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ON WEAKLY COMPLETELY QUASI PRIMARY AND COMPLETELY QUASI PRIMARY IDEALS IN TERNARY SEMIRINGS

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Abstract. In this investigation we studied completely quasi primary and weakly completely quasi primary ideals in ternary semirings. Some characterizations of completely quasi primary and weakly completely quasi primary ideals were obtained. Moreover, we investigated relationships between completely quasi primary and weakly completely quasi primary ideals in ternary semirings. Finally, we obtained necessary and sufficient conditions for a weakly completely quasi primary ideal to be a completely quasi primary ideal.

1. Introduction

There is a large literature dealing with ternary operations. The notion of a ternary semigroup is a natural generalization of a ternary group. Algebraic structures play a prominent role in mathematics with wide applications in many disciplines, such as computer sciences, information sciences, engineering, physics, etc. S. Banach (cf. J. Los [17]) found some applications in a ternary semigroup. He gave an example to show that a ternary semigroup does not necessarily reduce to an ordinary semigroup. J. Los [17] studied some properties of a ternary semigroup and proved that every ternary semigroup can be embedded in a semigroup.

In 1932, D. H. Lehmer introduced the concept of a ternary semigroup [16]. The algebraic structures of ternary semigroups were also studied by some authors, for example, Sioson studied ideals in ternary semigroups [20]. A nonempty set S is called a ternary semigroup if there exists a ternary operation

$$\cdot : S \times S \times S \to S,$$

written as $(x_1, x_2, x_3) \mapsto x_1x_2x_3$, satisfying the following identity for any $x_1, x_2, x_3, x_4, x_5 \in S$,

$$(x_1x_2x_3)x_4x_5 = x_1(x_2x_3x_4)x_5 = x_1x_2(x_3x_4x_5).$$

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In 2003, S. Kar and T. K. Dutta [8] introduced notion of a ternary semiring which was the generalization of a ternary ring. They also introduced the concept of a regular ternary semiring and studied some of its properties. In 2005 [8, 9] they studied prime ideals, semiprime ideals, irreducible ideals and the prime radical of a ternary semiring. In 2014, D. Madhusudhana Rao and G. Srinivasa Rao [18] investigated and studied about the classification of ternary semirings and some special elements in ternary semirings.

In this investigation we studied completely quasi primary and weakly completely quasi primary ideals in ternary semirings. Some characterizations of completely quasi primary and weakly completely quasi primary ideals were obtained. Moreover, we investigated relationships between completely quasi primary and weakly completely quasi primary ideals in ternary semirings. Finally, we obtained necessary and sufficient conditions for a weakly completely quasi primary ideal to be a completely quasi primary ideal.

2. Basic results

In this section we refer to [2] for some elementary aspects and quote a number of theorems and lemmas that are essential to step up this study. For more details we refer to the papers in the references.

Definition ([2])**.** A nonempty set S together with a binary operation called addition and ternary multiplication, denoted by juxtaposition is said to be a ternary semiring if S is an additive commutative semigroup satisfying the following conditions:

(1) $(abc)de = a(bcd)e = ab(cde)$,
(2) $(a + b)cd = acd + bcd$,
(3) $a(b + c)d = abd + acd$,
(4) $ab(c + d) = abc + abd$ for all $a, b, c, d, e \in S$.

Let S be a ternary semiring. If there exists an element $0 \in S$ such that $0 + x = x$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$, then 0 is called the zero element or simply the zero of the ternary semiring S. In this case we say that S is a ternary semiring with zero.

Definition ([2])**.** A nonempty subset I of a ternary semiring S is said to be a ternary left (resp., right, lateral) ideal or simply a left (resp., right, lateral) ideal of S if

(1) $a, b \in I$ implies $a + b \in I$,
(2) $b, c \in S, a \in I$ implies $bca \in I$ (resp., $bac \in I, abc \in I$).

If I is a left, lateral and right ideal of S, then S is called an ideal of S. An ideal I of S is called a k-ideal if $x + y \in I, x \in S, y \in I$ implies $x \in I$.

Definition. Let I be a proper ideal of a ternary semiring S. Then the congruence on S, denoted by ρ_I and defined by $s\rho_I\acute{s}$ if and only if $s + a_1 = \acute{s} + a_2$, for some $a_1, a_2 \in I$, is called the Bourne congruence on S defined by the ideal I.

We denote the Bourne congruence (ρ_I) class of an element r of S by r/ρ_I or simply by r/I and denote the set of all such congruence classes of S by $S/\rho I$ or simply by S/I. For any proper ideal I of S the factor ternary semiring [13], is defined under the following addition and ternary multiplication on S/I by $a/I + b/I = (a+b)/I$ and $(a/I)(b/I)(c/I) = (abc)/I$ for all $a, b, c \in S$. With these two operations S/I forms a ternary semiring and is called the Bourne factor ternary semiring or simply the factor ternary semiring.

Definition. If S and R are ternary semirings, then a function f from S to R is a ternary semiring homomorphism if and only if the following conditions are satisfied:

(1) $f(a+b) = f(a) + f(b)$ for all $a, b \in S$;
(2) $f(abc) = f(a)f(b)f(c)$ for all $a, b, c \in S$.

A homomorphism $f : S \to R$ is called an isomorphism if f is both injective and surjective. In that case, S is said to be isomorphic to R and is denoted by $S \cong R$.

Lemma 2.1. *Let A, B and C be ideals of a ternary semiring S. Then*

$$(A + B + C)/A \cong B/(A \cap B \cap C).$$

Definition. Let S be a ternary semiring and A be a subset of S. We write $\sqrt{A} = \{a \in S : a^n \in A$ for some positive odd integer $n\}$.

Remark 2.2. Let S be a ternary semiring and let I be an ideal of S. It is easy to verify that $I \subseteq \sqrt{I}$.

3. Main results

We start with the following theorem that gives a relation between completely quasi primary and weakly completely quasi primary ideals in a ternary semirings. Our starting point is the following lemma:

Lemma 3.1. *Let S be a ternary semiring and let A be a left ideal of R. Then $(A : b : c)_L$ is a left ideal in S, where $(A : b : c)_L = \{r \in R : rcb \in A\}$.*

Proof. Let S be a ternary semiring and let A be a left ideal of S. Then for $a_1, a_2, a_3 \in (A : b : c)_L, r, s \in R$ consider

$$\begin{aligned}(a_1 + a_2 + a_3)cb &= a_1cb + a_2cb + a_3cb \\ &\in A + A + A \\ &\subseteq A\end{aligned}$$

and

$$\begin{aligned}(sra_1)cb &= sr(a_1cb) \\ &\in srA\end{aligned}$$

$$\subseteq A,$$

that is $a_1 + a_2 + a_3, (sra_1)cb \in A$. Therefore $(A : b : c)_L$ is left ideal of S. $\square$

Corollary 3.2. *Let S be a ternary semiring and let A be a left ideal of R. Then $(A : B : C)_L$ is a left ideal in S, where $(A : B : C)_L = \{r \in R : rCB \subseteq A\}$.*

Proof. This follows from Lemma 3.1. $\square$

Lemma 3.3. *Let S be a ternary semiring and let A be a right ideal of R. Then $(A : b : c)_R$ is a right ideal in S, where $(A : b : c)_R = \{r \in R : cbr \in A\}$.*

Proof. This follows from Lemma 3.1. $\square$

Corollary 3.4. *Let S be a ternary semiring and let A be a right ideal of R. Then $(A : B : C)_R$ is a right ideal in S, where $(A : B : C)_R = \{r \in R : CBr \subseteq A\}$.*

Proof. This follows from Lemma 3.1. $\square$

Remark 3.5.

(1) Let S be a ternary semiring and let A be a left (right) ideal of S. It is easy to verify that $A \subseteq (A : r : s)_L (A \subseteq (A : r : s)_R)$, where $(A : r : s)_L = \{a \in R : asr \in A\}$ $((A : r : s)_R = \{a \in R : sra \in A\})$.
(2) Let S be a ternary semiring with identity e, and let A be a proper left (right) ideal of S. By Lemma 3.1, we have $e \notin (A : r : s)_L (e \notin (A : r : s)_R)$, where $r, s \in S - A$.
(3) Let S be a ternary semiring and let A, B, C, D be left (right) ideals of S. It is easy to verify that $(A : C : D)_L \subseteq (A : B : D)_L ((A : C : D)_R \subseteq (A : B : D)_R)$, where $B \subseteq C$.

Definition. A left ideal P of a ternary semiring S is called a completely quasi primary ideal if for $a, b, c \in S$ such that $abc \in P$ implies that $a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n.

Example 3.6. Let $S = Z_0^-$ be the ternary semiring of all non-positive integers with the usual addition and ternary multiplication and $P = \left\{x : x \in 30Z_0^-\right\}$. Then P is a completely quasi primary left ideal of S.

Definition. A left ideal P of a ternary semiring S is called a weakly completely quasi primary ideal if for $a, b, c \in S$ such that $0 \neq abc \in P$ implies that $a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n.

Example 3.7. Let $S = Z_0^- \times Z_0^-$ be a ternary semiring and $P = \{0\} \times 27Z_0^-$. Then P is a completely quasi primary left ideal of S. But P is not a weakly completely quasi primary ideal of S since $(0, -3)(-3, -3)(-3, -3) \in P$ such that $(0, -3) \notin P, (-3, -3) \notin P$ and $(-3, -3)^n \notin P$ for all positive odd integer n.

Theorem 3.8. *Let S be a ternary semiring, and let A be a two sided ideal of S. If A is a weakly completely quasi primary (completely quasi primary) ideal of S, then $(A : b : c)_L$ is a weakly completely quasi primary (completely quasi primary) ideal in S, where $b \in \sqrt{S - A}$.*

Proof. Let S be a ternary semiring, and let A be a weakly completely quasi primary ideal of S. Suppose that $0 \neq a_1a_2a_3 \in (A : b : c)_L$ such that $a_3^n \notin (A : b : c)_L$, for all positive odd integer n. Then $0 \neq (a_1a_2a_3)cb = a_1a_2(a_3cb) \in A$. By the definition of a weakly completely quasi primary ideal we get $a_1cb \in Acb \subseteq A$ or $a_2^ncb \in Acb \subseteq A$, for some positive odd integer n, so that $a_1 \in (A : b : c)_L$ or $a_2^n \in (A : b : c)_L$, for some positive odd integer n. Hence $(A : b : c)_L$ is a weakly completely quasi primary ideal in S. Clearly, each completely quasi primary ideal in S, is weakly completely quasi primary, i.e., in view of the previous result we infer that $(A : b : c)_L$ is a completely quasi primary ideal in S. □

Lemma 3.9. *Let P be a k-ideal of ternary semiring S and let P be a weakly completely quasi primary ideal but not a completely quasi primary ideal of S. If $abc = 0$ for some $a, b, c \notin \sqrt{P}$, then $aPc = abP = Pbc = 0$.*

Proof. Suppose that $apc \neq 0$. Then

$$\begin{aligned} 0 &\neq apc \\ &= 0 + apc \\ &= abc + apc \\ &= a(p + b)c \in P. \end{aligned}$$

Since P is a weakly completely quasi primary ideal of S, we have $a \in P$ or $c + p^n = (b + p)^n \in P$ or $c^n \in P$, for some positive odd integer n, that is, $a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n, a contradiction. Therefore $aPc = 0$. Similarly, we can show that $abP = Pbc = 0$. □

Theorem 3.10. *Let S be a ternary semiring and let P be a k-ideal of S. If P is a weakly completely quasi primary ideal that is not completely quasi primary, then $\sqrt{P} = \sqrt{0}$.*

Proof. Let S be a ternary semiring. First, we prove that $P^3 = 0$. Suppose that $P^3 \neq 0$ and we show that P is a completely quasi primary ideal in S. Let $abc \in P$, where $a, b, c \in S$. If $abc \neq 0$, then $a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n, since P is a weakly completely quasi primary ideal. So suppose that $abc = 0$. If $Pbc \neq 0$, then there is an element $\acute{p}$ of P such that $\acute{p}bc \neq 0$ so that

$$\begin{aligned} 0 &\neq \acute{p}bc \\ &= \acute{p}bc + 0 \\ &= \acute{p}bc + abc \\ &= (\acute{p} + a)bc \in P \end{aligned}$$

and hence P as a weakly completely quasi primary ideal gives either $\acute{p}+a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n. As $\acute{p}+a \in P$ and $p \in P$ we have $a \in P$ or $b^n \in P$ or $c^n \in P$, for some positive odd integer n. So we can assume that $Pbc = 0$. Similarly, we can assume that $aPc = abP = 0$. Since P^3, there exist $p_1, p_2, p_3 \in P$ such that $p_1p_2p_3 \neq 0$. Then

$$\begin{aligned} 0 &\neq abc \\ &= abc + ap_2c + p_1bc + p_1p_2c + abp_3 + ap_2p_3 + p_1bp_3 + p_1p_2p_3 \\ &= (a+p_1)(b+p_2)(c+p_3) \in P \end{aligned}$$

so $a+p_1 \in P$ or $(b+p_2)^n \in P$ or $(c+p_3)^n \in P$, for some positive odd integer n, and hence $a \in P$ or $b \in \sqrt{P}$ or $c \in \sqrt{P}$. Thus P is a completely quasi primary ideal. Clearly, $\sqrt{0} \subseteq \sqrt{P}$. Hence, $\sqrt{P} = \sqrt{0}$ as required. □

Corollary 3.11. *Let S be a ternary semiring and let P be a k-ideal of S. If $\sqrt{P} \neq \sqrt{0}$, then P is completely quasi primary if and only if P is weakly completely quasi primary.*

Proof. This follows from Theorem 3.10. □

Lemma 3.12. *Let S be a ternary semiring, and let P be a proper k-ideal of S. If P is a weakly completely quasi primary ideal of S, then*

$$(P : a : b)_L = P \cup (0 : a : b)_L,$$

where $a, b \in S - \sqrt{P}$.

Proof. Let S be a ternary semiring, and let P be a proper k-ideal of S. Clearly, $P \cup (0 : a : b)_L \subseteq (P : a : b)_L$. For the other inclusion, suppose that $x \in (P : a : b)_L$ so that $xba \in P$. If $xba \neq 0$ and P is a weakly completely quasi primary ideal of S, then $x \in P$. If $xba = 0$, then $x \in (0 : a : b)_L$. So we have the equality. □

Theorem 3.13. *Let S be a ternary semiring, P be a proper k-ideal of S and let $a, b \in S - \sqrt{P}$. If $(P : a : b)_L = P$ or $(0 : a : b)_L = P$, then P is a weakly completely quasi primary ideal of S.*

Proof. Let S be a ternary semiring and let P be a proper k-ideal of S. Suppose that $0 \neq xyz \in P$, where $x, y, z \in S$. Then $x \in (P : y : z)_L$ by Lemma 3.12, we have $(P : y : z)_L = P \cup (0 : y : z)_L$. Thus $x \in P$ or $x \in (0 : y : z)_L$ hence $x \in P$ since $xyz \neq 0$, as required. □

Lemma 3.14. *Let $S = S_1 \times S_2 \times S_3$, where each S_i is a ternary semiring. Then the following holds*:

(1) *If I is an ideal of S_1, then $\sqrt{I \times S_2 \times S_3} = \sqrt{I} \times S_2 \times S_3$.*
(2) *If I is an ideal of S_2, then $\sqrt{S_1 \times I \times S_3} = S_1 \times \sqrt{I} \times S_3$.*
(3) *If I is an ideal of S_3, then $\sqrt{S_1 \times S_3 \times I} = S_1 \times S_2 \times \sqrt{I}$.*

Proof. The proof is straightforward. □

Theorem 3.15. *Let $S = S_1 \times S_2$, where each S_i is a ternary semiring. If $P \times S_2$ is a weakly completely quasi primary ideal of S, then P is a weakly completely quasi primary ideal of S_1.*

Proof. Suppose that $S = S_1 \times S_2$, where each S_i is a ternary semiring and $P \times S_2$ is a weakly completely quasi primary ideal of S. Let $0 \neq a_1a_2a_3 \in P$, where $a_1, a_2, a_3 \in S_1$ so that $0 \neq (a_1b_1c_1, a_2b_2c_2) = (a_1, a_2)(b_1, b_2)(c_1, c_2) \in P \times S_2$. Since $P \times S_2$ is weakly completely quasi primary, we have $(a_1, a_2) \in P \times S_2$ or $(b_1, b_2)^n \in P \times S_2$ or $(c_1, c_2)^n \in P \times S_2$, for some positive odd integer n. It follows that $a_1 \in P$ or $b_1^n \in P$ or $c_1^n \in P$, for some positive odd integer n. By the definition of a weakly completely quasi primary ideal, we have P is a weakly completely quasi primary ideal of S_1. □

Theorem 3.16. *Let $S = S_1 \times S_2$, where each S_i is a ternary semiring. Then P is a completely quasi primary ideal of S_1 if and only if $P \times S_2$ is a completely quasi primary ideal of S.*

Proof. ($\Rightarrow$) Suppose that $S = S_1 \times S_2$, where each S_i is a ternary semiring and P is a completely quasi primary ideal of S_1. Let $(a_1b_1c_1, a_2b_2c_2) = (a_1, a_2)(b_1, b_2)(c_1, c_2) \in P \times S_2$, where $(a_1, a_2), (b_1, b_2), (c_1, c_2) \in S$ so that $a_1 \in P$ or $b_1^n \in P$ or $c_1^n \in P$, for some positive odd integer n, since P is completely quasi primary. It follows that $(a_1, a_2) \in P \times S_2$ or $(b_1, b_2)^n \in P \times S_2$ or $(c_1, c_2)^n \in P \times S_2$, for some positive odd integer n. By the definition of a completely quasi primary ideal, we have $P \times S_2$ is a completely quasi primary ideal of S.

($\Leftarrow$) This follows from Theorem 3.15. □

Corollary 3.17. *Let $S = S_1 \times S_2$, where each S_i is a ternary semiring. If P is a weakly completely quasi primary (completely quasi primary) ideal of S_2, then $S_1 \times P$ is a weakly completely quasi primary (completely quasi primary) ideal of S.*

Proof. This follows from Theorem 3.15. □

Corollary 3.18. *Let $S = \prod_{i=1}^{n} S_i$, where each S_i is a ternary semiring. If P is a weakly completely quasi primary (completely quasi primary) ideal of S_j, then $S_1 \times S_2 \times \cdots \times S_{j-1} \times P \times S_{j+2} \times \cdots \times S_n$ is a weakly completely quasi primary (completely quasi primary) ideal of S.*

Proof. This follows from Theorem 3.15 and Corollary 3.17. □

Theorem 3.19. *Let $S = S_1 \times S_2 \times S_3$, where each S_i is a ternary semiring with identity. If P is a weakly completely quasi primary ideal of S, then either $P = 0$ or P is completely quasi primary.*

Proof. Let $S = S_1 \times S_2 \times S_3$, where each S_i is a ternary semiring with identity and let $P = P_1 \times P_2 \times P_3$ be a weakly completely quasi primary ideal of S.

We can assume that $P \neq 0$. So there is an element (a_1, a_2, a_3) of P, with $(a_1, a_2, a_3) \neq (0, 0)$. Then $(0, 0, 0) \neq (a_1, a_2, a_3) = (a_1, 1, 1)(1, a_2, 1)(1, 1, a_3) \in P$ gives $(a_1, 1, 1) \in P$ or $(1, a_2, 1)^n \in P$ or $(1, 1, a_3)^n \in P$, for some positive odd integer n. If $(a_1, 1, 1) \in P$, then $P = P_1 \times S_2 \times S_3$. We will show that P_1 is completely quasi primary hence P is weakly completely quasi primary by Theorem 3.16. Let $xyz \in P_1$, where $x, y, z \in S_1$. Then

$$0 \neq (xyz, 1, 1) = (x, 1, 1)(y, 1, 1)(z, 1, 1) \in P_1 \times S_2 \times S_3 = P,$$

so $(x, 1, 1) \in P$ or $(y, 1, 1) \in \sqrt{P} = \sqrt{P_1 \times S_2 \times S_3} = \sqrt{P_1} \times S_2 \times S_3$ or $(z, 1, 1) \in \sqrt{P} = \sqrt{P_1 \times S_2 \times S_3} = \sqrt{P_1} \times S_2 \times S_3$ and hence $x \in P_1$ or $y^n \in P_1$ or $z^n \in P_1$, for some positive odd integer n. If $(1, a_2, 1)^n \in P$, then $(1, a_2^n, 1) \in P$, for some positive odd integer n, so $P = S_1 \times P_2 \times S_3$. By a similar argument, $S_1 \times P_2 \times S_3$ is completely quasi primary. If $(1, 1, a_3^n) = (1, 1, a_3)^n \in P$, then $P = S_1 \times S_2 \times P_3$. By a similar argument, $S_1 \times S_2 \times P_3$ is completely quasi primary. □

Theorem 3.20. *Let P be proper ideals of a ternary semiring S and $A \subseteq P$. Then the following holds*:

(1) *If P is weakly completely quasi primary (completely quasi primary), then P/A is weakly completely quasi primary (completely quasi primary).*

(2) *If A and P/A are weakly completely quasi primary (completely quasi primary), then P is weakly completely quasi primary (completely quasi primary).*

Proof. (1) Let $0 \neq (x + A)(y + A)(z + A) = xyz + A \in P/A$, where $x, y, z \in S$ so $xyz \in P$. If $xyz = 0 \in A$, then $(x+A)(y+A)(z+A) = xyz+A = 0+A = A$ a contradiction. So if P is weakly completely quasi primary, then $x \in P$ or $y^n \in P$ or $z^n \in P$, for some positive odd integer n, hence $x + A \in P/A$ or $(y + A)^n = y^n + A \in P/A$ or $(z + A)^n = z^n + A \in P/A$, for some positive odd integer n, as required.

(2) Let $0 \neq xyz \in P$ where $x, y, z \in S$, so $(x+A)(y+A)(z+A) = xyz+A \in P/A$. For $xyz \in A$ if A is weakly completely quasi primary, then $x \in A \subseteq P$ or $y^n \in A \subseteq P$ or $z^n \in A \subseteq P$, for some positive odd integer n. So we may assume that $xyz \notin A$. Then $x + A \in P/A$ or $y^n + A = (y + A)^n \in P/A$ or $z^n + A = (z + A)^n \in P/A$, for some positive odd integer n. It follows that $x \in P$ or $y^n \in P$ or $z^n \in P$, for some positive odd integer n, as needed. □

Theorem 3.21. *Let P, Q and R be weakly completely quasi primary ideals of a ternary semiring S that are not completely quasi primary. Then $P + Q + R$ is a weakly completely quasi primary ideal of S.*

Proof. Since $(P + Q + R)/Q \cong Q/(P \cap Q \cap R)$ we get that $(P + Q + R)/Q$ is weakly completely quasi primary by Theorem 3.20(1). Now the assertion follows from Theorem 3.20(2). □

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