Quantitative Comparisons on the Intrinsic Features of Foreign Exchange Rates Between the 1920s and the 2010s: Case of the USD-GBP Exchange Rate*

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This paper quantitatively compares the intrinsic features of the daily USD-GBP exchange rates in two different periods, the 1920s and the 2010s, under the same freely floating exchange rate system. Even though the foreign exchange markets in the 1920s seem to be much less organized and developed than in the 2010s, this paper finds that both the long memory volatility property and the structural break appear to be the common intrigue features of the exchange rates in the two periods by using the FIGARCH model. In particular, the long memory volatility properties in the two periods are found to be upward biased and overstated because of the structural breaks in the exchange markets. Thus this paper applies the Adaptive-FIGARCH model to consider the long memory volatility property and the structural breaks jointly. The main finding is that the structural breaks in the exchange markets affect the long memory volatility property significantly in the two periods but the degree of the long memory volatility property in the 1920s is reduced more remarkably than in the 2010s after the structural breaks are accounted for; thus implying that the structural breaks in the foreign exchange markets in the 1920s seem to be more significant.

I. INTRODUCTION

As pointed by Baillie and Bailey (1984), many economists have been fascinated for a long time with the floating exchange rates that occurred in the 1920s. In this context, the floating exchange rate in the 1920s appears to be very worthy of study because it provides a good opportunity to collaborate evidence from the current floating rate in the 2010s. In particular, the currency market in the early 1920s experienced one of the most turbulent periods in the history of foreign exchange markets as the markets adjusted to the post-WWI and non-gold standard conditions. The problems associated with the hyperinflation in Germany and the budget deficit in France spilled over to affect several neighboring currencies including the British Pound. Einzig (1937, 1962) has documented many of the main economic and political events of this period and their impacts on the currency markets. Thus, the period of the 1920s is a very interesting period of history since it is the earliest period of freely floating exchange rates that were remarkably turbulent because of the political and economic conditions in Europe, and it constitutes the other main source of information on the behavior of the floating exchange rates since it could be well documented from a data perspective (e.g. Matthews, 1986; Taylor and McMahon, 1988; Smith and Smith, 1990; Taylor, 1992; Baillie et al., 1993).

The exchange markets in the 1920s seem to be very different from those in the 2010s in several aspects. Although relatively little precise information is known about the extent of capital movements in the 1920s markets, it seems that there was a very low level of capital movements and arbitrage. Hence, the total volume of foreign exchange market transactions would be only marginally more than the volume of trade. And, the exchange markets in the 1920s were clearly less well organized and developed, and they were in the less sophisticated telecommunications system compared with the 2010s, which have more innovative market structures with more advanced computer technology and better developed financial instruments like options and futures. These facts distinguish the 1920s from the 2010s.

Despite the relatively primitive market conditions, the 1920s foreign exchange markets seem to be similar in character to the current markets in the 2010s in terms of the world economic situations. The world economy in the 1920s was recovering from the devastating effects of the post WWI with the turmoil of war reparations and hyperinflation in Germany (Baillie et al., 1993). This also led to concerted speculative attacks on various currencies. These situations in the 1920s are quite

similar to those in the 2010s which were overcoming the global financial crisis with a worldwide credit crunch caused by the collapse of the US subprime mortgage industry in 2007 so that most of exchange rates changed very volatilely in foreign exchange markets with severe speculations on several currencies occurred (Melvin and Taylor, 2009).

Hence, the main purpose of this paper is to quantitatively compare the intrinsic features of the exchange rates in the 2010s with those in the 1920s. For the comparison, this paper focuses on the two key features, the long memory volatility property and the structural breaks of the exchange rate returns in the periods of the 1920s and the 2010s. In particular, this paper uses the daily exchange rates of US Dollar (USD)-Great British Pound (GBP) which is globally traded in the both periods, in order to investigate the dynamics of the long memory volatility property and the structural breaks in the daily exchange returns. This analysis seems warranted for the reason that this issue has not been previously investigated and it is thus important to expand the range of empirical comparison studies.

The quantitative comparison in this paper finds that the daily USD-GBP exchange returns in the 1920s contain surprisingly similar intrinsic features to those in the 2010s in terms of the long memory volatility property and the structural breaks. First, the extreme turbulence in the markets is seen to induce the heavy tailed variance of unconditional returns in both the 1920s and the 2010s as studied by Koedijk et al. (1990). In particular, the daily USD-GBP exchange returns in the 1920s are found to exhibit the widespread long memory property in the volatility process of the exchange returns with quite persistent and hyperbolic decaying autocorrelations, which is extremely similar to that in the 2010s. In order to estimate the degree of the long memory volatility property of the exchange returns, this papers uses the FIGARH model of Baillie et al. (1996) as well as the GARCH model of Bollerslev (1986) for the comparison. The magnitude of the long memory volatility property in the daily USD-GBP exchange returns in the 1920s appears to be much greater than that in the 2010s.

Second, this paper finds that there exist several structural breaks in the daily USD-GBP exchange returns in the both periods of the 1920s and 2010s, which appear to be closely related to the long memory volatility property (Granger and Terasvirta, 1999; Diebold and Inoue, 2001). Some previous papers have suggested that the observed long memory property in conditional variance process may be generated by the presence of various types of structural breaks or regime switches,

and they have conjectured that the long memory persistence of the conditional variance process may be overstated due to the presence of the structural breaks. Also, they have suggested that the appropriate model for the conditional variance process of financial time series data should include both long memory property and structural breaks (Granger and Hyung, 2004; Morana and Beltratti, 2004; Martens et al., 2004; Choi and Zivot, 2007). In this context, the exchange returns in the 1920s is found to contain more significant structural breaks than in the 2010s, which implies that the structural breaks, which occurred more frequently in the foreign exchange markets in the 1920s seem to affect the long memory volatility property in the 1920s more significantly than in the 2010s. Thus the greater long memory volatility property in the exchange returns in the 1920s could be because of the more frequent structural breaks in the exchange markets in the 1920s.

   Thus, it could be necessary to consider both the structural breaks and the long memory property in the conditional variance process. This paper examines the two features, the structural breaks and the long memory property, together in the volatility process of the daily USD-GBP exchange returns by applying the Adaptive FIGARCH (A-FIGARCH) model of Baillie and Morana (2009) with the Adaptive GARCH (A-GARCH) model for the comparison. The adaptive-(FI) GARCH model augments the standard (FI)GARCH model with a deterministic component following Gallant's (1984) flexible function form. Thus, the A-(FI) GARCH model appears quite useful in analyzing the volatility process of the daily exchange returns by allowing for both the stochastic long memory component and the deterministic structural break component. Furthermore, the A-(FI)GARCH model has a good advantage of being computationally easy and straightforward since the model does not require pre-testing for the numbers of structural break points nor does it require any smooth transition between the volatility regimes.

   This paper finds that the A-(FI)GARCH model outperforms the standard (FI)GARCH model in the estimation of the long memory property in both periods when the structural breaks are present. As in the A-GARCH model, the degree of the long memory property in the volatility process of the daily returns is reduced in both periods after the structural breaks are accounted for in the A-FIGARCH model, thereby indicating that the structural break is another key intrigue feature of the exchange returns in both periods and that the part of the observed long memory property in the volatility process of the daily exchange returns in both periods could be upward biased and overstated by the structural breaks. In particular, the

long memory volatility property in the 1920s is reduced more remarkably, hence suggesting that the long memory property in the 1920 appears to be mostly a spurious feature because of the more significant structural breaks in the exchange markets in the 1920s.

The rest of this paper is organized as follows. Section II presents the descriptive statistics of the daily USD-GBP exchange returns in the periods of the 1920s and the 2010s; and provides the results from the estimation of the FIGARCH model as well as the GARCH model for the comparison in order to represent the long memory volatility property in the exchange returns. Section III reports the estimation results of the augmented A-FIGARCH model to account for the structural breaks and the long memory property jointly in the volatility process of the exchange returns together with the results of the A-GARCH model for the comparison. Section IV provides the brief conclusion.

II. DESCRIPTIVE STATISTICS AND LONG MEMORY VOLATILITY PROPERTY

1. Descriptive Statistics

Before embarking on the statistical and econometric analysis, it could be worthwhile visually examining the general patterns of the exchange rates under consideration. For the purpose, this section is concerned with the basic descriptive statistics and the long memory volatility property in the daily USD-GBP exchange rates in the periods of the 1920s and the 2010s. For the primary dataset in the 1902s, this paper uses the daily exchange rate data, which was originally collected from *Manchester Guardian* newspapers for the London market with sampling from May 1, 1922 through May 30, 1925.[1] Since the market was open on Saturdays, there are six observations per week and hence a total of 966 observations for this sample period. And, the dataset in the 2010s are obtained from the Olsen & Associates

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[1] Even though the 1920s data includes four exchange rates of Belgium Franc (BF), France Franc (FF), Italy Lila (IL) and US Dollar (USD) against the British Pound (BP), only the USD-GBP exchange rates is currently trading in the world exchange markets while the other exchange rates are not trading any more after the Euro currency was introduced in 1999. Furthermore, the credibility of the 1920s data has been proved by the paper of Phillips et al. (1996) which used the same data to test whether the forward rate is an unbiased predictor of the future spot rate in the 1920s.

with the sample period of May 3, 2010 through May 31, 2013, which is almost the same period as the 1920s data. In particular, the each quotation of the 2010s data consists of a bid price and an ask price and is recorded in time to the nearest second. Following the procedures of Baillie et al. (2000, 2004), the spot exchange rate for each daily interval is obtained by the average of the log bid and the log ask. The weekend data with much lower trading activities are excluded, thereby resulting in five observations per a week since they cannot provide any economic implications (Bollerslev and Domowitz, 1993). Thus, the exchange rates realize a sample of total 805 observations for the 2010s data.

The realizations of the daily USD-GBP exchange rates in the 1920s and the 2010s are plotted in Figures 1 (a) and (b) respectively. The figures show the similar movements of the exchange rates with significant changes in the mean over time in the two periods. In particular, the movements of the exchange rates in the 1920s generally appear to be more abrupt with several significant structural breaks in the market than those in the 2010s. After the WWI, the UK foreign exchange markets in the early 1920s experienced the most turbulent periods in the history as the markets adjusted to post war and non-gold standard conditions (Einzig, 1962). In particular, the hyperinflation in Germany and the large budget deficit in France affected the UK. Thus, the values of the GBP had become increasingly appreciated against USD during the periods of 1921 as well as the periods of early 1924 and mid-1925. In these periods, the UK monetary authorities were actively engaged in a return to gold policy given that in the latter part of this sample period (Taylor, 1992). But the GBP was depreciated steeply against the USD after October 1923 when the British government urged more expansionary fiscal and monetary policies to meet growing unemployment, which thus caused the outflows of capital from UK and more turbulence in the foreign exchange market (Aliber, 1962; Baillie and Bailey, 1984). Also, the period beginning in early 1924, witnessed speculative attacks on the European currencies including the UK pound. This led the some European governments including the UK to use apparently sterilized intervention in the hope of deterring future speculation. These kinds of the policy changes and the interventions by the UK government in order to adjust to post war and non-gold standard conditions affected the foreign exchange markets and caused the significant structural breaks in the USD-GBP exchange market.

Similarly, the movements of the USD-GBP exchange rate in the 2010s can be also characterized by several significant structural breaks in the exchange market. Generally the structural breaks in the market in the period of the 2010s may be closely related to the changes in the monetary policy by the US and the UK due to the global financial crisis. Since the culmination of the global financial crisis in 2008 which is caused by the US subprime mortgage crisis, major developed economies including the US and UK have experienced significant changes in the design and implementation of economic policies. The central banks including the Federal Reserve (Fed) and the Bank of England (BOE) have adopted unconventional monetary policy measures to money supply, called as "Quantitative Easing (QE)", which involved not only the active management of the size and composition of central bank balance sheets but also non-traditional mechanisms for central bank operations. This QE policy appears to depart from the standard procedure which would react to changes in inflation and output by changing short term interest rates. The QE policies implemented by the Fed and the BOE in the 2010s have taken many forms, but the most common one contains massive efforts to influence interest rates and exchange rates. In particular, the Fed and the BOE's QE policies are found to cause direct and great impacts on their exchange rates causing some significant structural breaks in the exchange markets (Joyce et al., 2011).

In order to analyze the patterns of the volatility process, the returns data of the daily exchange rates are defined in the conventional manner as continuously compounded rates of return and calculated as the first difference of the natural logarithm of prices. Figures 2 (a) and (b) shows that both daily exchange returns are all centered on zero and tremble by different intensity during the sample periods with volatility clustering revealing the presence of heteroskedasticity and strong ARCH effects. But more extreme changes and turbulences of the exchange returns at the 1920s markets are seen to induce much heavier tailed and undefined variance of unconditional returns phenomenon compared with the 2010s markets.

And, this paper uses the correlograms to investigate the inherent time series properties of the daily exchange returns data. Figures 3 (a) and (b) present the autocorrelation function of the returns, the squared returns and the absolute returns of the daily USD-GBP exchange rates in the 1920s and the 2010s with the dotted lines representing the band in which there is no serial correlation at the 95% confidence level. The first order autocorrelations in the two returns are all small whereas higher order autocorrelations of the two raw returns are not significant

indicating there is no serial correlations in the conditional mean process. Thus, the autocorrelation patterns of the mean process in the two exchange returns appear to be quite similar. However, the autocorrelations of the squared returns and the absolute returns for the two exchange rates are found to be very great showing the high level of serial correlations in the conditional variance process, and they decay very slowly at the hyperbolic rate, which is typical of freely floating nominal spot exchange rates and the feature of the long memory property. This long memory volatility property is very significant in the autocorrelations of the squared and absolute returns in both the 1920s and the 2010s and is more apparent in the autocorrelation functions of the absolute returns as presented by Ding and Granger (1996). Furthermore, the degree of the long memory volatility property seems to be more significant in the 1920s than in the 2010s.

Figure 1 (a). Daily USD-GBP Exchange Rate in the 1920s
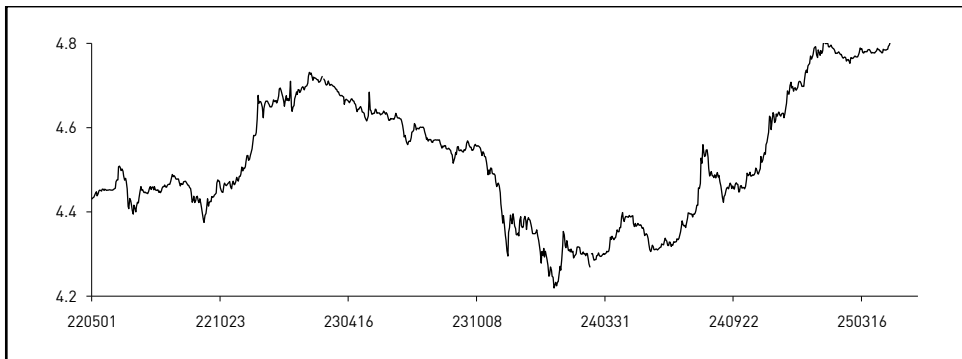
Figure 1 (b). Daily USD-GBP Exchange Rate in the 2010s

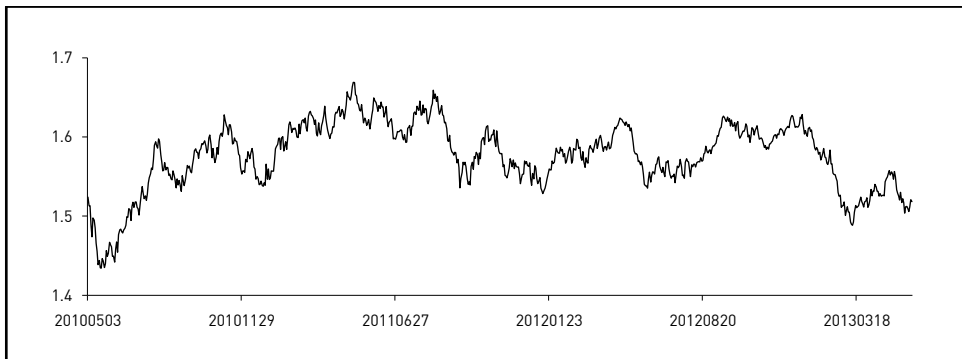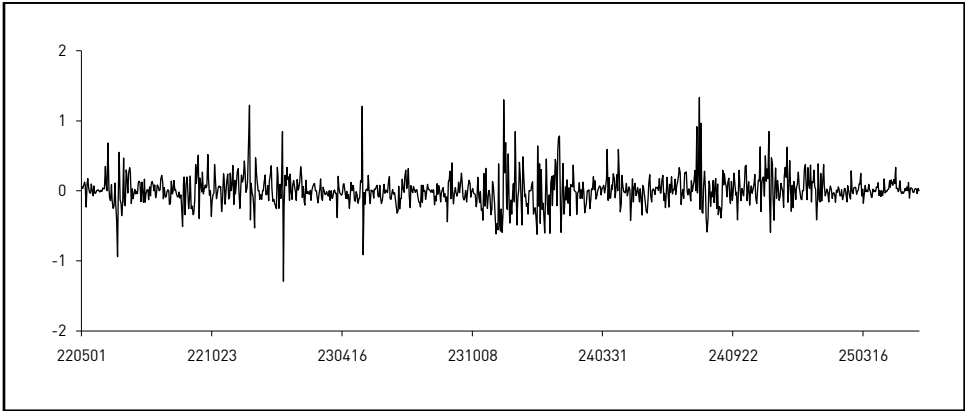Figure 2 (a). Daily USD-GBP Exchange Returns in the 1920s

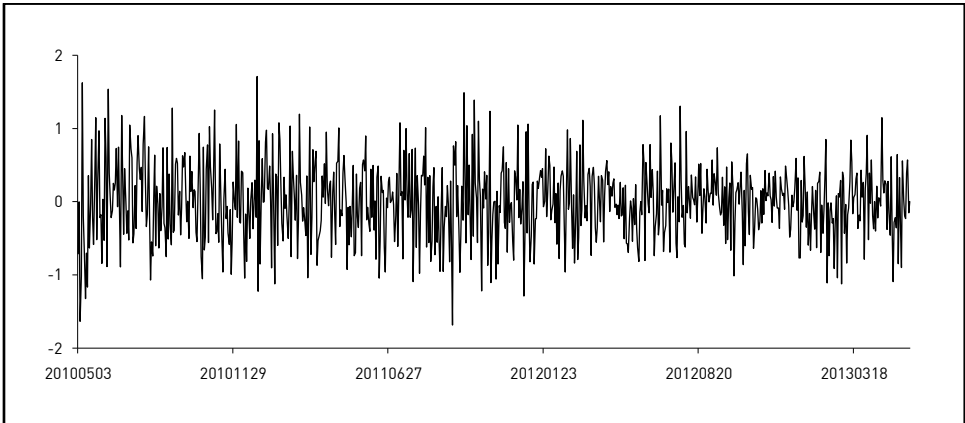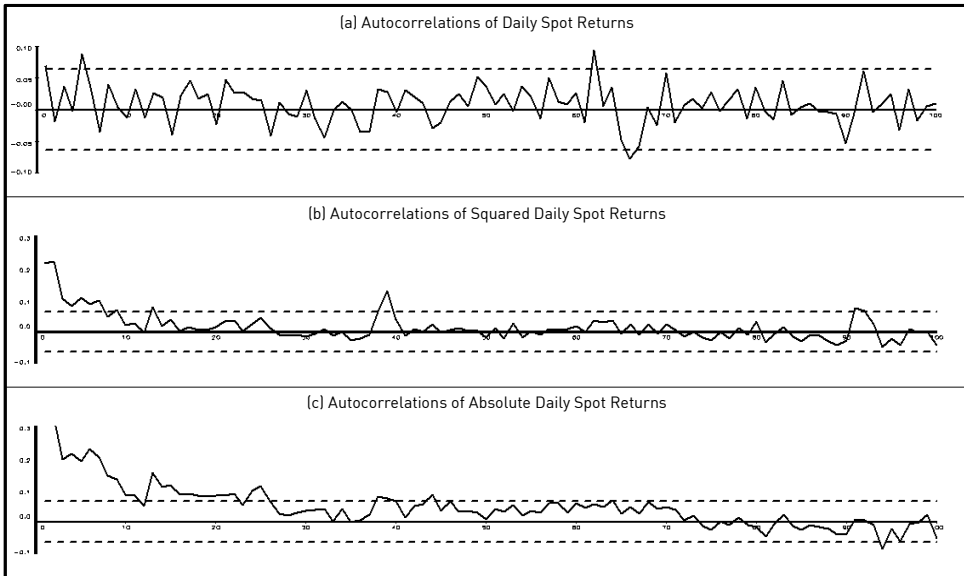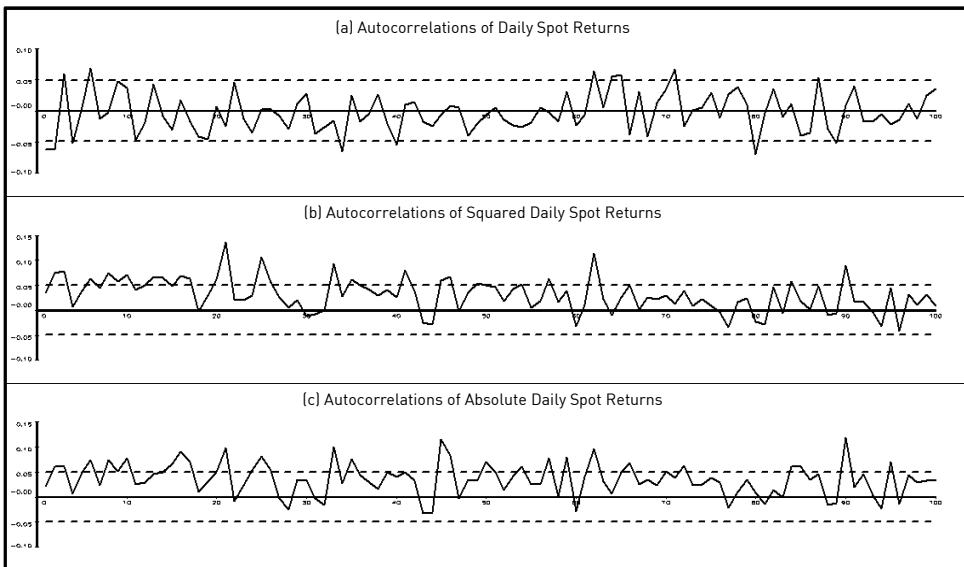Figure 2 (b). Daily USD-GBP Exchange Returns in the 2010s

Figure 3 (a). Correlograms of Daily USD-GBP Exchange Returns in the 1920s

Key: The dotted lines represent the band in which there is no serial correlation at the 95% confidence level.

Figure 3 (b). Correlograms of Daily USD-GBP Exchange Returns in the 2010s

Key: the same as the Figure 3(a).

In addition, the details of the descriptive statistics for the two daily USD-GBP exchange returns in the 1920s and the 2010s are provided in Table 1. The sample means of the daily returns in the 1920s and the 2010s are found to be 0.0097 and -0.0005, respectively, which are very close to zero and indistinguishable at the standard significance level, given the sample deviations of 0.227 and 0.520. In particular, the daily returns in the 1920s appear not to be normally distributed since the value of the skewness is 0.82 and the value of the kurtosis is 9.47, which are greater than the levels of the normal distribution, and they are all statistically significant.[2] The more substantial excess kurtosis in the 1920s is consistent with the more systematic occurrence of tranquil and volatile periods than in the 2010s, as presented in Figures 1 and 2.

The modified Ljung-Box test statistics for the test of the serial correlations, $Q_m(20)$, calculated from the returns in the 1920s and the 2010s are found to be 24.96 and 25.80, which imply that there are not any significant autocorrelations in the conditional mean process of the returns in the 1920s and the 2010s. But, the test statistics of $Q_m^2(20)$ for the squared returns are 151.54 and 50.99, which are statistically significant, thereby indicating the existence of highly persistent autocorrelations in the conditional variance process.[3] The serial correlation seems to be more significant in the volatility process of the 1920s returns due to the more significant structural breaks in the 1920s as presented by Figures 1 and 2. These patterns appear to be quite consistent with the correlograms in the Figure 3. Despite the more primitive market conditions in the 1920s compared with the current markets in 2010s, the exchange returns in the 1920s appear as a remarkably similar pattern to the current returns in the 2010s but with more persistent volatility process.

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[2] According to Jarque and Bera (1987), the standard errors of the sample skewness and the sample kurtosis in their corresponding normal distributions are $(6/T)^{1/2}$ and $(24/T)^{1/2}$.

[3] Following the suggestion of one referee, this paper uses the modified Ljung-Box tests for the serial correlations in order to avoid the distortions caused by possible outliers in the data instead of the standard Ljung-Box test. The values of the modified test statistics are found to be quite similar to those of the standard test statistics indicating that there is no evidence of serious outliers in the data. Even though the values of the standard Ljung-Box test statistics are not presented in this paper to reserve the space, they are available by the request on the author.

Table 1. Descriptive Statistics for the Daily USD-GBP Returns

	1920s	2010s
Mean	0.0097	-0.0005
Standard Deviation	0.2269	0.5214
Qm(20)	24.9892	25.8035
Qm2(20)	151.5376	50.9875
Skewness	0.8152	0.0729
Kurtosis	9.4747	3.1085
ρ1	0.0676	-0.0626

Key: The Q(20) and $Q_m^2(20)$ are the modified Ljung-Box test statistics at 20 degrees of freedom based on the returns and the squared returns. ρ_1 is the first order of autocorrelation.

2. Long Memory Volatility Process

In order to represent the basic stylized properties of the daily exchange returns defined previously, the ARMA (*m,n*)-FIGARCH (*p, d, q*) process is introduced for the econometric analysis,

$$y_t = \mu + \phi(L)y_{t-1} + \theta(L)\varepsilon_t \tag{1}$$

$$\varepsilon_t^2 = z_t\sigma_t^2 \tag{2}$$

$$[1-\beta(L)]\sigma_t^2 = \omega + [1-\beta(L)-\varphi(L)(1-L)^d]\varepsilon_t^2 \tag{3}$$

where y_t is the returns of the daily USD-GBP exchange rates, $z_t \sim i.i.d.(0,1)$, μ and ω are scalars, $\varphi(L), \theta(L), \beta(L)$ and $\phi(L)$ are polynomials in the lag operator and *d* represents the long memory parameter.

The FIGARCH model in equation (3) is motivated by noting that the standard GARCH (*p, q*) model of Bollerslev (1986) can be expressed as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \tag{4}$$

And, the FIGARCH (*p, d, q*) process can be specified as:

$$\phi(L)(1-L)^{d}\varepsilon_{t}^{2} = \omega + \left[1 - \beta(L)\right]\upsilon_{t}, \tag{5}$$

where $\varphi(L) = [1 - \alpha(L) - \beta(L)]$ is a polynomial in the lag operator of order max (p, q). Equation (5) can be easily shown to transform to equation (3), which is the standard representation for the conditional variance in the FIGARCH (p, d, q) process. The parameter (d) characterizes the long memory property which represents the hyperbolic decay in volatility because it allows for autocorrelations to decay at a very slow hyperbolic rate. When d is between 0 and 1, the FIGARCH model has an undefined unconditional variance, thereby suggesting the long memory pattern and is strictly stationary and ergodic (Baillie et al., 1996; Baillie and Morana, 2009). However, the process does posses a finite sum to its cumulative impulse response weights. This makes the FIGARCH model different from the other models of the long memory ARCH models proposed by Karanassos et al. (2004). Thus, the most advantage for the importance of the long memory process is that it could avoid the knife-edge distinction between I (0) and I (1) process and that it could explain different long run predictions and effects of shocks. See Baillie (1996) for the further surveys of the long memory property.

In particular, the equation (3) can reduce to the standard GARCH (1,1) model when d = 0, p = q = 1; and the equation (3) changes to the IGARCH (1,1) model when d = p = q = 1 with the complete persistence of the conditional variance to a shock in squared returns. The FIGARCH process has impulse response weights, $\sigma^{2}{}_{t} = \omega/(1 - \beta) + \lambda(L)\varepsilon^{2}{}_{t}$, where $\lambda_{k} \approx k^{d-1}$, which is essentially the long memory property of hyperbolic decay. The key advantage of the FIGARCH process is that it is flexible enough to allow for intermediate ranges of persistence when 0 < d < 1. The simpler version can be specified as the FIGARCH (1, d, 0) process, $\sigma^{2}{}_{t} = \omega + \beta\sigma^{2}{}_{t-1} + [1 - \beta L - (1 - L)^{d}]\varepsilon^{2}{}_{t}$, and the form of the corresponding impulse response weights is, $\sigma^{2}{}_{t} = \omega/(1 - \beta) + \lambda(L)\varepsilon^{2}{}_{t}$; and for large lag k, $\lambda_{k} \approx [(1-\beta)/\Gamma(d)]k^{d-1}$.

The equations (1) through (3) are estimated by using non-linear optimization procedures to maximize the Gaussian log likelihood function:

$$\ln(L;\Theta) = -(\frac{T}{2})\ln(2\pi) - (\frac{1}{2})\sum_{t=1}^{T}[\ln(\sigma_{t}^{2}) + \varepsilon_{t}^{2}\sigma_{t}^{-2}] \tag{6}$$

where Θ is a vector with the parameters to be estimated. However, it has long been recognized that most asset returns are not well represented by assuming z_t in equation (2) is normally distributed; for example see McFarland et al. (1982). And, the consistency and asymptotic normality of the QMLE for the conditional variance process can be determined based on the available results from the estimation of GARCH processes as pointed out by Baillie and Morana (2009). Thus, the inference is specified by using the QMLE of Bollerslev and Wooldridge (1992), which is valid when z_t is non-Gaussian. Providing the vector of parameter estimates obtained from maximizing (6) based on the sample in equations (1), (2) and (3) with z_t being non-normal by $\hat{\Theta}_T$, the limiting distribution of $\hat{\Theta}_T$ is:

$$T^{1/2}(\hat{\Theta}_T - \Theta_0) \rightarrow N[0, A(\Theta_0)^{-1}B(\Theta_0)A(\Theta_0)^{-1}], \qquad (7)$$

where A(.) and B(.) represent the Hessian and outer product gradient respectively, and Θ_0 denotes the vector of true parameter values. And, equation (7) is used to calculate the robust standard errors that are reported in the subsequent results in this paper.

This section of the paper represents an extensive analysis of the volatility properties of the two USD-GBP returns in the 1920s and the 2010s, by using the FIGARCH model of Baillie et al. (1996) and the GARCH model of Bollerslev (1986) for the comparison. The orders of the ARMA and (FI)GARCH polynomials in the lag operator are selected to be parsimonious and provide a proper model for the autocorrelation structure of the daily exchange returns data. In particular, this paper uses the basic portmanteau test statistic for the model specification in the mean process, and the similar degrees of freedom adjustments are used for the portmanteau test statistic based on the squared standardized residuals when testing for omitted conditional heteroscedasticity and ARCH effects. This adjustment is in the spirit of the suggestions by Diebold (1988) and others. And, the sample skewness and kurtosis of the standardized residuals (m_3 and m_4) are also considered. The exact parametric specification of the model, which best represents the degree of autocorrelation in the conditional mean and conditional variance of the daily returns are found to be the MA (1)-FIGARCH (1, d, 0) model and MA(1)-GARCH (1,1) model.

The estimation results are reported in Table 2 applying the above models for the USD-GBP exchange returns in the 1920s and the 2010s. In the case of the GARCH

model, the sum of the estimated values of the volatility persistence parameters (β and φ) in the GARCH model is equally found to almost close to 1, thereby implying the complete persistence of the IGARCH model. A consequence of neglecting structural breaks is that the GARCH model tends to produce results consistent with the data being generated by an IGARCH process. But the GRACH model may not provide any difference in the persistence of the volatility process of the daily returns in the two different periods.

Table 2. Estimation of GARCH and FIGARCH Model for the Daily USD-GBP Returns

	1920s		2010s	
	GARCH Model	FIGARCH Model	GARCH Model	FIGARCH Model
μ	-0.0005 (0.0062)	-0.0004 (0.0058)	0.0003 (0.0167)	0.0033 (2033)
θ	0.1410 (0.0492)	0.1386 (0.0483)	-0.0675 (0.0391)	-0.0578 (0.0388)
d	-	**0.8644 (0.1728)**	-	**0.2121 (0.0613)**
ω	0.0039 (0.0014)	0.0043 (0.0014)	0.0025 (0.0020)	0.0520 (0.0246)
β	0.3816 (0.1001)	0.4932 (0.1307)	0.0309 (0.0137)	0.2079 (0.0696)
φ	0.6097 (0.0695)	-	0.9592 (0.0184)	-
ln(L)	220.186	229.808	-600.227	-594.264
m_1	0.054	0.053	-0.007	-0.010
m_2	0.998	0.998	1.009	1.002
m_3	1.076	1.088	-0.040	-0.027
m_4	8.771	8.923	2.964	3.014
$Q_m(20)$	23.674	24.548	18.942	18.613
$Q_m^2(20)$	7.492	7.003	10.191	13.894
$W_{d=0}$		19.244		12.026

Key: Robust standard errors are in parentheses below the corresponding parameter estimates. The symbol ln (L) refers to the value of the maximized log likelihood function. The values of m_1 and m_2 are the mean and standard deviations of the standardized residuals while m_3 and m_4 are the skewness and kurtosis respectively of the standardized residuals. $Q_m(20)$ and $Q_m^2(20)$ are the modified Ljung-Box test statistics with 20 degrees of freedom also based on the standardized residuals and squared standardized residuals. The statistic $W_{d=0}$ is the robust Wald test for the GARCH (1,1) model against the FIGARCH (1,d,0) alternative.

However, the estimation result of the FIGARCH model which accounts for the long memory property shows that the long memory parameters (d) in the volatility process of the daily returns are estimated to be 0.86 and 0.21 for the 1920s and the

2010s returns and they are all the statistically significant at the conventional level, thereby implying that the degree of the persistence in the volatility process of the two returns are quite different depending on the periods. It presents strong support that there exists significant long memory property in the volatility process of the daily USD-GBP returns for the two periods and that the long memory volatility property in the 1920s appears to be much greater than that in the 2010s. This result confirms the fact represented in Figure 3, which shows the apparent autocorrelations decaying more slowly at the hyperbolic rate in the squared and the absolute returns in the 1920s than those in the 2010s. As some papers show that the time series with structural breaks can induce strong persistence in the autocorrelations (Diebold and Inoue, 2001; Granger and Hyung, 2004; Perron and Qu, 2006), the more significant long memory volatility property in the 1920s could be closely related to the more apparent and frequent structural breaks in the 1920s exchange markets, as presented in Figures 1 and 2. Thus, the long memory volatility property is one of key intrigue features in the daily USD-GBP returns for the 1920s and the 2010s, but it is much more significant in the 1920s than in 2010s.

Based on the robust Wald test, of the stationary GARCH (1,1) null hypothesis versus a FIGARCH (1,d,0) alternative, being overwhelmingly rejected, the FIGARCH model, which accounts for the long memory property generally yields an improvement in specification in all the cases considered for the GARCH model. And, the estimated values of the $Q_m(20)$ and the $Q_m^2(20)$ which are the modified Ljung-Box test statistics calculated from the standardized residuals show that the FIGARCH model specified for the daily returns performs a good job of capturing the autocorrelations in the conditional mean and the conditional variance of the daily USD-GBP return series. In each case there is no evidence of additional autocorrelation in the standardized residuals or squared standardized residuals. And, the estimated values of the mean (m_1) and the standard deviation (m_2) of the standardized residuals appear to be quite similar between the GARCH model and the FIGARCH model in the two periods. Also, a sequence of diagnostic portmanteau tests on the standardized residuals and squared standardized residuals failed to detect any need to further complicate the model, thereby indicating that the chosen model specification provides an adequate fit. Thus, the FIGARCH model matches the long memory volatility property of the daily USD-GBP returns in the 1920s and the 2010s more appropriately than the GARCH model. This finding is consistent with the papers of Andersen et al. (2003) and Bhardwaj and Swanson (2006), in

the fact that the long memory process model provide significantly better out of sample prediction than the GARCH model.

III. LONG MEMORY VOLATILITY PROPERTY AND STRUCTURAL BREAKS

This section considers the relation of the structural breaks with the long memory volatility property in the daily USD-GBP exchange returns by applying the Adaptive FIGARCH (A-FIGARCH) model of Baillie and Morana (2009).[4] As presented in Section I, many previous studies have provided abundant motivations to allow for the possibility of the structural breaks in the volatility process of financial time series data including foreign exchange rates. One of the quite powerful approaches is to allow the intercept to be time varying in order to account for the structural breaks as suggested by Baillie and Morana (2009). They have provided that the A-FIGARCH model can derived from the FIGARCH model of Baillie et al. (1996) by directly allowing the intercept in the conditional variance equation to be time varying according to the Gallant's (1984) flexible functional form. Thus, the A-FIGARCH can allow for a very efficient modeling of various types of structural breaks without requiring any pretests to determine the actual location of break points and adding estimation complexity. Also, the joint presence of the long memory and the structural break can be assessed by standard hypothesis tests of the fractional differencing parameter and the deterministic trigonometric components. Another advantage of this model is the simplicity of computation, thus adding no additional burden to the estimation of the usual FIGARCH model. Moreover, Baillie and Morana (2009) have found that the A-FIGARCH model shows a superior performance, relative to the usual FIGARCH model in terms of bias and root mean square error (RMSE).

In this context, this paper adopts the A-FIGARCH model together with the A-GARCH model for the comparison in order to account for jointly the long memory volatility property and the structural breaks in the two daily returns. The mean process of the daily returns is still specified as following an MA (1) process as in

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[4] There are different types of models allowing to model time varying unconditional moments such as the flexible coefficient GARCH model of Medeiros and Veiga (2004), the spine GARCH model of Engle and Rangel (2008) and the smooth transition model of Terasvirta and Gonzalez (2006).

Section II, whereas the volatility process is represented by the A-FIGARCH (1,d,0,k) model with the trigonometric term (k) for the Gallant's flexible functional form, which is the simplest version and appears to be quite useful in practice as suggested by Baillie and Morana (2009). This model can be written as:

$$y_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t \tag{8}$$

$$\varepsilon_t^2 = z_t\sigma_t^2 \tag{9}$$

$$(1-\beta L)\sigma_t^2 = \omega_t + [1-\beta L-(1-L)^d]\varepsilon_t^2 \tag{10}$$

$$\omega_t = \omega_0 + \sum_{j=1}^{k}[\gamma_j \sin(2\pi jt/T) + \delta_j \cos(2\pi jt/T)] \tag{11}$$

And, the Gaussian loglikelihood function of the model is the same as the MA (1)-FIGARCH (1, d, 0) model in Section II. Also, the estimation and inference for the parameters of the above model can be facilitated by the same method of QMLE by numerically maximizing the loglikelihood function with respect to the parameters as in Section II. The procedure can implement the simultaneous estimation of all the model's parameters, including those in the flexible function form which specify the time varying intercept in the conditional variance process. One important consideration is the determination of the trigonometric terms (k) in the Gallant flexible functional form for the practical implementation of the model. In this paper, the trigonometric terms (k) are selected 9 for the 1920s returns and 2 for the 2010s returns as based on the Akaike Information criterion (AIC) and the Schwartz Information criterion (SIC).

   The estimation results of the above model for the daily USD-GBP exchange returns are reported in Table 3. Once the structural breaks and the long memory volatility property are jointly modeled, an improvement in fit can be noted as well as a reduction in the long memory parameter, thus indicating the structural break is also one key intrigue feature of the daily returns in the two periods. In particular, the estimated parameters of the long memory volatility property in the daily returns are found to be 0.008 and 0.162 for the 1920s and the 2010s returns, and they are all statistically significant. As already found in the A-GARCH model, it can be noted that an upward and overstated bias in the long memory property is imparted by neglecting

the structural breaks in both cases by comparing the estimated long memory parameters. This finding is in line with Choi and Zivot (2006), in which allowing for structural breaks reduces the persistence, but there is still evidence of the long memory property in the forward discount series. Thus, the long memory volatility and the structural breaks could be the key intrigue features of the exchange returns in both cases.

Table 3. Estimation of Adaptive-GARCH and Adaptive-FIGARCH Model for the Daily USD-GBP Returns

	1920s		2010s	
	Adaptive-GARCH Model	Adaptive-FIGARCH Model	Adaptive-GARCH Model	Adaptive-FIGARCH Model
μ	0.0055 (0.0052)	-0.0001 (0.0047)	0.0016 (0.0167)	-0.0003 (0.0162)
θ	0.1525 (0.0448)	0.1237 (0.0484)	-0.0672 (0.0386)	-0.0646 (0.0392)
d	-	**0.00797** **(0.0020)**	-	**0.1620** **(0.0713)**
β	0.2504 (0.0653)	0.5683 (0.0878)	0.0257 (0.0145)	0.0781 (0.0661)
φ	0.4290 (0.0941)	-	0.9054 (0.0307)	-
ω_0	0.0165 (0.0046)	0.0165 (0.0046)	0.0182 (0.0085)	0.0603 (0.0226)
γ_1	-0.0037 (0.0024)	0.0003 (0.0020)	0.0061 (0.0031)	0.0271 (0.0205)
δ_1	0.0034 (0.0020)	0.0026 (0.0015)	0.0019 (0.0021)	0.0051 (0.0187)
γ_2	-0.0040 (0.0022)	-0.0012 (0.0014)	-0.0002 (0.0018)	0.0370 (0.0187)
δ_2	0.0065 (0.0031)	0.0010 (0.0016)	0.0032 (0.0025)	0.0681 (0.0585)
γ_3	-0.0055 (0.0029)	-0.0018 (0.0015)	-	-
δ_3	0.0070 (0.0031)	0.0015 (0.0016)	-	-
γ_4	-0.0022 (0.0021)	0.0013 (0.0016)	-	-
δ_4	0.0009 (0.0019)	0.0003 (0.0011)	-	-
γ_5	-0.0036 (0.0018)	-0.0015 (0.0013)	-	-
δ_5	0.0043 (0.0020)	-0.0018 (0.0014)	-	-

Table 3. Continued

	1920s		2010s	
	Adaptive-GARCH Model	Adaptive-FIGARCH Model	Adaptive-GARCH Model	Adaptive-GARCH Model
γ_6	-0.0070 (0.0028)	-0.0021 (0.0020)	-	-
δ_6	0.0004 (0.0026)	-0.0051 (0.0019)	-	-
γ_7	-0.0101 (0.0034)	0.0014 (0.0014)	-	-
δ_7	0.0041 (0.0025)	0.0015 (0.0014)	-	-
γ_8	-0.0006 (0.0016)	0.0021 (0.0014)	-	-
δ_8	0.0036 (0.0020)	-0.0015 (0.0013)	-	-
γ_9	-0.0007 (0.0016)	-0.0014 (0.0011)	-	-
δ_9	0.0009 (0.0017)	1.1188 (0.1344)	-	-
ln(L)	289.147	293.266	-595.443	-589.232
m_1	0.028	0.048	-0.006	-0.003
m_2	0.999	0.959	1.007	1.000
m_3	0.598	0.898	-0.023	-0.014
m_4	5.424	7.236	2.921	2.913
$Q_m(20)$	16.807	20.511	19.176	19.631
$Q_m^2(20)$	12.227	13.265	12.560	14.987
AIC	-528.294	-480.531	1208.886	1208.464
SIC	-406.491	-368.472	1251.093	1250.670
Nyb	0.307	0.227	0.079	0.068
Wf	-	126.916	-	10.064

Key: The same as Table 2 except that the trigonometric terms k =9 for the 1920s returns and k=2 for the 2010s returns, which is selected based on the AIC (Akaike Information Criterion) and the SIC (Schwarz Information Criterion. The values of Nyb are the Nyblom stability test statistics for the unconditional variance carried out on the standardized residuals. The values of W_f are the robust Wald test statistics for the FIGARCH model against the Adaptive-FIGARCH model alternative.

   The long memory property in the 2010s returns is still strong even after the structural breaks are eliminated, thereby suggesting that the long memory property in the 2010s returns appears to be a truly intrigue feature in the exchange markets.

But the long memory property in the 1920s returns is found to be reduced more significantly and quite small when the structural breaks are accounted for. This result indicates that the 1920s returns with the significant structural breaks may induce a strong persistence in the volatility process and hence the long memory property seems to be a spurious feature (Diebold and Inoue, 2001; Granger and Hyung, 2004; Perron and Qu; 2006). This may be because the long memory volatility property of the exchange returns in the 1920s could be easily confused with the structural breaks in foreign exchange markets so that it may be very difficult to distinguish between the intrigue and the spurious long memory property, as pointed by Shimotsu (2006), in which the long memory property and the structural breaks are almost observationally equivalent so that the long memory may fall into an "empty box" category.

In addition, the robust Wald test statistics of the FIGARCH null hypothesis versus the Adaptive-FIGARCH alternative overwhelmingly rejected the basic FIGARCH model supporting the facts that the inclusion of the trigonometric components makes an important improvement to the general goodness of fit of the model and furthermore the A-FIGARCH is superior to the basic FIGARCH whenever the structural breaks are presented, which is consistent with the findings of Baillie and Morana (2009). And, this paper also uses the Nyblom (1989) test in order to test the constancy of parameters by detecting possible changes in the estimates over time following Baillie and Morana (2009). All of the A-GARCH and the A-FIGARCH models accounting for the structural breaks do not present any significant Nyblom stability statistics, which cannot reject the null hypothesis of no breaks in variance for the estimates of the models at the conventional significant level. No evidence of instability in variance can be detected once the long memory and the structural breaks are allowed for. And, the estimated values of the mean (m_1) and the standard deviation (m_2) of the standardized residuals appear to be quite similar between the A-GARCH model and the A-FIGARCH model in the two periods, and the values are also very similar to those estimated from the basic GARCH and FIGARCH models in the two periods as presented in Table 2. These results suggest there is not any further evidence of the model mis-specification. Thus, this paper found improvement in specification fit and the reduction in the long memory parameter once the structural breaks and the long

memory property are jointly modeled.[5] And, these findings are quite consistent with the view that the long memory and the structural breaks are the features which can be easily confounded as pointed out by Baillie and Morana (2009).[6]

IV. CONCLUSION

The period of the 1920s is one of very interesting periods of history and the floating exchange rates in the 1920s are worthy of study because they can provide useful chances to compare the some evidence from the current floating rates in the 2010s, and they can constitute the other main source of information on the behavior of the floating exchange rates. Further, the 1920s foreign exchange markets, even with the relatively primitive market conditions are found to be quite similar to the markets in the 2010s. Hence, this paper quantitatively compares the intrigue features of the daily USD-GBP exchange rates in the 1920s with those in the 2010s. Special attention is devoted to account for both the structural breaks and the long memory volatility property of the daily exchange returns in both periods.

This paper first uses the FIGARCH model of Baillie et al. (1996) with the GARCH model of Bollerslev (1986) for the comparison in order to figure out the long memory volatility property of the daily exchange returns series in the periods of the 1920s and the 2010s. This paper finds strong evidence for the hyperbolic decay and significant persistence of the autocorrelations in the volatility process of the daily returns in the two periods, which is the typical feature of the long memory property. Thus, the long memory volatility property is found to be one of key intrigue features in the volatility process of the daily returns in the two periods. Moreover, the standard FIGARCH model is found to provide an adequate fit and match the dynamics of the daily returns in the two periods. In particular, the long memory volatility property in the 1920s returns appears to be much greater than

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[5] The values of the skewness (m3) and the kurtosis (m4) are still found to be different from the normal values of 0 and 3 indicating the A-FIGARCH model appears not to be enough to consider all the other factors except the structural breaks which affect the exchange rates in the foreign exchange markets. It could be improved by adding the nonparametric models like the jump process together. But the issue will be left for the future study.

[6] The findings in this paper appear to be quite consistent with the previous papers including Granger and Hyung (2004) and Starica and Granger (2005) which have investigated the presence of structural breaks in S&P 500 returns.

that in the 2010s returns, which could be closely related to the significant structural breaks in foreign exchange markets in the 1920s.

Following many previous studies that have allowed for the possibility of the structural breaks in the volatility process of financial time series data including foreign exchange rates, this paper then applies the Adaptive-FIGARCH (A-FIGARCH) model of Baillie and Morana (2009) with the Adaptive-GARCH (A-GARCH) model for the comparison, which is designed to model the structural beaks and the long memory property jointly in the volatility process of the daily exchange returns in the two periods. The main finding of this paper is that the A-FIGARCH model outperforms the standard FIGARCH model when the structural breaks are present; and furthermore it can provide significant gains in terms of bias and efficiency in estimating the long memory property in the volatility process. It can be seen that the long memory parameters are significantly reduced under the A-FIGARCH model compared with the estimated parameters under the FIGARCH model. Thus, the observed upward biased and overstated long memory property in the volatility process of the daily returns in the two periods could be imparted by neglecting the structural breaks, thereby indicating that both the long memory volatility property and the structural breaks are the key intrigue features of the daily returns in the two periods. In particular, the long memory property in the 1920s returns is found to be quite small when the structural breaks are accounted for in the specification model. This result implies that the significant structural breaks in the foreign exchange markets in the 1920s may induce a strong persistence in the volatility process of the daily returns and hence produce the more significant long memory property.

Hence, this paper should be important to expand the range of empirical studies since it seems warranted for the reason that this issue has not been investigated before and it could help us to understand the dynamic mechanism of the foreign exchange rates in terms of the structural breaks and the long memory volatility property. In particular, this paper suggests that it is possible to distinguish between the underlying long memory property in the volatility process and the effects of the structural breaks in the foreign exchange markets through the empirical analysis of the exchange rate in the different periods.

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