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TRIPLED COINCIDENCE AND COMMON TRIPLED FIXED POINT THEOREM FOR HYBRID PAIR OF MAPPINGS SATISFYING NEW CONTRACTIVE CONDITION

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Abstract. We establish a tripled coincidence and common tripled fixed point theorem for hybrid pair of mappings satisfying new contractive condition. To find tripled coincidence points, we do not use the continuity of any mapping involved therein. An example is also given to validate our result. We improve, extend and generalize several known results.

1. Introduction and Preliminaries

Let $(X,\ d)$ be a metric space. We denote by 2^X the class of all nonempty subsets of X, by $CL(X)$ the class of all nonempty closed subsets of X, by $CB(X)$ the class of all nonempty closed bounded subsets of X and by $K(X)$ the class of all nonempty compact subsets of X. A functional $H : CL(X) \times CL(X) \to \mathbb{R}_+ \cup \{+\infty\}$ is said to be the Pompeiu-Hausdorff generalized metric induced by d is given by

$$H(A,\ B) = \left\{ \begin{array}{c} \max\left\{\sup_{a\in A} D(a,\ B),\ \sup_{b\in B} D(b,\ A)\right\},\ \text{if maximum exists,} \\ +\infty,\ \text{otherwise,} \end{array} \right.$$

for all $A,\ B \in CL(X)$, where $D(x,\ A) = \inf_{a\in A} d(x,\ a)$ denote the distance from x to $A \subset X$. For simplicity, if $x \in X$, we denote $g(x)$ by gx.

Markin [31] initiated the study of fixed points for multivalued contractions and non-expansive mappings using the Hausdorff metric, which was further studied by several authors under different conditions. The multivalued theory has found application in control theory, convex optimization, differential inclusion and economics.

In [26], Guo and Lakshmikantham given the notion of coupled fixed point. Gnana-Bhaskar and Lakshmikantham [12] proved some results on the existence and uniqueness of coupled fixed points. Later on, Lakshmikantham and Ciric [27] generalized these results for nonlinear contraction mappings by introducing

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the notions of coupled coincidence point and mixed g-monotone property. These results are applied for proving the existence and uniqueness of the solution for periodic boundary value problems. Berinde and Borcut [10] introduced the concept of tripled fixed point for single valued mappings in partially ordered metric spaces and established the existence of tripled fixed point of single-valued mappings in partially ordered metric spaces. Samet et al. [34] claimed that most of the coupled fixed point theorems for single valued mappings on ordered metric spaces are consequences of well-known fixed point theorems. As a continuation of this work, several results of a coupled and tripled fixed point have been discussed in the recent literature including [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 25, 27, 29, 30, 32, 36].

The concepts related to coupled fixed point theory for multivalued mappings were extended by Abbas et al. [3] and obtained coupled coincidence point and common coupled fixed point theorems involving hybrid pair of mappings satisfying generalized contractive conditions in complete metric spaces.

These concepts were extended by Deshpande et al. [22] to multivalued mappings and obtained tripled coincidence points and common tripled fixed point theorems involving hybrid pair of mappings under generalized nonlinear contraction. Very few researchers focused on tripled fixed point theorems for hybrid pair of mappings including [1, 2, 3, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 35].

In [22], Deshpande et al. introduced the following for multivalued mappings:

Definition 1. Let X be a nonempty set, $F : X \times X \times X \to 2^X$ and g be a self-mapping on X. An element $(x, y, z) \in X \times X \times X$ is called

(1) a tripled fixed point of F if $x \in F(x, y, z)$, $y \in F(y, z, x)$ and $z \in F(z, x, y)$.

(2) a tripled coincidence point of hybrid pair $\{F, g\}$ if $gx \in F(x, y, z)$, $gy \in F(y, z, x)$ and $gz \in F(z, x, y)$.

(3) a common tripled fixed point of hybrid pair $\{F, g\}$ if $x = gx \in F(x, y, z)$, $y = gy \in F(y, z, x)$ and $z = gz \in F(z, x, y)$.

We denote the set of tripled coincidence points of mappings F and g by $C(F, g)$. Note that if $(x, y, z) \in C(F, g)$, then (y, z, x) and (z, x, y) are also in $C(F, g)$.

Definition 2. Let $F : X \times X \times X \to 2^X$ be a multivalued mapping and g be a self-mapping on X. The hybrid pair $\{F, g\}$ is called w-compatible if $gF(x, y, z) \subseteq F(gx, gy, gz)$ whenever $(x, y, z) \in C(F, g)$.

Definition 3. Let $F : X \times X \times X \to 2^X$ be a multivalued mapping and g be a self-mapping on X. The mapping g is called F-weakly commuting at some point $(x, y, z) \in X \times X \times X$ if $g^2x \in F(gx, gy, gz)$, $g^2y \in F(gy, gz, gx)$ and $g^2z \in F(gz, gx, gy)$.

Lemma 1.1. [33]. *Let $(X,\ d)$ be a metric space. Then, for each $a \in X$ and $B \in K(X)$, there is $b_0 \in B$ such that $D(a,\ B) = d(a,\ b_0)$, where $D(a,\ B) = \inf_{b\in B} d(a,\ b)$.*

In this paper, we establish a tripled coincidence and common tripled fixed point theorem for hybrid pair of mappings satisfying new contractive condition. To find tripled coincidence points, we do not use the continuity of any mapping involved therein. Our result improve, extend and generalize the results of Gnana-Bhaskar and Lakshmikantham [12] and Lakshmikantham and Ciric [27]. An example is also given to validate our result.

2. Main results

Let Φ denote the set of all functions $\varphi : [0,\ +\infty) \to [0,\ +\infty)$ satisfying
(i_φ) φ is non-decreasing,
(ii_φ) $\varphi(t) < t$ for all $t > 0$,
(iii_φ) $\lim_{r\to t+} \varphi(r) < t$ for all $t > 0$,
and Ψ denote the set of all functions $\psi : [0,\ +\infty) \to [0,\ +\infty)$ which satisfies
(i_ψ) ψ is continuous,
(ii_ψ) $\psi(t) < t$ for all $t > 0$.
Note that, by (i_ψ) and (ii_ψ) we have that $\psi(t) = 0$ if and only if $t = 0$.

For simplicity, we define the following:

$$\begin{aligned} &M(x,\ y,\ z,\ u,\ v,\ w) \\ =\ &\min\left\{\begin{array}{c} D(gx,\ F(x,\ y,\ z)),\ D(gu,\ F(u,\ v,\ w)), \\ D(gy,\ F(y,\ z,\ x)),\ D(gv,\ F(v,\ w,\ u)), \\ D(gz,\ F(z,\ x,\ y)),\ D(gw,\ F(w,\ u,\ v)), \\ D(gx,\ F(u,\ v,\ w)),\ D(gu,\ F(x,\ y,\ z)), \\ D(gy,\ F(v,\ w,\ u)),\ D(gv,\ F(y,\ z,\ x)), \\ D(gz,\ F(w,\ u,\ v)),\ D(gu,\ F(z,\ x,\ y)) \end{array}\right\} \end{aligned}$$

and

$$\begin{aligned} &m(x,\ y,\ z,\ u,\ v,\ w) \\ =\ &\min\left\{\begin{array}{c} D(x,\ F(x,\ y,\ z)),\ D(u,\ F(u,\ v,\ w)), \\ D(y,\ F(y,\ z,\ x)),\ D(v,\ F(v,\ w,\ u)), \\ D(z,\ F(z,\ x,\ y)),\ D(w,\ F(w,\ u,\ v)), \\ D(x,\ F(u,\ v,\ w)),\ D(u,\ F(x,\ y,\ z)), \\ D(y,\ F(v,\ w,\ u)),\ D(v,\ F(y,\ z,\ x)), \\ D(z,\ F(w,\ u,\ v)),\ D(u,\ F(z,\ x,\ y)) \end{array}\right\}. \end{aligned}$$

Theorem 2.1. *Let $(X,\ d)$ be a metric space, $F : X \times X \times X \to K(X)$ and $g : X \to X$ be two mappings. Assume there exist some $\varphi \in \Phi$ and $\psi \in \Psi$ such* *that*

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \qquad (1) \\ \leq\ & \varphi\left(\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}\right) + \psi\left(M(x,\ y,\ z,\ u,\ v,\ w)\right), \end{aligned}$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$. *Furthermore assume that* $F(X \times X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subset of* X. *Then* F *and* g *have a tripled coincidence point. Moreover,* F *and* g *have a common tripled fixed point, if one of the following conditions holds:*

(*a*) F *and* g *are* $w-$*compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some* $(x,\ y,\ z) \in C(F,\ g)$ *and for some* $u,\ v,\ w \in X$ *and* g *is continuous at* u, v *and* w.

(*b*) g *is* $F-$*weakly commuting for some* $(x,\ y,\ z) \in C(F,\ g)$ *and* gx, gy *and* gz *are fixed points of* g, *that is,* $g^2x = gx$, $g^2y = gy$ *and* $g^2z = gz$.

(*c*) g *is continuous at* x, y *and* z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some* $(x,\ y,\ z) \in C(F,\ g)$ *and for some* $u,\ v,\ w \in X$.

(*d*) $g(C(F,\ g))$ *is a singleton subset of* $C(F,\ g)$.

Proof. Let $x_0,\ y_0,\ z_0 \in X$ be arbitrary. Then $F(x_0,\ y_0,\ z_0)$, $F(y_0,\ z_0,\ x_0)$ and $F(z_0,\ x_0,\ y_0)$ are well defined. Choose $gx_1 \in F(x_0,\ y_0,\ z_0)$, $gy_1 \in F(y_0,\ z_0,\ x_0)$ and $gz_1 \in F(z_0,\ x_0,\ y_0)$, because $F(X \times X \times X) \subseteq g(X)$. Since $F : X \times X \times X \to K(X)$, therefore by Lemma 1.1, there exist $u_1 \in F(x_1,\ y_1,\ z_1)$, $u_2 \in F(y_1,\ z_1,\ x_1)$ and $u_3 \in F(z_1,\ x_1,\ y_1)$ such that

$$\begin{aligned} d(gx_1,\ u_1) &\leq H(F(x_0,\ y_0,\ z_0),\ F(x_1,\ y_1,\ z_1)), \\ d(gy_1,\ u_2) &\leq H(F(y_0,\ z_0,\ x_0),\ F(y_1,\ z_1,\ x_1)), \\ d(gz_1,\ u_3) &\leq H(F(z_0,\ x_0,\ y_0),\ F(z_1,\ x_1,\ y_1)). \end{aligned}$$

Since $F(X \times X \times X) \subseteq g(X)$, there exist $x_2,\ y_2,\ z_2 \in X$ such that $u_1 = gx_2$, $u_2 = gy_2$ and $u_3 = gz_2$. Thus

$$\begin{aligned} d(gx_1,\ gx_2) &\leq H(F(x_0,\ y_0,\ z_0),\ F(x_1,\ y_1,\ z_1)), \\ d(gy_1,\ gy_2) &\leq H(F(y_0,\ z_0,\ x_0),\ F(y_1,\ z_1,\ x_1)), \\ d(gz_1,\ gz_2) &\leq H(F(z_0,\ x_0,\ y_0),\ F(z_1,\ x_1,\ y_1)). \end{aligned}$$

Continuing this process, we obtain sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X such that for all $n \in \mathbb{N}$, we have $gx_{n+1} \in F(x_n,\ y_n,\ z_n)$, $gy_{n+1} \in F(y_n,\ z_n,\ x_n)$ and $gz_{n+1} \in F(z_n,\ x_n,\ y_n)$ such that

$$\begin{aligned} & d(gx_n,\ gx_{n+1}) \\ \leq\ & H(F(x_{n-1},\ y_{n-1},\ z_{n-1}),\ F(x_n,\ y_n,\ z_n)) \\ \leq\ & \varphi\left(\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_{n-1},\ gz_n)\right\}\right) \\ & + \psi\left(M(x_{n-1},\ y_{n-1},\ z_{n-1},\ x_n,\ y_n,\ z_n)\right). \end{aligned}$$

Thus, by (i_ψ) and (ii_ψ), we get

$$d(gx_n,\ gx_{n+1}) \leq \varphi\left(\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_{n-1},\ gz_n)\right\}\right).$$

Similarly

$$\begin{aligned} d(gy_n,\ gy_{n+1}) &\leq \varphi\left(\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_{n-1},\ gz_n)\right\}\right), \\ d(gz_n,\ gz_{n+1}) &\leq \varphi\left(\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_{n-1},\ gz_n)\right\}\right). \end{aligned}$$

Combining them, we get

$$\begin{aligned} &\max\left\{d(gx_n,\ gx_{n+1}),\ d(gy_n,\ gy_{n+1}),\ d(gz_n,\ gz_{n+1})\right\} \\ \leq\ &\varphi\left(\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_{n-1},\ gz_n)\right\}\right), \end{aligned} \tag{2}$$

which implies, by (ii_φ), that

$$\begin{aligned} &\max\left\{d(gx_n,\ gx_{n+1}),\ d(gy_n,\ gy_{n+1}),\ d(gz_n,\ gz_{n+1})\right\} \\ <\ &\max\left\{d(gx_{n-1},\ gx_n),\ d(gy_{n-1},\ gy_n),\ d(gz_n,\ gz_{n+1})\right\}. \end{aligned}$$

This shows that the sequence $\{\delta_n\}_{n=0}^{\infty}$ defined by

$$\delta_n = \max\left\{d(gx_n,\ gx_{n+1}),\ d(gy_n,\ gy_{n+1}),\ d(gz_n,\ gz_{n+1})\right\},$$

is a decreasing sequence of positive numbers. Then there exists $\delta \geq 0$ such that

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty} \max\left\{d(gx_n,\ gx_{n+1}),\ d(gy_n,\ gy_{n+1}),\ d(gz_n,\ gz_{n+1})\right\} = \delta. \tag{3}$$

We shall prove that $\delta = 0$. Suppose that $\delta > 0$. Letting $n \to \infty$ in (2), by using (3) and (iii_φ), we get

$$\delta \leq \lim_{n\to\infty} \varphi(\delta_n) = \lim_{\delta_n\to\delta+} \varphi(\delta_n) < \delta,$$

which is a contradiction. Hence

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty} \max\left\{d(gx_n,\ gx_{n+1}),\ d(gy_n,\ gy_{n+1}),\ d(gz_n,\ gz_{n+1})\right\} = 0. \tag{4}$$

We now prove that $\{gx_n\}_{n=0}^{\infty}$, $\{gy_n\}_{n=0}^{\infty}$ and $\{gz_n\}_{n=0}^{\infty}$ are Cauchy sequences in $(X,\ d)$. Suppose, to the contrary, that one of the sequences is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ for which we can subsequences $\{gx_{n(k)}\}$, $\{gx_{m(k)}\}$ of $\{gx_n\}_{n=0}^{\infty}$, $\{gy_{n(k)}\}$, $\{gy_{m(k)}\}$ of $\{gy_n\}_{n=0}^{\infty}$ and $\{gz_{n(k)}\}$, $\{gz_{m(k)}\}$ of $\{gz_n\}_{n=0}^{\infty}$ such that

$$\max\left\{d(gx_{n(k)}, gx_{m(k)}), d(gy_{n(k)}, gy_{m(k)}), d(gz_{n(k)}, gz_{m(k)})\right\} \geq \varepsilon,\ k = 1, 2, ... \tag{5}$$

We can choose $n(k)$ to be the smallest positive integer satisfying (5). So

$$\max\left\{d(gx_{n(k)-1}, gx_{m(k)}), d(gy_{n(k)-1}, gy_{m(k)}), d(gz_{n(k)-1}, gz_{m(k)})\right\} < \varepsilon. \tag{6}$$

By (5), (6) and the triangle inequality, we have

$$\begin{aligned} \varepsilon &\leq r_k = \max\left\{d(gx_{n(k)}, gx_{m(k)}), d(gy_{n(k)}, gy_{m(k)}), d(gz_{n(k)}, gz_{m(k)})\right\} \\ &\leq \max\left\{d(gx_{n(k)}, gx_{n(k)-1}), d(gy_{n(k)}, gy_{n(k)-1}), d(gz_{n(k)}, gz_{n(k)-1})\right\} \\ &\quad + \max\left\{d(gx_{n(k)-1}, gx_{m(k)}), d(gy_{n(k)-1}, gy_{m(k)}), d(gz_{n(k)-1}, gz_{m(k)})\right\} \\ &< \max\left\{d(gx_{n(k)}, gx_{n(k)-1}), d(gy_{n(k)}, gy_{n(k)-1}), d(gz_{n(k)}, gz_{n(k)-1})\right\} + \varepsilon. \end{aligned}$$

Letting $k \to \infty$ in the above inequality and using (4), we get

$$\lim_{k\to\infty} r_k = \lim_{k\to\infty} \max\left\{d(gx_{n(k)}, gx_{m(k)}), d(gy_{n(k)}, gy_{m(k)}), d(gz_{n(k)}, gz_{m(k)})\right\} = \varepsilon. \tag{7}$$

By the triangle inequality, we have

$$\begin{aligned} & \max\left\{d(gx_{n(k)}, gx_{m(k)}), d(gy_{n(k)}, gy_{m(k)}), d(gz_{n(k)}, gz_{m(k)})\right\} \\ \leq\ & \max\left\{d(gx_{n(k)}, gx_{n(k)+1}), d(gy_{n(k)}, gy_{n(k)+1}), d(gz_{n(k)}, gz_{n(k)+1})\right\} \\ & + \max\left\{d(gx_{n(k)+1}, gx_{m(k)+1}), d(gy_{n(k)+1}, gy_{m(k)+1}), d(gz_{n(k)+1}, gz_{m(k)+1})\right\} \\ & + \max\left\{d(gx_{m(k)+1}, gx_{m(k)}), d(gy_{m(k)+1}, gy_{m(k)}), d(gz_{m(k)+1}, gz_{m(k)})\right\} \\ \leq\ & \delta_{n(k)} + \delta_{m(k)} + \max\left\{\begin{array}{c} d(gx_{n(k)+1},\ gx_{m(k)+1}), \\ d(gy_{n(k)+1},\ gy_{m(k)+1}), \\ d(gz_{n(k)+1},\ gz_{m(k)+1}) \end{array}\right\}. \end{aligned}$$

Thus

$$r_k \leq \delta_{n(k)} + \delta_{m(k)} + \max\left\{\begin{array}{c} d(gx_{n(k)+1},\ gx_{m(k)+1}), \\ d(gy_{n(k)+1},\ gy_{m(k)+1}), \\ d(gz_{n(k)+1},\ gz_{m(k)+1}) \end{array}\right\}. \tag{8}$$

Since $gx_{n(k)+1} \in F(x_{n(k)},\ y_{n(k)},\ z_{n(k)})$, $gx_{m(k)+1} \in F(x_{m(k)},\ y_{m(k)},\ z_{m(k)})$, $gy_{n(k)+1} \in F(y_{n(k)},\ z_{n(k)},\ x_{n(k)})$, $gy_{m(k)+1} \in F(y_{m(k)},\ z_{m(k)},\ x_{m(k)})$, $gz_{n(k)+1} \in F(z_{n(k)},\ x_{n(k)},\ y_{n(k)})$, $gz_{m(k)+1} \in F(z_{m(k)},\ x_{m(k)},\ y_{m(k)})$, therefore by (1), we have

$$\begin{aligned} & d(gx_{n(k)+1},\ gx_{m(k)+1}) \\ \leq\ & H(F(x_{n(k)},\ y_{n(k)},\ z_{n(k)}),\ F(x_{m(k)},\ y_{m(k)},\ z_{m(k)})) \\ \leq\ & \varphi\left(\max\left\{d(gx_{n(k)},\ gx_{m(k)}),\ d(gy_{n(k)},\ gy_{m(k)}),\ d(gz_{n(k)},\ gz_{m(k)})\right\}\right) \\ & +\psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right) \\ \leq\ & \varphi(r_k) +\ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right). \end{aligned}$$

Thus

$$\begin{aligned} & d(gx_{n(k)+1},\ gx_{m(k)+1}) \\ \leq\ & \varphi(r_k) +\ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right). \end{aligned}$$

Similarly

$$\begin{aligned} & d(gy_{n(k)+1},\ gy_{m(k)+1}) \\ \leq\ & \varphi(r_k) +\ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right), \end{aligned}$$

and

$$\begin{aligned} & d(gz_{n(k)+1},\ gz_{m(k)+1}) \\ \leq\ & \varphi(r_k) +\ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right). \end{aligned}$$

Combining them, we get

$$\max\left\{\begin{array}{c} d(gx_{n(k)+1},\ gx_{m(k)+1}), \\ d(gy_{n(k)+1},\ gy_{m(k)+1}), \\ d(gz_{n(k)+1},\ gz_{m(k)+1}) \end{array}\right\} \tag{9}$$
$$\leq \quad \varphi(r_k) + \ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right).$$

By (8) and (9), we get

$$r_k \leq \delta_{n(k)} + \delta_{m(k)} + \varphi(r_k) + \ \psi\left(M(x_{n(k)},\ y_{n(k)},\ z_{n(k)},\ x_{m(k)},\ y_{m(k)},\ z_{m(k)})\right).$$

Letting $k \to \infty$ in the above inequality, by using (4), (7), (i_ψ), (ii_ψ) and (iii_φ), we get

$$\varepsilon \leq 0 + 0 + \lim_{k\to\infty} \varphi(r_k) + 0 \ \leq \lim_{r_k\to\varepsilon+} \varphi(r_k) \quad < \varepsilon,$$

which is a contradiction. This shows that $\{gx_n\}_{n=0}^{\infty}$, $\{gy_n\}_{n=0}^{\infty}$ and $\{gz_n\}_{n=0}^{\infty}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, therefore there exist x, y, $z \in X$ such that

$$\lim_{n\to\infty} gx_n = gx, \quad \lim_{n\to\infty} gy_n = gy \text{ and } \lim_{n\to\infty} gz_n = gz. \tag{10}$$

Now, since $gx_{n+1} \in F(x_n,\ y_n,\ z_n)$, $gy_{n+1} \in F(y_n,\ z_n,\ x_n)$ and $gz_{n+1} \in F(z_n,$ $x_n,\ y_n)$, therefore by using condition (1), we get

$$\begin{aligned} & D(gx_{n+1},\ F(x,\ y,\ z)) \\ \leq\ & H(F(x_n,\ y_n,\ z_n),\ F(x,\ y,\ z)) \\ \leq\ & \varphi\left(\max\{d(gx_n,\ gx),\ d(gy_n,\ gy),\ d(gz_n,\ gz)\}\right) \\ & +\psi\left(M\{x_n,\ y_n,\ z_n,\ x,\ y,\ z\}\right). \end{aligned}$$

Letting $n \to \infty$ in the above inequality, by using (10), (i_ψ), (ii_ψ) and (iii_φ), we get

$$D(gx,\ F(x,\ y,\ z)) \leq \lim_{t\to 0+} \varphi(t) + 0 = 0 + 0 = 0,$$

which implies that

$$D(gx,\ F(x,\ y,\ z)) = 0.$$

Similarly, we can get

$$D(gy,\ F(y,\ z,\ x)) = 0 \text{ and } D(gz,\ F(z,\ x,\ y)) = 0.$$

It follows that

$$gx \in F(x,\ y,\ z),\ gy \in F(y,\ z,\ x) \text{ and } gz \in F(z,\ x,\ y),$$

that is $(x,\ y,\ z)$ is a tripled coincidence point of F and g.

Suppose now that (a) holds. Assume that for some $(x,\ y,\ z) \in C(F,\ g)$,

$$\lim_{n\to\infty} g^n x = u, \quad \lim_{n\to\infty} g^n y = v \text{ and } \lim_{n\to\infty} g^n z = w, \tag{11}$$

where u, v, $w \in X$. Since g is continuous at u, v and w, we have, by (11), that u, v and w are fixed points of g, that is,

$$gu = u,\ gv = v \text{ and } gw = w. \tag{12}$$

As F and g are $w-$compatible, so for all $n \geq 1$

$$\begin{array}{rcl} g^n x & \in & F(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z), \\ g^n y & \in & F(g^{n-1}y,\ g^{n-1}z,\ g^{n-1}x), \\ g^n z & \in & F(g^{n-1}z,\ g^{n-1}x,\ g^{n-1}y). \end{array} \tag{13}$$

By using (1) and (13), we obtain

$$\begin{array}{rl} & D(g^n x,\ F(u,\ v,\ w)) \\ \leq & H(F(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z),\ F(u,\ v,\ w)) \\ \leq & \varphi\left(\max\{d(g^n x,\ gu),\ d(g^n y,\ gv),\ d(g^n z,\ gw)\}\right) \\ & +\psi\left(M(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z,\ u,\ v,\ w)\right). \end{array}$$

On taking limit as $n \to \infty$ in the above inequality, by using (11), (12), (i_ψ), (ii_ψ) and (iii_φ), we get

$$D(gu,\ F(u,\ v,\ w)) \leq \lim_{t \to 0+} \varphi(t) + 0 = 0 + 0 = 0,$$

which implies that

$$D(gu,\ F(u,\ v,\ w)) = 0.$$

Similarly, we can get

$$D(gv,\ F(v,\ w,\ u)) = 0 \text{ and } D(gw,\ F(w,\ u,\ v)) = 0.$$

It follows that

$$gu \in F(u,\ v,\ w),\ gv \in F(v,\ w,\ u) \text{ and } gw \in F(w,\ u,\ v). \tag{14}$$

By (12) and (14), we get

$$u = gu \in F(u,\ v,\ w),\ v = gv \in F(v,\ w,\ u) \text{ and } w = gw \in F(w,\ u,\ v),$$

that is, $(u,\ v,\ w)$ is a common tripled fixed point of F and g.

Suppose now that (b) holds. Assume that for some $(x,\ y,\ z) \in C(F,\ g)$, g is $F-$weakly commuting, that is, $g^2x \in F(gx,\ gy,\ gz)$, $g^2y \in F(gy,\ gz,\ gx)$ and $g^2z \in F(gz,\ gx,\ gy)$ and $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$. Thus $gx = g^2x \in F(gx,\ gy,\ gz)$, $gy = g^2y \in F(gy,\ gz,\ gx)$ and $gz = g^2z \in F(gz,\ gy,\ gx)$, that is, $(gx,\ gy,\ gz)$ is a common tripled fixed point of F and g.

Suppose now that (c) holds. Assume that for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$,

$$\lim_{n\to\infty} g^n u = x,\ \lim_{n\to\infty} g^n v = y \text{ and } \lim_{n\to\infty} g^n w = z.$$

Since g is continuous at x, y and z. Therefore x, y and z are fixed points of g, that is,

$$gx = x,\ gy = y \text{ and } gz = z.$$

Since $(x,\ y,\ z) \in C(F,\ g)$, therefore, we obtain

$$x = gx \in F(x,\ y,\ z),\ y = gy \in F(y,\ z,\ x) \text{ and } z = gz \in F(z,\ x,\ y),$$

that is, $(x,\ y,\ z)$ is a common tripled fixed point of F and g.

Finally, suppose that (d) holds. Let $g(C(F,\ g)) = \{(x,\ x,\ x)\}$. Then $\{x\} = \{gx\} = F(x,\ x,\ x)$. Hence $(x,\ x,\ x)$ is tripled fixed point of F and g. ∎

Example 1. *Suppose that* $X = [0,\ 1]$, *equipped with the metric* $d : X \times X \to [0,\ +\infty)$ *defined by* $d(x,\ y) = \max\{x,\ y\}$ *and* $d(x,\ x) = 0$ *for all* $x,\ y \in X$. *Let* $F : X \times X \times X \to K(X)$ *be defined as*

$$F(x,\ y,\ z) = \begin{cases} \{0\}, & \text{for } x,\ y,\ z = 1, \\ \left[0,\ \frac{x^2+y^2+z^2}{6}\right], & \text{for } x,\ y,\ z \in [0,\ 1), \end{cases}$$

and $g : X \to X$ *be defined as*

$$g(x) = x^2,\ \text{for all } x \in X.$$

Define $\varphi : [0,\ +\infty) \to [0,\ +\infty)$ *by*

$$\varphi(t) = \begin{cases} \frac{t}{2}, & \text{for } t \neq 1, \\ \frac{3}{4}, & \text{for } t = 1, \end{cases}$$

and $\psi : [0,\ +\infty) \to [0,\ +\infty)$ *by*

$$\psi(t) = \frac{t}{4},\ \text{for all } t \geq 0.$$

Now, for all $x,\ y,\ z,\ u,\ v,\ w \in X$ *with* $x,\ y,\ z,\ u,\ v,\ w \in [0,\ 1)$, *we have*

Case (a) If $x^2 + y^2 + z^2 = u^2 + v^2 + w^2$, *then*

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\ = \ & \frac{u^2+v^2+w^2}{6} \\ \leq \ & \frac{1}{6}\max\{x^2,\ u^2\} + \frac{1}{6}\max\{y^2,\ v^2\} + \frac{1}{6}\max\{z^2,\ w^2\} \\ \leq \ & \frac{1}{6}d(gx,\ gu) + \frac{1}{6}d(gy,\ gv) + \frac{1}{6}d(gz,\ gw) \\ \leq \ & \frac{1}{2}\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) \\ \leq \ & \varphi\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) + \psi\left(M(x,\ y,\ z,\ u,\ v,\ w)\right). \end{aligned}$$

Case (b) If $x^2+y^2+z^2 \neq u^2+v^2+w^2$ *with* $x^2+y^2+z^2 < u^2+v^2+w^2$, *then*

$$\begin{aligned}
& H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\
= \quad & \frac{u^2+v^2+w^2}{6} \\
\leq \quad & \frac{1}{6}\max\{x^2,\ u^2\} + \frac{1}{6}\max\{y^2,\ v^2\} + \frac{1}{6}\max\{z^2,\ w^2\} \\
\leq \quad & \frac{1}{6}d(gx,\ gu) + \frac{1}{6}d(gy,\ gv) + \frac{1}{6}d(gz,\ gw) \\
\leq \quad & \frac{1}{2}\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) \\
\leq \quad & \varphi\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) + \psi\left(M(x,\ y,\ z,\ u,\ v,\ w)\right).
\end{aligned}$$

Similarly, we obtain the same result for $u^2+v^2+w^2 < x^2+y^2+z^2$. *Thus the contractive condition (1) is satisfied for all* $x,\ y,\ z,\ u,\ v,\ w \in X$ *with* $x,\ y,\ z,\ u,\ v,\ w \in [0,\ 1)$. *Again, for all* $x,\ y,\ z,\ u,\ v,\ w \in X$ *with* $x,\ y,\ z \in [0,\ 1)$ *and* $u,\ v,\ w = 1$, *we have*

$$\begin{aligned}
& H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\
= \quad & \frac{x^2+y^2+z^2}{6} \\
\leq \quad & \frac{1}{6}\max\{x^2,\ u^2\} + \frac{1}{6}\max\{y^2,\ v^2\} + \frac{1}{6}\max\{z^2,\ w^2\} \\
\leq \quad & \frac{1}{6}d(gx,\ gu) + \frac{1}{6}d(gy,\ gv) + \frac{1}{6}d(gz,\ gw) \\
\leq \quad & \frac{1}{2}\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) \\
\leq \quad & \varphi\left(\max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}\right) + \psi\left(M(x,\ y,\ z,\ u,\ v,\ w)\right).
\end{aligned}$$

Thus the contractive condition (1) is satisfied for all $x,\ y,\ z,\ u,\ v,\ w \in X$ *with* $x,\ y,\ z \in [0,\ 1)$ *and* $u,\ v,\ w = 1$. *Similarly, we can see that the contractive condition (1) is satisfied for all* $x,\ y,\ z,\ u,\ v,\ w \in X$ *with* $x,\ y,\ z,\ u,\ v,\ w = 1$. *Hence, the hybrid pair* $\{F,\ g\}$ *satisfies the contractive condition (1), for all* $x,\ y,\ z,\ u,\ v,\ w \in X$. *In addition, all the other conditions of Theorem 2.1 are satisfied and* $z = (0,\ 0,\ 0)$ *is a common tripled fixed point of hybrid pair* $\{F,\ g\}$. *The function* $F : X \times X \times X \to K(X)$ *involved in this example is not a continuous function on* $X \times X \times X$.

Corollary 2.2. *Let* $(X,\ d)$ *be a metric space,* $F : X \times X \times X \to K(X)$ *and* $g : X \to X$ *be two mappings. Assume there exist some* $\varphi \in \Phi$ *and* $\psi \in \Psi$ *such that*

$$\begin{aligned}
& H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\
\leq \quad & \varphi\left(\frac{d(gx,\ gu) + d(gy,\ gv) + d(gz,\ gw)}{3}\right) + \psi\left(M(x,\ y,\ z,\ u,\ v,\ w)\right),
\end{aligned}$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$. *Furthermore assume that* $F(X\times X\times X)\subseteq g(X)$ *and* $g(X)$ *is a complete subset of* X. *Then* F *and* g *have a tripled coincidence point. Moreover,* F *and* g *have a common tripled fixed point, if one of the following conditions holds:*

(a) F *and* g *are* $w-$*compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some* $(x,\ y,\ z)\in C(F,\ g)$ *and for some* $u,\ v,\ w\in X$ *and* g *is continuous at* u, v *and* w.

(b) g *is* $F-$*weakly commuting for some* $(x,\ y,\ z)\in C(F,\ g)$ *and* gx, gy *and* gz *are fixed points of* g, *that is,* $g^2x = gx$, $g^2y = gy$ *and* $g^2z = gz$.

(c) g *is continuous at* x, y *and* z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some* $(x,\ y,\ z)\in C(F,\ g)$ *and for some* $u,\ v,\ w\in X$.

(d) $g(C(F,\ g))$ *is a singleton subset of* $C(F,\ g)$.

Proof. Since

$$\begin{aligned} & \frac{d(gx,\ gu)+d(gy,\ gv)+d(gz,\ gw)}{3} \\ \leq\ & \max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\}. \end{aligned}$$

Then, we apply Theorem 2.1, since φ is non-decreasing. ∎

If we put $g = I$ (the identity mapping) in the Theorem 2.1, we get the following result:

Corollary 2.3. *Let* $(X,\ d)$ *be a complete metric space,* $F: X\times X\times X\to K(X)$ *be a mapping. Assume there exist some* $\varphi\in\Phi$ *and* $\psi\in\Psi$ *such that*

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\ \leq\ & \varphi\left(\max\{d(x,\ u),\ d(y,\ v),\ d(z,\ w)\}\right)+\psi\left(m(x,\ y,\ z,\ u,\ v,\ w)\right), \end{aligned}$$

for all $x,\ y,\ z,\ u,\ v,\ w\in X$. *Then* F *has a tripled fixed point.*

If we put $g = I$ (the identity mapping) in the Corollary 2.2, we get the following result:

Corollary 2.4. *Let* $(X,\ d)$ *be a complete metric space,* $F: X\times X\times X\to K(X)$ *be a mapping. Assume there exist some* $\varphi\in\Phi$ *and* $\psi\in\Psi$ *such that*

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\ \leq\ & \varphi\left(\frac{d(x,\ u)+d(y,\ v)+d(z,\ w)}{3}\right)+\psi\left(m(x,\ y,\ z,\ u,\ v,\ w)\right), \end{aligned}$$

for all $x,\ y,\ z,\ u,\ v,\ w\in X$. *Then* F *has a tripled fixed point.*

If we put $\psi(t) = 0$ in Theorem 2.1, we get the following result:

Corollary 2.5. *Let $(X,\ d)$ be a metric space, $F : X \times X \times X \to K(X)$ and $g : X \to X$ be two mappings. Assume there exists some $\varphi \in \Phi$ such that*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le \varphi\left(\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}\right),$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$. Furthermore assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(*a*) *F and g are $w-$compatible. $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ and $\lim_{n\to\infty} g^n z = w$ for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$ and g is continuous at u, v and w.*

(*b*) *g is $F-$weakly commuting for some $(x,\ y,\ z) \in C(F,\ g)$ and gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.*

(*c*) *g is continuous at x, y and z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$.*

(*d*) *$g(C(F,\ g))$ is a singleton subset of $C(F,\ g)$.*

If we put $\psi(t) = 0$ in Corollary 2.2, we get the following result:

Corollary 2.6. *Let $(X,\ d)$ be a metric space, $F : X \times X \times X \to K(X)$ and $g : X \to X$ be two mappings. Assume there exists some $\varphi \in \Phi$ such that*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le \varphi\left(\frac{d(gx,\ gu),\ d(gy,\ gv) + d(gz,\ gw)}{3}\right),$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$. Furthermore assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(*a*) *F and g are $w-$compatible. $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ and $\lim_{n\to\infty} g^n z = w$ for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$ and g is continuous at u, v and w.*

(*b*) *g is $F-$weakly commuting for some $(x,\ y,\ z) \in C(F,\ g)$ and gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.*

(*c*) *g is continuous at x, y and z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$.*

(*d*) *$g(C(F,\ g))$ is a singleton subset of $C(F,\ g)$.*

If we put $g = I$ (the identity mapping) in the Corollary 2.5, we get the following result:

Corollary 2.7. *Let $(X,\ d)$ be a complete metric space, $F : X \times X \times X \to K(X)$ be a mapping. Assume there exists some $\varphi \in \Phi$ such that*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le \varphi\left(\max\left\{d(x,\ u),\ d(y,\ v),\ d(z,\ w)\right\}\right),$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$. Then F has a tripled fixed point.

If we put $g = I$ (the identity mapping) in the Corollary 2.6, we get the following result:

Corollary 2.8. *Let* (X, d) *be a complete metric space,* $F : X\times X\times X \to K(X)$ *be a mapping. Assume there exists some* $\varphi \in \Phi$ *such that*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le \varphi\left(\frac{d(x,\ u)+d(y,\ v)+d(z,\ w)}{3}\right),$$

for all $x, y, z, u, v, w \in X$. *Then* F *has a tripled fixed point.*

If we put $\varphi(t) = kt$ where $0 < k < 1$ in Corollary 2.5, we get the following result:

Corollary 2.9. *Let* (X, d) *be a metric space. Assume* $F : X\times X\times X \to K(X)$ *and* $g : X \to X$ *be two mappings satisfying*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le k \max\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\},$$

for all $x, y, z, u, v, w \in X$, *where* $0 < k < 1$. *Furthermore assume that* $F(X\times X\times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subset of* X. *Then* F *and* g *have a tripled coincidence point. Moreover,* F *and* g *have a common tripled fixed point, if one of the following conditions holds:*

(a) F *and* g *are* $w-$*compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some* $(x, y, z) \in C(F, g)$ *and for some* $u, v, w \in X$ *and* g *is continuous at* u, v *and* w.

(b) g *is* $F-$*weakly commuting for some* $(x, y, z) \in C(F, g)$ *and* gx, gy *and* gz *are fixed points of* g, *that is,* $g^2x = gx$, $g^2y = gy$ *and* $g^2z = gz$.

(c) g *is continuous at* x, y *and* z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some* $(x, y, z) \in C(F, g)$ *and for some* $u, v, w \in X$.

(d) $g(C(F, g))$ *is a singleton subset of* $C(F, g)$.

If we put $\varphi(t) = kt$ where $0 < k < 1$ in Corollary 2.6, we get the following result:

Corollary 2.10. *Let* (X, d) *be a metric space. Assume* $F : X\times X\times X \to K(X)$ *and* $g : X \to X$ *be two mappings satisfying*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \le \frac{k}{3}\left(d(gx,\ gu)+d(gy,\ gv)+d(gz,\ gw)\right),$$

for all $x, y, z, u, v, w \in X$ *where* $0 < k < 1$. *Furthermore assume that* $F(X\times X\times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subset of* X. *Then* F *and* g *have a tripled coincidence point. Moreover,* F *and* g *have a common tripled fixed point, if one of the following conditions holds:*

(a) F *and* g *are* $w-$*compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some* $(x, y, z) \in C(F, g)$ *and for some* $u, v, w \in X$ *and* g *is continuous at* u, v *and* w.

(b) g is $F-$weakly commuting for some $(x,\ y,\ z) \in C(F,\ g)$ and gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.

(c) g is continuous at x, y and z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$.

(d) $g(C(F,\ g))$ is a singleton subset of $C(F,\ g)$.

If we put $g = I$ (the identity mapping) in the Corollary 2.9, we get the following result:

Corollary 2.11. *Let $(X,\ d)$ be a complete metric space, $F : X\times X\times X \to K(X)$ be a mapping satisfying*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \leq k \max\left\{d(x,\ u),\ d(y,\ v),\ d(z,\ w)\right\},$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$, where $0 < k < 1$. Then F has a tripled fixed point.

If we put $g = I$ (the identity mapping) in the Corollary 2.10, we get the following result:

Corollary 2.12. *Let $(X,\ d)$ be a complete metric space, $F : X\times X\times X \to K(X)$ be a mapping satisfying*

$$H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \leq \frac{k}{3}\left(d(x,\ u) + d(y,\ v) + d(z,\ w)\right),$$

for all $x,\ y,\ z,\ u,\ v,\ w \in X$, where $0 < k < 1$. Then F has a tripled fixed point.

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