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Parametric Blind Restoration of Bi-level Images with Unknown Intensities

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** Short Paper*

Abstract: We propose a parametric blind deconvolution method for bi-level images with unknown intensity levels that estimates unknown parameters for point spread functions and images by minimizing a penalized nonlinear least squares objective function based on normalized correlation coefficients and two regularization functions. Unlike conventional methods, the proposed method does not require knowledge about true intensity values. Moreover, the objective function of the proposed method can be effectively minimized, since it has the special structure of nonlinear least squares. We demonstrate the effectiveness of the proposed method through simulations and experiments.

Keywords: Inverse problems, Blind deconvolution, Image reconstruction, Image restoration

1. Introduction

We study the blind restoration of a blurred, noisy, bi-level image, which is frequently required in barcode decoding, text image processing, etc. [1-3]. The blurred, noisy image is often modeled by the convolution of a true bi-level image and an *unknown* point spread function (PSF), plus additive noise, as follows [1]:

$$y(t_i) = x(t_i) * g(t_i) + n(t_i), \; i = 0,1,\ldots,N-1 \tag{1}$$

where $t_i, i = 0,.1,\ldots,N-1$ denotes spatial locations for one-dimensional (1D) or 2D images, * is the convolution operator, $y(t_i)$ denotes a blurred, noisy, observed image, $g(t_i)$ is the value of the *unknown* PSF, $x(t_i)$ is the value of the true bi-level image, and $n(t_i)$ denotes the value of additive white Gaussian noise. True image x has only two *unknown* intensity values. For out-of-focus blur, the PSF is often parameterized using the Gaussian function with *unknown* standard deviation $\sigma > 0$, as follows [1]:

$$g_\sigma(t_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t_i^2}{2\sigma^2}}, \; i = 0,1,\ldots,N-1 \tag{2}$$

Under the Gaussian noise assumption, the maximum likelihood estimation of x would be the minimizer of the sum of square errors between $y(t_i)$ and $x(t_i) * g_\sigma(t_i)$. Since minimization of the sum of square errors is an *ill-posed* problem, in the sense that the solution is not unique, incorporation of regularization functions based on *a priori* information about a *true* image is essential to determine a unique solution. One of the most frequently used regularization methods is the total variation (TV) penalty method, which is based on *a priori* information that the true image has little variation in the spatial domain [4]. Since the TV penalty function does not fully utilize the important *a priori* information that the true image has only two intensity values, a previous investigation proposed a method that incorporates an additional double-well penalty function that encourages a restored image to be a bi-level of 0 and 1, which is defined as follows [1]:

$$\begin{aligned}(\hat{\sigma}, \hat{x}) = argmin_{\sigma>0,x} \sum_{i=0}^{N-1} \left(y(t_i) - x(t_i) * g_\sigma(t_i) \right)^2 \\ + \lambda \sum_{i=0}^{N-1} | x(t_{i+1}) - x(t_i) | + \gamma \sum_{i=0}^{N-1} x(t_i)^2 (1 - x(t_i))^2\end{aligned} \tag{3}$$

where λ and γ are regularization parameters for TV and the double-well function, respectively. Although the TV with known intensities method defined in (3) is useful for theoretical study of bi-level image restoration, it is not practically applicable, since the method assumes that the true bi-level image has intensity values of 0 and 1, which is not true, in general. Note that it is not only the true intensity values that are not known in practice, but estimation of the true intensity values from the observed image is challenging, too, due to blur and noise. Other methods for bi-level image restoration also assume that the two intensity values are known [5]. Although there is an investigation that can be used for bi-level images with unknown intensity values, the method is limited for invertible PSF [6]. Note that the out-of-focus blur defined in (2) is not invertible.

2. Method

In this short paper, we propose parametric blind deconvolution of bi-level images with *unknown* intensity values. We estimate image x and σ by minimizing penalized nonlinear least squares as follows:

$$\left(\hat{x},\hat{\sigma}\right) = argmin_{\sigma>0,x} F\left(x,\sigma\right) + \lambda_1 R_1\left(x\right) + \lambda_2 R_2\left(x\right) \quad (4)$$

where $R_1\left(x\right)$ and $R_2\left(x\right)$ are regularization functions, λ_1 and λ_2 are associated regularization parameters, and $F\left(x,\sigma\right)$ is a data fidelity term defined as follows:

$$F\left(x,\sigma\right) = \frac{1}{2}\sum_{i=0}^{N-1}\left[f_i\left(x,\sigma\right)\right]^2 \quad (5)$$

where $f_i\left(x,\sigma\right)$ is

$$f_i\left(x,\sigma\right) = \left(\frac{y\left(t_i\right)-m_y}{\gamma_y}\right) - \left(\frac{x\left(t_i\right)*g_\sigma\left(t_i\right)-m_{x*g\sigma}}{\gamma_{x*g_\sigma}}\right) \quad (6)$$

where m_y and m_{x*g_σ} are the sample means, and γ_y^2 and $\gamma_{x*g_\sigma}^2$ are the sample variances of $y\left(t_i\right)$ and $x\left(t_i\right)*g_\sigma\left(t_i\right)$, $i=0,1,\ldots,N-1$, which are defined as follows:

$$\gamma_y^2 = \frac{1}{N}\sum_{j=0}^{N-1}\left(y\left(t_j\right)-m_y\right)^2 \quad (7)$$

$$\gamma_{x*g_\sigma}^2 = \frac{1}{N}\sum_{j=0}^{N-1}\left(x\left(t_j\right)*g_\sigma\left(t_j\right)-m_y\right)^2 \quad (8)$$

Note that the data fidelity term in the objective function in (5) is closely related to the normalized correlation coefficient, as shown in the following:

$$F\left(x,\sigma\right) = 1-\sum_{i=0}^{N-1}\left(\frac{y\left(t_i\right)-m_y}{\gamma_y}\right)\left(\frac{x\left(t_i\right)*g_\sigma\left(t_i\right)-m_{x*g\sigma}}{\gamma_{x*g_\sigma}}\right) = 1-C\left(y,x*g_\sigma\right) \quad (9)$$

where $C\left(y,x*g_\sigma\right)$ denotes the normalized correlation coefficient between y and $x*g_\sigma$ .Therefore, minimizing $F\left(x,\sigma\right)$ in (5) is equivalent to maximizing $C\left(y,x*g_\sigma\right)$. Since the normalized correlation coefficient has the maximum value of unity when two signals have the same shape, regardless of scale and levels, we use (5) as a data fidelity term that is independent of true intensity values. Since restored bi-level images are binarized for further processing, such as decoding or character recognition, restoring a more bi-level–like image that can be more correctly binarized is sufficient for the restoration of bi-level images. The advantage in using the proposed nonlinear least squares type of data fidelity term in (5) over the normalized correlation coefficient is from fast convergence using specialized algorithms for nonlinear least squares, such as Gauss-Newton and Levenberg-Marquardt [7]. In particular, when a residual term of the objective function is small, the optimization methods for nonlinear least squares converge approximately quadratically [7]. We incorporate two regularization functions: a quadratic roughness penalty function to suppress noise, and another penalty function to encourage an estimated image to be bi-level. We design the roughness penalty function in (4) as follows:

$$R_1\left(x\right) = \sum_{i=0}^{N-1}\left(x\left(t_{i+1}\right)-x\left(t_i\right)\right)^2 \quad (10)$$

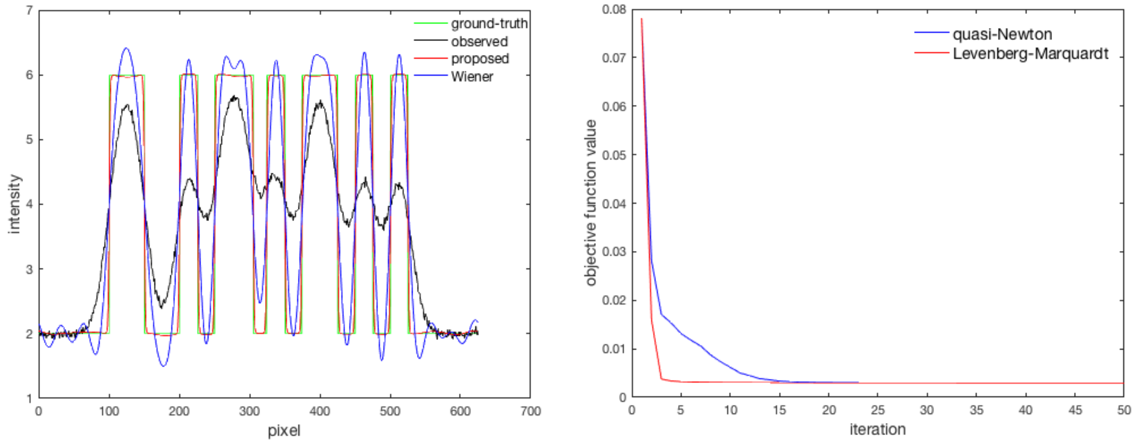
The roughness penalty function in (10) encourages the estimated image to be smooth, thereby making the estimated image robust against noise. In conventional methods, to reduce the noise effect while preserving edges, a TV method is often used [1]. However, we use the quadratic roughness penalty function in (10). Since we have another penalty function that encourages the estimated image to be bi-level, we are able to preserve edges while using the quadratic smoothness penalty function. We designed the penalty function for estimated images to have two intensities (+1 and -1) as follows:

$$R_2\left(x\right) = \sum_{i=0}^{N-1}\left(x\left(t_i\right)^2-1\right)^2 \quad (11)$$

Note that the objective function of the proposed method, which is the combination of (5), (10), and (11), has the special structure of nonlinear least squares.

3. Experimental Results

To demonstrate the performance of the proposed method, we conducted simulations with a 1D bi-level signal and its noisy, blurred signal (shown in Fig. 1). We synthesized the blurred and noisy observed signal by convolving a true bi-level signal with a Gaussian PSF ($\sigma = 16$ pixels). The size of the signal was 625 pixels, the intensity values of the true bi-level signal were {2,6}, and the signal-to-noise ratio (SNR) of the blurred, noisy signal



(a) (b)

Fig. 1. (a) Noisy, blurred signals and restored signals from the three methods, (b) convergence of Gauss–Newton and quasi-Newton methods in the proposed method.

Table 1. Correlation Coefficient of Restored Image.

SNR (dB)	Blurred				Wiener				Proposed			
	σ				σ				σ			
	13	16	19	22	13	16	19	22	13	16	19	22
35	0.84	0.78	0.73	0.68	0.94	0.94	0.91	0.86	0.99	0.99	0.99	0.96
30	0.84	0.78	0.73	0.68	0.94	0.93	0.89	0.85	0.99	0.98	0.96	0.97
25	0.83	0.78	0.72	0.68	0.94	0.91	0.86	0.83	0.99	0.99	0.99	0.98
20	0.83	0.77	0.72	0.68	0.93	0.89	0.84	0.80	0.99	0.99	0.97	0.92

Table 2. Bit Error Rates of Restored Image (%).

SNR (dB)	Blurred				Wiener				Proposed			
	σ				σ				σ			
	13	16	19	22	13	16	19	22	13	16	19	22
35	2.65	7.44	18.0	23.7	1.41	1.55	3.02	5.35	0.06	0.14	0.10	2.23
30	2.76	7.45	17.4	23.6	1.50	1.85	4.17	5.53	0.16	0.22	0.28	1.88
25	3.00	7.59	16.7	23.3	1.64	2.83	4.93	4.92	0.52	0.50	0.72	1.28
20	3.64	8.25	16.8	22.9	1.91	2.73	4.61	5.10	0.58	0.80	1.56	4.31

was 30 dB.

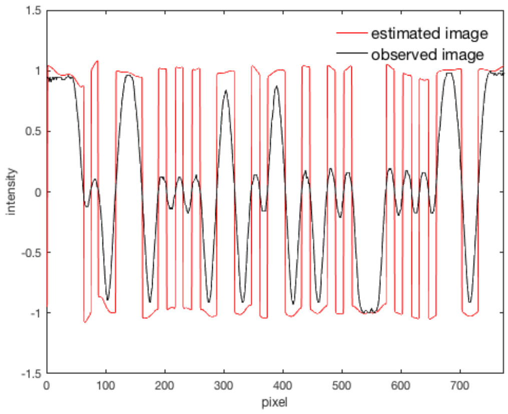
We restore the degraded image via the proposed method. For comparison purposes, we also attempted to restore the degraded image by Wiener filtering, which is the well-known linear least squares error filter if PSF and SNR are known. Although the Wiener filter cannot be applied for blind deconvolution, since neither the PSF nor the SNR are known, we report results using ideal Wiener filtering with true PSF and SNR for comparison purposes. Such ideal Wiener filtering was used for evaluating the performance of linear restoration in a previous investigation [6].

Fig. 1 shows restored signals via the proposed method and the Wiener filter. As shown in the figure, the restored signal from the proposed method was more similar to the true signal. We think the superiority of the proposed method originates from the two effective regularization functions.

We claimed that the advantage of the proposed method over maximizing the normalization coefficient is faster convergence, thanks to the special structure of nonlinear least squares in our objective function. To support the claim, we compared the convergence of the Levenberg-Marquardt method, which is for nonlinear least squares, with the quasi-Newton method for general optimization. Fig. 1(b) shows the convergence rates of the two optimization methods. As shown in Fig. 1(b), the Levenberg-Marquardt method converged a lot faster than the quasi-Newton method. Note that minimization using nonlinear least squares converged very quickly.

To evaluate the statistical properties of the proposed method, we repeated 50 simulations of different noise

(a)



(b)

Fig. 2. (a) Real barcode image, (b) one horizontal line of the real barcode and the restored signals from the proposed and TV-based methods.

realizations for different amounts of blur and noise. Then, we computed the correlation coefficient of the restored image and the true image, and the bit error rate of the restored image after binarization. We used the well-known Otsu method for the binarization [8]. Tables 1 and 2 show the correlation coefficients and bit error rates, respectively. As shown in the tables, the proposed method outperformed the Wiener filter method with true PSF and SNR. We believe this is due to the fact that our method is a nonlinear filtering method using effective regularization. Although the Wiener filter should have the least error among the linear filtering methods, there may exist nonlinear filters that can yield a smaller error than the Wiener filter.

To demonstrate the effectiveness of the proposed method for real images, we conducted experiments using a barcode image acquired by a Cannon EOS40D digital camera. Fig. 2(a) shows an acquired noisy, blurred barcode image, and Fig. 2(b) shows one line of the acquired image and restored signal using the proposed method. Note that we were not able to apply Wiener filtering to the real image since true PSF was not known. We normalized the acquired image in such a way that intensity values lie between -1 and 1. As shown in Fig. 2(b), the restored signal using the proposed method is more similar to the true bi-level signal. As shown in Fig. 2(b), the proposed method was able to restore small peaks in the central part of the signal, thereby making the restored signal correctly decodable.

4. Conclusion

We propose a parametric blind deconvolution method for bi-level images with unknown intensity levels. Unlike conventional methods, the proposed method does not require knowledge about true intensity values, since it is based on the normalized correlation coefficient. Moreover, the objective function of the proposed method can be more effectively minimized than the normalized correlation coefficient thanks to the special structure of nonlinear least squares. In simulations and experiments, the proposed method performed better than the ideal Wiener filtering method. We believe that the proposed method should be useful for parametric blind deconvolution of bi-level images, since true intensity values of bi-level images are not known in practice.

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