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nth-order q-derivatives of Srivastava's General Triple q-hypergeometric Series with Respect to Parameters

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Abstract. We obtain q–derivatives of Srivastava's general triple q-hypergeometric series with respect to its parameters. The particular cases leading to results for three Srivastava's triple q–hypergeometric series $H_{A,q}$, $H_{B,q}$ and $H_{C,q}$ are also considered.

1. Introduction

This paper continues the study of q–derivatives of multivariable q–hypergeometric series with respect to its parameters. Earlier, Ancarani and Gasaneo [1, 2, 3] have studied derivatives of hypergeometric series with respect to its numerator and denominator parameters followed by the work of Ghany [5] for derivatives of q–hypergeometric series with respect to its parameters. The results obtained were useful in the study of Hahn polynomials and certain problems in mathematical physics. These works were extended to derivatives of Appell functions with respect to parameters and q-derivatives of 3-variable q-Lauricella functions [6] as well as k-variable q-Lauricella functions and q-Kampé de Fériet function by Sahai and Verma [7, 8]. In the present paper, we obtain the nth-order q–derivatives of Srivastava's general triple q-hypergeometric series with respect to its parameters. We also consider particular cases leading to results for three Srivastava's triple q-hypergeometric series $H_{A,q}$, $H_{B,q}$ and $H_{C,q}$.

A q-extension of the Srivastava and Daoust multivariable hypergeometric function was given by Srivastava [10, 11] as follows:

$$\Phi^{A:B^{(1)};\dots;B^{(n)}}_{C:D^{(1)};\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta^{(1)},\dots,\theta^{(n)}]:[(b^{(1)}):\phi^{(1)}];\dots;[b^{(n)}:\phi^{(n)}];\\ [(c):\psi^{(1)},\dots,\psi^{(n)}]:[(d^{(1)}):\delta^{(1)}];\dots;[d^{(n)}:\delta^{(n)}];\end{matrix}\; q; x_1,\dots,x_n\right)$$

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$$= \sum_{m_1,\ldots,m_n=0}^{\infty} \frac{\prod_{j=1}^{A}(a_j;q)_{m_1\theta_j^{(1)}+\cdots+m_n\theta_j^{(n)}} \prod_{j=1}^{B^{(1)}}(b_j^{(1)};q)_{m_1\phi_j^{(1)}} \cdots \prod_{j=1}^{B^{(n)}}(b_j^{(n)};q)_{m_n\phi_j^{(n)}}}{\prod_{j=1}^{C}(c_j;q)_{m_1\psi_j^{(1)}+\cdots+m_n\psi_j^{(n)}} \prod_{j=1}^{D^{(1)}}(d_j^{(1)};q)_{m_1\delta_j^{(1)}} \cdots \prod_{j=1}^{D^{(n)}}(d_j^{(n)};q)_{m_n\delta_j^{(n)}}} \prod_{i=1}^{n} \frac{{x_i}^{m_i}}{(q;q)_{m_i}} \tag{1.1}$$

where the arguments $x_1, \ldots, x_n$, the complex parameters

$$a_j, j = 1, \ldots, A;\; b_j^{(k)}, j = 1, \ldots, B_j^{(k)};$$
$$c_j, j = 1, \ldots, C;\; d_j^{(k)}, j = 1, \ldots, D_j^{(k)}; k = 1, \ldots, n,$$

and the associated coefficients

$$\theta_j^{(k)}, j = 1, \ldots, A; \qquad \phi_j^{(k)}, j = 1, \ldots, B^{(k)};$$
$$\psi_j^{(k)}, j = 1, \ldots, C; \qquad \delta_j^{(k)}, j = 1, \ldots, D^{(k)};\; k = 1, \ldots, n,$$

are so constrained that the multiple series (1.1) converges.

The function (1.1) embodies a large number of various q-series of one and several variables. A particular case of (1.1), namely

$$\Phi_{l;s_1;\ldots;s_n}^{p;q_1;\ldots;q_n}\left({}^{(a_p);(b_{q_1}^{(1)});\ldots;(b_{q_n}^{(n)});}_{(c_l);(d_{s_1}^{(1)});\ldots;(d_{s_n}^{(n)});} q; x_1, \ldots, x_n \right)$$
$$= \sum_{m_1,\ldots,m_n=0}^{\infty} \wedge(q, m_1, \ldots, m_n) \frac{{x_1}^{m_1}}{[m_1]_q!} \cdots \frac{{x_n}^{m_n}}{[m_n]_q!} \tag{1.2}$$

where

$$\wedge(q, m_1, \ldots, m_n) = \frac{\prod_{j=1}^{p}(a_j;q)_{m_1+\cdots+m_n} \prod_{j=1}^{q_1}(b_j^{(1)};q)_{m_1} \ldots \prod_{j=1}^{q_n}(b_j^{(n)};q)_{m_n}}{\prod_{j=1}^{l}(c_j;q)_{m_1+\cdots+m_n} \prod_{j=1}^{s_1}(d_j^{(1)};q)_{m_1} \ldots \prod_{j=1}^{s_n}(d_j^{(n)};q)_{m_n}}$$

which follows by taking the coefficients $\theta_j^{(k)}$, $\phi_j^{(k)}$, $\psi_j^{(k)}$, $\delta_j^{(k)}$, in (1.1) equal to 1, has already been studied for q-derivatives with respect to parameters by Sahai and Verma, [7].

In the present paper, the authors discuss the q-derivatives of a generalized q-series, which is a q-extension of Srivastava's general triple hypergeometric function [9], given by

$$F^{(3)}[q, x_1, x_2, x_3] = F^{(3)}\left[{}^{(a)\,::\,(b);\,(b')\,;\,(b'')\,:\,(c)\,;\,(c')\,;\,(c'')\,;}_{(e)\,::\,(g)\,;\,(g')\,;\,(g'')\,:\,(h);\,(h');\,(h'')\,;} q, x_1, x_2, x_3 \right]$$
$$= \sum_{m_1,m_2,m_3=0}^{\infty} \wedge(q, m_1, m_2, m_3) \prod_{i=1}^{3} \frac{{x_i}^{m_i}}{[m_i]_q!} \tag{1.3}$$

where

$$\wedge(q, m_1, m_2, m_3) = \frac{\prod_{j=1}^{A}(a_j;q)_{m_1+m_2+m_3}\prod_{j=1}^{B}(b_j;q)_{m_1+m_2}\prod_{j=1}^{B'}(b'_j;q)_{m_2+m_3}\prod_{j=1}^{B''}(b''_j;q)_{m_3+m_1}}{\prod_{j=1}^{E}(e_j;q)_{m_1+m_2+m_3}\prod_{j=1}^{G}(g_j;q)_{m_1+m_2}\prod_{j=1}^{G'}(g'_j;q)_{m_2+m_3}\prod_{j=1}^{G''}(g''_j;q)_{m_3+m_1}}$$

$$\times\frac{\prod_{j=1}^{C}(c_j;q)_{m_1}\prod_{j=1}^{C'}(c'_j;q)_{m_2}\prod_{j=1}^{C''}(c''_j;q)_{m_3}}{\prod_{j=1}^{H}(h_j;q)_{m_1}\prod_{j=1}^{H'}(h'_j;q)_{m_2}\prod_{j=1}^{H''}(h''_j;q)_{m_3}} \tag{1.4}$$

and (a) abbreviates the array of A parameters $a_1, a_2, \ldots, a_A$, etc and $(a;q)_n$ is the q-shifted factorial defined by [4]

$$(a;q)_n = \prod_{m=0}^{n-1}(1-aq^m), \qquad (a;q)_0 = 1.$$

It is interesting to point out that the series (1.3) is not a particular case of the series (1.1).

The q-derivative operator $D_{x,q}$ is defined for fixed q by

$$D_{x,q}f(x) = \frac{f(x)-f(xq)}{(1-q)x}, \qquad q \neq 1. \tag{1.5}$$

Indeed, as $q \to 1$, the q–derivative operator becomes ordinary differential operator, provided f is differentiable at x.

The q–multinomial coefficient is defined by

$$\begin{bmatrix} n \\ k_1, \ldots, k_n \end{bmatrix}_q = \frac{[n]_q!}{[k_1]_q! \ldots [k_n]_q! [n-(k_1+\cdots+k_n)]_q!}$$

for $k = 0, 1, \ldots, n$, where

$$[n]_q = \sum_{k=1}^{n} q^{k-1}, \qquad [0]_q = 0$$

and

$$[n]_q! = \prod_{k=1}^{n}[k]_q, \qquad [0]_q! = 1.$$

The q-analogues of three Srivastava's triple hypergeometric functions are defined as follows

$$H_{A,q}(a, b_1, b_2; c_1, c_2; q, x_1, x_2, x_3)$$

$$= \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a;q)_{m_1+m_3}(b_1;q)_{m_1+m_2}(b_2;q)_{m_2+m_3}}{(c_1;q)_{m_1}(c_2;q)_{m_2+m_3}\prod_{i=1}^{3}[m_i]_q!} x_1^{m_1}x_2^{m_2}x_3^{m_3}; \tag{1.6}$$

$$H_{B,q}(a,b_1,b_2;c_1,c_2,c_3;q,x_1,x_2,x_3)$$
$$= \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a;q)_{m_1+m_3}(b_1;q)_{m_1+m_2}(b_2;q)_{m_2+m_3}}{(c_1;q)_{m_1}(c_2;q)_{m_2}(c_3;q)_{m_3}\prod_{i=1}^{3}[m_i]_q!} x_1^{m_1}x_2^{m_2}x_3^{m_3}; \tag{1.7}$$

$$H_{C,q}(a,b_1,b_2;c;q,x_1,x_2,x_3)$$
$$= \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a;q)_{m_1+m_3}(b_1;q)_{m_1+m_2}(b_2;q)_{m_2+m_3}}{(c;q)_{m_1+m_2+m_3}\prod_{i=1}^{3}[m_i]_q!} x_1^{m_1}x_2^{m_2}x_3^{m_3}. \tag{1.8}$$

For brevity, we write (a^i) to denote array of parameters $a_1,\ldots,a_{i-1},a_{i+1},\ldots,a_A$, $i=1,\ldots,A$, and $(q^n a)$ to denote parameters $q^n a_1,\ldots,q^n a_A$, etc.

Also, we use the notation $F^{(3)}[q,q^n e_i,q^{n_1}x_1,q^{n_2}x_2]$ for the series

$$F^{(3)}\left[\begin{matrix}(a) :: (b);\ (b');\ (b'') : (c);\ (c');\ (c'');\\ (e^i),q^n e_i :: (g);\ (g');\ (g'') : (h);\ (h');\ (h'');\end{matrix} q,q^{n_1}x_1,q^{n_2}x_2,x_3\right].$$

2. Main Results

The nth-order q-derivatives of Srivastava's general triple q-hypergeometric function with respect to variable x_j, $j=1,2,3$, can be easily obtained. We have,

$$D_{x_1,q}^{(n)}F^{(3)}[q,ux_1,x_2,x_3]$$
$$= \frac{\prod_{i=1}^{A}(a_i;q)_n\prod_{i=1}^{B}(b_i;q)_n\prod_{i=1}^{B''}(b_i'';q)_n\prod_{i=1}^{C}(c_i;q)_n u^n}{\prod_{i=1}^{E}(e_i;q)_n\prod_{i=1}^{G}(g_i;q)_n\prod_{i=1}^{G''}(g_i'';q)_n\prod_{i=1}^{H}(h_i;q)_n(1-q)^n}$$
$$\times F^{(3)}\left[\begin{matrix}(q^n a) :: (q^n b);\ (b');\ (q^n b'') : (q^n c);\ (c');\ (c'');\\ (q^n e) :: (q^n g);\ (g');\ (q^n g'') : (q^n h);\ (h');\ (h'');\end{matrix} q,ux_1,x_2,x_3\right], \tag{2.1}$$

$$D_{x_2,q}^{(n)}F^{(3)}[q,x_1,ux_2,x_3]$$
$$= \frac{\prod_{i=1}^{A}(a_i;q)_n\prod_{i=1}^{B}(b_i;q)_n\prod_{i=1}^{B'}(b_i';q)_n\prod_{i=1}^{C'}(c_i';q)_n u^n}{\prod_{i=1}^{E}(e_i;q)_n\prod_{i=1}^{G}(g_i;q)_n\prod_{i=1}^{G'}(g_i';q)_n\prod_{i=1}^{H'}(h_i';q)_n(1-q)^n}$$
$$\times F^{(3)}\left[\begin{matrix}(q^n a) :: (q^n b);\ (q^n b');\ (b'') : (c);\ (q^n c');\ (c'');\\ (q^n e) :: (q^n g);\ (q^n g');\ (g'') : (h);\ (q^n h');\ (h'');\end{matrix} q,x_1,ux_2,x_3\right], \tag{2.2}$$

$$D_{x_3,q}^{(n)}F^{(3)}[q,x_1,x_2,ux_3]$$
$$= \frac{\prod_{i=1}^{A}(a_i;q)_n\prod_{i=1}^{B'}(b_i';q)_n\prod_{i=1}^{B''}(b_i'';q)_n\prod_{i=1}^{C''}(c_i'';q)_n u^n}{\prod_{i=1}^{E}(e_i;q)_n\prod_{i=1}^{G'}(g_i';q)_n\prod_{i=1}^{G''}(g_i'';q)_n\prod_{i=1}^{H''}(h_i'';q)_n(1-q)^n}$$
$$\times F^{(3)}\left[\begin{matrix}(q^n a) :: (b);\ (q^n b');\ (q^n b'') : (c);\ (c');\ (q^n c'');\\ (q^n e) :: (g);\ (q^n g');\ (q^n g'') : (h);\ (h');\ (q^n h'');\end{matrix} q,x_1,x_2,ux_3\right]. \tag{2.3}$$

In the following, we present q-derivatives of $F^{(3)}[q,x_1,x_2,x_3]$ with respect to a parameter in terms of its q-derivatives with respect to variables.

We start with the following lemmas whose proofs are straightforward computations [8].

Lemma 2.1. *If m_1, m_2, m_3 are non-negative integers, then*

$$\begin{aligned}[m_1+m_2+m_3]_q &= \frac{1}{3}\{(1+q^{m_3}+q^{m_2+m_3})[m_1]_q+(1+q^{m_1}+q^{m_3+m_1})[m_2]_q \\ &\quad +(1+q^{m_2}+q^{m_1+m_2})[m_3]_q\}. \end{aligned} \tag{2.4}$$

Equivalently,

$$\begin{aligned}[m_1+m_2+m_3]_q &= \frac{1}{3}\{(1+q^{m_2}+q^{m_2+m_3})[m_1]_q+(1+q^{m_3}+q^{m_3+m_1})[m_2]_q \\ &\quad +(1+q^{m_1}+q^{m_1+m_2})[m_3]_q\}. \end{aligned} \tag{2.5}$$

Let $N_j=\sum_{i=1}^{j} n_i$, where $j=1,2$.

Lemma 2.2. *If m_1, m_2, m_3, n_1, n_2 are non-negative integers, then for $N_2 \le n$*

$$\begin{aligned} &[m_1+m_2+m_3-n]_q \\ &= \frac{1}{q^{N_2m_1+n_1m_2}+q^{(n-n_2)m_2+(n-N_2)m_3}+q^{(n-n_1)m_3+n_2m_1}} \\ &\Big[(q^{N_2m_1+n_1m_2}+q^{(n+1-n_2)m_2+(n+1-N_2)m_3-N_2}+q^{(n+1-n_1)m_3+n_2m_1-n_1})[m_1-(n-N_2)]_q \\ &+(q^{(N_2+1)m_1+n_1m_2-n+N_2}+q^{(n-n_2)m_2+(n-N_2)m_3}+q^{(n+1-n_1)m_3+(n_2+1)m_1-n+n_2})[m_2-n_2]_q \\ &+(q^{(N_2+1)m_1+(n_1+1)m_2-n+n_1}+q^{(n+1-n_2)m_2+(n-N_2)m_3-n_2}+q^{(n-n_1)m_3+n_2m_1})[m_3-n_1]_q\Big]. \end{aligned} \tag{2.6}$$

Lemma 2.3. *If m_1, m_2, m_3, n_1, n_2 are non-negative integers, then for $N_2 \le n$*

$$\begin{aligned} &[m_1+m_2+m_3+n]_q \\ &= \frac{1}{q^{N_2m_1+n_1m_2}+q^{(n-n_2)m_2+(n-N_2)m_3}+q^{(n-n_1)m_3+n_2m_1}} \\ &\Big[(q^{N_2m_1+n_1m_2}+q^{(n+1-n_2)m_2+(n+1-N_2)m_3+N_2}+q^{(n+1-n_1)m_3+n_2m_1+n_1})[m_1+(n-N_2)]_q \\ &+(q^{(N_2+1)m_1+n_1m_2+n-N_2}+q^{(n-n_2)m_2+(n-N_2)m_3}+q^{(n+1-n_1)m_3+(n_2+1)m_1+n-n_2})[m_2+n_2]_q \\ &+(q^{(N_2+1)m_1+(n_1+1)m_2+n-n_1}+q^{(n+1-n_2)m_2+(n-N_2)m_3+n_2}+q^{(n-n_1)m_3+n_2m_1})[m_3+n_1]_q\Big]. \end{aligned} \tag{2.7}$$

Theorem 2.4. *The following nth-order $q-$derivative formula holds true for $F^{(3)}[q,x_1,x_2,x_3]$:*

$$D_{a_i,q}^{(n)} F^{(3)}[q,x_1,x_2,x_3] = \frac{(-1)^n q^{\binom{n}{2}}}{3(a_i;q)_n} \sum_{N_2 \le n} \begin{bmatrix} n \\ n_1, n_2 \end{bmatrix}_{q^{-1}} x_1^{n-N_2} x_2^{n_2} x_3^{n_1} D_{x_1,q}^{(n-N_2)} D_{x_2,q}^{(n_2)}$$

$$D^{(n_1)}_{x_3,q}\Big\{F^{(3)}[q,q^{N_2}x_1,q^{n_1}x_2]+F^{(3)}[q,q^{n-n_2}x_2,q^{n-N_2}x_3]+F^{(3)}[q,q^{n_2}x_1,q^{n-n_1}x_3]\Big\},$$
(2.8)

where $i=1,\dots,A$.

Proof. We first prove the result for $n=1$.

$$\begin{aligned}&D_{a_i,q}\,F^{(3)}[q,x_1,x_2,x_3]\\ =\ & -\frac{1}{(1-a_i)}\sum_{m_1,m_2,m_3=0}^{\infty}[m_1+m_2+m_3]_q\times\frac{\wedge(q,m_1,\,m_2,\,m_3)}{\prod_{i=1}^{3}[m_i]_q!}x_1^{m_1}x_2^{m_2}x_3^{m_3}\end{aligned}$$

Using (2.4) of Lemma 2.1, this gives

$$\begin{aligned}&D_{a_i,q}F^{(3)}[q,x_1,x_2,x_3]\\ =\ & -\frac{1}{3(1-a_i)}\Big\{x_1D_{x_1,q}\Big(F^{(3)}[q,x_1,x_2,x_3]+F^{(3)}[q,x_1,x_2,qx_3]+F^{(3)}[q,x_1,qx_2,qx_3]\Big)\\ &+x_2D_{x_2,q}\Big(F^{(3)}[q,x_1,x_2,x_3]+F^{(3)}[q,qx_1,x_2,x_3]+F^{(3)}[q,qx_1,x_2,qx_3]\Big)\\ &+x_3D_{x_3,q}\Big(F^{(3)}[q,x_1,x_2,x_3]+F^{(3)}[q,x_1,qx_2,x_3]+F^{(3)}[q,qx_1,qx_2,x_3]\Big)\Big\}\end{aligned}$$

which equals R. H. S of (2.8) for $n=1$. To prove the general case, we apply induction. Let the result be true for $n=m$, then for $n=m+1$ we have:

$$\begin{aligned}&D^{(m+1)}_{a_i,q}F^{(3)}[q,x_1,x_2,x_3]\\ =\ & D_{a_i,q}\frac{(-1)^m q^{\binom{m}{2}}}{3(a_i;q)_m}\sum_{N_2\le m}\begin{bmatrix}m\\ n_1,n_2\end{bmatrix}_{q^{-1}}x_1^{m-N_2}x_2^{n_2}x_3^{n_1}D^{(m-N_2)}_{x_1,q}D^{(n_2)}_{x_2,q}\\ &D^{(n_1)}_{x_3,q}\Big\{F^{(3)}[q,q^{N_2}x_1,q^{n_1}x_2]+F^{(3)}[q,q^{m-n_2}x_2,q^{m-N_2}x_3]+F^{(3)}[q,q^{n_2}x_1,q^{m-n_1}x_3]\Big\}\\ =\ & \frac{(-1)^m\,q^{\binom{m}{2}}}{3}\sum_{N_2\le m}\sum_{m_1,m_2,m_3=0}^{\infty}\begin{bmatrix}m\\ n_1,n_2\end{bmatrix}_{q^{-1}}\\ &\times\Big\{[(q^{N_2m_1+n_1m_2}+q^{(m-n_2)m_2+(m-N_2)m_3}+q^{(m-n_1)m_3+n_2m_1})M_{1,m-N_2-1}\\ &M_{2,n_2-1}M_{3,n_1-1}]\times D_{a_i,q}\frac{\wedge(q,m_1,m_2,m_3)}{(a_i;q)_m}\prod_{i=1}^{3}\frac{x_i^{m_i}}{[m_i]_q!}\Big\}\end{aligned}$$

where $M_{i,l}=[m_i]_q[m_i-1]_q\cdots[m_i-l]_q$. This gives

$$\begin{aligned}&D^{(m+1)}_{a_i,q}F^{(3)}[q,x_1,x_2,x_3]\\ =\ & \frac{(-1)^{m+1}q^{\binom{m+1}{2}}}{3(a_i;q)_{m+1}}\sum_{N_2\le m}\sum_{m_1,m_2,m_3=0}^{\infty}\begin{bmatrix}m\\ n_1,n_2\end{bmatrix}_{q^{-1}}\Big\{[(q^{N_2m_1+n_1m_2}\\ &+q^{(m-n_2)m_2+(m-N_2)m_3}+q^{(m-n_1)m_3+n_2m_1})M_{1,m-N_2-1}M_{2,n_2-1}M_{3,n_1-1}]\\ &\times[m_1+m_2+m_3-m]_q\frac{\wedge(q,m_1,\,m_2,\,m_3)}{\prod_{i=1}^{3}[m_i]_q!}x_1^{m_1}x_2^{m_2}x_3^{m_3}\Big\}\end{aligned}$$

Using Lemma 2.2, this leads to

$$
\begin{aligned}
&D_{a_i,q}^{(m+1)} F^{(3)}[q,x_1,x_2,x_3] \\
= \;& \frac{(-1)^{m+1} q^{\binom{m+1}{2}}}{3(a_i;q)_{m+1}} \sum_{N_2 \le m} \sum_{m_1,m_2,m_3=0}^{\infty} \begin{bmatrix} m \\ n_1, n_2 \end{bmatrix}_{q^{-1}} \Big\{ [(q^{N_2 m_1 + n_1 m_2} \\
&+ q^{(m+1-n_2)m_2 + (m+1-N_2)m_3 - N_2} + q^{(m+1-n_1)m_3 + n_2 m_1 - n_1}) M_{1,m-N_2} M_{2,n_2-1} M_{3,n_1-1} \\
&+ (q^{(N_2+1)m_1 + n_1 m_2 - m + N_2} + q^{(m-n_2)m_2 + (m-N_2)m_3} + q^{(m+1-n_1)m_3 + (n_2+1)m_1 - m + n_2}) \\
&\times M_{1,m-N_2-1} M_{2,n_2} M_{3,n_1-1} + (q^{(N_2+1)m_1 + (n_1+1)m_2 - m + n_1} + q^{(m+1-n_2)m_2 + (m-N_2)m_3 - n_2} \\
&+ q^{(m-n_1)m_3 + n_2 m_1}) M_{1,m-N_2-1} M_{2,n_2-1} M_{3,n_1}] \times \frac{\wedge(q,m_1,\ m_2,\ m_3)}{\prod_{i=1}^{3} [m_i]_q!} x_1^{m_1} x_2^{m_2} x_3^{m_3} \Big\} \\
= \;& \frac{(-1)^{m+1} q^{\binom{m+1}{2}}}{3(a_i;q)_{m+1}} \sum_{m_1,m_2,m_3=0}^{\infty} \sum_{N_2 \le m+1} \frac{[m]_{q^{-1}}!}{[n_1]_{q^{-1}}! [n_2]_{q^{-1}}! [m+1-N_2]_{q^{-1}}!} \Big\{ [([m+1-N_2]_{q^{-1}} \\
&+ [n_2]_{q^{-1}} q^{-m+N_2-1} + [n_1]_{q^{-1}} q^{-m+n_1-1}) q^{N_2 m_1 + n_1 m_2} + ([m+1-N_2]_{q^{-1}} q^{-N_2} + [n_2]_{q^{-1}} \\
&+ [n_1]_{q^{-1}} q^{-n_2}) q^{(m-n_2+1)m_2 + (m-N_2+1)m_3} + ([m+1-N_2]_{q^{-1}} q^{-n_1} + [n_2]_{q^{-1}} q^{-m+n_2-1} \\
&+ [n_1]_{q^{-1}}) q^{(m-n_1+1)m_3 + n_2 m_1}] M_{1,m-N_2} M_{2,n_2-1} M_{3,n_1-1} \Big\} \times \frac{\wedge(q,m_1,\ m_2,\ m_3)}{\prod_{i=1}^{3} [m_i]_q!} x_1^{m_1} x_2^{m_2} x_3^{m_3} \\
= \;& \frac{(-1)^{m+1} q^{\binom{m+1}{2}}}{3(a_i;q)_{m+1}} \sum_{m_1,m_2,m_3=0}^{\infty} \sum_{N_2 \le m+1} \begin{bmatrix} m+1 \\ n_1, n_2 \end{bmatrix}_{q^{-1}} (q^{N_2 m_1 + n_1 m_2} \\
&+ q^{(m-n_2+1)m_2 + (m-N_2+1)m_3} + q^{(m-n_1+1)m_3 + n_2 m_1}) M_{1,m-N_2} M_{2,n_2-1} M_{3,n_1-1} \\
&\times \frac{\wedge(q,m_1,\ m_2,\ m_3)}{\prod_{i=1}^{3} [m_i]_q!} x_1^{m_1} x_2^{m_2} x_3^{m_3}
\end{aligned}
$$

which equals R. H. S of (2.8) for $n = m+1$. This completes the proof. □

Note that if we use (2.5) in place of (2.4), we will get the following expression for $D_{a_i,q}^{(n)} F^{(3)}[q,x_1,x_2,x_3]$:

$$
\begin{aligned}
&D_{a_i,q}^{(n)} F^{(3)}[q,x_1,x_2,x_3] \\
&= \frac{(-1)^n q^{\binom{n}{2}}}{3(a_i;q)_n} \sum_{N_2 \le n} \begin{bmatrix} n \\ n_1, n_2 \end{bmatrix}_{q^{-1}} x_1^{n-N_2} x_2^{n_2} x_3^{n_1} D_{x_1,q}^{(n-N_2)} D_{x_2,q}^{(n_2)} \\
&D_{x_3,q}^{(n_1)} \Big\{ F^{(3)}[q, q^{N_2} x_1, q^{n_2} x_3] + F^{(3)}[q, q^{n_1} x_1, q^{n-n_2} x_2] \\
&+ F^{(3)}[q, q^{n-N_2} x_2, q^{n-n_1} x_3] \Big\}.
\end{aligned} \tag{2.9}
$$

We now give the q-derivatives of $F^{(3)}[q,x_1,x_2,x_3]$ with respect to remaining numerator parameters. The proofs are omitted.

Theorme 2.5. *The following nth-order q-derivative formulas hold true for* $F^{(3)}[q,x_1,x_2,x_3]$:

$$
D_{b_i,q}^{(n)} F^{(3)}[q,x_1,x_2,x_3] = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b_i;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_1^{n-n_1} x_2^{n_1}
$$

$$D_{x_1,q}^{(n-n_1)} D_{x_2,q}^{(n_1)} \Big\{ F^{(3)}[q, q^{n_1} x_1] + F^{(3)}[q, q^{n-n_1} x_2] \Big\}, \tag{2.10}$$

where $i = 1, \ldots, B$;

$$D_{b'_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b'_i; q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_2^{n-n_1} x_3^{n_1}$$
$$D_{x_2,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ F^{(3)}[q, q^{n_1} x_2] + F^{(3)}[q, q^{n-n_1} x_3] \Big\}, \tag{2.11}$$

where $i = 1, \ldots, B'$;

$$D_{b''_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b''_i; q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_1^{n-n_1} x_3^{n_1}$$
$$D_{x_1,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ F^{(3)}[q, q^{n_1} x_1] + F^{(3)}[q, q^{n-n_1} x_3] \Big\}, \tag{2.12}$$

where $i = 1, \ldots, B''$.

Theorem 2.6. *The following nth-order q-derivative formulas hold true for* $F^{(3)}[q, x_1, x_2, x_3]$:

$$D_{c_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{(-1)^n q^{\binom{n}{2}} x_1^n}{(c_i; q)_n} D_{x_1,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3], i = 1, \ldots, C; \tag{2.13}$$

(2.14)

$$D_{c'_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{(-1)^n q^{\binom{n}{2}} x_2^n}{(c'_i; q)_n} D_{x_2,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3], i = 1, \ldots, C';$$

(2.15)

$$D_{c''_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{(-1)^n q^{\binom{n}{2}} x_3^n}{(c''_i; q)_n} D_{x_3,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3], i = 1, \ldots, C''.$$

Theorem 2.7. *The following nth-order* $q-$*derivative formula holds true for* $F^{(3)}[q, x_1, x_2, x_3]$:

$$D_{e_i,q}^{(n)} F^{(3)}[q, x_1, x_2, x_3] = \frac{1}{3\,(e_i; q)_n} \sum_{N_2 \le n} \begin{bmatrix} n \\ n_1, n_2 \end{bmatrix}_q (x_1 x_2 x_3 D_{x_1}^{(n-N_2)} D_{x_2}^{(n_2)} D_{x_3}^{(n_1)}$$
$$x_1^{n-N_2-1} x_2^{n_2-1} x_3^{n_1-1}) \Big\{ F^{(3)}[q, q^n e_i, q^{N_2} x_1, q^{n_1} x_2] + F^{(3)}[q, q^n e_i, q^{n-n_2} x_2, q^{n-N_2} x_3]$$
$$+ F^{(3)}[q, q^n e_i, q^{n_2} x_1, q^{n-n_1} x_3] \Big\},$$

(2.16)

where $i = 1, \ldots, E$.

Proof. We first prove the result for $n = 1$. Clearly

$$\begin{aligned} & D_{e_i,q}\, F^{(3)}[q, x_1, x_2, x_3] \\ = \; & \frac{1}{(1-e_i)} \sum_{m_1,m_2,m_3=0}^{\infty} [m_1+m_2+m_3]_q \frac{(e_i;q)_{m_1+m_2+m_3}}{(e_i q;q)_{m_1+m_2+m_3}} \\ & \times \wedge (q, m_1, m_2, m_3) \prod_{i=1}^{3} \frac{x_i^{m_i}}{[m_i]_q!}. \end{aligned}$$

By Lemma 2.1, this gives

$$\begin{aligned} & D_{e_i,q}\, F^{(3)}[q, x_1, x_2, x_3] = \frac{1}{3(1-e_i)} \\ & \times \Big\{ x_1 D_{x_1,q} \left(F^{(3)}[q, qe_i, x_1, x_2, x_3] + F^{(3)}[q, qe_i, x_1, x_2, qx_3] + F^{(3)}[q, qe_i, x_1, qx_2, qx_3] \right) \\ & + x_2 D_{x_2,q} \left(F^{(3)}[q, qe_i, x_1, x_2, x_3] + F^{(3)}[q, qe_i, qx_1, x_2, x_3] + F^{(3)}[q, qe_i, qx_1, x_2, qx_3] \right) \\ & + x_3 D_{x_3,q} \left(F^{(3)}[q, qe_i, x_1, x_2, x_3] + F^{(3)}[q, qe_i, x_1, qx_2, x_3] + F^{(3)}[q, qe_i, qx_1, qx_2, x_3] \right) \Big\}, \end{aligned}$$

which equals R.H.S. of (2.16) for $n = 1$. Assuming that the result be true for $n = m$, we have

$$\begin{aligned} & D_{e_i,q}^{(m+1)} F^{(3)}[q, x_1, x_2, x_3] \\ = \; & D_{e_i,q} \frac{1}{3\,(e_i;q)_m} \sum_{N_2 \le m} \begin{bmatrix} m \\ n_1, n_2 \end{bmatrix}_q (x_1 x_2 x_3 D_{x_1}^{(m-N_2)} D_{x_2}^{(n_2)} D_{x_3}^{(n_1)} \\ & x_1^{m-N_2-1} x_2^{n_2-1} x_3^{n_1-1}) \Big\{ F^{(3)}[q, q^m e_i, q^{n_1+n_2} x_1, q^{n_1} x_2] \\ & + F^{(3)}[q, q^m e_i, q^{n-n_2} x_2, q^{m-N_2} x_3] + F^{(3)}[q, q^m e_i, q^{n_2} x_1, q^{n-n_1} x_3] \Big\} \\ = \; & \frac{1}{3} \sum_{N_2 \le m} \sum_{m_1,m_2,m_3=0}^{\infty} \begin{bmatrix} m \\ n_1, n_2 \end{bmatrix}_q \Big\{ (q^{N_2 m_1 + n_1 m_2} + q^{(m-n_2)m_2 + (m-N_2)m_3} \\ & + q^{(m-n_1)m_3 + n_2 m_1}) \overline{M}_{1,m-N_2-1} \overline{M}_{2,n_2-1} \overline{M}_{3,n_1-1} \Big\} \times D_{e_i,q} \frac{\wedge(q, m_1, m_2, m_3)}{(e_i;q)_m} \\ & \times \frac{(e_i;q)_{m_1+m_2+m_3}}{(e_i q^m;q)_{m_1+m_2+m_3}} \prod_{i=1}^{3} \frac{x_i^{m_i}}{[m_i]_q!}. \end{aligned}$$

where $\overline{M}_{i,l} = [m_i]_q [m_i+1]_q \cdots [m_i+l]_q$. This gives

$$\begin{aligned} & D_{e_i,q}^{(m+1)} F^{(3)}[q, x_1, x_2, x_3] \\ = \; & \frac{1}{3(e_i;q)_{m+1}} \sum_{N_2 \le m} \sum_{m_1,m_2,m_3=0}^{\infty} \begin{bmatrix} m \\ n_1, n_2 \end{bmatrix}_q \Big\{ (q^{N_2 m_1 + n_1 m_2} + q^{(m-n_2)m_2 + (m-N_2)m_3} \end{aligned}$$

$$+q^{(m-n_1)m_3+n_2m_1})\overline{M}_{1,m-N_2-1}\overline{M}_{2,n_2-1}\overline{M}_{3,n_1-1}\Big\}\times[m_1+m_2+m_3+m]_q$$
$$\times\wedge(q,m_1,m_2,m_3)\frac{(e_i;q)_{m_1+m_2+m_3}}{(q^{m+1}e_i;q)_{m_1+m_2+m_3}}\prod_{i=1}^{3}\frac{x_i^{m_i}}{[m_i]_q!}.$$

By Lemma 2.3, we get

$$\begin{aligned}
&D_{e_i,q}^{(m+1)}F^{(3)}[q,x_1,x_2,x_3]\\
=\;&\frac{1}{3(e_i;q)_{m+1}}\sum_{N_2\le m}\sum_{m_1,m_2,m_3=0}^{\infty}\begin{bmatrix}m\\n_1,n_2\end{bmatrix}_q\Big\{(q^{N_2m_1+n_1m_2}\\
&+q^{(m+1-n_2)m_2+(m+1-N_2)m_3+N_2}+q^{(m+1-n_1)m_3+n_2m_1+n_1})\overline{M}_{1,m-N_2}\\
&\times\overline{M}_{2,n_2-1}\overline{M}_{3,n_1-1}+(q^{(N_2+1)m_1+n_1m_2+m-N_2}+q^{(m-n_2)m_2+(m-N_2)m_3}\\
&+q^{(m+1-n_1)m_3+(n_2+1)m_1+m-n_2})\overline{M}_{1,m-N_2-1}\overline{M}_{2,n_2}\overline{M}_{3,n_1-1}\\
&+(q^{(N_2+1)m_1+(n_1+1)m_2+m-n_1}+q^{(m+1-n_2)m_2+(m-N_2)m_3+n_2}\\
&+q^{(m-n_1)m_3+n_2m_1})\overline{M}_{1,m-N_2-1}\overline{M}_{2,n_2-1}\overline{M}_{3,n_1}\Big\}\\
&\times\wedge(q,m_1,m_2,m_3)\frac{(e_i;q)_{m_1+m_2+m_3}}{(q^{m+1}e_i;q)_{m_1+m_2+m_3}}\prod_{i=1}^{3}\frac{x_i^{m_i}}{[m_i]_q!}.\\
=\;&\frac{1}{3(e_i;q)_{m+1}}\sum_{m_1,m_2,m_3=0}^{\infty}\Big\{\sum_{N_2\le m+1}\frac{[m]_q![m+1-N_2]_q}{[n_1]_q![n_2]_q![m+1-N_2]_q!}(q^{N_2m_1+n_1m_2}\\
&+q^{(m+1-n_2)m_2+(m+1-N_2)m_3+N_2}+q^{(m+1-n_1)m_3+n_2m_1+n_1})\overline{M}_{1,m-N_2}\overline{M}_{2,n_2-1}\\
&\times\overline{M}_{3,n_1-1}+\sum_{N_2\le m+1}\frac{[m]_q!}{[n_1]_q![n_2-1]_q![m+1-N_2]_q!}(q^{(N_2m_1+n_1m_2+m-N_2+1)}\\
&+q^{((m+1-n_2)m_2+(m-N_2+1)m_3)}+q^{(m+1-n_1)m_3+n_2m_1+m+1-n_2})\overline{M}_{1,m-N_2}\overline{M}_{2,n_2-1}\\
&\times\overline{M}_{3,n_1-1}+\sum_{N_2\le m+1}\frac{[m]_q!}{[n_1-1]_q![n_2]_q![m+1-N_2]_q!}(q^{(N_2m_1+n_1m_2+m-n_1+1)}\\
&+q^{(m+1-n_2)m_2+(m-N_2+1)m_3+n_2}+q^{(m-n_1+1)m_3+n_2m_1})\overline{M}_{1,m-N_2}\overline{M}_{2,n_2-1}\\
&\times\overline{M}_{3,n_1-1}\Big\}\times\wedge(q,m_1,m_2,m_3)\frac{(e_i;q)_{m_1+m_2+m_3}}{(q^{m+1}e_i;q)_{m_1+m_2+m_3}}\prod_{i=1}^{3}\frac{x_i^{m_i}}{[m_i]_q!}.\\
=\;&\frac{1}{3(e_i;q)_{m+1}}\sum_{m_1,m_2,m_3=0}^{\infty}\sum_{N_2\le m+1}\begin{bmatrix}m+1\\n_1,n_2\end{bmatrix}_q(q^{N_2m_1+n_1m_2}\\
&+q^{(m-n_2+1)m_2+(m-N_2+1)m_3}+q^{(m-n_1+1)m_3+n_2m_1})\overline{M}_{1,m-N_2}\overline{M}_{2,n_2-1}\overline{M}_{3,n_1-1}\\
&\times\wedge(q,m_1,m_2,m_3)\frac{(e_i;q)_{m_1+m_2+m_3}}{(q^{m+1}e_i;q)_{m_1+m_2+m_3}}\prod_{i=1}^{3}\frac{x_i^{m_i}}{[m_i]_q!}.
\end{aligned}$$

which equals R.H.S of (2.16) for $n = m + 1$. This completes the proof of (2.16).

Note that if we use (2.5) instead of (2.4), we will arrive at the following expression for $D^{(n)}_{e_i,q}F^{(3)}[q,x_1,x_2,x_3]$ given by

$$\begin{aligned} &D^{(n)}_{e_i,q}F^{(3)}[q,x_1,x_2,x_3] \\ = \ &\frac{1}{3\,(e_i;q)_n}\sum_{N_2\le n}\begin{bmatrix} n \\ n_1,n_2\end{bmatrix}_q (x_1x_2x_3D^{(n-N_2)}_{x_1}D^{(n_2)}_{x_2}D^{(n_1)}_{x_3}x_1^{n-N_2-1}x_2^{n_2-1} \\ &x_3^{n_1-1})\Big\{F^{(3)}[q,q^ne_i,q^{N_2}x_1,q^{n_2}x_3]+F^{(3)}[q,q^ne_i,q^{n_1}x_1,q^{n-n_2}x_2] \\ &+F^{(3)}[q,q^ne_i,q^{n-N_2}x_2,q^{n-n_1}x_3]\Big\}, \end{aligned} \tag{2.17}$$

We now give the $q-$derivatives of $F^{(3)}[q,x_1,x_2,x_3]$ with respect to remaining denominator parameters. The proofs are omitted.

Theorem 2.8. *The following nth-order $q-$derivative formulas hold true for* $F^{(3)}[q,x_1,x_2,x_3]$:

$$\begin{aligned} D^{(n)}_{g_i,q}F^{(3)}[q,x_1,x_2,x_3] &= \frac{1}{2\,(g_i;q)_n}\sum_{n_1\le n}\begin{bmatrix} n \\ n_1\end{bmatrix}_q x_1x_2D^{(n-n_1)}_{x_1}D^{(n_1)}_{x_2}x_1^{n-n_1-1} \\ &x_2^{n_1-1}\Big\{F^{(3)}[q,q^ng_i,q^{n_1}x_1]+F^{(3)}[q,q^ng_i,q^{n-n_1}x_2]\Big\}, \end{aligned} \tag{2.18}$$

where $i = 1,\ldots,G$;

$$\begin{aligned} D^{(n)}_{g'_i,q}F^{(3)}[q,x_1,x_2,x_3] &= \frac{1}{2\,(g'_i;q)_n}\sum_{n_1\le n}\begin{bmatrix} n \\ n_1\end{bmatrix}_q x_2x_3D^{(n-n_1)}_{x_2}D^{(n_1)}_{x_3}x_2^{n-n_1-1} \\ &x_3^{n_1-1}\Big\{F^{(3)}[q,q^ng'_i,q^{n_1}x_2]+F^{(3)}[q,q^ng'_i,q^{n-n_1}x_3]\Big\}, \end{aligned} \tag{2.19}$$

where $i = 1,\ldots,G'$;

$$\begin{aligned} D^{(n)}_{g''_i,q}F^{(3)}[q,x_1,x_2,x_3] &= \frac{1}{2\,(g''_i;q)_n}\sum_{n_1\le n}\begin{bmatrix} n \\ n_1\end{bmatrix}_q x_3x_1D^{(n-n_1)}_{x_3}D^{(n_1)}_{x_1}x_3^{n-n_1-1} \\ &x_1^{n_1-1}\Big\{F^{(3)}[q,q^ng''_i,q^{n_1}x_3]+F^{(3)}[q,q^ng''_i,q^{n-n_1}x_1]\Big\}, \end{aligned} \tag{2.20}$$

where $i = 1,\ldots,G''$;

Theorem 2.9. *The following nth-order $q-$derivative formulas hold true for* $F^{(3)}[q,x_1,x_2,x_3]$:

$$D^{(n)}_{h_j,q}F^{(3)}[q,x_1,x_2,x_3] = \frac{x_1}{(h_j;q)_n}D^{(n)}_{x_1,q}\Big\{x_1^{n-1}F^{(3)}[q,q^nh_j]\Big\}, \tag{2.21}$$

where $j=1,\ldots,H$;

$$D^{(n)}_{h'_j,q}\,F^{(3)}[q,x_1,x_2,x_3] \;=\; \frac{x_2}{\left(h'_j;q\right)_n} D^{(n)}_{x_2,q}\Big\{x_2^{n-1}F^{(3)}[q,q^n h'_j]\Big\}, \tag{2.22}$$

where $j=1,\ldots,H'$;

$$D^{(n)}_{h''_j,q}\,F^{(3)}[q,x_1,x_2,x_3] \;=\; \frac{x_3}{\left(h''_j;q\right)_n} D^{(n)}_{x_3,q}\Big\{x_3^{n-1}F^{(3)}[q,q^n h''_j]\Big\}, \tag{2.23}$$

where $j=1,\ldots,H''$.

The $q-$derivatives of the three Srivastava's triple $q-$hypergeometric functions with respect to their parameters can now be deduced from the above results. Indeed, we have

Theorem 2.10. *The following nth-order* $q-$*derivative formulas hold true for* $H_{A,q}$:

$$\begin{aligned} D^{(n)}_{a,q}\,H_{A,q} \;&=\; \frac{(-1)^n\,q^{\binom{n}{2}}}{2\,(a;q)_n}\sum_{n_1\le n}\begin{bmatrix} n\\ n_1\end{bmatrix}_{q^{-1}} x_1^{n-n_1}x_3^{n_1}\\ &\qquad D^{(n-n_1)}_{x_1,q}D^{(n_1)}_{x_3,q}\Big\{H_{A,q}(q^{n_1}x_1)+H_{A,q}(q^{n-n_1}x_3)\Big\}, \end{aligned} \tag{2.24}$$

$$\begin{aligned} D^{(n)}_{b_1,q}\,H_{A,q} \;&=\; \frac{(-1)^n\,q^{\binom{n}{2}}}{2\,(b_1;q)_n}\sum_{n_1\le n}\begin{bmatrix} n\\ n_1\end{bmatrix}_{q^{-1}} x_1^{n-n_1}x_2^{n_1}\\ &\qquad D^{(n-n_1)}_{x_1,q}D^{(n_1)}_{x_2,q}\Big\{H_{A,q}(q^{n_1}x_1)+H_{A,q}(q^{n-n_1}x_2)\Big\}, \end{aligned} \tag{2.25}$$

$$\begin{aligned} D^{(n)}_{b_2,q}\,H_{A,q} \;&=\; \frac{(-1)^n\,q^{\binom{n}{2}}}{2\,(b_2;q)_n}\sum_{n_1\le n}\begin{bmatrix} n\\ n_1\end{bmatrix}_{q^{-1}} x_2^{n-n_1}x_3^{n_1}\\ &\qquad D^{(n-n_1)}_{x_2,q}D^{(n_1)}_{x_3,q}\Big\{H_{A,q}(q^{n_1}x_2)+H_{A,q}(q^{n-n_1}x_3)\Big\}, \end{aligned} \tag{2.26}$$

$$D^{(n)}_{c_1,q}\,H_{A,q} \;=\; \frac{x_1}{(c_1;q)_n}D^{(n)}_{x_1,q}\{x_1^{n-1}H_{A,q}(q^n c_1)\}, \tag{2.27}$$

$$\begin{aligned} D^{(n)}_{c_2,q}\,H_{A,q} \;&=\; \frac{1}{2\,(c_2;q)_n}\sum_{n_1\le n}\begin{bmatrix} n\\ n_1\end{bmatrix}_{q} x_2x_3D^{(n-n_1)}_{x_2}D^{(n_1)}_{x_3}x_2^{n-n_1-1}\\ &\qquad x_3^{n_1-1}\{H_{A,q}(q^n c_2,q^{n_1}x_2)+H_{A,q}(q^n c_2,q^{n-n_1}x_3)\}. \end{aligned} \tag{2.28}$$

where $H_{A,q}(q^n x_1)$ *stands for* $H_{A,q}(a,b_1,b_2;c_1,c_2;q,q^n x_1,x_2,x_3)$ *etc.*

Theorem 2.11. *The following nth-order $q-$derivative formulas hold true for $H_{B,q}$:*

$$D_{a,q}^{(n)} H_{B,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(a;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_1^{n-n_1} x_3^{n_1} D_{x_1,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ H_{B,q}(q^{n_1}x_1) + H_{B,q}(q^{n-n_1}x_3) \Big\}, \tag{2.29}$$

$$D_{b_1,q}^{(n)} H_{B,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b_1;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_1^{n-n_1} x_2^{n_1} D_{x_1,q}^{(n-n_1)} D_{x_2,q}^{(n_1)} \Big\{ H_{B,q}(q^{n_1}x_1) + H_{B,q}(q^{n-n_1}x_2) \Big\}, \tag{2.30}$$

$$D_{b_2,q}^{(n)} H_{B,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b_2;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_2^{n-n_1} x_3^{n_1} D_{x_2,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ H_{B,q}(q^{n_1}x_2) + H_{B,q}(q^{n-n_1}x_3) \Big\}, \tag{2.31}$$

$$D_{c_j,q}^{(n)} H_{B,q} = \frac{x_j}{(c_j;q)_n} D_{x_j,q}^{(n)} \Big\{ x_j^{n-1} H_{B,q}(q^n c_j) \Big\}, j = 1,2,3, \tag{2.32}$$

where $H_{B,q}(q^n x_1)$ stands for $H_{B,q}(a, b_1, b_2; c_1, c_2, c_3; q, q^n x_1, x_2, x_3)$ etc.

Theorem 2.12. *The following nth-order $q-$derivative formulas hold true for $H_{C,q}$:*

$$D_{a,q}^{(n)} H_{C,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(a;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q} x_1^{n-n_1} x_3^{n_1} D_{x_1,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ H_{C,q}(q^{n_1}x_1) + H_{C,q}(q^{n-n_1}x_3) \Big\}, \tag{2.33}$$

$$D_{b_1,q}^{(n)} H_{C,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b_1;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_1^{n-n_1} x_2^{n_1} D_{x_1,q}^{(n-n_1)} D_{x_2,q}^{(n_1)} \Big\{ H_{C,q}(q^{n_1}x_1) + H_{C,q}(q^{n-n_1}x_2) \Big\}, \tag{2.34}$$

$$D_{b_2,q}^{(n)} H_{C,q} = \frac{(-1)^n q^{\binom{n}{2}}}{2\,(b_2;q)_n} \sum_{n_1 \le n} \begin{bmatrix} n \\ n_1 \end{bmatrix}_{q^{-1}} x_2^{n-n_1} x_3^{n_1} D_{x_2,q}^{(n-n_1)} D_{x_3,q}^{(n_1)} \Big\{ H_{C,q}(q^{n_1}x_2) + H_{C,q}(q^{n-n_1}x_3) \Big\}, \tag{2.35}$$

$$
\begin{aligned}
&D_{c,q}^{(n)} H_{C,q} \\
= \quad & \frac{1}{3\,(c;q)_n} \sum_{N_2 \le n} \begin{bmatrix} n \\ n_1, n_2 \end{bmatrix}_q x_1 x_2 x_3 D_{x_1}^{(n-N_2)} D_{x_2}^{(n_2)} D_{x_3}^{(n_1)} x_1^{n-N_2-1} \\
& x_2^{n_2-1} x_3^{n_1-1} \Big\{ H_{C,q}(q^n c, q^{n_1+n_2} x_1, q^{n_1} x_2) + H_{C,q}(q^n c, q^{n-n_2} x_2, q^{n-n_1-n_2} x_3) \\
& + H_{C,q}(q^n c, q^{n_2} x_1, q^{n-n_1} x_3) \Big\}. \qquad (2.36)
\end{aligned}
$$

where $H_{C,q}(q^n x_1)$ *stands for* $H_{C,q}(a, b_1, b_2; c; q, q^n x_1, x_2, x_3)$ *etc.*

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