

FMEA에서 고장 심각도의 탐지시간에 따른 위험성 평가

장현애 · 윤원영* · 권혁무***

사단법인 부산품질혁신센터 · *부산대학교 산업공학과 · **부경대학교 시스템경영공학과
(2016. 4. 25. 접수 / 2016. 7. 27. 수정 / 2016. 8. 9. 채택)

Risk Evaluation in FMEA when the Failure Severity Depends on the Detection Time

Hyeon Ae Jang · Won Young Yun* · Hyuck Moo Kwon***

Busan Innovation Center for Quality

*Department of Industrial Engineering, Pusan National University

**Department of System Management and Engineering, Pukyong National University

(Received April 25, 2016 / Revised July 27, 2016 / Accepted August 9, 2016)

Abstract : The FMEA is a widely used technique to pre-evaluate and avoid risks due to potential failures for developing an improved design. The conventional FMEA does not consider the possible time gap between occurrence and detection of failure cause. When a failure cause is detected and corrected before the failure itself occurs, there will be no other effect except the correction cost. But, if its cause is detected after the failure actually occurs, its effects will become more severe depending on the duration of the uncorrected failure. Taking this situation into account, a risk metric is developed as an alternative to the RPN of the conventional FMEA. The severity of a failure effect is first modeled as linear and quadratic severity functions of undetected failure time duration. Assuming exponential probability distribution for occurrence and detection time of failures and causes, the expected severity is derived for each failure cause. A new risk metric REM is defined as the product of a failure cause occurrence rate and the expected severity of its corresponding failure. A numerical example and some discussions are provided for illustration.

Key Words : FMEA, RPN, REM, failure risk, failure severity

1. Introduction

The FMEA(Failure mode and effect analysis) is extensively used as a powerful tool for system safety and reliability analysis of products and processes in a wide range of industries, including the aerospace, nuclear, automotive, electronics and medical industries¹⁾. In conventional FMEA, the risk of a failure mode or cause is evaluated with RPN(Risk priority number), which is the mathematical product of its occurrence, severity and detection. Each value of the three components is usually determined as to the guidelines of FMEA handbook²⁾ or similar documents. Occurrence and detection basically include the probability concepts in their nature, but severity is merely the relative

rank associated with the most serious effect of a failure within the scope of the individual FMEA.

In traditional FMEA, the 1 to 10 scale for the three risk components depends largely on the past experience and intuition of the FMEA team³⁾. And its application has a number of limitations to be more carefully addressed for further improvement. Some important drawbacks are; (i) the subjective and inconsistent process of evaluating RPN, (ii) unreasonable allocation of the same weight on its three risk components, (iii) totally different contexts of risk possible with the same value of RPN, (iv) and dependence on intuition and experience rather than scientific method for estimating three risk components. Many authors have attempted to improve the RPN of FMEA. Abdelgawad and

* Corresponding Author : Hyuck Moo Kwon, Tel : +82-51-629-6480, E-mail : iehmkwon@pknu.ac.kr
Department of Systems Management and Engineering, Pukyong National University, 365, Yongso-ro, Nam-gu, Busan 48513, Korea

Fayek⁴⁾ used a combination of fuzzy FMEA and fuzzy AHP (Analytic hierarchy process) for risk management in the construction industries. Kumru and Kumru⁵⁾ directly evaluate the linguistic assessment of factors based on fuzzy theory to obtain the RPN value. Liu et al.⁶⁾ employed an intuitionistic fuzzy hybrid weighted Euclidean distance operator to overcome the limitations of traditional FMEA. Liu et al.⁷⁾ proposed a risk priority model based on fuzzy set theory, treating the risk factors and their weights as fuzzy variables. And Liu et al.⁸⁾ proposed a risk priority model based on the grey relational projection method.

There are few, if any, studies that consider the role of time in FMEA. Kwon et al.⁹⁾ constructed a time dependent model with a quadratic loss function for the unfulfilled mission period, presenting an optimal monitoring policy. Kwon et al.³⁾ proposed a time dependent expected loss model, assuming a homogeneous Poisson process for occurrence of failures and causes. Rhee and Ishii¹⁰⁾ introduced a life cost-based FMEA for analyzing design alternatives of a particular system with Monte Carlo simulation.

In this paper, we consider the time sequence of failure occurrence, i.e. cause - failure - effect. It is quite natural to assume that a failure occurs after at least one of its causes has occurred. And also, it may take time to detect the failure after its occurrence. We consider a situation where the severity of a failure effect depends on the length of undetected time duration of failure. Generally, the detection of a failure cause prior to the actual failure will only require a constant cost of fixing the cause. If the failure is detected after its actual occurrence, then the loss may become significantly larger as to the undetected time duration of the failure gets longer. We consider the linear and quadratic severity functions of undetected time duration. Assuming exponential probability law for failure occurrence and detection times, we suggest a risk evaluation model for improving the conventional FMEA.

2. Failure Risk Components

2.1 Time delay between occurrence and detection

In the conventional FMEA, RPN is obtained by

$$RPN = O \times S \times D, \quad (1)$$

where O, S and D are the numbers corresponding to

occurrence of a failure cause, severity of the failure effect and detection of the failure or its cause, respectively. In general, the three components are scored by experts using a numeric scale from 1 to 10, which is based on commonly agreed evaluation criteria¹⁾. A failure mode with a larger RPN carries a greater risk, and hence, special consideration should be made for such mode. Since the evaluation of the three components depends on subjectivity of the FMEA team, the RPN is quantitative but hardly objective. This is especially true of severity, as it is naturally linked with the size of a failure effect, unlike the probability-oriented components of occurrence and detection.

In FMEA, severity has quite a different meaning in the estimation of risk than those of the other two components. The numbers for ranking occurrence and detection are closely related with probability, whereas the number for ranking severity is far from probability concept¹¹⁾. Severity is merely a relative rank, being determined by individual FMEA teams. FMEA teams reach a consensus on severity ratings using the severity rating table²⁾. This means that severity ratings might differ, depending on the team to carry out the assessment in the conventional FMEA. To complement these shortcomings, the process of failure occurrence and detection is first examined.

Suppose that an item is exposed to random events leading to a failure and any failure occurs after occurrence of its corresponding cause. Then the occurrence process of failure mode F_i , $i = 1, 2, \dots, m$ can be described by the elapsed time T_{ij} , $j = 1, 2, \dots, n_i$ from the occurrence of the j^{th} cause to the actual failure occurrence. There may also be a time delay D_{ij} from the occurrence of the j^{th} cause of failure mode F_i until its detection. This situation is depicted in Fig. 1. Thus, when an item is exposed to random events leading

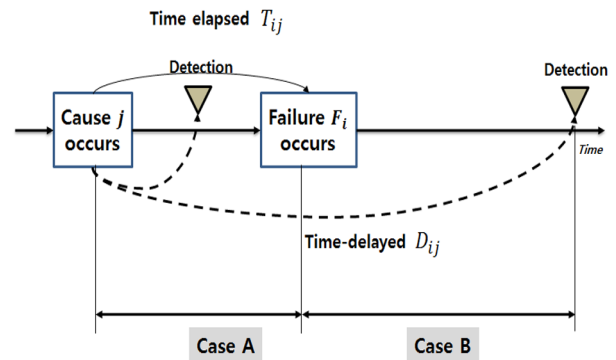


Fig. 1. Time delay between occurrence and detection.

to failure, a stochastic model will be appropriate to describe the failure occurrence, detection and its severity.

2.2 Occurrence and detection of a failure cause

Suppose that the events leading to failure occur as to a HPP (homogeneous Poisson process) with a constant rate. Then the occurrence rate of the j^{th} cause of failure mode F_i can be described by τ_{ij} .

With most systems, occurrence of failure cause is not usually detected instantaneously. There usually be a time gap between occurrence and detection of a failure cause. Let D_{ij} be the elapsed time from the occurrence of the j^{th} cause of failure mode F_i to its detection. Under HPP assumption with a constant detection rate μ_{ij} over time, the probability density function of D_{ij} will be

$$f_{D_{ij}}(d) = \mu_{ij}e^{-\mu_{ij}d}, d > 0. \quad (2)$$

2.3 Occurrence of a failure

Let T_{ij} be the time elapsed from the occurrence of the j^{th} cause to the occurrence of failure mode F_i . Under HPP assumption with constant occurrence rate λ_{ij} , T_{ij} will be exponentially distributed with mean $1/\lambda_{ij}$ and its probability density function will be

$$f_{T_{ij}}(t) = \lambda_{ij}e^{-\lambda_{ij}t}, t > 0. \quad (3)$$

2.4 Severity

Note that the failure cause may be detected before or after its corresponding failure actually occurs, as described by case A and B of Fig. 1. Under case A where the a failure cause is detected prior to the actual failure occurrence, the severity of its effect will be small. The only cost incurred in this case will be attributable to corrective action for the failure cause and will not be so much dependent on the elapsed time to the cause detection. Under case B where a failure is detected after its occurrence, the losses due to the failure may be accumulated until the corrective action is taken.

Assuming that an immediate corrective action is taken on detection of failure cause, the severity of the failure effect will naturally depend on the elapsed time to detection of the corresponding failure cause. Thus, the

severity of each failure mode may reasonably be estimated based on the length of undetected time duration after its occurrence.

Considering two possible situations in Fig. 1, it is reasonable to assume that the severity of a failure effect is constant when a cause is detected before its corresponding failure occurs. Let a_{ij} be the constant severity coefficient due to the j^{th} cause of failure mode F_i .

When a failure is detected after its occurrence, its severity will depend on the time difference of $|D_{ij} - T_{ij}|$. Let S_{ij} denote the severity for the effect of failure mode F_i in relation with its j^{th} cause. S_{ij} will reasonably be a non-decreasing function of $|D_{ij} - T_{ij}|$. We consider the linear and quadratic functions of $|D_{ij} - T_{ij}|$ for S_{ij} .

For linear type, S_{ij} will be proportional to $|D_{ij} - T_{ij}|$ with constant severity coefficient b_i . Thus, S_{ij} is defined by

$$S_{ij} = \begin{cases} a_{ij}, & 0 < D_{ij} \leq T_{ij}, \\ a_{ij} + b_i(D_{ij} - T_{ij}), & T_{ij} < D_{ij}. \end{cases} \quad (4)$$

When the risk increases dramatically as time goes by with a failure left uncorrected, a quadratic S_{ij} will be more appropriate. For quadratic type, S_{ij} is defined by

$$S_{ij} = \begin{cases} a_{ij}, & 0 < D_{ij} \leq T_{ij}, \\ a_{ij} + b_i(D_{ij} - T_{ij})^2, & T_{ij} < D_{ij}. \end{cases} \quad (5)$$

3. Risk evaluation

3.1 The expected failure severity

For linear S_{ij} , its expected value was obtained by using Equation (2), (3) and (4) as follows:

$$\begin{aligned} E[S_{ij}] &= a_{ij} \int_0^\infty \int_d^\infty f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\ &\quad + a_{ij} \int_0^\infty \int_0^d f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\ &\quad + b_i \int_0^\infty \int_0^d (d_{ij} - t_{ij}) f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\ &= a_{ij} + b_i \left(\frac{\lambda_{ij}}{\mu_{ij}(\mu_{ij} + \lambda_{ij})} \right). \end{aligned} \quad (6)$$

See Appendix I for detailed derivation.

For quadratic S_{ij} , its expected value was obtained by

using Equation (2), (3) and (5) as follows:

$$\begin{aligned}
 E[S_{ij}] &= a_{ij} \int_0^\infty \int_0^\infty f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\
 &+ a_{ij} \int_0^\infty \int_0^d f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\
 &+ b_i \int_0^\infty \int_0^d (d_{ij} - t_{ij})^2 f_{T_{ij}}(t) f_{D_{ij}}(d) dt dd \\
 &= a_{ij} + b_i \left(\frac{2\lambda_{ij}}{\mu_{ij}^2(\mu_{ij} + \lambda_{ij})} \right). \quad (7)
 \end{aligned}$$

See Appendix II for detailed derivation.

3.2 The risk evaluation metric

Since FMEA is a prevention-oriented technique, the risk linked with each failure cause is necessary to be evaluated. We incorporate the occurrence rate of each failure cause with the expected severity of the corresponding failure into one risk metric, i.e., the REM (Risk evaluation metric). The REM of to the j^{th} cause of failure mode F_i is simply defined by the mathematical product of the failure cause occurrence rate τ_{ij} and $E[S_{ij}]$.

$$REM_{ij} = \tau_{ij} E[S_{ij}]. \quad (8)$$

Thus, REM_{ij} is obtained by

$$REM_{ij} = \tau_{ij} \left[a_{ij} + b_i \left(\frac{\lambda_{ij}}{\mu_{ij}(\mu_{ij} + \lambda_{ij})} \right) \right], \text{ for linear } S_{ij}, \quad (9)$$

$$REM_{ij} = \tau_{ij} \left[a_{ij} + b_i \left(\frac{2\lambda_{ij}}{\mu_{ij}^2(\mu_{ij} + \lambda_{ij})} \right) \right], \text{ for quadratic } S_{ij}. \quad (10)$$

Note that a_{ij} reflects the cost of correcting the j^{th} failure cause of failure mode F_i , while b_i is a constant coefficient reflecting the failure effect including the cost for repairing failure mode F_i .

Table 1. The modified FMEA sheet for risk evaluation

Failure mode		Failure effect		Failure cause			REM
Fail -ure	Occur -rence rate (λ_{ij})	Severity coefficient		Causes	Detection rate (μ_{ij})	Occurrence rate (τ_{ij})	
		(a_{ij})	(b_i)				
...
F_i	λ_{i1}	a_{i1}	b_i	c_{i1}	μ_{i1}	τ_{i1}	REM_{i1}
	λ_{i2}	a_{i2}	b_i	c_{i2}	μ_{i2}	τ_{i2}	REM_{i2}
	λ_{i3}	a_{i3}	b_i	c_{i3}	μ_{i3}	τ_{i3}	REM_{i3}
...

The REM of all failure causes can be obtained using Equation (9) and (10). Thus, we can evaluate the risk of the j^{th} cause of failure mode F_i based on REM_{ij} for every i and j . A modified FMEA sheet of Table 1 can be used to facilitate the evaluation procedure.

4. Numerical example

4.1 An example

An example of a dual clutch transmission is excerpted from Zhang et al.¹²⁾ to demonstrate application of the proposed FMEA model. Some numerical figures are supplemented or slightly changed as necessary to fit our models. Table 2 shows a part of conventional FMEA sheet. The failure mode ‘clutch disengaging’ may result in ‘shifting failure’ or ‘loss of acceleration’, i.e., reduced level of performance. Thus, severity 7 is assigned to this failure mode according to the conventional FMEA guide. Since the failure mode ‘incorrect gear position’ may result in ‘damage of gear box’ or ‘possible collision’ which affects safe vehicle operation, its severity is ranked as 9. Other numbers for occurrence and detection are given in Table 2.

Table 2 uses RPN for evaluating the risk linked with each failure cause according to the conventional FMEA. Based on this table, the modified FMEA sheet is constructed to demonstrate a practical application of REM. Numerical figures are assigned for a_{ij} , b_i , λ_{ij} , μ_{ij} and τ_{ij} considering those numbers given in Table 2.

Table 2. An illustrative example

Failure Mode	Potential Effects	S	Failure Causes	O	D	RPN	Rank
clutch dis-engaging	shifting failure	7	dual clutch mechatronic assembly fault	4	3	84	5
	loss of acceleration		shift mechatronic assembly fault	4	5	140	1
incorrect gear position	damage of gearbox possible collision	9	sensor fault	3	5	135	2
			ECU fault	2	5	90	4
			actuator fault	4	3	108	3

The severity of failure modes, 7 and 9 were used as the severity coefficient b_i . The occurrence ranking of each failure cause is converted into the occurrence rate τ_{ij} with reference to the FMEA handbook. For example,

Table 3. The corresponding numerical figures for the proposed model

Failure mode		Failure effect		Failure cause		
Failure	Occurrence rate (λ_{ij})	Severity coefficient		Cause	Detection rate (μ_{ij})	Occurrence rate (τ_{ij})
		(a_{ij})	(b_i)			
clutch disengaging	0.01	1	7	dual clutch mechatronic assembly fault	0.02	0.001
	0.005	1	7	shift mechatronic assembly fault	0.005	0.001
incorrect gear position	0.003	1	9	sensor fault	0.005	0.0005
	0.0001	1	9	ECU fault	0.005	0.0001
	0.00001	1	9	actuator fault	0.02	0.001

‘dual clutch mechatronic assembly fault’ has 4 for occurrence ranking, which corresponds to the occurrence of 1 in 1,000. Numbers are also assigned to λ_{ij} and μ_{ij} . Similarly, all the necessary numerical values are provided in Table 3.

4.2 Risk evaluation

To illustrate the effectiveness of the proposed FMEA approach, we used the above example to analyze and compare with the conventional FMEA. Table 4 provides the REM value for each failure cause. Comparing it with Table 2, we can see that the failure cause “shift mechatronic assembly fault” is ranked as 1 consistently with both the conventional FMEA and the proposed model. But this is not always the case. For example, the failure cause ‘dual clutch mechatronic assembly fault’ is ranked as 3 for both linear and quadratic severity functions in Table 4, while ranked as 5 in Table 2. Under such an inconsistent situation, REM should be considered to be more important. In fact, the RPN value of the conventional FMEA has no practical meaning except priority order, which may also be doubtful. For example, the RPN difference between the two failure causes ‘dual clutch mechatronic assembly fault’ and ‘ECU fault’ is $90 - 84 = 6$, while the difference between ‘ECU fault’ and ‘actuator fault’ is $108 - 90 = 18$. But it does not mean the difference 18 is three times as large as 6 from the aspects of risk. On the other hand, with quadratic severity function, the corresponding REM differences $11.67 - 1.10 = 10.57$ and $1.10 - 0.02 = 1.08$ have practical meaning, that is, the risk difference 10.57 is about ten times as large as 1.08. Thus, the FMEA team will have more specific and

Table 4. REM values for each failure cause

Failure Causes	Expected severity		REM	
	Linear	Quadratic	Linear	Quadratic
dual clutch mechatronic assembly fault	117.667	11,668	0.118	11.67
shift mechatronic assembly fault	701.000	280,001	0.701	280.00
Sensor fault	526.000	210,001	0.263	105.00
ECU fault	28.451	10,981	0.003	1.10
actuator fault	1.175	18	0.001	0.02

realistic risk information with REM than the traditional method. The proposed FMEA model may also lead the FMEA team to anticipate the severity of the risk if sufficient data is available. In addition, the proposed model enables the FMEA team to analyze various situations by assigning different values to the cost or distribution parameters.

4.3 The sensitivity of REM to the distribution parameters

REM of every failure cause may be examined on sensitivity. The failure cause ‘dual clutch mechatronic assembly fault’ will be studied here since the patterns of variation in REM of the others will be similar. Sensitivity of REM may be analyzed for any of a_{ij} , b_i , τ_{ij} , λ_{ij} and μ_{ij} . The effect of variation in a_{ij} or b_i or τ_{ij} can easily be predicted if the other parameters are fixed. Sensitivity analysis of REM is carried out only for λ_{ij} and μ_{ij} , which are the distribution parameters mostly affecting the severity of the failure cause. The sensitivity of REM is analyzed for each linear and quadratic severity function, assuming that the severity coefficients a_{ij} and b_i are fixed as 1 and 7, respectively, and that the occurrence rate τ_{ij} of failure cause was fixed as 0.1.

The REM values for linear and quadratic severity functions with different values of λ_{ij} and μ_{ij} are shown in Fig. 2 and Fig. 3, respectively. The results show that; (i) REM with the quadratic severity function is more sensitive than REM with the linear severity function, (ii) smaller values of μ_{ij} make REM more sensitive, and (iii) if λ_{ij} takes extremely small or large values, REM is relatively insensitive to λ_{ij} .

Fig. 2 and Fig. 3 show that REM has its maximum value when μ_{ij} and λ_{ij} are 0.005 and 0.5 respectively. It simply means that the risk is maximized with a potential failure

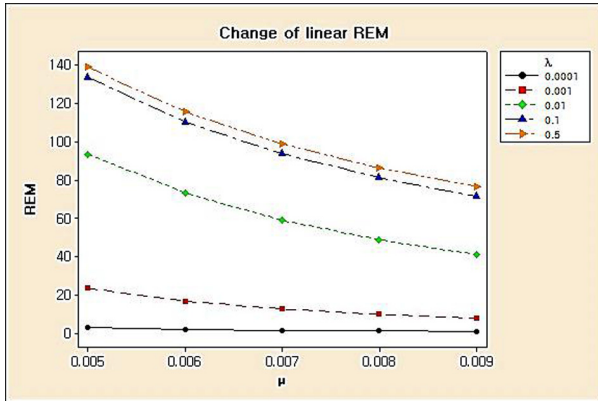


Fig. 2. Sensitivity of REM for linear model.

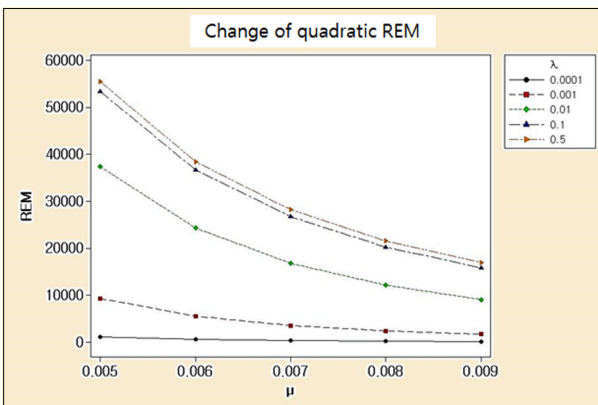


Fig. 3. Sensitivity of REM for quadratic model.

cause which is hardly detectable but highly possible to incur a failure. There are big differences of REM values with $\lambda_{ij} = 0.001, 0.01$, and 0.1 , while showing small differences with $\lambda_{ij} = 0.1$ and 0.5 or $\lambda_{ij} = 0.0001$ and 0.001 . This implies that REM does not show much difference when the possibility of a failure is very high or very low. It means that (i) every cause that incurs a failure with a high possibility should be taken care of and (ii) on the other hand, a cause that hardly incurs a failure need not to be addressed so much. For the failure causes with medium possibility of incurring actual failures, the priority of corrective actions should be carefully determined.

5. Conclusion

A new risk metric REM is developed to compensate the shortcomings of RPN of the conventional FMEA. Assuming a homogeneous Poisson process for occurrence and detection of failure and its cause, linear and quadratic functions of the undetected failure time duration are used

for evaluating severity. The REM of each failure cause is obtained by multiplying its occurrence rate with the expected severity of its corresponding failure mode.

Risk evaluation based on the REM provides a more meaningful metric than the conventional RPN. REM represents the size of risk induced by a failure cause, while RPN only represents the risk priority. That is, REM provides a meaningful quantitative measure, while RPN provides only comparative priority information with rarely practical meaning with its numerical value. Thus, REM not only provides priority information of potential failure cause to be corrected but also enables to evaluate effectiveness of corrective actions quantitatively.

Numerical analyses show that (i) REM provides the risk priority information that may be different from that of the conventional RPN, (ii) the difference in values of REM reflects the difference in size of practical risks, (iii) smaller detection rate makes REM more sensitive to failure occurrence rate, (iv) the priority of corrective actions should be rightly determined on the cause with medium possibility of incurring actual failures, while causes rarely incurring failure need not to be so carefully addressed. These results are consistent with our intuition.

This study assumes exponential probability models and independency of occurrence and detection times of a failure and its cause for analytic simplicity. The real situation, however, may be more complicated. Further studies are expected for the cases where (i) the time variable is not exponentially distributed, (ii) occurrence of a failure affects detection of its cause, (iii) some causes are interrelated and dependent on each other, and (iv) there exists a hierarchical relationship among some failure causes. In fact, these are only some examples of possible situations and there may be many others to be covered in the future study.

Acknowledgement: This work was supported by a Research Grant of Pukyong National University(2015 Year).

References

- 1) H. Liu, L. Liu and N. Liu, "Risk evaluation approaches in failure mode and effects analysis: A literature review", Expert Systems with Applications, Vol. 40, pp. 828-838,

2013.

- 2) Ford Design Institute, "FMEA HANDBOOK", VERSION 4.1, 2004.
- 3) H. M. Kwon, S. H. Hong, M. K. Lee and A. Sutrisno, "Risk Evaluation Based on the Time Dependence Expected Loss Model in FMEA", Journal of Korean Society of Safety, Vol. 26, No. 6, pp. 104-110, 2011.
- 4) M. Abdelgawad and A. R. Fayek, "Risk management in the construction industry using combined Fuzzy FMEA and Fuzzy AHP", Journal of Construction Engineering and management, Vol. 136, pp. 1028-1036, 2010.
- 5) M. Kumru and P. Y. Kumru, "Fuzzy FMEA application to improve purchasing process in a public hospital", Applied Soft Computing, Vol. 13, No. 1, pp. 721~733, 2013.
- 6) H. C. Liu, L. Liu and P. Li, "Failure mode and effects analysis using intuitionistic fuzzy hybrid weighted Euclidean distance operator", International Journal of Systems Science, Vol. 45, No. 10, pp. 2012-2030, 2014.
- 7) H. C. Liu, X. J. Fan, P. Li and Y. Z. Chen, "Evaluating the risk of failure modes with extended MULTIMOORA method under fuzzy environment", Engineering Applications of Artificial Intelligence, Vol. 34, pp. 168-177, 2014.
- 8) H. C. Liu, J. X. You, X. J. Fan and Q. L. Liu, "Failure mode and effects analysis using D numbers and grey relational projection method", Expert Systems with Applications, Vol. 41, pp. 4670-4679, 2014.
- 9) H. M. Kwon, S. H. Hong and M. K. Lee, "An expected loss model for FMEA under periodic monitoring of failure causes", Journal of the Korean Institute of Industrial Engineers, Vol. 39, No. 2, pp. 143-148, 2013.
- 10) S. J. Rhee and K. Ishii, "Using cost based FMEA to enhance reliability and serviceability", Advanced Engineering Informatics, Vol. 17, pp. 179-188, 2003.
- 11) M. S. Baek, H. A. Jang and H. M. Kwon, "A modified Metric of FMEA for Risk Evaluation Based on ASIL of Safety System", Journal Korean Society Quality Management, Vol. 42, No. 4, pp. 543-562, 2014.
- 12) H. Zhang, W. Li and J. Quin, "Model-based functional safety analysis method for automotive embedded system application", International Conference on Intelligent Control and Information Proceedings, China: Dalian, pp. 13-15, 2010.

Appendix

I. Proof of Equation (6) related with b_i

$$\begin{aligned}
 & \int_0^\infty \int_0^d (d-t) f_{D_{ij}}(d) f_{T_{ij}}(t) dt dd \\
 &= \int_0^\infty \left[\int_0^d (d-t) \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} e^{-\lambda_{ij}t} dt \right] dd \\
 &= \int_0^\infty \left[d \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} \int_0^d e^{-\lambda_{ij}t} dt - \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} \int_0^d t e^{-\lambda_{ij}t} dt \right] dd \\
 &= \int_0^\infty \left[d \mu_{ij} e^{-\mu_{ij}d} - d \mu_{ij} e^{-(\mu_{ij} + \lambda_{ij})d} + d \mu_{ij} e^{-(\mu_{ij} + \lambda_{ij})d} \right. \\
 &\quad \left. + \frac{1}{\lambda_{ij}} \mu_{ij} e^{-(\mu_{ij} + \lambda_{ij})d} - \frac{1}{\lambda_{ij}} \mu_{ij} e^{-\mu_{ij}d} \right] dd \\
 &= \mu_{ij} \int_0^\infty d e^{-\mu_{ij}d} dd + \frac{\mu_{ij}}{\lambda_{ij}} \int_0^\infty e^{-(\mu_{ij} + \lambda_{ij})d} dd - \frac{\mu_{ij}}{\lambda_{ij}} \int_0^\infty e^{-\mu_{ij}d} dd \\
 &= \frac{\lambda_{ij}}{\mu_{ij}(\mu_{ij} + \lambda_{ij})}
 \end{aligned}$$

II. Proof of Equation (7) related with b_i

$$\begin{aligned}
 & \int_0^\infty \int_0^d (d-t)^2 f_{D_{ij}}(d) f_{T_{ij}}(t) dt dd \\
 &= \int_0^\infty \left[\int_0^d (d-t)^2 \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} e^{-\lambda_{ij}t} dt \right] dd \\
 &= \int_0^\infty \left[d^2 \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} \int_0^d e^{-\lambda_{ij}t} dt - 2d \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} \int_0^d t e^{-\lambda_{ij}t} dt \right. \\
 &\quad \left. + \mu_{ij} e^{-\mu_{ij}d} \lambda_{ij} \int_0^d t^2 e^{-\lambda_{ij}t} dt \right] dd \\
 &= \int_0^\infty \left[d^2 \mu_{ij} e^{-\mu_{ij}d} - \frac{2d \mu_{ij}}{\lambda_{ij}} e^{-\mu_{ij}d} - \frac{2 \mu_{ij}}{\lambda_{ij}^2} e^{-(\mu_{ij} + \lambda_{ij})d} + \frac{2 \mu_{ij}}{\lambda_{ij}^2} e^{-\mu_{ij}d} \right] dd \\
 &= \mu_{ij} \int_0^\infty d^2 e^{-\mu_{ij}d} dd - \frac{2 \mu_{ij}}{\lambda_{ij}} \int_0^\infty d e^{-\mu_{ij}d} dd - \frac{2 \mu_{ij}}{\lambda_{ij}^2} \int_0^\infty e^{-(\mu_{ij} + \lambda_{ij})d} dd \\
 &\quad + \frac{2 \mu_{ij}}{\lambda_{ij}^2} \int_0^\infty e^{-\mu_{ij}d} dd \\
 &= \frac{2 \lambda_{ij}}{\mu_{ij}^2(\mu_{ij} + \lambda_{ij})}
 \end{aligned}$$