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Monodromy Maps of Fibered 2-Bridge Knots as Elements in Automorphism Groups of Free Groups

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Abstract. In this note, we study a monodromy map of a fibered 2-bridge knot. We show the monodromy map of a fibered 2-bridge knot as an element in the automorphism group of a free group.

1. Introduction

Let $\Sigma_{g,1}$ be a compact oriented connected surface of genus g with 1 boundary components. Let $\mathcal{M}_{g,1}$ be the mapping class group of $\Sigma_{g,1}$:

$$\mathcal{M}_{g,1} = \pi_0(\mathrm{Diff}^+(\Sigma_{g,1}, \partial\Sigma_{g,1})).$$

Namely,

$$\mathcal{M}_{g,1} = \{\varphi : \Sigma_{g,1} \to \Sigma_{g,1} : \text{orientation preserving diffeomorphism such that } \varphi|_{\partial\Sigma_{g,1}} = \mathrm{id}_{\partial\Sigma_{g,1}}\}$$
$$/\text{isotopy fixing } \partial\Sigma_{g,1} \text{ pointwise}$$

The mapping class group $\mathcal{M}_{g,1}$ acts on $\pi := \pi_1(\Sigma_{g,1}, *)$, where $*$ is a point in $\partial\Sigma_{g,1}$. That is to say, an element of $\mathcal{M}_{g,1}$ can be regarded as an element of $\mathrm{Aut}(\pi)$. To be precise, it is known (eg. [12]) that , we have

$$\mathcal{M}_{g,1} \cong \{\varphi \in \mathrm{Aut}(\pi) \mid \varphi(\zeta) = \zeta\},$$

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where ζ is an element in π, which is parallel to $\partial\Sigma_{g,1}$ ignoring the orientation.

Let K be an oriented link in the 3-sphere S^3, and $\tilde{R}$ a Seifert surface of K. The exterior of K, say $E(K)$, is $S^3 - \mathrm{Int}N(K)$ where $N(K)$ means a regular neighborhood of K in S^3 and Int is its interior. Set $R = \tilde{R} \cap E(K)$. If $E(K)$ is a fiber bundle over the circle S^1 with fiber R, then K is called a *fibered knot*. It is known that R is a minimal genus Seifert surface in such case. Thus, if K is a fibered knot, $E(K)$ is $R \times [0, 2\pi]$ with an identification according to a diffeomorphism $h : R \to R$ by which $R \times \{0\}$ is attached to $R \times \{2\pi\}$. This map is called a *monodromy* of K. Set $R := \Sigma_{g,1}$, then we may regard h as an element of $\mathcal{M}_{g,1}$.

In this note, we focus on fibered 2-bridge knots. We study the monodromy maps of a fibered 2-bridge knot as an element of $\mathrm{Aut}(\pi)$. Torus knots are fibered, and torus knots of type $T(p, 2)$ (p : odd) are 2-bridge. We study these knots in Section 3. General case is discussed in Section 4. We put a list of the monodromies of fibered 2-bridge knots with up to 12 crossings in Section 5.

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2. Fibered 2-Bridge Links

Let L be a 2-bridge link $S(q, p)$ in Schubert's notation. Here, p and q are coprime integers, and q is odd. It is known that $S(q, p)$ and $S(q', p')$ are equivalent if and only if $q' = q$ and $p' \equiv p^{\pm 1} \pmod{q}$, and that $S(q, -p)$ gives the mirror image of $S(q, p)$.

Consider a *subtractive* continued fraction expansion of p/q (see [5]):

$$\frac{p}{q} = r + [b_1, b_2, \dots, b_n] = r + \cfrac{1}{b_1 - \cfrac{1}{b_2 - \cfrac{1}{b_3 - \cfrac{1}{\ddots - \cfrac{1}{b_n}}}}},$$

where $r, b_i \in \mathbb{Z}$ and $b_i \neq 0$. The *length* of this expansion is n. Then L is the boundary of the surface obtained by plumbing n bands in a row, the i-th band has b_i half-twists (right-handed if $b_i > 0$ and left-handed if $b_i < 0$). If some b_i is odd, the expansion is said to be of *odd type*. Otherwise, it is said to be *even type*. Any fraction is presented by both odd type and even type. In this paper, an expansion always means a subtractive one. We note that the following equality:

$$r + [b_1, -b_2, b_3, -b_4, \dots, (-1)^{n-1} b_n] = r + \cfrac{1}{b_1 + \cfrac{1}{b_2 + \cfrac{1}{b_3 + \cfrac{1}{\ddots + \cfrac{1}{b_n}}}}}.$$

In what follows, we suppose that an expansion is even type. Then, if the length of an expansion is even (odd resp.), L becomes a knot (2-component link resp.) and the genus of L is equal to $n/2$ ($(n-1)/2$ resp.) by [9]. Further, we suppose $p < q$, then $r = 0$.

It is known that L is fibered if and only if $|b_i| = 2$ ([2], [10]), namely L is obtained from a disk by n Hopf plumbings. The monodromy of the right-handed Hopf band (left-handed Hopf band resp.) is right Dehn twist D_R (left Dehn twist D_L resp.), and it is known that the monodromy of L is $D_n D_{n-1} \cdots D_1$ where D_i is D_R or D_L according to $b_i = 2$ or $b_i = -2$ for each i, see Proposition 2 in [4]. Thus it is not difficult to treat the monodromy maps of fibered 2-bridge knots. In particular, if all b_i are the same sign, L becomes a torus knot of type $T(p, \pm 2)$, which we study in the next section.

Example 2.1. The knot 8_{12} is $S(29, 12)$ in Schubert's notation. Since 12/29 has the following continued fraction expansion:

$$\frac{12}{29} = \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}} = [2, -2, 2, -2],$$

the knot 8_{12} can be regarded as the knot obtained by plumbings of the right-handed Hopf band, the left-handed Hopf band, the right-handed Hopf band, and the left-handed Hopf band, successively. See Figure 1.

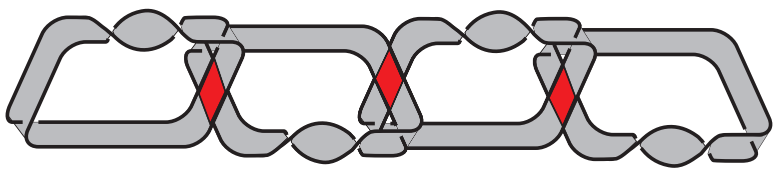

Figure 1

3. Torus Knots

In this section, we study the monodromy of the torus knots of type $T(p, 2)$ ($p \geq 3$, odd). The torus knot of type $T(5, 2)$ has a projection as illustrated in Figure 2, and we denote by K the knot. Its minimal genus Seifert surface $\tilde{R}$ has genus 2, then we may take a set of generators of $\pi_1(\tilde{R}, \tilde{*})$, say $\{a_1, a_2, a_3, a_4\}$, as in the right-hand side figure in Figure 2. Note that $\tilde{*}$ is on the knot K. Let $E = S^3 - \mathrm{Int}N(K)$, and we denote by D the meridian disk of the solid torus $N(K)$ containing $\tilde{*}$. Further, set $R = \tilde{R} \cap E$. Note that R and ∂D intersects one point. Let $*$ be this point. One can view the monodromy as the automorphism of $\pi_1(R, *)$ obtained by pushing generators (with basepoint on ∂R) off the $+$ side of R through $S^3 - R$ and onto the $-$ side of R where the basepoint travels along the meridian. We draw the product manifold $R \times [-\pi/2, \pi/2]$ in E as illustrated in Figure 3. Here we regard $R \times \{0\}$ as R. Then we can consider the set of generators of $\pi_1(R \times \{\pi/2\}, *^+)$

($\pi_1(R \times \{-\pi/2\}, *^-)$ resp.) corresponding to $\{a_1, a_2, a_3, a_4\}$, where $*^+$ ($*^-$ resp.) is the point in $\partial(R \times \{\pi/2\})$ ($\partial(R \times \{-\pi/2\}$ resp.), corresponding to $* \times \{\pi/2\}$ ($* \times \{-\pi/2\}$ resp.). As illustrated in Figure 3, we denote the set by $\{a_1^+, a_2^+, a_3^+, a_4^+\}$ ($\{a_1^-, a_2^-, a_3^-, a_4^-\}$ resp.).

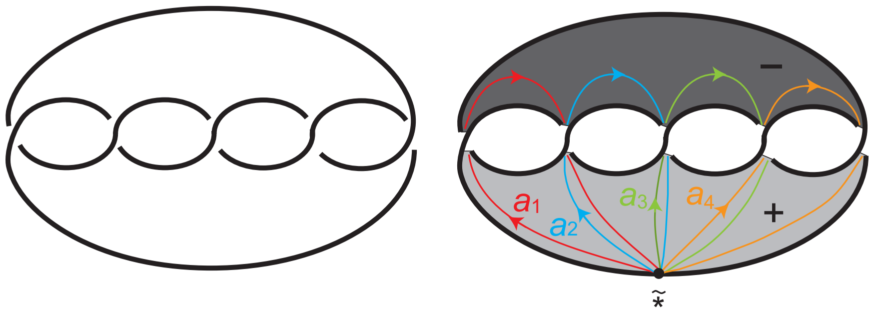

Figure 2

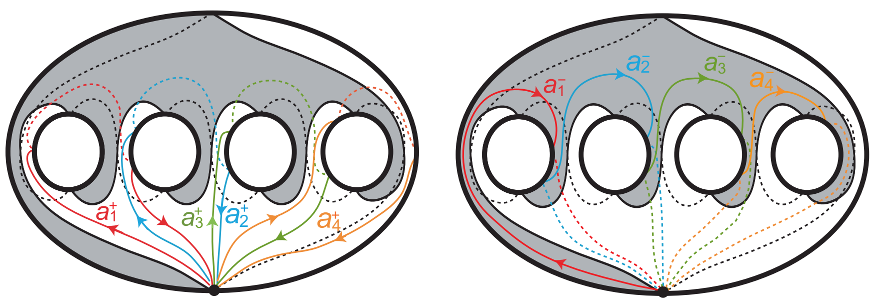

Figure 3

Since the torus knot K of type $T(p, 2)$ is a fibered knot and R is the minimal genus Seifert surface of K, $E - \mathrm{Int}N(R)$ is homeomorphic to $R \times [\pi/2, 3\pi/2]$, so that we may suppose that $R \times \{3\pi/2\} = R \times \{-\pi/2\}$. Now, we move a generator $a_i^+(\subset R \times \{\pi/2\})$ toward $R \times \{3\pi/2\}$ by an isotopy, in $R \times [\pi/2, 3\pi/2]$. Note that the starting and ending points of a_i^+ are in $*^+$ and move on $\partial D - R$ so that arrive at $*^-$ by this isotopy. The movement of a_1^+, a_2^+, a_3^+ are similar, so we will demonstrate the cases of a_2^+ and a_4^+. It is not difficult to see there is an isotopy that we have $a_2^+ \subset R \times (\pi/2, 3\pi/2)$ as in the left-hand side figure in Figure 4. Denote by ι this isotopy. We suppose that this a_2^+ is in $R \times \{\pi\}$. Moreover, we may see a_2^+ in $R \times \{3\pi/2\}$ by the next isotopy, say τ, as illustrated in the right-hand side figure of Figure 4. Let $a_2^{+,\pi/2}, a_2^{+,\pi}, a_2^{+,3\pi/2}$ be a_2^+ in $R \times \{\pi/2\}, R \times \{\pi\}, R \times \{3\pi/2\}$ respectively, then we can see that $\iota(a_2^{+,\pi/2}) = a_2^{+,\pi} = \tau^{-1}(a_2^{+,3\pi/2})$. Since

$\tau \circ \iota(a_2^{+,\pi/2}) = a_2^{+,3\pi/2}$ is a simple closed curve on $R \times \{-\pi/2\}$, it can be presented by the set of generator $\{a_1^-, a_2^-, a_3^-, a_4^-\}$ as an element of $\pi_1(R \times \{-\pi/2\}, *^-)$, so that we have $a_2^{+,3\pi/2} = a_1^- a_2^- (a_3^-)^{-1}(a_2^-)^{-1}(a_1^-)^{-1}$. By the same argument, we have : $a_1^{+,3\pi/2} = a_1^-(a_2^-)^{-1}(a_1^-)^{-1}$, $a_3^{+,3\pi/2} = a_1^- a_2^- a_3^- (a_4^-)^{-1}(a_3^-)^{-1}(a_2^-)^{-1}(a_1^-)^{-1}$, $a_4^{+,3\pi/2} = a_1^- a_2^- a_3^- a_4^-$. This means that the monodromy map of K may have the presentation as an element of $\mathrm{Aut}(\pi_1(R,*))$ as follows: $a_1 \mapsto a_1 a_2^{-1} a_1^{-1}$, $a_2 \mapsto a_1 a_2 a_3^{-1} a_2^{-1} a_1^{-1}$, $a_3 \mapsto a_1 a_2 a_3 a_4^{-1} a_3^{-1} a_2^{-1} a_1^{-1}$, $a_4 \mapsto a_1 a_2 a_3 a_4$.

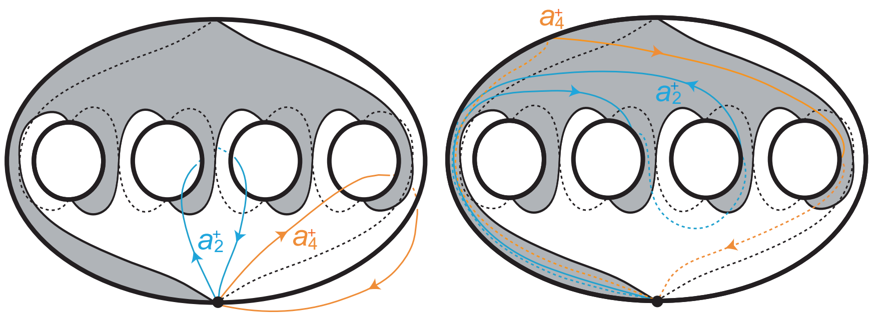

Figure 4

By the same argument, we have:

Theorem 3.1. *Let K be the torus knot of type $T(p,2)$ and R the minimal genus Seifert surface. Let $\{a_i\}$ $(i = 1, 2, \ldots, p-1)$ be a set of generators of $\pi_1(R,*)$. Then the monodromy is given as follows:*

$$a_i \longmapsto \prod_{j=1}^{i} a_j \prod_{k=0}^{i} a_{i+1-k}^{-1} \qquad (1 \leq i \leq p-2)$$

$$a_{p-1} \longmapsto \prod_{j=1}^{p-1} a_j$$

4. The Monodromies of Fibered 2-Bridge Knots

Let K be the 2-bridge knot of type (p,q), which has the following subtractive continued fraction:

$$\frac{p}{q} = r + [b_1, b_2, \ldots, b_n] = r + \cfrac{1}{b_1 - \cfrac{1}{b_2 - \cfrac{1}{b_3 - \cfrac{1}{\ddots - \frac{1}{b_n}}}}}, \qquad (b_i : \text{even}).$$

Then $n = 2g$, where g is the genus of K. If the knot is fibered, $|b_i| = 2$. Let R be the minimal genus Seifert surface of K as illustrated in Figure 1. We take a set of generators of $\pi_1(R,*)$ $\langle a_1, a_2, \ldots, a_{2g}\rangle$ as follows. Set the base point, a_1 and a_2 as in Figure 5.

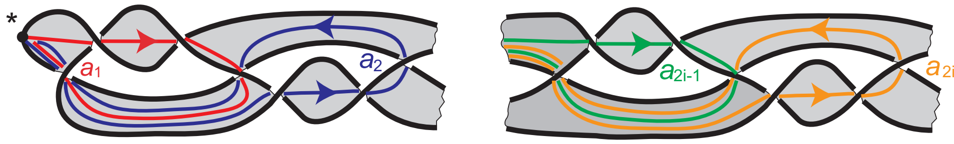

Figure 5

Similarly, we define the generators a_{2i-1} and a_{2i} $(2 \le i \le g)$ as in Figure 5. (This figure is the case of $b_{2i-1} = 2, b_{2i} = -2$.) The loop a_k starts the base point, goes along the untwisted parts of the Hopf bands corresponding to b_j $(j = 1, 2, \ldots, k-1)$ successively and the twisted part of the Hopf band corresponding to b_k. Then it goes back to the base point via the untwisted parts of the Hopf bands corresponding to b_j $(j = k-1, k-2, \ldots, 1)$ again.

Since $S^3 - \mathrm{Int}N(R)$ is the genus $2g$ handlebody, we may take $\langle \gamma_1, \gamma_2, \ldots, \gamma_{2g}\rangle$ a set of generators of $\pi_1(S^3 - \mathrm{Int}N(R))$ where γ_i is the loop that goes from the base point to upper side and goes through the disk $\tilde{\gamma}_i$ and comes back to the base point. Here $\gamma_i \cap \tilde{\gamma}_i$ $(i = 1, 2, \ldots, 2g)$ is one point and $\gamma_i \cap \tilde{\gamma}_j = \emptyset$ if $i \neq j$. See Figure 6.

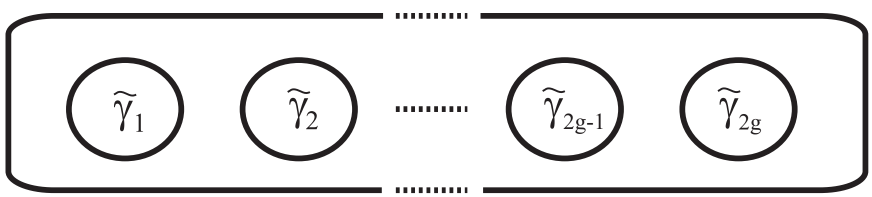

Figure 6

For example, in the case of the knot 8_{12}, the generators of $\pi_1(R,*)$ can be seen in Figure 7, then we have $a_2^+ \sim \gamma_1^{-1}\gamma_2\gamma_3$ and $a_4^+ \sim \gamma_3^{-1}\gamma_4$ as in Figure 8 by the same argument in Section 3. Here $A \sim B$ means A is homotopic to B in $\pi_1(S^3 - \mathrm{Int}N(R))$.

In general, we have the following lemma as seen in Figure 9 where we draw the case of $c_{2i-1} = b_{2i-1}/2 = 1$ and $c_{2i} = b_{2i}/2 = -1$. Since the parts of $a_j^{\pm}$ $(j = 2i-1$ or $2i)$ which are not drawn in the figure run untwisted part of each Hopf band, they do not affect the result.

Lemma 4.1.(Section 5 in [8]) *Let K be a fibered 2-bridge knot of type $S(q,p)$ and $\frac{p}{q}$ has the subtractive continued fraction $[b_1, b_2, \ldots, b_{2g}]$. Set $c_i = b_i/2$, i.e., $c_i = \pm 1$. For $1 \le i \le 2g$, we have:*

$$a_{2i-1}^+ \sim \gamma_{2i-1}^{c_{2i-1}}, \qquad a_{2i}^+ \sim \gamma_{2i-1}^{-1}\gamma_{2i}^{-c_{2i}}\gamma_{2i+1},$$
$$a_{2i-1}^- \sim \gamma_{2i-2}^{-1}\gamma_{2i-1}^{c_{2i-1}}\gamma_{2i}, \qquad a_{2i}^- \sim \gamma_{2i}^{-c_{2i}},$$

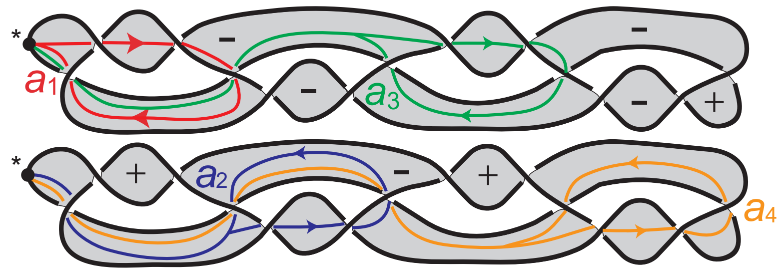

Figure 7

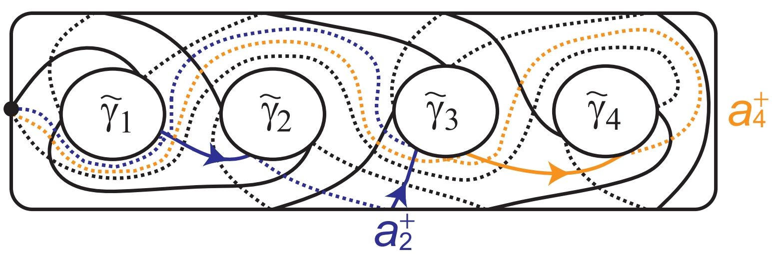

Figure 8

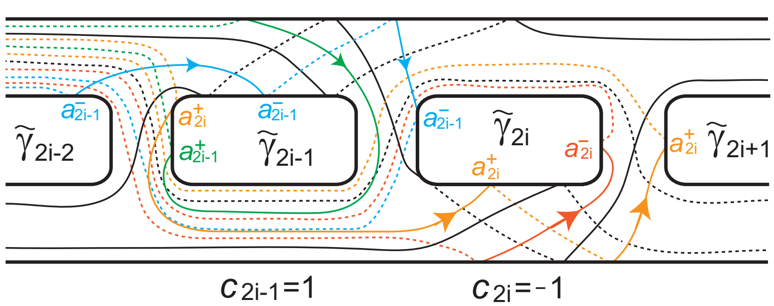

Figure 9

where $\gamma_0 = \gamma_{2g+1} = 1$.

Theorem 4.2. *Under the same notations as in Lemma 4.1, the monodromy of K is given explicitly by the following. If* $g = 1$, *then* $a_1 \mapsto a_1 a_2^{c_2}$, $a_2 \mapsto (a_1 a_2^{c_2})^{-c_1} a_2$. *For* $g \geq 2$,

$$\begin{aligned}
a_1 &\mapsto a_1 a_2^{c_2} \\
a_2 &\mapsto (a_1 a_2^{c_2})^{-c_1} a_2 (a_2^{-c_2} a_3 a_4^{c_4})^{c_3} \\
a_{2i-1} &\mapsto a_{2i-2}^{-c_{2i-2}} a_{2i-1} a_{2i}^{c_{2i}} \qquad (2 \leq i \leq g) \\
a_{2i} &\mapsto (a_{2i-2}^{-c_{2i-2}} a_{2i-1} a_{2i}^{c_{2i}})^{-c_{2i-1}} a_{2i} (a_{2i}^{-c_{2i}} a_{2i+1} a_{2i+2}^{c_{2i+2}})^{c_{2i+1}} \qquad (2 \leq i \leq g-1) \\
a_{2g} &\mapsto (a_{2g-2}^{-c_{2g-2}} a_{2g-1} a_{2g}^{c_{2g}})^{-c_{2g-1}} a_{2g}.
\end{aligned}$$

Proof. Just calculations using Lemma 4.1. Suppose $g = 1$. By Lemma 4.1, $a_1^+ \sim \gamma_1^{c_1}, a_2^+ \sim \gamma_1^{-1}\gamma_2^{-c_2}, a_1^- \sim \gamma_1^{c_1}\gamma_2$ and $a_2^- \sim \gamma_2^{-c_2}$. Then we have $a_1^+ \sim a_1^-(a_2^-)^{c_2}$ and $a_2^+ \sim (a_1^-(a_2^-)^{c_2})^{-c_1} a_2^-$, so that $a_1 \mapsto a_1 a_2^{c_2}$ and $a_2 \mapsto (a_1 a_2^{c_2})^{-c_1} a_2$. Suppose $g \geq 2$. We have $a_1^+ \mapsto a_1 a_2^{c_2}$ via the same argument. Since $a_3^- \sim \gamma_2^{-1}\gamma_3^{c_3}\gamma_4$ and $a_4^- \sim \gamma_4^{-c_4}$, we have $\gamma_3^{c_3} \sim \gamma_2 a_3^-(a_4^-)^{c_4} \sim (a_2^-)^{-c_2} a_3^-(a_4^-)^{c_4}$. Because $a_2^+ \sim \gamma_1^{-1}\gamma_2^{-c_2}\gamma_3 \sim (a_1^+)^{-c_1}(a_2^-)((a_2^-)^{-c_2} a_3^-(a_4^-)^{c_4})^{c_3}$, then $a_2 \mapsto (a_1 a_2^{c_2})^{-c_1} a_2 (a_2^{-c_2} a_3 a_4^{c_4})^{c_3}$. For i $(2 \leq i \leq g)$, $a_{2i-1}^+ \sim \gamma_{2i-2} a_{2i-1}^- \gamma_{2i}^{-1} \sim (a_{2i-2}^-)^{-c_{2i-2}} a_{2i-1}^- (a_{2i}^-)^{c_{2i}}$. Hence $a_{2i-1} \mapsto a_{2i-2}^{-c_{2i-2}} a_{2i-1} a_{2i}^{c_{2i}}$. For $i\,(2 \leq i \leq g-1)$, $a_{2i}^+ \sim (a_{2i-1}^+)^{-c_{2i-1}} a_{2i}^- (a_{2i+1}^+)^{c_{2i+1}}$. So we have the conclusion via the presentation of a_{2i-1}^+ and a_{2i+1}^+. Similarly we have the case of a_{2g}. □

Let K be a fibered 2-bridge knot of type $S(q,p)$ and $\frac{p}{q}$ has the subtractive continued fraction $[b_1, b_2, \ldots, b_{2g}]$, and R the minimal genus (genus $2g$) Seifert surface of K as above. From Theorem 4.2, we obtain the following matrix A as the transformation matrix : $H_1(R;\mathbb{Z}) \to H_1(R;\mathbb{Z})$:

$$A = \begin{pmatrix} A_0 & & & & \\ & A_2 & & & \\ & & \ddots & & \\ & & & A_{2g-4} & \\ & & & & A_{2g} \end{pmatrix} \text{ where } A_0 = \begin{pmatrix} 1 & c_2 & 0 & 0 \\ -c_1 & 1 - c_1c_2 - c_2c_3 & c_3 & c_3c_4 \end{pmatrix},$$

$A_{2i} = \begin{pmatrix} -c_{2i} & 1 & c_{2i+2} & 0 & 0 \\ c_{2i}c_{2i+1} & -c_{2i+1} & 1 - c_{2i+1}c_{2i+2} - c_{2i+2}c_{2i+3} & c_{2i+3} & c_{2i+3}c_{2i+4} \end{pmatrix}$ $(1 \leq i \leq g-2)$, and $A_{2g-2} = \begin{pmatrix} -c_{2g-2} & 1 & c_{2g} \\ c_{2g-2}c_{2g-1} & -c_{2g-1} & 1 - c_{2g-1}c_{2g} \end{pmatrix}$. By Milnor [7], $\Delta_K(t) = \det(tI - A)$, where I is the $2g \times 2g$ identity matrix. By standard arguments of the linear algebra, we have:

Corollary 4.3. *Let K be a fibered* 2*-bridge knot as above. Then the coefficient of* t *of the Alexander polynomial of K is* $-2g + \sum_{1 \leq i \leq 2g-1} c_i c_{i+1}$. *That of the term* t^m

$(2 \leq m \leq g)$ *of the Alexander polynomial of* K *is given by the following:*

$$
\begin{aligned}
&\frac{(-1)^m}{m!} \prod_{i=0}^{m-1} (2g-i) \\
&+ \sum_{j=1}^{m-1} \Big(\frac{(-1)^{m-j}}{(m-j)!} \prod_{i=0}^{m-1-j} (2(g-j)-i) \sum_{1 \leq i_1 < i_2 < \cdots < i_j \leq 2g-j} c_{i_1} c_{i_1+1} c_{i_2+1} c_{i_2+2} \cdots c_{i_j+j-1} c_{i_j+j} \Big) \\
&+ \sum_{1 \leq i_1 < i_2 < \cdots < i_m \leq 2g-m} c_{i_1} c_{i_1+1} c_{i_2+1} c_{i_2+2} \cdots c_{i_m+m-1} c_{i_m+m}
\end{aligned}
$$

Remark 4.4. In [6], Kanenobu gave an algorithm for calculating the Alexander polynomial of a 2-bridge link using the continued fraction expansion. Fukuhara [3] gave an explicit formula for the Alexander polynomial of a 2-bridge link using elementary number theoretical functions.

Appendix: List of Monodromies

We put a list of the monodromies of fibered 2-bridge knots with up to 12 crossings according to Theorem 4.2. A projection of each knot can be found in [1] and [11]. We use the Schubert's notation in Table I [1].

Knot	Images of $a_1, a_2, \ldots, a_{2g}$
3_1	$a_1a_2^{-1}, a_1$
4_1	$a_1a_2^{-1}, a_2a_1^{-1}a_2$
5_1	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3$
6_2	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3$
6_3	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_4$
7_1	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1}, a_4a_5$
7_6	$a_1a_2^{-1}, a_1a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4$
7_7	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_4$
8_2	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1}, a_4a_5$
8_7	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6, a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6$
8_9	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6, a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6$
8_{12}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4$
9_1	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7$
9_{11}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6, a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6$
9_{17}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1}, a_4a_5a_6, a_4a_5a_6a_6$
9_{20}	$a_1a_2^{-1}, a_1a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1}, a_4a_5$
9_{26}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6, a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6$
9_{27}	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6^{-1}, a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6$
9_{31}	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_4a_6a_5^{-1}a_4, a_4^{-1}a_5a_6^{-1}, a_4^{-1}a_5$
10_2	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7$

10_5	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_7a_8, a_6^{-1}a_7a_8, a_8^{-1}a_7^{-1}a_6a_8$
10_9	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_7a_8, a_6^{-1}a_7a_8, a_8^{-1}a_7^{-1}a_6a_8$
10_{17}	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$ $a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_7a_8, a_6^{-1}a_7a_8, a_8^{-1}a_7^{-1}a_6a_8$
10_{29}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1}, a_4a_5$
10_{41}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6^{-1}, a_4a_5a_6^{-1}, a_6a_5^{-1}a_4^{-1}a_6$
10_{42}	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_4a_6^{-1}a_5^{-1}a_4, a_4^{-1}a_5a_6, a_4^{-1}a_5a_6a_6$
10_{43}	$a_1a_2^{-1}, a_1a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_4a_5a_6, a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_6$
10_{44}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_4a_6a_5^{-1}a_4, a_4^{-1}a_5a_6^{-1}, a_4^{-1}a_5$
10_{45}	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6^{-1}, a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6$
$11a_{96}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_6^{-1}a_5^{-1}a_4^{-1}, a_4a_5a_6, a_4a_5a_6a_6$
$11a_{121}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_4a_6^{-1}a_5^{-1}a_4, a_4^{-1}a_5a_6, a_4^{-1}a_5a_6a_6$
$11a_{159}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_4a_5a_6, a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_6$
$11a_{174}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$11a_{175}$	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$11a_{176}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$11a_{177}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$ $a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$11a_{179}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_8$
$11a_{180}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$11a_{182}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1},$ $a_4a_5a_6, a_4a_5a_6a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$11a_{184}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$ $a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$11a_{203}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1},$ $a_4a_5a_6, a_4a_5a_6a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$11a_{206}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_6a_7a_8^{-1}, a_6a_7a_8^{-1}, a_8a_7^{-1}a_6^{-1}a_8$
$11a_{306}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6^{-1},$ $a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7$
$11a_{308}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6^{-1},$ $a_4a_5a_6^{-1}, a_6a_5^{-1}a_4^{-1}a_6a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7$
$11a_{367}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1},$ $a_4a_5a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7a_{10}a_9^{-1}a_8^{-1}, a_8a_9a_{10}^{-1}, a_8a_9$
$12a_{477}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_4a_5a_6^{-1}, a_4a_5a_6^{-1}, a_6a_5^{-1}a_4^{-1}a_6$
$12a_{497}$	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6^{-1},$ $a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_8$
$12a_{498}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$

	$a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$12a_{499}$	$a_1a_2^{-1}, a_1a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_4a_6a_5^{-1}a_4,$ $a_4^{-1}a_5a_6^{-1}, a_4^{-1}a_5a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$12a_{500}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1},$ $a_4a_5a_6, a_4a_5a_6a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$12a_{501}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6^{-1},$ $a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_8$
$12a_{506}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$ $a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$12a_{512}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$12a_{517}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1},$ $a_4a_5a_6, a_4a_5a_6a_6a_8a_7^{-1}a_6, a_6^{-1}a_7a_8^{-1}, a_6^{-1}a_7$
$12a_{521}$	$a_1a_2^{-1}, a_2a_1^{-1}a_2a_2a_3a_4, a_2a_3a_4, a_4^{-1}a_3^{-1}a_2^{-1}a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_7a_8^{-1}, a_6^{-1}a_7a_8^{-1}, a_8a_7^{-1}a_6a_8$
$12a_{528}$	$a_1a_2^{-1}, a_1a_2a_3a_4^{-1}, a_2a_3a_4^{-1}, a_4a_3^{-1}a_2^{-1}a_4a_6a_5^{-1}a_4^{-1},$ $a_4a_5a_6^{-1}, a_4a_5a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$12a_{535}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_4a_6a_5^{-1}a_4,$ $a_4^{-1}a_5a_6^{-1}, a_4^{-1}a_5a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$12a_{536}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6^{-1},$ $a_4a_5a_6^{-1}, a_6a_5^{-1}a_4^{-1}a_6a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_8$
$12a_{541}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_4a_6a_5^{-1}a_4,$ $a_4^{-1}a_5a_6^{-1}, a_4^{-1}a_5a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_8$
$12a_{579}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6^{-1},$ $a_4^{-1}a_5a_6^{-1}, a_6a_5^{-1}a_4a_6a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$12a_{583}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_5a_6,$ $a_4^{-1}a_5a_6, a_6^{-1}a_5^{-1}a_4a_6a_8^{-1}a_7^{-1}a_6, a_6^{-1}a_7a_8, a_6^{-1}a_7a_8a_8$
$12a_{584}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6,$ $a_4a_5a_6, a_6^{-1}a_5^{-1}a_4^{-1}a_6a_8^{-1}a_7^{-1}a_6, a_6^{-1}a_7a_8, a_6^{-1}a_7a_8a_8$
$12a_{649}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_4a_5a_6^{-1},$ $a_4a_5a_6^{-1}, a_6a_5^{-1}a_4^{-1}a_6a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_8$
$12a_{651}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1},$ $a_4a_5a_6, a_4a_5a_6a_6a_8^{-1}a_7^{-1}a_6, a_6^{-1}a_7a_8, a_6^{-1}a_7a_8a_8$
$12a_{716}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1},$ $a_4a_5a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7a_8a_9a_{10}, a_8a_9a_{10}, a_{10}^{-1}a_9^{-1}a_8^{-1}a_{10}$
$12a_{722}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1},$ $a_4a_5a_8a_7^{-1}a_6^{-1}, a_6a_7a_8^{-1}, a_6a_7a_{10}^{-1}a_9^{-1}a_8^{-1}, a_8a_9a_{10}, a_8a_9a_{10}a_{10}$
$12a_{1039}$	$a_1a_2^{-1}, a_1a_4^{-1}a_3^{-1}a_2^{-1}, a_2a_3a_4, a_2a_3a_4a_4a_6^{-1}a_5^{-1}a_4,$ $a_4^{-1}a_5a_6, a_4^{-1}a_5a_6a_7a_8, a_6^{-1}a_7a_8, a_8^{-1}a_7^{-1}a_6a_8$
$12a_{1128}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1},$ $a_4a_5a_6a_7a_8, a_6a_7a_8, a_8^{-1}a_7^{-1}a_6^{-1}a_9a_{10}, a_8^{-1}a_9a_{10}, a_{10}^{-1}a_9^{-1}a_8a_{10}$
$12a_{1134}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6a_5^{-1}a_4^{-1}, a_4a_5a_6^{-1},$ $a_4a_5a_8^{-1}a_7^{-1}a_6^{-1}, a_6a_7a_8, a_6a_7a_8a_9a_{10}, a_8^{-1}a_9a_{10}, a_{10}^{-1}a_9^{-1}a_8a_{10}$
$12a_{1273}$	$a_1a_2^{-1}, a_1a_4a_3^{-1}a_2^{-1}, a_2a_3a_4^{-1}, a_2a_3a_6^{-1}a_5^{-1}a_4^{-1}, a_4a_5a_6,$ $a_4a_5a_6a_7a_8, a_6^{-1}a_7a_8, a_8^{-1}a_7^{-1}a_6a_9a_{10}, a_8^{-1}a_9a_{10}, a_{10}^{-1}a_9^{-1}a_8a_{10}$

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