

FRACTIONAL CALCULUS FORMULAS INVOLVING \bar{H} -FUNCTION AND SRIVASTAVA POLYNOMIALS

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ABSTRACT. Here, in this paper, we aim at establishing some new unified integral and differential formulas associated with the \bar{H} -function. Each of these formula involves a product of the \bar{H} -function and Srivastava polynomials with essentially arbitrary coefficients and the results are obtained in terms of two variables \bar{H} -function. By assigning suitably special values to these coefficients, the main results can be reduced to the corresponding integral formulas involving the classical orthogonal polynomials including, for example, Hermite, Jacobi, Legendre and Laguerre polynomials. Furthermore, the \bar{H} -function occurring in each of main results can be reduced, under various special cases.

1. Introduction and preliminaries

The Fox's H -function is defined and represented in the following manner [13]:

$$(1.1) \quad H_{p,q}^{m,n} \left[z \left| \begin{smallmatrix} (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{smallmatrix} \right. \right] = \frac{1}{2\pi i} \int_L \theta(s) z^s ds,$$

where $i = \sqrt{-1}$ and

$$(1.2) \quad \theta(s) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j s) \prod_{j=1}^n \Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^q \Gamma(1 - b_j + \beta_j s) \prod_{j=n+1}^p \Gamma(a_j - \alpha_j s)}.$$

The nature of the contour L of the integral (1.1), the conditions of existence of the H -function defined by (1.1) and other details can be found in the book Kilbas and Saigo [8].

A lot of research work has been recently come up on the study and development of a function that is more general than Fox's H -function, known as \bar{H} -function. The \bar{H} -function was introduced by Inayat-Hussain [6, 7], which is

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a generalization of familiar Fox H -function, it is defined and represented in the following manner [2]:

$$(1.3) \quad \overline{H}_{P,Q}^{M,N}[z] = \overline{H}_{P,Q}^{M,N} \left[z \left| \begin{smallmatrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{smallmatrix} \right. \right] = \frac{1}{2\pi i} \int_L \overline{\theta}(\xi) z^\xi d\xi, \quad (z \neq 0),$$

where $i = \sqrt{-1}$ and

$$(1.4) \quad \overline{\theta}(\xi) = \frac{\prod_{j=1}^M \Gamma(b_j - \beta_j \xi) \prod_{j=1}^N \{\Gamma(1 - a_j + \alpha_j \xi)\}^{A_j}}{\prod_{j=M+1}^Q \{\Gamma(1 - b_j + \beta_j \xi)\}^{B_j} \prod_{j=N+1}^P \Gamma(a_j - \alpha_j \xi)}.$$

It may be noted that the $\overline{\theta}(\xi)$ contains fractional powers of some of the gamma function and M, N, P, Q are integers such that $1 \leq M \leq Q$, $1 \leq N \leq P$. Also, $(a_j)_{1,P}$ and $(b_j)_{1,Q}$ are complex parameters; $(\alpha_j)_{1,P}$, $(\beta_j)_{1,Q}$ are positive real numbers (not all zero simultaneously) and the exponents $(A_j)_{1,N}$ and $(B_j)_{M+1,Q}$ may take non-integer values, which we assume to be positive for standardization purpose. The nature of contour L , sufficient conditions of convergence of defining integral (1.4) and other details about the \overline{H} -function, reader can also refer the papers [6, 7, 10] for more detail.

The behavior of the \overline{H} -function for small values of $|z|$ and sufficient conditions for the absolute convergence of the defining integral follows easily [2, 17]

$$(1.5) \quad \overline{H}_{P,Q}^{M,N}[z] = o(|z|^\alpha),$$

where

$$(1.6) \quad \alpha = \min_{1 \leq j \leq M} \operatorname{Re} \left(\frac{b_j}{\beta_j} \right), \quad |z| \rightarrow 0,$$

$$(1.7) \quad \Omega = \sum_{j=1}^M |\beta_j| + \sum_{j=1}^N |A_j \alpha_j| - \sum_{j=M+1}^Q |B_j \beta_j| - \sum_{j=N+1}^P |\alpha_j| > 0,$$

and $|\arg z| < \frac{1}{2}\Omega \pi$.

Recently, the \overline{H} -function of two variable is defined and represented by Singh and Mandia [27] in the following manner:

$$(1.8) \quad \begin{aligned} \overline{H}[x, y] &= \overline{H} \left[\begin{array}{c} x \\ y \end{array} \right] \\ &= \overline{H}_{p_1, q_1; p_2, q_2; p_3, q_3}^{0, n_1: m_2, n_2: m_3, n_3} \left[\begin{array}{c} x \\ y \end{array} \left| \begin{smallmatrix} (a_j, \alpha_j; A_j)_{1,p_1}, (c_j, \gamma_j; K_j)_{1,n_2}, (c_j, \gamma_j)_{n_2+1, p_2}, (e_j, \eta_j; R_j)_{1, n_3}, (e_j, \eta_j)_{n_3+1, p_3} \\ (b_j, \beta_j; B_j)_{1, q_1}, (d_j, \delta_j)_{1, m_2}, (d_j, \delta_j; L_j)_{m_2+1, q_2}, (f_j, \lambda_j)_{1, m_3}, (f_j, \lambda_j; S_j)_{m_3+1, q_3} \end{smallmatrix} \right. \right] \\ &= -\frac{1}{4\pi^2} \int_{L_1} \int_{L_2} \theta_1(\xi, \mu) \theta_2(\xi) \theta_3(\mu) x^\xi y^\mu d\xi d\mu, \quad (x, y \neq 0), \end{aligned}$$

$$(1.9) \quad \theta_1(\xi, \mu) = \frac{\prod_{j=1}^{n_1} \Gamma(1 - a_j + \alpha_j \xi + A_j \mu)}{\prod_{j=n_1+1}^{p_1} \Gamma(a_j - \alpha_j \xi - A_j \mu) \prod_{j=1}^{q_1} \Gamma(1 - b_j + \beta_j \xi + B_j \mu)},$$

$$(1.10) \quad \theta_2(\xi) = \frac{\prod_{j=1}^{n_2} \{\Gamma(1 - c_j + \gamma_j \xi)\}^{K_j} \prod_{j=1}^{m_2} \Gamma(d_j - \delta_j \xi)}{\prod_{j=n_2+1}^{p_2} \Gamma(c_j - \gamma_j \xi) \prod_{j=m_2+1}^{q_2} \{\Gamma(1 - d_j + \delta_j \xi)\}^{L_j}},$$

$$(1.11) \quad \theta_3(\mu) = \frac{\prod_{j=1}^{n_3} \{\Gamma(1 - e_j + \eta_j \mu)\}^{R_j} \prod_{j=1}^{m_3} \Gamma(f_j - \lambda_j \mu)}{\prod_{j=n_3+1}^{p_3} \Gamma(e_j - \eta_j \mu) \prod_{j=m_3+1}^{q_3} \{\Gamma(1 - f_j + \lambda_j \mu)\}^{S_j}},$$

The general class of polynomials (Srivastava polynomials) $S_n^m[x]$ will be defined and represented as follows [28, p. 1, eq. 1]:

$$(1.12) \quad S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k, \quad n = 0, 1, 2, \dots$$

where m is an arbitrary positive integer and the coefficient $A_{n,k}$ ($n, k \geq 0$) are arbitrary constants, real or complex. Where, $(\lambda)_n$ denotes the Pochhammer symbol defined by

$$(1.13) \quad (\gamma)_n = \frac{\Gamma(\gamma + n)}{\Gamma(\gamma)} = \begin{cases} 1, & (n = 0, \gamma \neq 0) \\ \gamma(\gamma + 1) \cdots (\gamma + n - 1), & (n \in N, \gamma \in C), \end{cases}$$

By suitable specializing the coefficient $A_{n,k}$, the polynomials $S_n^m(x)$ can easily be reduced to the classical orthogonal polynomials including, for example, the Hermite polynomials $H_n(x)$, the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$, and the Laguerre polynomials $L_n^{(\alpha)}(x)$, and also to several familiar particular cases of the Jacobi polynomials such as the Gegenbauer polynomials $C_n^\nu(x)$, the Legendre polynomials $P_n(x)$, and the Tchebycheff polynomials $T_n(x)$ and $U_n(x)$ (see, for detail [30]).

Other interesting special cases of the Srivastava polynomials (1.12) include such generalized hypergeometric polynomials as the Bessel polynomials, the generalized Hermite polynomials, and the Brafman polynomials as a particular case.

2. Generalized fractional differintegral operators

If $\alpha, \alpha', \beta, \beta', \gamma \in C$ and $[\operatorname{Re}(\gamma) > 0]$, $x > 0$, then the generalized fractional calculus operators involving Appell function F_3 given by Saigo and Maeda [20] are defined by

$$(2.1) \quad \begin{aligned} & \left(I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} f \right) (x) \\ &= \frac{x^{-\alpha}}{\Gamma(\gamma)} \int_0^x t^{-\alpha'} (x-t)^{\gamma-1} F_3 \left(\alpha, \alpha', \beta, \beta'; \gamma; 1 - \frac{t}{x}, 1 - \frac{x}{t} \right) f(t) dt \\ &= \left(\frac{d}{dx} \right)^k \left(I_{0+}^{\alpha, \alpha'+k, \beta+k, \beta'+k, \gamma+k} f \right) (x) \quad (\operatorname{Re}(\gamma) \leq 0; k = [-\operatorname{Re}(\gamma)] + 1), \\ & \left(I_{-}^{\alpha, \alpha', \beta, \beta', \gamma} f \right) (x) \end{aligned}$$

$$\begin{aligned}
&= \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty t^{-\alpha} (t-x)^{\gamma-1} F_3 \left(\alpha, \alpha', \beta, \beta'; \gamma; 1 - \frac{x}{t}, 1 - \frac{x}{t} \right) f(t) dt \\
(2.2) \quad &= \left(-\frac{d}{dx} \right)^k \left(I_{0+}^{\alpha, \alpha' \beta, \beta' + k, \gamma + k} f \right) (x) \quad (\operatorname{Re}(\gamma) \leq 0; k = [-\operatorname{Re}(\gamma)] + 1),
\end{aligned}$$

$$\begin{aligned}
&\left(D_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} f \right) (x) \\
&= \left(I_{0+}^{-\alpha', -\alpha, -\beta', -\beta, -\gamma} f \right) (x) \quad (\operatorname{Re}(\gamma) > 0) \\
&= \left(\frac{d}{dx} \right)^k \left(I_{0+}^{-\alpha', -\alpha, -\beta' + k, -\beta, -\gamma + k} f \right) (x) \quad (\operatorname{Re}(\gamma) > 0; k = [\operatorname{Re}(\gamma)] + 1) \\
&= \frac{1}{\Gamma(n-\gamma)} \left(\frac{d}{dx} \right)^n (x)^{\alpha'} \int_0^x (x-t)^{n-\gamma-1} t^\alpha \\
(2.3) \quad &\times F_3 \left(-\alpha', -\alpha, n - \beta', -\beta, n - \gamma; 1 - \frac{t}{x}, 1 - \frac{x}{t} \right) f(t) dt,
\end{aligned}$$

$$\begin{aligned}
&\left(D_{-}^{\alpha, \alpha', \beta, \beta', \gamma} f \right) (x) \\
&= \left(I_{-}^{-\alpha', -\alpha, -\beta', -\beta, -\gamma} f \right) (x) \quad (\operatorname{Re}(\gamma) > 0) \\
&= \left(-\frac{d}{dx} \right)^k \left(I_{-}^{-\alpha', -\alpha, -\beta', -\beta + k, -\gamma + k} f \right) (x) \quad (\operatorname{Re}(\gamma) > 0; k = [\operatorname{Re}(\gamma)] + 1) \\
&= \frac{1}{\Gamma(n-\gamma)} \left(-\frac{d}{dx} \right)^n (x)^\alpha \int_x^\infty (t-x)^{n-\gamma-1} t^{\alpha'} \\
(2.4) \quad &\times F_3 \left(-\alpha', -\alpha, -\beta', n - \beta, n - \gamma; 1 - \frac{x}{t}, 1 - \frac{t}{x} \right) f(t) dt.
\end{aligned}$$

Here $F_3(\alpha, \alpha', \beta, \beta'; \gamma; z, \xi)$ is the familiar Appell hypergeometric function of two variables defined by

$$\begin{aligned}
&F_3(\alpha, \alpha', \beta, \beta'; \gamma; z, \xi) \\
(2.5) \quad &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha)_m (\alpha')_n (\beta)_m (\beta')_n}{(\gamma)_{m+n}} \frac{z^m \xi^n}{m! n!} \quad (|z| < 1 \text{ and } |\xi| < 1),
\end{aligned}$$

where $(\lambda)_n$ denotes the Pochhammer symbol defined by

$$(\lambda)_n = \begin{cases} \lambda(\lambda+1) \cdots (\lambda+n-1) & (n \in \mathbb{N}) \\ 1 & (n=0). \end{cases}$$

It is noted that the series in (2.5) is absolutely convergent for all $z, \xi \in \mathbb{C}$ with $|z| < 1$ and $|\xi| < 1$, and for all $z, \xi \in \mathbb{C} \setminus \{1\}$ with $|z| = 1$ and $|\xi| = 1$.

The generalized fractional calculus operators due to Saigo-Maeda defined in [3, 12, 16, 21, 25] reduce to the following generalized fractional calculus

operators due to Saigo [18, 24, 26]:

$$(2.6) \quad \left(I_{0+}^{\alpha, 0, \beta, \beta', \gamma} f \right) (x) = \left(I_{0+}^{\gamma, \alpha-\gamma, -\beta} f \right) (x), \quad (\gamma \in C) \quad (x > 0)$$

$$(2.7) \quad \left(I_{-}^{\alpha, 0, \beta, \beta', \gamma} f \right) (x) = \left(I_{-}^{\gamma, \alpha-\gamma, -\beta} f \right) (x), \quad (\gamma \in C) \quad (x > 0)$$

$$(2.8) \quad \left(D_{0+}^{0, \alpha', \beta, \beta', \gamma} f \right) (x) = \left(D_{0+}^{\gamma, \alpha'-\gamma, \beta'-\gamma} f \right) (x), \quad (\operatorname{Re}(\gamma) > 0) \quad (x > 0)$$

$$(2.9) \quad \left(D_{-}^{0, \alpha', \beta, \beta', \gamma} f \right) (x) = \left(D_{-}^{\gamma, \alpha'-\gamma, \beta'-\gamma} f \right) (x), \quad (\operatorname{Re}(\gamma) > 0) \quad (x > 0).$$

Further from [20, p. 394, eq. (4.18) and (4.19)], we have:

Lemma 1. *The next formula holds for $\alpha, \alpha', \beta, \beta', \gamma \in C$, $\operatorname{Re}(\alpha) > 0$:*

$$(2.10) \quad \begin{aligned} & \left(I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} t^{\rho-1} \right) (x) \\ &= \frac{\Gamma(\rho) \Gamma(\rho + \gamma - \alpha - \alpha' - \beta) \Gamma(\rho + \beta' - \alpha')}{\Gamma(\rho + \gamma - \alpha - \alpha') \Gamma(\rho + \gamma - \alpha' - \beta) \Gamma(\rho + \beta')} x^{\rho - \alpha - \alpha' + \gamma - 1}, \end{aligned}$$

where $\operatorname{Re}(\gamma) > 0$, $\operatorname{Re}(\rho) > \max[0, \operatorname{Re}(\alpha + \alpha' + \beta - \gamma), \operatorname{Re}(\alpha' - \beta')]$, $(x > 0)$.

In particular, if $\alpha' = 0$, $\beta = -\eta$, $\gamma = \alpha$, and α is replaced by $\alpha + \beta$ in (2.10) then Saigo-Maeda operator becomes Saigo operator and we obtain [20]

$$(2.11) \quad \left(I_{0+}^{\alpha, \beta, \eta} t^{\rho-1} \right) (x) = \frac{\Gamma(\rho) \Gamma(\rho + \eta - \beta)}{\Gamma(\rho - \beta) \Gamma(\rho + \alpha + \eta)} x^{\rho - \beta - 1}, \quad (x > 0).$$

Lemma 2. *The next formula holds for $\alpha, \alpha', \beta, \beta', \gamma \in C$, $\operatorname{Re}(\alpha) > 0$:*

$$(2.12) \quad \begin{aligned} & \left(I_{-}^{\alpha, \alpha', \beta, \beta', \gamma} t^{\rho-1} \right) (x) \\ &= \frac{\Gamma(1 + \alpha + \alpha' - \gamma - \rho) \Gamma(1 + \alpha + \beta' - \gamma - \rho) \Gamma(1 - \beta - \rho)}{\Gamma(1 - \rho) \Gamma(1 + \alpha + \alpha' + \beta' - \gamma - \rho) \Gamma(1 + \alpha - \beta - \rho)} x^{\rho - \alpha - \alpha' + \gamma - 1}, \end{aligned}$$

where

$\operatorname{Re}(\gamma) > 0$, $x > 0$, $\operatorname{Re}(\rho) < 1 + \min[\operatorname{Re}(-\beta), \operatorname{Re}(\alpha + \alpha' - \gamma), \operatorname{Re}(\alpha + \beta' - \gamma)]$.

In particular, if $\alpha' = 0$, $\beta = -\eta$, $\gamma = \alpha$, α is replaced by $\alpha + \beta$ in (2.12), then

$$(2.13) \quad \left(I_{-}^{\alpha, \beta, \eta} t^{\rho-1} \right) (x) = \frac{\Gamma(1 + \beta - \rho) \Gamma(1 + \eta - \rho)}{\Gamma(1 - \rho) \Gamma(1 + \alpha + \beta + \eta - \rho)} x^{\rho - \beta - 1}, \quad (x > 0),$$

Lemma 3. *The next formula holds for $\alpha, \alpha', \beta, \beta', \gamma \in C$, $\operatorname{Re}(\alpha) > 0$:*

$$(2.14) \quad \begin{aligned} & \left(D_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} t^{\rho-1} \right) (x) \\ &= \frac{\Gamma(\rho) \Gamma(\rho + \alpha - \beta) \Gamma(\rho + \alpha + \alpha' + \beta' - \gamma)}{\Gamma(\rho - \beta) \Gamma(\rho + \alpha + \alpha' - \gamma) \Gamma(\rho + \alpha + \beta' - \gamma)} x^{\rho + \alpha + \alpha' - \gamma - 1}, \end{aligned}$$

where $\operatorname{Re}(\rho) > 0$, $\operatorname{Re}(\rho + \alpha - \beta) > 0$, $\operatorname{Re}(\rho + \alpha + \alpha' + \beta' - \gamma) > 0$, $(x > 0)$.

Lemma 4. *The next formula holds for $\alpha, \alpha', \beta, \beta', \gamma \in C$, $\operatorname{Re}(\alpha) > 0$:*

$$(2.15) \quad \begin{aligned} & \left(D_{-}^{\alpha, \alpha', \beta, \beta', \gamma} t^{\rho-1} \right) (x) \\ &= (-1)^n \frac{\Gamma(1-\alpha-\alpha'+\gamma-\rho)\Gamma(1-\alpha'-\beta+\gamma-\rho)\Gamma(1+\beta'-\rho)}{\Gamma(1-\rho)\Gamma(1-\alpha-\alpha'-\beta+\gamma-\rho)\Gamma(1-\alpha'+\beta'-\rho)} x^{\rho+\alpha+\alpha'-\gamma-1}, \end{aligned}$$

where $\operatorname{Re}(1+\beta-\rho) > 0$, $\operatorname{Re}(1-\rho-\alpha'-\beta+\gamma) > 0$, $\operatorname{Re}(1-\rho-\alpha-\alpha'+\gamma) > 0$, $n = [\operatorname{Re}(\gamma)+1]$, $(x > 0)$.

3. Fractional differintegral formulas

In this section we will establish four fractional differintegral formulas for the product of \overline{H} -function and Srivastava polynomials.

Theorem 1. *Let $\alpha, \alpha', \beta, \beta', \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$, $\lambda, \sigma > 0$ and $\operatorname{Re}(\gamma) > 0$. Then we have*

$$(3.1) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\ & \times \left. \left. \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \right) \right\} (x) \\ &= b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{4,4:P,Q:0,1}^{0,4:M,N:1,0} \left[\begin{array}{c} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{array} \left| \begin{array}{c} E_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ E_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right], \end{aligned}$$

where

$$\begin{aligned} E_1 &= (1-\eta-\delta k, v; 1), (1-\mu-\lambda k, \sigma; 1), \\ &\quad (1-\mu-\gamma+\alpha+\alpha'+\beta-\lambda k, \sigma; 1), (1-\mu+\alpha'-\beta'-\lambda k, \sigma; 1); \text{ and} \\ E_2 &= (1-\eta-\delta k, v; 0), (1-\mu-\gamma+\alpha+\alpha'-\lambda k, \sigma; 1), \\ &\quad (1-\mu-\gamma+\alpha'+\beta-\lambda k, \sigma; 1), (1-\mu-\beta'-\lambda k, \sigma; 1). \end{aligned}$$

Also, satisfy the following conditions:

$$(3.2) \quad \begin{aligned} \text{(i)} \quad & |\arg z| < \frac{1}{2}\Omega\pi, \quad \Omega > 0, \quad \Omega = \sum_{j=1}^M |\beta_j| + \sum_{j=1}^N |A_j \alpha_j| - \sum_{j=M+1}^Q |B_j \beta_j| - \sum_{j=N+1}^P |\alpha_j|. \end{aligned}$$

(ii) $|\frac{a}{b}x| < 1$, also we have

$$\operatorname{Re}(\mu) + \sigma \min_{1 \leq j \leq M} \operatorname{Re} \left(\frac{b_j}{\beta_j} \right) > \max [0, \operatorname{Re}(\alpha' - \beta'), \operatorname{Re}(\alpha + \alpha' + \beta - \gamma)],$$

$$\operatorname{Re}(\eta) + v \min_{1 \leq j \leq M} \operatorname{Re} \left(\frac{b_j}{\beta_j} \right) > \max [0, \operatorname{Re}(\alpha' - \beta'), \operatorname{Re}(\alpha + \alpha' + \beta - \gamma)].$$

Proof. To prove the fractional integral formula (3.1), we first express the general class of polynomials occurring on its left-hand side in the series from given by (1.12), replace the \overline{H} -function occurring therein by its well-known Mellin-Barnes contour integral given by (1.3), interchange the order of summations and make a little simplification.

$$(3.3) \quad \begin{aligned} \mathcal{I}_1 = & \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(\sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \frac{1}{2\pi i} \int_{L_1} \overline{\theta}(s) z^s ds \right. \right. \\ & \times b^{-\eta-\delta k-v s} \left(1 - \frac{a}{b} t \right)^{-(\eta+\delta k+v s)} t^{\mu+\lambda k+\sigma s-1} \left. \right) \} (x). \end{aligned}$$

Now, interchanging the order of integrals and summations, we obtain the following form after a little simplification:

$$(3.4) \quad \begin{aligned} \mathcal{I}_1 = & b^{-\eta} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} \left(\frac{1}{2\pi i} \right)^2 \int_{L_1} \int_{L_2} \overline{\theta}(s) z^s b^{-v s} \left(-\frac{a}{b} \right)^\xi ds d\xi \\ & \times \frac{\Gamma(\eta+\delta k+v s+\xi)}{\Gamma(\eta+\delta k+v s)\Gamma(1+\xi)} \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} (t^{\mu+\lambda k+\sigma s+\xi-1}) \right\} (x) \\ = & b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \frac{1}{2\pi i} \int_{L_1} \overline{\theta}(s) (z b^{-v} x^\sigma)^s ds \\ & \times \frac{1}{2\pi i} \int_{L_2} \frac{1}{\Gamma(1+\xi)} \left(-\frac{a}{b} x \right)^\xi d\xi \frac{\Gamma(\eta+\delta k+v s+\xi)\Gamma(\mu+\lambda k+\sigma s+\xi)}{\Gamma(\eta+\delta k+v s)\Gamma(\mu+\lambda k+\sigma s+\gamma-\alpha-\alpha'+\xi)} \\ (3.5) \quad & \times \frac{\Gamma(\mu+\lambda k+\sigma s+\gamma-\alpha-\alpha'-\beta+\xi)\Gamma(\mu+\lambda k+\sigma s-\alpha'+\beta'+\xi)}{\Gamma(\mu+\lambda k+\sigma s+\gamma-\alpha'-\beta+\xi)\Gamma(\mu+\lambda k+\sigma s+\beta'+\xi)}, \end{aligned}$$

by re-interpreting the Mellin-Barnes counter integral in terms of the \overline{H} -function of two variables defined by (1.8), we obtain the right-hand side of (3.1) after little simplifications. This completes proof of Theorem 1. \square

In view of the relation (2.6), then we get the following corollary concerning left-sided Saigo fractional integral operator [18, 19, 20].

Corollary 1.1. *Let $\alpha, \beta, \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$; $\lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then the following result holds true:*

$$(3.6) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha, \beta, \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\ & \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \mid \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right] \left. \right) \} (x) \\ = & b^{-\eta} x^{\mu-\beta-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{3,3:P,Q:0,1}^{0,3:M,N:1,0} \left[\begin{array}{c|c} z x^\sigma b^{-v} & E'_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ -\frac{a}{b} x & E'_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right], \end{aligned}$$

where $E'_1 = (1 - \eta - \delta k, v; 1), (1 - \mu - \lambda k, \sigma; 1), (1 - \mu - \lambda k + \beta - \gamma, \sigma; 1)$; and $E'_2 = (1 - \eta - \delta k, v; 0), (1 - \mu - \lambda k + \beta, \sigma; 1), (1 - \mu - \lambda k - \alpha - \gamma, \sigma; 1)$. Also, satisfy the following conditions:

$$\operatorname{Re}(\mu) + \sigma \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) > \max[0, \operatorname{Re}(\beta - \gamma)],$$

$$\operatorname{Re}(\eta) + v \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) > \max[0, \operatorname{Re}(\beta - \gamma)],$$

the conditions (i) and (ii) given in Theorem 1 are also satisfied.

Next, if we set $\beta = -\alpha$ in (3.6), we obtain the following result concerning left-sided Riemann-Liouville fractional integral operator [14, 18]:

Corollary 1.2. *Let $\alpha, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$; $\lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then we have*

$$(3.7) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \right. \right. \\ & \times \overline{H}_{P,Q}^{M,N} \left[z t^{\sigma} (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \} (x) \\ & = b^{-\eta} x^{\mu+\alpha-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{2,2:P,Q:0,1}^{0,2:M,N:1,0} \left[\begin{array}{c|c} zx^{\sigma} b^{-v} & E''_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ -\frac{a}{b}x & E''_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right], \end{aligned}$$

where $E''_1 = (1 - \eta - \delta k, v; 1), (1 - \mu - \lambda k, \sigma; 1)$; and $E''_2 = (1 - \eta - \delta k, v; 0), (1 - \mu - \alpha - \lambda k, \sigma; 1)$, and the existence conditions of the above corollary easily follows with the help of (3.6).

Theorem 2. *Let $\alpha, \alpha', \beta, \beta', \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$, $\lambda, \sigma > 0$ and $\operatorname{Re}(\gamma) > 0$. Then we have*

$$(3.8) \quad \begin{aligned} & \left\{ I_{-}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \right. \right. \\ & \times \overline{H}_{P,Q}^{M,N} \left[z t^{\sigma} (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \} (x) \\ & = b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{4,4:P,Q:0,1}^{0,4:M,N:1,0} \left[\begin{array}{c|c} zx^{\sigma} b^{-v} & F_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ -\frac{a}{b}x & F_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right], \end{aligned}$$

where

$$F_1 = (1 - \eta - \delta k, v; 1), (\mu + \lambda k + \gamma - \alpha - \alpha', -\sigma; -1),$$

$$(\mu + \gamma - \alpha - \beta' + \lambda k, -\sigma; -1), (\mu + \beta + \lambda k, -\sigma; -1); \text{ and}$$

$$F_2 = (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1),$$

$$(\mu + \gamma - \alpha - \alpha' - \beta' + \lambda k, -\sigma; -1), (\mu - \alpha + \beta + \lambda k, -\sigma; -1).$$

Also, satisfy the following conditions:

$$\operatorname{Re}(\mu) - \sigma \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) < 1 + \min[\operatorname{Re}(-\beta), \operatorname{Re}(\alpha + \alpha' - \gamma), \operatorname{Re}(\alpha + \beta' - \gamma)],$$

$$\operatorname{Re}(\eta) - v \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) < 1 + \min[\operatorname{Re}(-\beta), \operatorname{Re}(\alpha + \alpha' - \gamma), \operatorname{Re}(\alpha + \beta' - \gamma)].$$

and the conditions (i) and (ii) in Theorem 1 are also satisfied.

Proof. Using the series definition (1.12) and replacing the \overline{H} -function occurring therein by its well-known Mellin-Barnes contour integral given by (1.3) to the integrand of (3.8) and then interchanging the order of the integral sign and the summation, and finally applying the (1.8) to the resulting integrals, we can get the expression as in the right-hand side of (3.8). \square

In view of the relation (2.7), then we get the following corollary concerning right-sided Saigo fractional integral operator [18, 19, 20].

Corollary 2.1. *Let $\alpha, \beta, \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}; \lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then the following result holds true:*

$$(3.9) \quad \begin{aligned} & \left\{ I_{-}^{\alpha, \beta, \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\ & \times \left. \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \right) \left. \right\} (x) \\ & = b^{-\eta} x^{\mu-\beta-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{3,3:P,Q:0,1}^{0,3:M,N:1,0} \left[\begin{array}{c} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{array} \left| \begin{array}{c} F'_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ F'_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right], \end{aligned}$$

where $F'_1 = (1 - \eta - \delta k, v; 1), (\mu - \beta + \lambda k, -\sigma; -1), (\mu - \gamma + \lambda k, -\sigma; -1)$;
and $F'_2 = (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1), (\mu + \lambda k - \alpha - \beta - \gamma, -\sigma; -1)$.

Also, satisfy the following conditions:

$$\operatorname{Re}(\mu) - \sigma \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) < 1 + \min[\operatorname{Re}(\beta), \operatorname{Re}(\gamma)],$$

$$\operatorname{Re}(\eta) - v \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) < 1 + \min[\operatorname{Re}(\beta), \operatorname{Re}(\gamma)],$$

the conditions (i) and (ii) given in Theorem 1 are also satisfied.

If we set $\beta = -\alpha$ in (3.9), we obtain the following result concerning right-sided Riemann-Liouville fractional integral operator [14, 18]:

Corollary 2.2. *Let $\alpha, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}; \lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then*

$$\left\{ I_{-}^{\alpha} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right.$$

$$\begin{aligned}
& \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] \Bigg\} (x) \\
& = b^{-\eta} x^{\mu+\alpha-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\
(3.10) \quad & \times \overline{H}_{2,2:P,Q:0,1}^{0,2:M,N:1,0} \left[\begin{matrix} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{matrix} \left| \begin{matrix} F_1'', (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ F_2'', (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{matrix} \right. \right],
\end{aligned}$$

where

$$\begin{aligned}
F_1'' &= (1 - \eta - \delta k, v; 1), (\mu + \alpha + \lambda k, -\sigma; -1); \text{ and} \\
F_2'' &= (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1),
\end{aligned}$$

and the existence conditions of the above corollary easily follows with the help of (3.9).

Theorem 3. Let $\alpha, \alpha', \beta, \beta', \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$, $\lambda, \sigma > 0$ and $\operatorname{Re}(\gamma) > 0$. Then we have

$$\begin{aligned}
& \left\{ D_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\
& \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] \Bigg\} (x) \\
& = b^{-\eta} x^{\mu+\alpha+\alpha'-\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\
(3.11) \quad & \times \overline{H}_{4,4:P,Q:0,1}^{0,4:M,N:1,0} \left[\begin{matrix} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{matrix} \left| \begin{matrix} L_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ L_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{matrix} \right. \right],
\end{aligned}$$

where

$$\begin{aligned}
L_1 &= (1 - \eta - \delta k, v; 1), (1 - \mu - \lambda k, \sigma; 1), \\
&\quad (1 - \mu - \alpha - \alpha' - \beta' + \gamma - \lambda k, \sigma; 1), (1 - \mu - \alpha + \beta - \lambda k, \sigma; 1); \text{ and} \\
L_2 &= (1 - \eta - \delta k, v; 0), (1 - \mu - \alpha - \alpha' + \gamma - \lambda k, \sigma; 1), \\
&\quad (1 - \mu - \alpha - \beta' + \gamma - \lambda k, \sigma; 1), (1 - \mu + \beta - \lambda k, \sigma; 1).
\end{aligned}$$

Satisfy the following conditions:

$$\operatorname{Re}(\mu) + \sigma \min_{1 \leq j \leq M} \operatorname{Re} \left(\frac{b_j}{\beta_j} \right) + \max [0, \operatorname{Re}(\alpha - \beta), \operatorname{Re}(\alpha + \alpha' + \beta' - \gamma)] > 0,$$

$$\operatorname{Re}(\eta) + v \min_{1 \leq j \leq M} \operatorname{Re} \left(\frac{b_j}{\beta_j} \right) + \max [0, \operatorname{Re}(\alpha - \beta), \operatorname{Re}(\alpha + \alpha' + \beta' - \gamma)] > 0,$$

and the conditions (i) and (ii) as given in Theorem 1 are also satisfied.

Proof. To prove the fractional differential formula (3.11), we first express the general class of polynomials occurring on its left-hand side in the series from given by (1.12), replace the \overline{H} -function occurring therein by its well-known

Mellin-Barnes contour integral given by (1.3), interchange the order of summations and make a little simplification.

$$(3.12) \quad \mathcal{D}_1 = \left\{ D_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(\sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} \frac{1}{2\pi i} \int_{L_1} \bar{\theta}(s) z^s ds \right. \right. \\ \times b^{-\eta-\delta k-v s} \left(1 - \frac{a}{b} t \right)^{-(\eta+\delta k+v s)} t^{\mu+\lambda k+\sigma s-1} \left. \right) \right\} (x).$$

Now, interchanging the order of integrals and summations, we obtain the following form after a little simplification:

$$(3.13) \quad \mathcal{D}_1 = b^{-\eta} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} \left(\frac{1}{2\pi i} \right)^2 \int_{L_1} \int_{L_2} \bar{\theta}(s) (zb^{-v})^s \left(-\frac{a}{b} \right)^\xi ds d\xi \\ \times \frac{\Gamma(\eta+\delta k+v s+\xi)}{\Gamma(\eta+\delta k+v s)\Gamma(1+\xi)} \left\{ D_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} (t^{\mu+\lambda k+\sigma s+\xi-1}) \right\} (x).$$

Now, using Lemma 3, we arrive at the following:

$$(3.14) \quad \mathcal{D}_1 = b^{-\eta} x^{\mu+\alpha+\alpha'-\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \frac{1}{2\pi i} \int_{L_1} \bar{\theta}(s) (zb^{-v} x^\sigma)^s ds \\ \times \frac{1}{2\pi i} \int_{L_2} \frac{1}{\Gamma(1+\xi)} \left(-\frac{a}{b} x \right)^\xi \frac{\Gamma(\eta+\delta k+v s+\xi)\Gamma(\mu+\lambda k+\sigma s+\xi)}{\Gamma(\eta+\delta k+v s)\Gamma(\mu+\lambda k+\sigma s-\gamma+\alpha+\alpha'+\xi)} \\ \times \frac{\Gamma(\mu+\lambda k+\sigma s-\gamma+\alpha+\alpha'+\beta'+\xi)\Gamma(\mu+\lambda k+\sigma s+\alpha-\beta+\xi)}{\Gamma(\mu+\lambda k+\sigma s-\gamma+\alpha+\beta'+\xi)\Gamma(\mu+\lambda k+\sigma s-\beta+\xi)} d\xi,$$

by re-interpreting the Mellin-Barnes contour integral in terms of the \overline{H} -function of two variables defined by (1.8), we obtain the right-hand side of (3.11) after little simplifications. This completes proof of Theorem 3. \square

In view of the relation (2.8), then we get the following corollary concerning left-sided Saigo fractional derivative operator [18, 19, 20].

Corollary 3.1. *Let $\alpha, \beta, \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}$; $\lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then we have the following result:*

$$(3.15) \quad \left\{ D_{0+}^{\alpha, \beta, \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\ \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \right\} (x) \\ = b^{-\eta} x^{\mu+\beta-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ \times \overline{H}_{3,3:P,Q:0,1}^{0,3:M,N:1,0} \left[\begin{array}{c} zx^\sigma b^{-v} \\ -\frac{a}{b} x \end{array} \left| \begin{array}{l} L'_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ L'_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right],$$

where $L'_1 = (1 - \eta - \delta k, v; 1), (1 - \mu - \lambda k, \sigma; 1), (1 - \mu - \lambda k - \alpha - \beta - \gamma, \sigma; 1)$; and $L'_2 = (1 - \eta - \delta k, v; 0), (1 - \mu - \lambda k - \beta, \sigma; 1), (1 - \mu - \lambda k - \gamma, \sigma; 1)$.

Also, satisfy the following conditions:

$$\operatorname{Re}(\mu) + \sigma \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) > \max[0, \operatorname{Re}(-\alpha - \beta - \gamma)],$$

$$\operatorname{Re}(\eta) + v \min_{1 \leq j \leq M} \operatorname{Re}\left(\frac{b_j}{\beta_j}\right) > \max[0, \operatorname{Re}(-\alpha - \beta - \gamma)],$$

the conditions (i) and (ii) given in Theorem 1 are also satisfied.

Further, if we set $\beta = -\alpha$ in the above result, then we obtain the following result concerning left-sided Riemann-Liouville fractional derivative operator [14, 18]:

Corollary 3.2. Let $\alpha, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}; \lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then

$$(3.16) \quad \begin{aligned} & \left\{ D_{0+}^{\alpha} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \right. \right. \\ & \times \overline{H}_{P,Q}^{M,N} \left[z t^{\sigma} (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \Big\} (x) \\ & = b^{-\eta} x^{\mu-\alpha-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{2,2:P,Q:0,1}^{0,2:M,N:1,0} \left[\begin{array}{c} zx^{\sigma} b^{-v} \\ -\frac{a}{b} x \end{array} \left| \begin{array}{l} L''_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ L''_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right], \end{aligned}$$

where $L''_1 = (1 - \eta - \delta k, v; 1), (1 - \mu - \lambda k, \sigma; 1)$; and $L''_2 = (1 - \eta - \delta k, v; 0), (1 - \mu + \alpha - \lambda k, \sigma; 1)$, and the conditions of existence of the above corollary follow easily with the help of (3.15).

Theorem 4. Let $\alpha, \alpha', \beta, \beta', \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}, \lambda, \sigma > 0$ and $\operatorname{Re}(\gamma) > 0$. Then we have

$$(3.17) \quad \begin{aligned} & \left\{ D_{-}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \right. \right. \\ & \times \overline{H}_{P,Q}^{M,N} \left[z t^{\sigma} (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \Big\} (x) \\ & = b^{-\eta} x^{\mu+\alpha+\alpha'-\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \times \overline{H}_{4,4:P,Q:0,1}^{0,4:M,N:1,0} \left[\begin{array}{c} zx^{\sigma} b^{-v} \\ -\frac{a}{b} x \end{array} \left| \begin{array}{l} W_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ W_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right], \end{aligned}$$

where

$$W_1 = (1 - \eta - \delta k, v; 1), (\mu + \lambda k + \alpha + \alpha' - \gamma, -\sigma; -1),$$

$$(\mu + \alpha' + \beta - \gamma + \lambda k, -\sigma; -1), (\mu - \beta' + \lambda k, -\sigma; -1); \text{ and}$$

$$W_2 = (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1), (\mu + \alpha + \alpha' + \beta - \gamma + \lambda k, -\sigma; -1),$$

$$(\mu + \alpha' - \beta' + \lambda k, -\sigma; -1).$$

Also, satisfy the following conditions:

$$\begin{aligned} & \operatorname{Re}(\mu) + \sigma \max_{1 \leq j \leq N} \operatorname{Re}\left(\frac{\operatorname{Re}(a_j)-1}{\alpha_j}\right) \\ & < 1 + \min [\operatorname{Re}(-\beta), \operatorname{Re}(\gamma - \alpha - \alpha' - k), \operatorname{Re}(\gamma - \alpha' - \beta)], \\ & \operatorname{Re}(\eta) + v \max_{1 \leq j \leq N} \operatorname{Re}\left(\frac{\operatorname{Re}(a_j)-1}{\alpha_j}\right) \\ & < 1 + \min [\operatorname{Re}(-\beta), \operatorname{Re}(\gamma - \alpha - \alpha' - k), \operatorname{Re}(\gamma - \alpha' - \beta)], \end{aligned}$$

here $k = [\operatorname{Re}(\gamma)] + 1$, and the conditions (i) and (ii) given in Theorem 1 are also satisfied.

Proof. The result (3.17) can easily be obtained by following similar lines of Theorem 3 and take into account relation (2.4). \square

In view of the relation (2.9), then we get the following corollary concerning right-sided Saigo fractional derivative operator [18, 19, 20].

Corollary 4.1. Let $\alpha, \beta, \gamma, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}; \lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then the following result holds true:

$$\begin{aligned} & \left\{ D_{-}^{\alpha, \beta, \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\ & \quad \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{array}{l} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{array} \right. \right] \left. \right) \Big\} (x) \\ & = b^{-\eta} x^{\mu+\beta-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ (3.18) \quad & \times \overline{H}_{3,3:P,Q:0,1}^{0,3:M,N:1,0} \left[\begin{array}{c} z x^\sigma b^{-v} \\ -\frac{a}{b} x \end{array} \left| \begin{array}{c} W'_1, (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ W'_2, (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{array} \right. \right], \end{aligned}$$

where $W'_1 = (1 - \eta - \delta k, v; 1), (\mu + \beta + \lambda k, -\sigma; -1), (\mu - \alpha - \gamma + \lambda k, -\sigma; -1)$; and $W'_2 = (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1), (\mu + \beta - \gamma + \lambda k, -\sigma; -1)$.

Also, satisfy the following conditions:

$$\operatorname{Re}(\mu) + \sigma \max_{1 \leq j \leq N} \left(\frac{\operatorname{Re}(a_j)-1}{\alpha_j} \right) < 1 + \min [0, [\operatorname{Re}(\alpha)] - \operatorname{Re}(\beta) - 1, \operatorname{Re}(\alpha + \gamma)],$$

$$\operatorname{Re}(\eta) + v \max_{1 \leq j \leq N} \left(\frac{\operatorname{Re}(a_j)-1}{\alpha_j} \right) < 1 + \min [0, [\operatorname{Re}(\alpha)] - \operatorname{Re}(\beta) - 1, \operatorname{Re}(\alpha + \gamma)],$$

the conditions (i) and (ii) given in Theorem 1 are also satisfied.

If we set $\beta = -\alpha$ in (3.19), we obtain the following result concerning right-sided Riemann-Liouville fractional derivative operator [14, 18]:

Corollary 4.2. Let $\alpha, \mu, \eta, \delta, v, z, a, b \in \mathbb{C}; \lambda, \sigma > 0$ and $\operatorname{Re}(\alpha) > 0$. Then

$$\left\{ D_{-}^{\alpha} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right.$$

$$\begin{aligned}
& \times \overline{H}_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] \Big) \Big\} (x) \\
& = b^{-\eta} x^{\mu-\alpha-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\
(3.19) \quad & \times \overline{H}_{2,2:P,Q:0,1}^{0,2:M,N:1,0} \left[\begin{matrix} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{matrix} \left| \begin{matrix} W_1'', (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P}, - \\ W_2'', (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (0, 1; 1) \end{matrix} \right. \right],
\end{aligned}$$

where $W_1'' = (1 - \eta - \delta k, v; 1), (\mu - \alpha + \lambda k, -\sigma; -1)$; and $W_2'' = (1 - \eta - \delta k, v; 0), (\mu + \lambda k, -\sigma; -1)$, and the existence conditions of the above corollary easily follows with the help of (3.19).

4. Special cases

The \overline{H} -function reduces to a large number of special function [7, 9, 10, 13, 23] and so from Theorems 1-4, we can further obtain various fractional calculus results involving a number of special function. Here, we provide a few special cases of our main findings (see also, [1, 3, 24, 29, 30]).

(i) If we set $M = 1, N = P$ and $Q = Q + 1$ in Theorem 1, the \overline{H} -function occurring in L.H.S. breaks up into the generalized Wright hypergeometric function ${}_P\overline{\psi}_Q$ [2], then Theorem 1 takes the following form after a little simplification:

$$\begin{aligned}
& \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\
& \times {}_P\overline{\psi}_Q \left[\begin{matrix} -z t^\sigma (b-at)^{-v} \\ -\frac{a}{b}x \end{matrix} \left| \begin{matrix} (1-a_j, \alpha_j; A_j)_{1,P} \\ (1-b_j, \beta_j; B_j)_{1,Q} \end{matrix} \right. \right] \Big) \Big\} (x) \\
& = b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\
(4.1) \quad & \times \overline{H}_{4,4:P,Q+1:0,1}^{0,4:1,P:1,0} \left[\begin{matrix} zx^\sigma b^{-v} \\ -\frac{a}{b}x \end{matrix} \left| \begin{matrix} E_1, (1-a_j, \alpha_j; A_j)_{1,P}, - \\ E_2, (1-b_j, \beta_j; B_j)_{1,Q}, (0, 1; 1) \end{matrix} \right. \right],
\end{aligned}$$

where E_1 and E_2 are same as given in Theorem 1. The conditions of validity of the above result easily follow from (3.1).

(ii) If we take $A_j = 1$ ($j = 1, \dots, N$), $B_j = 1$ ($j = M + 1, \dots, Q$), the \overline{H} -function reduces to the H -function. Then Theorem 1 takes the following form:

$$\begin{aligned}
& \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] \right. \right. \\
& \times H_{P,Q}^{M,N} \left[z t^\sigma (b-at)^{-v} \left| \begin{matrix} (a_j, \alpha_j)_{1,P} \\ (b_j, \beta_j)_{1,Q} \end{matrix} \right. \right] \Big) \Big\} (x) \\
& = b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k}
\end{aligned}$$

$$(4.2) \quad \times H_{4,4:P,Q:0,1}^{0,4:M,N:1,0} \left[\begin{array}{c|c} zx^{\sigma}b^{-v} & E_1, (a_j, \alpha_j)_{1,P}, - \\ -\frac{a}{b}x & E_2, (b_j, \beta_j)_{1,Q}, (0, 1) \end{array} \right],$$

where E_1 and E_2 are same as given in Theorem 1. The conditions of validity of the above result easily follow from (3.1).

Remark 4.1. We can also obtain results for ordinary Bessel function of the first kind $J_v(z)$, modified Bessel functions $K_v(z)$ (Macdonald function) and $Y_v(z)$ (Neumann function), generalized Bessel-Maitland function $J_{v,\lambda}^{\mu}$, Kummer's confluent hypergeometric function $\phi(a; d; -z)$, Gauss's hypergeometric function

$${}_2F_1(b, a; d; -zt^{\mu}),$$

MacRobert's E -function, Whittaker function by using the relation with H -function and these functions.

(iii) If we set $M = 1, N = P = 0$ and $Q = 2$ in (3.1), the \overline{H} -function of one variable occurring in L.H.S. breaks up into the generalized Wright-Bessel function $\overline{J}_b^{\omega,B}(z)$ [5], then Theorem 1 takes the following form:

$$(4.3) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \right. \right. \\ & \quad \times \left. \overline{J}_b^{\omega,B} \left[-z t^{\sigma} (b-at)^{-v} \right] \right) \Big\} (x) \\ & = b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \quad \times \overline{H}_{4,4:0,2:0,1}^{0,4:1,0:1,0} \left[\begin{array}{c|c} zx^{\sigma}b^{-v} & E_1, -, - \\ -\frac{a}{b}x & E_2, (0, 1), (-b, \omega; B), (0, 1; 1) \end{array} \right], \end{aligned}$$

where E_1 and E_2 are the same as given in (3.1). The conditions of validity of the above result easily follow from (3.1).

(iv) If we reduce the \overline{H} -function of one variable occurring in L.H.S. of (3.1) to a generalized Riemann-Zeta function [4] given by

$$(4.4) \quad \phi(z, l, \rho) = \sum_{r=0}^{\infty} \frac{z^r}{(\rho+r)^l} = \overline{H}_{2,2}^{1,2} \left[-z \left| \begin{array}{l} (0, 1; 1), (1-\rho, 1; l) \\ (0, 1), (-\rho, 1; l) \end{array} \right. \right],$$

then we arrive at the following result:

$$(4.5) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \phi \left[z t^{\sigma} (b-at)^{-v}, l, \rho \right] \right) \right\} (x) \\ & = b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \quad \times \overline{H}_{4,4:2,2:0,1}^{0,4:1,2:1,0} \left[\begin{array}{c|c} -zx^{\sigma}b^{-v} & E_1, (0, 1; 1), (1-\rho, 1; l), - \\ -\frac{a}{b}x & E_2, (0, 1), (-\rho, 1; l), (0, 1; 1) \end{array} \right], \end{aligned}$$

where E_1 and E_2 are the same as given in Theorem 1. The conditions of validity of the above result can easily be followed directly from (3.1).

(v) If we take $M = 1$ and $N = P = Q = 2$ in (3.1), then the \overline{H} -function of one variable occurring in L.H.S. reduces into the poly-logarithm of complex order ν , denoted by $L^\nu(z)$ [22, eq. (1.12)], then (3.1) takes the following form after simplification:

$$(4.6) \quad \begin{aligned} & \left\{ I_{0+}^{\alpha, \alpha', \beta, \beta', \gamma} \left(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^\lambda (b-at)^{-\delta} \right] L^\nu \left[z t^\sigma (b-at)^{-\nu} \right] \right) \right\} (x) \\ &= b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} b^{-\delta k} x^{\lambda k} \\ & \quad \times \overline{H}_{4,4:2,2:0,1}^{0,4:1,2:1,0} \left[\begin{array}{c} -zx^\sigma b^{-\nu} \\ -\frac{a}{b}x \end{array} \middle| \begin{array}{c} E_1, (0, 1; 1), (1, 1; \nu), - \\ E_2, (0, 1), (0, 1; \nu-1), (0, 1; 1) \end{array} \right], \end{aligned}$$

where E_1 and E_2 are the same as given in (3.1). The conditions of validity of the above result easily follow from Theorem 1.

5. Concluding remarks

In this paper, we have studied and given new unified fractional differintegral formulas involving a product of the \overline{H} -function and Srivastava polynomials. The results have been given in terms of the product of one variable \overline{H} -function and Srivastava polynomials in a compact and elegant form, the results are obtained in terms of two variables \overline{H} -function. A specimen of some of interesting special cases of main integral formula is presented briefly.

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