Trans. Korean Soc. Noise Vib. Eng., 26(6) : 635~643, 2016
http://dx.doi.org/10.5050/KSNVE.2016.26.6.635

한국소음진동공학회논문집 제 26 권 제 6 호, pp. 635~643, 2016
ISSN 1598-2785(Print), ISSN 2287-5476(Online)

Structural Damping Effects on Stability of a Cantilever Column under Sub-tangentially Follower Force
종동력을 받는 외팔기둥의 동적 안정성에 미치는 구조감쇠 효과

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(Received March 3, 2016 ; Revised June 24, 2016 ; Accepted June 24, 2016)

Key Words : Structural Damping(구조감쇠), Follower Force(종동력), Stability Map(안정성 지도), Flutter(플러터), Beck's Column(Beck 기둥)

ABSTRACT

A stability theory of a damped cantilever column under sub-tangential follower forces is first summarized based on the stability map. It is then demonstrated that internal and external damping can be exactly transformed to Rayleigh damping so that the damping coefficients can be effectively determined using proportional damping. Particularly a parametric study with variation of damping coefficients is performed in association with flutter loads of Beck's column and it is shown that two damping coefficients can be correctly estimated for real systems under the assumption of Rayleigh damping. Finally a frequency equation of a cantilever beam subjected to both a sub-tangentially follower force and two kinds of damping forces is presented in the closed-form and its stability maps are constructed and compared with FE solutions in the practical range of damping coefficients.

요 약

안정성 지도(stability map)을 이용하여 부분 종동력(sub-tangentailly follower force)를 받는 외팔기둥의 동적안정성 이론을 요약한다. Rayleigh 감쇠를 가정하여 내적 및 외적 감쇠효과를 2개의 감쇠비를 통하여 반영하고, 감쇠비 변화에 따른 플러터하중의 변화와 관련된 매개변수 연구를 수행한다. 또한, 종동력을 받는 외팔기둥에 대한 진동수 방정식의 엄밀해를 유도하고, 특정 감쇠비 범위에 대한 안정성 지도를 유한요소 해석결과와 함께 비교/분석한다.

1. Introduction

Dynamic stability problem of a cantilever column under to the circulatory force, i.e. purely rotation-dependent force, has been well known and interesting topics related to it have been intensively studied by many researchers since it was analytically solved by von Beck(1). Related to initial researches, it is worth referring the monographs by Ziegler(2), Bolotin(3), and Leipholz(4) who treated the static and dynamic stability of the

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A part of this paper was presented at the KSNVE 2016 Annual Spring Conference
‡ Recommended by Editor Hyung Jo Jung

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non-conservative system analytically.

According to the linear stability theory, it is well known that external damping tends to make flutter loads of the non-conservative system increased but the column under small internal damping loses its stability at drastically deceased flutter load. In this case, Beck's column subjected to follower forces a little bigger than internally damped flutter loads become unstable in form of oscillations with a slow growth of amplitude, which is sometimes called a quiet flutter. This destabilizing effect of small internal damping on the stability of a non-conservative system, Ziegler's paradox, has been one of attractive research topics[5~9]. Shear and rotary inertia effects on stability of Beck's column have been investigated by many scholars[10~14]. In addition, Langthjem and Sugiyama[15] and Elishakoff[16] published survey papers regarding dynamic stability of columns subjected to follower loads. And Lee et al.[17] investigated the stability of Beck's column with a tip mass.

In many texts on structural dynamics, it has been addressed that structural damping in FE analysis of beam structures can be modeled as a kind of Rayleigh damping. Recently some researchers[18~20] noticed that external and internal (visco-elastic) damping of a non-conservative system can be treated as Rayleigh damping and explored the destabilizing effects of internal damping using FE analysis.

However, to authors' knowledge, properties of the damping coefficients with relation to a non-conservative system have not been investigated. Besides, a closed-form solution of Beck's problem considering both damping effects and effects of shear deformation and rotary inertia has not been proposed and its stability behaviors have not been reported in the practical range. The important points presented in this paper are summarized as follows:

(1) Linear stability theory of sub-tangentially loaded and damped Beck's columns is first summarized based on so called stability map using an analytical approach.

(2) It is then demonstrated that internal and external damping can be exactly transformed to Rayleigh damping so that damping coefficients can be effectively determined using proportional damping. Particularly a parametric study with variation of damping coefficients is performed in association with flutter loads of Beck's column and it is shown that two damping coefficients can be exactly calculated for real systems under assumption of Rayleigh damping.

(3) Finally, a frequency equation of Timoshenko cantilever beams subjected to a sub-tangentially follower force and two kinds of damping forces is newly derived in the closed-form and its stability maps are constructed and compared with FE solutions in the practical range of damping coefficients and shear parameters.

2. Linear Stability Theory of Damped Beck's Column

In this section, a linear stability theory of damped Beck's columns is summarized using an analytical approach.

Figure 1 shows a prismatic cantilever column subjected to a follower force P at the tip end A in which the direction of the force changes sub-tangentially according to the deformed column axis. The coordinate x is measured along the centroidal axis of the column. The mass per unit length, the flexural rigidity, the sectional area, and

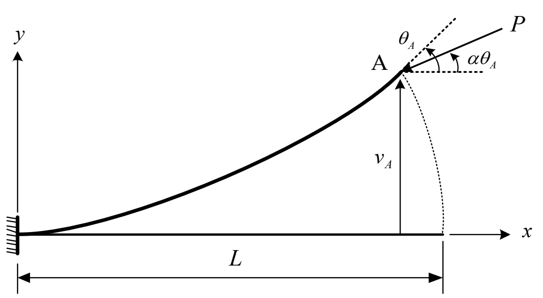

Fig. 1 Beck's column under sub-tangentially follower force

the compressive force P are constant throughout the column length. The extended Hamilton's principle for the externally and internally damped and shear-rigid Beck's column can be expressed by,

$$\int_{t_1}^{t_2}\left[\int_0^l \left\{\begin{matrix}\rho A\dot{v}\delta\dot{v} - EI\,v''\delta v'' + P\,v'\delta v' \\ -(\gamma_1\dot{v}\delta v + \gamma_2\dot{v}''\delta v'')\end{matrix}\right\} dx \atop -\alpha P v'(l)\delta v(l)\right] dt = 0 \quad (1)$$

where l = the total length of the column; v = the lateral displacement of the column; γ_1 and γ_2 = the external and internal damping coefficients, respectively; α = the non-conservativeness parameter denoting sub-tangentiality. The first three terms in the square bracket are the kinetic energy, the elastic strain energy of the system, and the potential energy due to the axial force P, respectively and the fourth and last terms denote the works done by non-conservative damping and the follower end force. Also, the symbol δ denotes the first variation, t represents time and t_1 & t_2 are the integration limits.

Now integrating by parts for the whole length of the column, the well-known equation of motion and boundary conditions are obtained:

$$\begin{aligned}&\rho A\ddot{v} + EI\,v'''' + P\,v'' + \gamma_1\dot{v} + \gamma_2\dot{v}'''' = 0\\ &v(0) = v'(0) = 0\\ &EI\,v''(l) + \gamma_2\dot{v}''(l) = 0\\ &EI\,v'''(l) + P(1-\alpha)v'(l) + \gamma_2\dot{v}'''(l) = 0\end{aligned} \quad (2)$$

For convenience, the following dimensionless variables are introduced:

$$v^* = \frac{v}{l},\ x^* = \frac{x}{l},\ t^* = \frac{t\sqrt{EI/(\rho A)}}{l^2},\ P^* = \frac{Pl^2}{EI}$$
$$\gamma_1^* = \frac{\gamma_1 l^2}{\sqrt{\rho AEI}},\ \gamma_2^* = \frac{\gamma_2}{l^2\sqrt{\rho AEI}} \quad (3a\sim f)$$

Finally, transforming Eq. (2) to the dimensionless equations and introducing the solution in the form of $v^*(x^*,t^*) = e^{\Omega t^*}V(x^*)$ leads to the following equation for a non-conservative system:

$$\begin{aligned}&V'''' + P^*V'' + \Omega^2 V + \Omega\gamma_1^* V + \Omega\gamma_2^* V'''' = 0\\ &V(0) = V'(0) = 0;\ (1+\Omega\gamma_2^*)V''(1) = 0;\\ &(1+\Omega\gamma_2^*)V'''(1) + P^*(1-\alpha)V'(1) = 0\end{aligned} \quad (4a\sim d)$$

where

$$\Omega\ (\equiv \eta^* + i\omega^*) = (\eta + i\omega)l^2\sqrt{\frac{\rho A}{EI}} \quad (4e)$$

In which ω and ω^* are the frequency and the dimensionless frequency, respectively.

Now general solution of Eq. (4a) is

$$\begin{aligned}V(x^*) &= A\cosh(\lambda_1 x^*) + B\sinh(\lambda_1 x^*)\\ &\quad + C\cos(\lambda_2 x^*) + D\sin(\lambda_2 x^*)\end{aligned} \quad (5)$$

where

$$\lambda_1 = \sqrt{\sqrt{\frac{P^{*2}}{4(1+\Omega\gamma_2^*)^2} - \frac{\Omega\gamma_1^* + \Omega^2}{1+\Omega\gamma_2^*}} - \frac{P^*}{2(1+\Omega\gamma_2^*)}}$$
$$\lambda_2 = \sqrt{\sqrt{\frac{P^{*2}}{4(1+\Omega\gamma_2^*)^2} - \frac{\Omega\gamma_1^* + \Omega^2}{1+\Omega\gamma_2^*}} + \frac{P^*}{2(1+\Omega\gamma_2^*)}}$$

Invoking boundary conditions of Eq. (4b)~(4d) leads to the following characteristic equation.

$$\begin{aligned}&(1+\Omega\gamma_2^*)(\lambda_1^4 + \lambda_2^4) + P^*(1-\alpha)(\lambda_1^2 - \lambda_2^2)\\ &+\left\{2\lambda_1^2\lambda_2^2(1+\Omega\gamma_2^*) + P^*(\alpha-1)(\lambda_1^2 - \lambda_2^2)\right\}\\ &\quad\times\cos(\lambda_2)\cosh(\lambda_1)\\ &+\lambda_1\lambda_2\left\{2P^*(\alpha-1) + (\lambda_2^2 - \lambda_1^2)(1+\Omega\gamma_2^*)\right\}\\ &\quad\times\sin(\lambda_2)\sinh(\lambda_1) = 0\end{aligned} \quad (6)$$

where the critical flutter and divergence loads can be determined by constructing eigencurves of P^*-Ω^2 based on Eq. (6). Detinko[21] presented a generalized version of Eq. (6) by considering effects of a tip mass additionally. On the other hand, the frequency is zero in the case of static divergence system. So the solution of Eq. (4a) is

$$V(x^*) = A\cos\left(\sqrt{P^*}x^*\right) + B\sin\left(\sqrt{P^*}x^*\right) + Cx^* + D \quad (7)$$

In which the divergence loads can be evaluated from the following buckling equation:

$$\alpha + (1-\alpha)\cos\sqrt{P^*} = 0 \tag{8}$$

Generally stability of Beck's column depends on the location of Ω on the complex plane. As the follower force P is increased, the system is stable if the Ω stays in the left-hand half-plane ($\eta \le 0$). In the undamped non-conservative system, the Ω remains on the pure imaginary axis ($\eta = 0$) at first but two frequencies coalesce and the real part η changes negative around the undamped flutter load. The η flutter loads of the damped system are calculated when the transits from the negative values to the positive with the oscillatory part ω being non-zero. On the other hand, static divergence occurs when the η becomes positive and the ω is equal to zero. Therefore, the divergence loads can be directly calculated from the root of Eq. (8).

3. FE Formulation of Damped Beck's Column

In this section, a FE formulation of Beck's column is presented based on the energy expression of Eq. (1) and Rayleigh damping is discussed with relation to the internal and external damping terms in the previous section.

Figure 2 shows a FE modeling of Beck's column using 20 two-node beam elements which are interpolated by cubic Hermitian polynomials. The resulting equations of motion are expressed as

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \left[\mathbf{K_E} - \mathrm{P}\mathbf{K_G} - \alpha \mathrm{P}\mathbf{K_{NC}}\right]\mathbf{U} = \mathbf{0} \\ &\mathbf{M} = \sum_e \mathbf{m_e}, \mathbf{K_E} = \sum_e \mathbf{k_e}, \mathbf{K_G} = \sum_e \mathbf{k_g} \end{aligned} \tag{9}$$

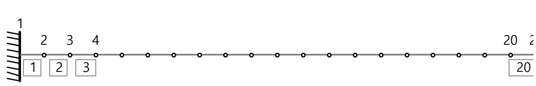

Fig. 2 FE modeling of Beck's column

where the well-known element mass matrix $\mathbf{m_e}$, elastic and geometric stiffness matrices $\mathbf{k_e}$, $\mathbf{k_g}$ are

$$\mathbf{m_e} = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ & 4l^2 & 13l & -3l^2 \\ & & 156 & -22l \\ symm. & & & 4l^2 \end{bmatrix}$$

$$\mathbf{k_e} = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ & 2l^2 & -3l & l^2 \\ & & 6 & -3l \\ symm & & & 2l^2 \end{bmatrix}$$

$$\mathbf{k_g} = \frac{1}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ & 4l^2 & -3l & -l^2 \\ & & 36 & -3l \\ symm & & & 4l^2 \end{bmatrix} \mathbf{k_{nc}} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{10}$$

The load correction stiffness matrix $\mathbf{k_{nc}}$ is not null only for the element including the tip end as shown in Eq. (10). In particular, it should be noted that the damping-related terms of Eq. (1) can be expressed using $\mathbf{m_e}$, $\mathbf{k_e}$ as

$$\begin{aligned} &\int_0^l \left(\gamma_1 \dot{v}\delta v + \gamma_2 \dot{v}''\delta v''\right)dx \\ &= \sum_e \delta\mathbf{u_e}\left(\frac{\gamma_1}{\rho A}\mathbf{m_e} + \frac{\gamma_2}{EI}\mathbf{k_e}\right)\dot{\mathbf{u}}_\mathbf{e} \end{aligned} \tag{11}$$

which means that the damping matrix in Eq. (9) due to external and internal damping terms can be expressed in a form of Rayleigh damping as follows:

$$\mathbf{C} = \frac{\gamma_1}{\rho A}\mathbf{M} + \frac{\gamma_2}{EI}\mathbf{K_E} \tag{12}$$

In other words, external and internal damping coefficients γ_1, γ_2 are directly connected to Rayleigh damping coefficients by Eq. (12). Hence the damping coefficients can be easily determined under assumption of being proportional damping as follows:

$$2\xi\omega_1 = \frac{\gamma_1}{\rho A} + \omega_1^2 \frac{\gamma_2}{EI}$$
$$2\xi\omega_2 = \frac{\gamma_1}{\rho A} + \omega_2^2 \frac{\gamma_2}{EI} \qquad (13)$$

where $\omega_1\left(=3.516\sqrt{\frac{EI}{\rho A l^4}}\right), \omega_2\left(=22.06\sqrt{\frac{EI}{\rho A l^4}}\right)$ denote the first and second natural frequencies of a cantilever beam, respectively, and γ_1, γ_2 are obtained by solving the simultaneous Eq. (13). Table 1 shows not only material and geometric properties of the Beck's column example used in this study but also the external and internal damping coefficients corresponding to proportional damping ξ of 0.1 % and 2 %, respectively. In this case, note that the ratio of dimensionless damping coefficients, γ_1^*/γ_2^* is about 77.5.

Table 1 Material and geometric properties of Beck's column

Property	Unit	
Cross sectional area, A	1.2×10^{-4} m^2	
Moment of inertia, I	1.0×10^{-9} m^4	
Elastic modulus, E	10.0 Gpa	
Poisson ratio, ν	0.3	
Mass density, ρ	1.0×10^{3} kgm^{-3}	
Total length of column, L	1.0 m	
Damping coefficients corresponding to proportional damping of 0.1 %	γ_1, γ_1^*	5.535×10^{-2} 6.064×10^{-3}
	γ_2, γ_2^*	8.575×10^{-6} 7.828×10^{-5}
Damping coefficients corresponding to proportional damping of 2 %	γ_1, γ_1^*	1.107 1.213×10^{-1}
	γ_2, γ_2^*	1.715×10^{-4} 1.566×10^{-3}

4. Estimation of Damping Coefficients in Beck's Column

In the previous section, it was concluded that internal and external damping forces of Eq. (2) are closely linked to Rayleigh damping forces in FE equation and their coefficients can be determined by solving the simultaneous Eq. (13) under the assumption of proportional damping. Also, a dimensionless form of Eq. (13) can be rewritten as

$$2\xi\omega^* = \gamma_1^* + \left(\omega^*\right)^2 \gamma_2^* \qquad (14)$$

In order to examine the effects of internal and internal damping coefficients on the damped flutter loads, Table 2 shows how the flutter loads of Beck's column vary with the increase of damping ratio for $\alpha = 1.0$. After two damping coefficients γ_1^*, γ_2^* are calculated for the given ξ, the second, the third and the fourth columns of Table 2 list the flutter loads calculated for $\gamma_1^*, \gamma_2^* = 0$, $\gamma_1^* = 0, \gamma_2^*$ and γ_1^*, γ_2^* respectively. Surprisingly, Table 2 demonstrates that flutter loads strongly depend on the damping ratio, γ_1^*/γ_2^* except for its last low.

Table 2 Damped flutter loads of Beck's column with increase of the damping ratio

Damping ratio (%) ξ	Damped flutter load, $P_{fl\_d}^*$		
	External damping only $\gamma_1^*, \gamma_2^* = 0$	Internal damping only $\gamma_1^* = 0, \gamma_2^*$	External & internal damping, γ_1^*, γ_2^*
0.01	20.06	10.94	17.03
0.1	20.06	10.94	17.03
2.0	20.06	10.94	17.03
5.0	20.06	10.95	17.06
8.0	20.07	10.95	17.10
10.0	20.08	10.96	17.15
100.0	21.83	12.6	26.22

From the second column, it is observed that the flutter loads with consideration of external damping only are practically the same as the undamped flutter load, 20.05. Also, the flutter loads of the third column are nearly equal to the damped load, 10.94 under the extreme condition of small internal damping only. Especially the fourth column shows damped flutter loads under both two damping coefficients, which are around 17.03. This means that the damped flutter loads of Beck's column under proportional damping in practical range will not be different from 17.03 greatly.

Table 3 shows variation of the flutter loads as two damping coefficients increase independently. From Table 3, it is observed that the damped flutter load tends to be deceased with the in-

Table 3 Damped flutter loads of Beck's column with variation of two damping coefficients

γ_2^* \ γ_1^*	0.0	0.001	0.01	0.1	1.0	10.0	100.0
0.0	20.05	20.05	20.05	20.05	20.11	24.27	37.21
0.0001	10.94	12.88	17.55	19.91	20.11	24.28	37.21
0.001	10.94	11.19	12.88	17.56	19.98	24.38	37.21
0.01	10.97	10.99	11.22	12.93	17.80	25.20	37.21
0.1	13.64	13.64	13.68	14.08	17.33	32.10	37.38
0.2	21.51	21.51	21.56	21.98	25.86	41.31	37.85

Table 4 Flutter and divergence loads of shear-rigid ($S = 0.0$) Beck's column with variation of α

Sub-tangentiality, α	0.0	0.1	0.2	0.3	0.321	0.354	0.4
1st divergence load, P_{d1}^*	2.467 (2.467)	2.830 (2.829)	3.325 (3.325)	4.055 (4.055)	4.257 (4.257)	4.623 (4.626)	5.292 (5.292)
2nd divergence load, P_{d2}^*	22.21 (22.21)	21.17 (21.17)	19.89 (19.90)	18.23 (18.23)	17.81 (17.81)	17.08 (17.08)	15.86 (15.86)
Undamped flutter load, P_{fl1}^*	-	-	-	-	19.42 (19.55)	17.08 (17.08)	16.40 (16.41)
Undamped flutter load, P_{fl2}^*					19.42 (19.55)	24.13 (24.13)	28.78 (28.78)
Damped flutter load 1, $P_{fl\_d1}^*$	-	-	-	-	19.42 (19.55)	17.08 (17.08)	15.86 (15.86)
Damped flutter load 2, $P_{fl\_d2}^*$	-	-	-	-	19.22 (19.45)	17.08 (17.08)	15.86 (15.86)
Sub-tangentiality, α	0.439	0.5	0.6	0.7	0.8	0.9	1.0
1st divergence load, P_{d1}^*	6.099 (6.099)	9.870 (9.867)	-	-	-	-	-
2nd divergence load, P_{d2}^*	14.55 (14.55)	9.870 (9.873)	-	-	-	-	-
Undamped flutter load, P_{fl1}^*	16.16 (16.16)	16.05 (16.05)	16.26 (16.29)	16.79 (16.82)	17.59 (17.59)	18.67 (18.69)	20.05 (20.06)
Undamped flutter load, P_{fl2}^*	33.34 (33.34)	49.35 (49.33)					
Damped flutter load 1, $P_{fl\_d1}^*$	14.55 (14.55)	9.870 (9.872)	9.379 (9.379)	9.305 (9.305	9.529 (9.529)	10.05 (10.05)	10.94 (10.94)
Damped flutter load 2, $P_{fl\_d2}^*$	14.55 (14.55)	14.20 (14.20)	14.12 (14.12)	14.40 (14.40)	14.98 (14.98)	15.85 (15.85)	17.03 (17.03)

troduction of internal damping but the external damping alleviates the tendency, which has been well known. Particularly, it should be noticed that flutter loads do not fluctuate significantly when the damping ratio γ_1^* / γ_2^* is kept as 100.0 and 10.0 similarly to the case of Table 2.

Based on these observations, the following two cases for internal and external damping are chosen in this study to examine damping effects on flutter loads effectively:

- Case 1: Small internal damping only

$$\gamma_1^* = 0, \gamma_2^* = 7.828 \times 10^{-5}$$

- Case 2: External and internal damping

$$\gamma_1^* = 1.213 \times 10^{-1}, \gamma_2^* = 1.566 \times 10^{-3}$$

in which case 1 has been taken to explore effects of small internal damping on flutter instability only while case 2 corresponding to 2 % proportional damping, has been chosen to represent realistic damping forces. Also, damped flutter loads calculated for case 1 and 2 are denoted as $P_{fl\_d1}^*$ and $P_{fl\_d2}^*$, respectively.

By solving Eq. (6) using Mathematica[22] and performing FE analysis using Eq. (9), a stability diagram of Beck's column has been constructed under the damping condition of two cases. Figure 3 and Table 4 show stability diagrams and flutter/divergence loads of sub-tangentially loaded and damped Beck's rods as α varies in the range of $0.0 \le \alpha \le 1.0$. It can be noticed that there exist divergence loads in the range of $\alpha \le 0.5$ from Eq. (8) and flutter instability governs in the range of $\alpha > 0.5$.

It is observed in Fig. 3 that the undamped Beck's system may be stable or unstable depending on the range of α as follows:

1. The range of $0.0 \le \alpha \le 0.32$: it simply loses its stability at the first divergence load.
2. The range of $0.32 \le \alpha \le 0.3543$: it is first stable with increase of the follower force but becomes unstable by divergence, flutter, and again divergence.
3. The range of $0.3543 \le \alpha \le 0.5$: it is stable initially and then loses its stability by divergence but it restores its stability again at the second divergence load (re-stabilization) and unstable finally by flutter.
4. The range of $0.5 \le \alpha \le 1.0$: it loses its stability at the undamped flutter load.

On the other hand, stability of the damped Beck's system(case 2) depending on α can be addressed as follows:

1. The range of $0.0 \le \alpha \le 0.321$: same as the range 1 of the undamped system.
2. The range of $0.321 \le \alpha \le 0.354$: same as the range 2.
3. The range of $0.354 \le \alpha \le 0.439$: it is stable initially and then loses its stability by divergence but it becomes unstable by quiet flutter at the second divergence load and finally by violent flutter. It is again governed by divergence at much higher follower loads.
4. The range of $0.439 \le \alpha \le 0.5$: overall it is the same as the range 3 of the damped system

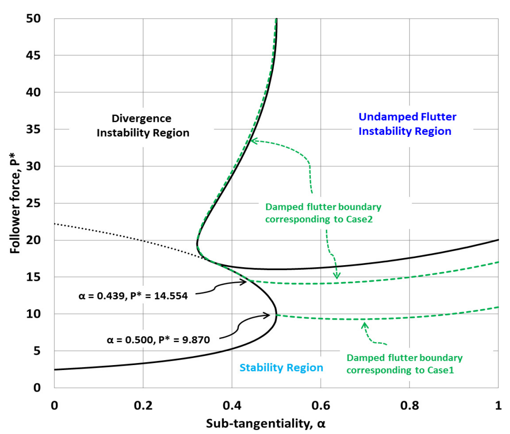

Fig. 3 Stability regions of shear-rigid Beck's column with variation of α

but it shows re-stabilization between the second divergence load and the damped flutter load and quiet flutter in stability between the damped and the undamped flutter load.

5. The range of $0.5 \le \alpha \le 1.0$: It loses its stability by quiet flutter at the damped flutter load and by violent flutter at the undamped flutter load.

5. Conclusions

A stability theory of Beck's column has been shortly summarized based on the stability map. Under the assumption of Rayleigh damping, both internal and external damping coefficients have been effectively determined by exploring the effects of proportional damping parameters on flutter instability in practical ranges. Particularly a frequency equation of cantilever beams subjected to both a sub-tangentially follower force and two kinds of damping forces has been derived in the closed-form and its stability maps of Beck's columns have been constructed with comparison of FE solutions. Resultantly, it is conformed that external damping compensates partly the destabilizing effect of internal damping and thus alleviates it in the pure flutter system.

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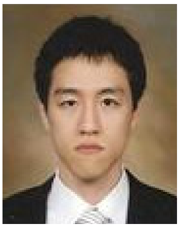

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