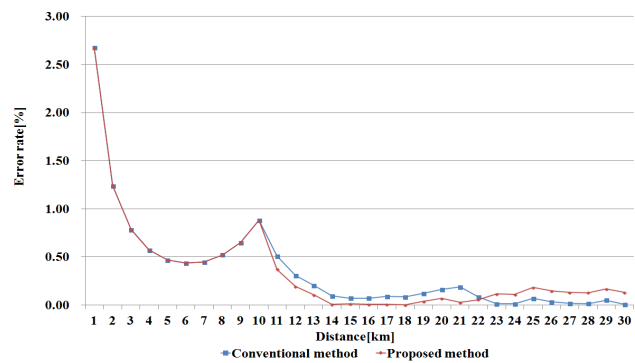
Table 7 is tables comparing the simulation results with short-circuit current value using the proposed numerical calculation and the maximum error rate in section 1 is 8.787 [%] of 10[km] point and minimum error rate is 0.099 [%] over 16[km] points.

The proposed formula is simpler but the results were more accurate as the average error rate is 1.888 [%].

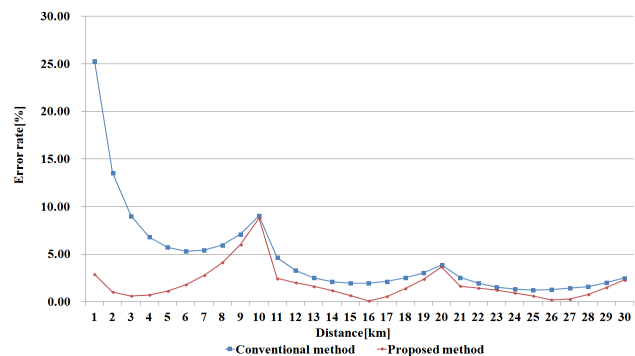
**2.6 Comparing the error rate on the basis of simulation**

Fig. 8 is a compares the proposed and conventional error rate calculation method. In section 1, error rates are the same. but error rate of the proposed calculation method is lower except after 23[Km] and It can be confirmed that the proposed calculation method is lower by as much as 0.006 [%].

Fig. 9 shows comparison of the proposed and conventional error rate calculation method when applying the leakage reactance. Error rate of the proposed calculation



**Fig. 8.** Comparison of error rate( $Z_{AT}=0.033j[\Omega]$ )



**Fig. 9.** Comparison of error rate( $Z_{AT}=0.45j[\Omega]$ )

method is lower and It can be confirmed that the proposed calculation method is lower by as much as 2.836 [%].

**3. Conclusion**

In this paper, short-circuit current is analyzed by numerical analysis and simulation for the correct operation of the protective relay and the formula has been proposed for simple and accurate calculation of the short-circuit current.

In order to confirm the validity of the proposed formula, the simulation error rate was analyzed compared with the conventional numerical methods.

The average error rate of comparing the short circuit current value using conventional calculation method and the value of the numerical simulation calculations was 0.363[%]; and the average error rate of comparing the short circuit current value using proposed calculation method with the value of the numerical simulation calculations was 0.357[%]. These results were similar but the error rate of the proposed calculation method is lower than the conventional one.

Compared to simulation applying the leakage reactance of the actual auto-transformer, error rate of the conventional method is 4.624% and error rate of the proposed method is 1.888%. It can be confirmed that the proposed calculation method is lower by as much as 2.836 [%]. It is considered

an error because the simulation results are not correct to 100%. Thus, it demonstrated the feasibility of the proposed calculation method.

In future research, the study to increase the accuracy of the protection relays is required taking into consideration the source impedance.

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