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FUZZY GENERAL NONLINEAR ORDERED RANDOM VARIATIONAL INEQUALITIES IN ORDERED BANACH SPACES

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Abstract. The main object of this work to introduced and studied a new class of *fuzzy general nonlinear ordered random variational inequalities* in ordered Banach spaces. By using the random B-restricted accretive mapping with measurable mappings $\alpha, \alpha^{'} : \Omega \to (0, 1)$, an existence of random solutions for this class of *fuzzy general nonlinear ordered random variational inequality* (equation) with fuzzy mappings is established, a random approximation algorithm is suggested for fuzzy mappings, and the relation between the first value $x_0(t)$ and the random solutions of *fuzzy general nonlinear ordered random variational inequality* is discussed.

1. Introduction

The variational inclusions, which was introduced and studied by Hassouni and Moudafi [19] is a useful and important extension of variational inequalities. In last decades, monotonicity techniques were extended and applied because of their importance in theory of variational inequality, complementarity problems and variational inclusions. Recently some systems of variational inequalities, variational inclusions, complementarity problems and equilibrium problems have been studied by some authors in recent years because of their close relations to Nash equilibrium problems. Huang and Fang [21] introduced a system of ordered complementarity problems and established some existence results for using fixed point theory. Verma [32] introduced and studied the some systems of variational inequalities and developed some random iterative algorithm for approximation of random solutions for system of variational inequalities.
On the other hand in 1972, the number of solutions of nonlinear equation has been introduced and studied by Amann [6] and recent years, the nonlinear mapping, fixed point theory and application have been extensively studied in ordered Banach spaces, [16, 17, 18]. Very recently Li [22, 23, 24] has studied

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the approximation solution for general nonlinear ordered variational inequalities and ordered equations in ordered Banach spaces.

Fuzzy sets were founded by Professor L. A. Zadeh in year 1965 [34]. The address of fuzzy set theory, since its introduction has been dramatic and breathtaking, several research papers have published in different journals devoted entirely to theoretical and application aspects of fuzzy sets. In 1989, Chang and Zhu [9] introduced the concept of variational inequalities in fuzzy mappings in abstract spaces and investigated existence theorem for some kinds of variational inequalities for fuzzy mappings. Very recently, the problems of random generalized fuzzy variational inclusions involving random nonlinear mapping have been studied by Zhang and Bi [33] in Hilbert spaces. Afterwards, on several kinds of variational inequalities, variational inclusions and complementarity problems for fuzzy mappings were considered and studied by many authors see for instance, Ahmad and Salahuddin [1, 2, 3], Ahmad and Bazan [4], Agarwal *et al.* [5], Anastassiou *et al.* [7], Chang and Huang [10], Chang and Salahuddin [11], Chang *et al.* [12], Cho *et al.* [14], Ding and Park [15], Huang [20], Lee *et al.* [25, 26, 27], Salahuddin [28], Salahuddin and Ahmad [30], Salahuddin *et al.* [31] and Salahuddin and Verma [29], *etc.*
Inspired and motivated by recent works, in this communication, *fuzzy general nonlinear ordered random variational inequalities* and an operator $\oplus$ is introduced and the qualities of an operator $\oplus$ is studied in ordered Banach spaces. Applying the random B-restricted accretive method of random mapping A with measurable operators $\alpha, \alpha^{'}$, an existence theorem of random solutions for this class of *fuzzy general nonlinear ordered variational inequalities* is established, a random approximation algorithm is suggested and the relation between the values $x_0(t)$ and the random solutions of the *fuzzy general nonlinear ordered random variational inequality* is discussed.

2. Preliminaries

Throughout this work, we assume that (Ω, Σ, μ) is a complete σ- finite measurable space and X is a separable real Banach space endowed with dual space X^*, the norm $\|\cdot\|$ and the dual pair $\langle\cdot,\cdot\rangle$ between X and X^*. We denote by $\mathfrak{B}(X)$ the class of Borel σ- field in X. Let 2^X and $CB(X)$ denote the family of all nonempty subset of X and the family of all nonempty bounded closed sets of X, respectively.

Definition 1. A mapping $x : \Omega \to X$ is said to be measurable if for any $\mathrm{B} \in \mathfrak{B}(X), \{t \in \Omega, x(t) \in \mathrm{B}\} \in \Sigma$.

Definition 2. A mapping $f : \Omega \times X \to X$ is called a random operator if for any $x \in X, f(t, x) = x(t)$ is a measurable. A random operator f is said to be continuous if for any $t \in \Omega$, the mapping $f(t, \cdot) : X \to X$ is continuous.

Definition 3. A set valued mapping $T : \Omega \times X \to 2^X$ is said to be measurable if for any $\mathrm{B} \in \mathfrak{B}(X), T^{-1}(\mathrm{B}) = \{t \in \Omega, T(t) \cap \mathrm{B} \neq \emptyset\} \in \Sigma$.

Definition 4. A mapping $u : \Omega \to X$ is called a measurable selection of a set valued measurable mapping $T : \Omega \to 2^X$ if u is a measurable and for any $t \in \Omega, u(t) \in T(t)$.

Definition 5. A mapping $T : \Omega \times X \to 2^X$ is called a random set valued mapping if for any $x \in X, T(\cdot, x)$ is a measurable. A random set valued mapping $T : \Omega \times X \to CB(X)$ is said to be $\mathcal{H}$-continuous if for any $t \in \Omega, T(t, \cdot)$ is continuous in Hausdorff metric.

Definition 6. A fuzzy mapping $F : \Omega \to \mathfrak{F}(X)$ is called measurable if for any $\alpha \in (0,1), (F(\cdot))_\alpha : \Omega \to 2^X$ is a measurable set valued mappings.

Definition 7. A fuzzy mapping $F : \Omega \times X \to \mathfrak{F}(X)$ is called a random fuzzy mapping if for any $x \in X, F(\cdot, x) \to \mathfrak{F}(X)$ is a measurable fuzzy mapping.

Let $\mathfrak{F}(X)$ be a collection of all fuzzy sets over X. A mapping F from X to $\mathfrak{F}(X)$ is called a fuzzy mapping on X. If F is a fuzzy mapping on X, the $F(x)$ (denote it by F_x, in the sequel) is a fuzzy set on X and $(F_x)(y)$ is the membership function of y in F_x. Let $N \in \mathfrak{F}(x), q \in [0,1]$, then the set

$$(N)_q = \{x \in X : N(x) \geq q\}$$

is called a q-cut set of N.

Let $T : \Omega \times X \to \mathfrak{F}(X)$ be the random fuzzy mapping satisfying the following condition (C):

(C): There exists a mapping $a : X \to [0,1]$ such that $(T_{t,x})_{a(x)} \in CB(X)\ \forall (t,x) \in \Omega \times X$. By using the random fuzzy mapping T we can define random set valued mapping $\tilde{T}$ as follows:

$\tilde{T} : \Omega \times X \to CB(X),\ x \to (T_{t,x})_{a(x)},\ \forall\ (t,x) \in \Omega \times X$ where $T_{t,x} = T(t, x(t))$.

Let X be a real ordered Banach space with a norm $\|\cdot\|$ and θ be a zero in the X. Let P be a normal cone of X and $\leq$ be a partial ordered relation defined by the cone P. Given a mapping $a : X \to [0,1]$, random fuzzy mapping $T : \Omega \times X \to \mathfrak{F}(X)$, let $A, g, f : \Omega \times X \to X$ be the single valued random nonlinear ordered comparison mappings and range $g(x(t),t) \cap \mathrm{dom}A(\cdot,t) = \emptyset\ \forall\ t \in \Omega$, we consider the following problem:

Find a measurable mapping $x, u : \Omega \to X$ such that for all $t \in \Omega, x(t) \in X, T_{(t,x(t))}(u(t)) \geq a(x(t))$ and $g(t, x(t)) \cap \mathrm{dom}\ A(\cdot, t) \neq \emptyset$ for $t \in \Omega$ such that

$$A(g(x(t),t),t) + f(u(t),t) \geq \theta. \tag{1}$$

The problem (2.1) is called a *fuzzy general nonlinear ordered random variational inequalities* in ordered Banach spaces.

Definition 8. [13] Let X be a real Banach space with a norm $\|\cdot\|$, θ be a zero element in the X. A nonempty closed convex subsets P of X is said to be a cone if

(a) for any $x \in P$, and any $\lambda > 0$, $\lambda x \in P$ holds;
(b) if $x \in P$ and $-x \in P$ then $x = \theta$.

Definition 9. [22] Let P be a cone of X. P is said to be a normal cone if and only if there exists a constant $N > 0$ such that for $\theta \leq x \leq y$, hold $\|x\| \leq N\|y\|$ where N is called normal constant of P.

Lemma 2.1. [13] *Let P be a cone in X, for arbitrary $x, y \in X, x \leq y$ if and only if $x - y \in P$, then the relation $\leq$ in X is a partial ordered relation in X where the Banach space X with an ordered relation $\leq$ defined by a normal cone P is called an ordered Banach space.*

Definition 10. [13] Let X be an ordered Banach space and P be a cone of X. The $\leq$ is a partial ordered relation defined by the cone P for all $x, y \in X$ if hold $x \leq y($ or $y \leq x)$ then x and y is said to be the comparison between each other (denoted by $x \propto y$ for $x \leq y$ and $y \leq x$).

Definition 11. [13] Let X be an ordered Banach space and P be a cone of X. The $\leq$ is a partial ordered relation defined by the cone P, for arbitrary $x, y \in X$, $lub\{x, y\}$ and $glb\{x, y\}$ express the least upper bound of the set $\{x, y\}$ and the greatest lower bound of the set $\{x, y\}$ on the partial ordered relation $\leq$ respectively. Suppose $lub\{x, y\}$ and $glb\{x, y\}$ exists some binary operator can be defined as follows:

(i) $x \vee y = lub\{x, y\}$;
(ii) $x \wedge y = glb\{x, y\}$;
(iii) $x \oplus y = (x - y) \vee (y - x)$.

$\vee, \wedge$ and $\oplus$ is called OR, AND and XOR operations, respectively. For arbitrary $x, y, w \in X$ then holds the following relations:

(1) if $x \leq y$ then $x \vee y = y$, $x \wedge y = x$;
(2) if x and y can be compared then $\theta \leq x \oplus y$;
(3) $(x + w) \vee (y + w)$ exists and $(x + w) \vee (y + w) = (x \vee y) + w$;
(4) $(x \wedge y) = (x + y) - (x \vee y)$;
(5) if $\lambda \geq 0$ then $\lambda(x \vee y) = \lambda x \vee \lambda y$;
(6) if $\lambda \leq 0$ then $\lambda(x \wedge y) = \lambda x \vee \lambda y$;
(7) if $x \neq y$ then the converse holds for (5) and (6);
(8) if for any $x, y \in X$, either $x \vee y$ and $x \wedge y$ exists, then X is a lattice;
(9) $(x + w) \wedge (y + w)$ exists and $(x + w) \wedge (y + w) = (x \wedge y) + w$;
(10) $(x \wedge y) = -(-x \vee -y)$;
(11) $(-x) \wedge (x) \leq \theta \leq (-x) \vee x$.

Lemma 2.2. [16] *If $x \propto y$ then $lub\{x, y\}$ and $glb\{x, y\}$ exist, $x - y \propto y - x$, and $\theta \leq (x - y) \vee (y - x)$.*

Lemma 2.3. [16] *If for any natural number n, $x \propto y_n$ and $y_n \to y^*$ $(n \to \infty)$ then $x \propto y^*$.*

Lemma 2.4. [22] *Let X be an ordered Banach space and P be a cone of X. The $\leq$ is a partial ordered relation defined by the cone P, if for $x, y, z, w \in X$ they can be compared each other, then holds the following relations:*

(1) $x \oplus y = y \oplus x$;
(2) $x \oplus x = \theta$;
(3) $\theta \leq x \oplus \theta$;
(4) *let λ be a real then* $(\lambda x) \oplus (\lambda y) = \mid \lambda \mid (x \oplus y)$;
(5) *if x, y and w can be comparative each other then* $(x \oplus y) \leq x \oplus w + w \oplus y$;
(6) *let $(x + y) \vee (u + v)$ exists and if $x \propto u, v$ and $y \propto u, v$ then;*

$$(x + y) \oplus (u + v) \leq (x \oplus u + y \oplus v) \wedge (x \oplus v + y \oplus u);$$

(7) *if x, y, z, w can be compared with each other then*

$$(x \wedge y) \oplus (z \wedge w) \leq ((x \oplus z) \vee (y \oplus w)) \wedge ((x \oplus w) \vee (y \oplus z));$$

(8) $\alpha x \oplus \beta x = \mid \alpha - \beta \mid x + (\alpha \oplus \beta)x$ *if* $x \propto \theta$.

Lemma 2.5. [8] *Let $T : \Omega \times X \to CB(X)$ be a $\mathcal{H}$-continuous random set valued mapping. Then for any measurable mapping $w : \Omega \to X$, the set valued mapping $T(\cdot, w(\cdot)) : \Omega \to CB(X)$ is a measurable.*

Lemma 2.6. [8] *Let $T, S : \Omega \to CB(X)$ be the two measurable set valued mappings, $\epsilon > 0$ be a constant and $\upsilon : \Omega \to H$ be a measurable selection of S then there exists a measurable selection $w : \Omega \to H$ of T such that for all $t \in \Omega$*

$$\|\upsilon(t) - w(t)\| \leq (1 + \epsilon)\mathcal{H}(S(t), T(t)).$$

Definition 12. Let X be a real ordered Banach space and let $A, B : \Omega \times X \to X$ be the two random mappings.

(i) $A(t)$ is said to be randomly comparison if for any $t \in \Omega$ and each $x(t), y(t) \in X$, $x(t) \propto y(t)$ then $A(x(t), t) \propto A(y(t), t)$, $x(t) \propto A(x(t), t)$ and $y(t) \propto A(y(t), t)$.
(ii) $A(t)$ and $B(t)$ are said to be randomly comparison with each other if for each $t \in \Omega, x(t) \in X, A(x(t), t) \propto B(x(t), t)$ (denoted by $A(t) \propto B(t)$).

Obviously, if $A(t)$ is a randomly comparison, then $A(t) \propto I(t)$ (where $I(t)$ is an random identity mapping on the X).

Definition 13. Let X be a real ordered Banach space, P be a normal cone with normal constant N in X, Ω be a set in X, $A : \Omega \times X \to X$ be a random mapping. A random mapping $A(t)$ is said to be randomly $\beta(t)$-order compression with respect to a measurable mapping $\beta : \Omega \to (0, 1)$ if $A(t)$ is a randomly comparative with respect to the measurable mapping $\beta : \Omega \to (0, 1)$ such that for any $t \in \Omega$,

$$A(x(t), t) \oplus A(y(t), t) \leq \beta(t)(x(t) \oplus y(t)),$$

holds.

Definition 14. Let X be a real ordered Banach space, P be a normal cone with normal constant N in the X, Ω be a nonempty open subset of X in which the t takes values, $A, B : \Omega \times X \to X$ be the two random mappings, I be an identity mapping on the $X \times X$.

(i) A mapping $A : \Omega \times X \to X$ is said to be randomly restricted accretive mapping if $A(t)$ is randomly comparative and there exists two measurable mappings $\alpha, \alpha^{'} : \Omega \to (0, 1)$ such that for all $t \in \Omega, x(t), y(t) \in X$,

$$(A(x(t), t) + I(x(t), t)) \oplus (A(y(t), t) + I(y(t), t))$$
$$\leq \alpha(t)(A(x(t), t) \oplus A(y(t), t)) + \alpha^{'}(t)(x(t) \oplus y(t))$$

holds where I is an random identity mapping on $\Omega \times X$.

(ii) A mapping $A : \Omega \times X \to X$ is said to be randomly $B(t)$-restricted accretive mapping, if $A(t), B(t) \in X,\ t \in \Omega$ and $A(t) \wedge B(t) : \Omega \times X \to A(x(t), t) \wedge B(x(t), t) \in X$ for all $t \in \Omega$ all are randomly comparative and they are randomly comparison for $t \in \Omega$ and there exists two measurable mappings $\alpha, \alpha^{'} : \Omega \to (0, 1)$ such that for any $t \in \Omega$, and an arbitrary $x(t), y(t) \in X$, holds

$$(A(x(t), t) \wedge B(x(t), t) + I(x(t), t)) \oplus (A(y(t), t) \wedge B(y(t), t) + I(y(t), t))$$
$$\leq \alpha(t)((A(x(t), t) \wedge B(x(t), t)) \oplus (A(y(t), t) \wedge B(y(t), t)))$$
$$+\alpha^{'}(t)(x(t) \oplus y(t)),$$

where $I(x(t), t) = x(t) : X \times \Omega \to X$ is a random identity mapping.

Lemma 2.7. [22] *Let X be an ordered Banach space, P be a normal cone with normal constant N in X, $A : X \to X$ be a comparative then for any $x, y \in X$*

(1) $\|\theta \oplus \theta\| = \|\theta\| = 0$,
(2) $\|x \vee y\| \leq \|x\| \vee \|y\| \leq \|x\| + \|y\|$,
(3) $\|x \oplus y\| \leq \|x - y\| \leq N\|x \oplus y\|$,
(4) *if $x \propto y$, then* $\|x \oplus y\| = \|x - y\|$,
(5) $\lim_{x \to x_0} \|A(x) - A(x_0)\| = 0$, *if and only if*

$$\lim_{x \to x_0} A(x) \oplus A(x_0) = \theta.$$

3. Main Results

In this section, we will show the convergence of the approximation of random sequences for finding random solutions of the problem (2.1) and discussed the relation between the initial random values $x_0(t)$ and the random solution of the problem (2.1).

Theorem 3.1. *Assume that (Ω, Σ, μ) is a complete σ-finite measurable space and X is a separable real Banach space, P a normal cone with normal constant N, in X, $\leq$ is an ordered relation defined by the cone P, Ω is a nonempty open subset of X in which the $t \in \Omega$, let $T : \Omega \times X \to \mathfrak{F}(X)$ be the random fuzzy mapping satisfying condition (C) and $\tilde{T} : \Omega \times X \to CB(X)$ be the random* *continuous set valued mapping induced by T respectively. Let a mapping $\tilde{T}$ be the random $\mathcal{H}$-continuous ordered compression mapping with the measure $\eta : \Omega \to (0,1)$. Let $A, g, f, \tilde{T}, B$ and $(A+f) : \Omega \times X \to X$ be some random comparison mappings to each others and $A(t), B(t)$ be the random comparison mapping. Let $A(t)$ be the random $\beta(t)$-ordered compression measurable mapping with measure $\beta : \Omega \to (0,1)$. Let f be a random $\sigma(t)$-ordered compression mapping with measure $\sigma : \Omega \to (0,1)$ and g be a random $\gamma(t)$-ordered compression mapping with measure $\gamma : \Omega \to (0,1)$. If $A+f$ is a random $B(t)$-restricted accretive mapping for two random measurable mappings $\alpha, \alpha' : \Omega \to (0,1)$ and $\rho : \Omega \to (0,1)$ is a any measure*

$$\rho(t)[\beta(t)\gamma(t) + \sigma(t)\eta(t)] < \frac{1-\alpha'(t)}{\alpha(t)} \tag{2}$$

holds. Then the fuzzy general nonlinear ordered random variational inequality problem

$$[A(g(x(t),t),t) + f(u(t),t)] \geq \theta, \forall\ x(t) \in X, u(t) \in \tilde{T}(x(t),t), t \in \Omega, \tag{3}$$

there exists a random solutions $x^*(t) \in X, u^*(t) \in \tilde{T}(x^*(t),t)$, for $t \in \Omega$ and for any $x_0(t) \in X$

$$\|x^*(t) - x_0(t)\| \leq \{1 + \frac{N(\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)) + \alpha'(t))}{1 - (\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)) + \alpha'(t))}\}\times$$
$$\|A(g(x_0(t),t),t) + f(u_0(t),t) - x_0(t)\|.$$

Moreover, $x_n(t) \to x^*(t), u_n(t) \to u^*(t)$ where $\{x_n(t)\}$ and $\{u_n(t)\}$ are the random sequences obtained by random iterative algorithm.

Proof. Let X be a real ordered Banach space, P be a normal cone with normal constant N in the X, $\leq$ be an ordered relation defined by the cone P, Ω be a nonempty open subset of X in which the t takes values. For any $t \in \Omega$ and $x_1(t), x_2(t) \in X$ and $\rho : \Omega \to (0,1)$ is a measurable mapping. Let $x_0(t) \propto x_1(t)$. Then for $x_0(t) \in X$ and

$$x_1(t) = \rho(t)[A(g(x_0(t),t),t) + f(u_0(t),t)] \wedge B(x_0(t),t) + I(x_0(t),t),$$
$$\text{for } u_0(t) \in \tilde{T}(x_0(t),t).$$

Since $A(t)$ and $B(t)$ be the randomly ordered comparison to each other so that $x_0(t) \propto x_1(t)$. Further we can have a random iterative algorithm for *fuzzy general nonlinear ordered random variational inequalities* (1), *i.e.*

$$A(g(x(t),t),t) + f(u(t),t) \geq 0,\ t \in \Omega, x(t) \in X, u(t) \in \tilde{T}(x(t),t)$$

in ordered Banach space X :

$$x_{n+1}(t) = \rho(t)[A(g(x_n(t),t),t) + f(u_n(t),t)] + I(x_n(t),t),$$

for a measurable mapping $\rho : \Omega \to (0,1)$ where $n = 0,1,2,\cdots$. It follows from the condition $A, g, f, B, \tilde{T}$ and $(A+f) : \Omega \times X \to X$ of the random comparison mappings and $t \in \Omega$, $x_0(t) \propto x_1(t)$ that is $x_n(t) \propto x_{n+1}(t)$. By using the random $B(t)$ restricted accretive mapping of $A+f$ and the randomly $\beta(t)$- ordered compression of $A(t)$, the random $\gamma(t)$-ordered compression of a random mapping $g : \Omega \times X \to X$ and $\tilde{T}$ be the random $\mathcal{H}$-continuous ordered compression mapping with the measurable mapping $\eta : \Omega \to (0,1)$ and Lemma 2.4(6), we have

$$\begin{aligned}
\theta &\leq x_{n+1}(t) \oplus x_n(t) \\
&\leq [\rho(t)(A(g(x_n(t),t),t) + f(u_n(t),t)) + I(x_n(t),t)] \\
&\quad \oplus [\rho(t)(A(g(x_{n-1}(t),t),t) + f(u_{n-1}(t),t)) + I(x_{n-1}(t),t)] \\
&\leq \alpha(t)[\rho(t)(A(g(x_n(t),t),t) + f(u_n(t),t)) \oplus (\rho(t)(A(g(x_{n-1}(t),t),t) \\
&\quad + f(u_{n-1}(t),t)))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \alpha(t)\rho(t)[(A(g(x_n(t),t),t) \oplus A(g(x_{n-1}(t),t),t)) \\
&\quad + (f(u_n(t),t) \oplus f(u_{n-1}(t),t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \alpha(t)\rho(t)[\beta(t)(g(x_n(t),t) \oplus g(x_{n-1}(t),t)) + \sigma(t)(u_n(t) \oplus u_{n-1}(t))] \\
&\quad + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \alpha(t)\rho(t)[\beta(t)\gamma(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\quad + \sigma(t)\mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x_{n-1}(t),t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \alpha(t)\rho(t)[\beta(t)\gamma(t)(x_n(t) \oplus x_{n-1}(t)) + \sigma(t)\eta(t)(1 + \frac{1}{n})(x_n(t) \oplus x_{n-1}(t))] \\
&\quad + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq (\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) + \alpha^{'}(t))(x_n(t) \oplus x_{n-1}(t)) \\
&\leq (\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) + \alpha^{'}(t))^n N(x_1(t) \oplus x_0(t)).
\end{aligned}$$

Since

$$u_n(t) \oplus u_{n-1}(t) = \mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x_{n-1}(t),t)) \leq (1 + \frac{1}{n})\eta(t)(x_n(t) \oplus x_{n-1}(t)).$$

By Lemma 2.2 and Definition 9, we obtain

$$\|x_n(t) - x_{n-1}(t)\| \leq \triangle^n N \|x_1(t) - x_0(t)\| \tag{4}$$

where $\triangle^n = \alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) + \alpha^{'}(t)$.
Let $\triangle = \alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)) + \alpha^{'}(t)$ with

$$\rho(t)[\beta(t)\gamma(t) + \sigma(t)\eta(t)] < \frac{1 - \alpha^{'}(t)}{\alpha(t)}.$$

Hence for any $m > n > 0$ we have

$$\|x_m(t) - x_n(t)\| \leq \sum_{i=n}^{m-1} \|x_{i+1}(t) - x_i(t)\| \leq N\|x_1(t) - x_0(t)\| \sum_{i=n}^{m-1} \triangle^i.$$

It follows from the condition (2) that $0 \leq \triangle \leq 1$ and

$$\|x_m(t) - x_n(t)\| \to 0, \text{ as } n \to \infty$$

and so $\{x_n(t)\}$ is a random Cauchy sequence in complete space X. Let $x_n(t) \to x^*(t)$ as $n \to \infty (x^*(t) \in X, t \in \Omega)$. From the condition that $A, g, \tilde{T}$ are randomly continuous and $\rho : \Omega \to (0, 1)$ is a measurable space, we can have

$$\begin{aligned}
x^*(t) &= \lim_{n\to\infty} x_{n+1}(t) \\
&= \lim_{n\to\infty} [\rho(t)(A(g(x_n(t),t),t) + f(u_n(t),t))] + I(x_n(t),t) \\
&= \lim_{n\to\infty} \rho(t)A(g(x_n(t),t),t) + \rho(t) \lim_{n\to\infty} f(u_n(t),t) + \lim_{n\to\infty} I(x_n(t),t) \\
&= \rho(t)A(g(\lim_{n\to\infty} x_n(t),t),t) + \rho(t)f(\lim_{n\to\infty} u_n(t),t) + I(\lim_{n\to\infty} x_n(t),t) \\
&= \rho(t)A(g(\lim_{n\to\infty} x_n(t),t),t) + \rho(t)f(\tilde{T}(\lim_{n\to\infty} x_n(t),t)) + I(\lim_{n\to\infty} x_n(t),t) \\
&= \rho(t)A(g(x^*(t),t),t) + \rho(t)f(u^*(t),t) + x^*(t).
\end{aligned}$$

Hence $x^*(t)$ is a solution of equation (1). By random $\mathcal{H}$-continuous order compression of $\tilde{T}$, we have

$$\begin{aligned}
u_n(t) \oplus u_{n-1}(t) &\leq \mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x_{n-1}(t),t)) \\
&\leq (1 + \frac{1}{n})\eta(t)(x_n(t) \oplus x_{n-1}(t)).
\end{aligned}$$

It follows that $\{u_n(t)\}$ is also a random Cauchy sequence in X and completeness of X, $u_n(t) \to u^*(t)$. Note that $u_n(t) \in \tilde{T}(x_n(t),t)$, we have

$$\begin{aligned}
\|u^*(t) - \tilde{T}(x^*(t),t)\| &\leq \|u^*(t) - u_n(t)\| + \mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x^*(t),t)) \\
&\leq \|u^*(t) - u_n(t)\| + (1 + \frac{1}{n})\eta(t)\|x_n(t) - x^*(t)\| \\
&\to 0 \text{ as } n \to \infty.
\end{aligned}$$

Hence $\|u^*(t) - u_n(t)\| = 0$ and therefore $u^*(t) \in \tilde{T}(x^*(t),t)$ is also a random solution of (1). We know that $(x^*(t), u^*(t))$ is a random solution set of equation (1). It follows that

$$A(g(x_n(t),t),t) + f(u_n(t),t) \propto x^*(t), \; n = 0, 1, 2, \cdots, t \in \Omega$$

from Lemma 2.3 and (4)

$$\begin{aligned}
\|x^*(t) - x_0(t)\| &= \lim_{n\to\infty} \|x_n(t) - x_0(t)\| \\
&\leq \lim_{n\to\infty} \sum_{i=1}^{n} \|x_{i+1}(t) - x_i(t)\| \\
&\leq \lim_{n\to\infty} N \sum_{i=2}^{n} \triangle^{n-1} \|x_1(t) - x_0(t)\| + \|x_1(t) - x_0(t)\| \\
&\leq (1 + \frac{N(\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)) + \alpha'(t))}{1 - (\alpha(t)\rho(t)(\beta(t)\gamma(t) + \sigma(t)\eta(t)) + \alpha'(t))}) \times
\end{aligned}$$

$$\|A(g(x_0(t),t),t) + f(u_0(t),t) - x_0(t)\|$$

holds. This complete the proof. □

Lemma 3.2. *Assume that (Ω, Σ, μ) is a complete σ-finite measurable space and X is a separable real Banach space, P a normal cone with normal constant N in the space X, $\leq$ is a partial ordered relation defined by the cone P, Ω is a nonempty open subset of X in which the $t \in \Omega$. Let $T : \Omega \times X \to \mathfrak{F}(X)$ be the random fuzzy mapping satisfy the conditions (C) and $\tilde{T} : \Omega \times X \to CB(X)$ be the random continuous set valued mapping induced by T respectively. Let a mapping $\tilde{T}$ be the random $\mathcal{H}$-continuous ordered compression mapping with measure $\eta : \Omega \to (0,1)$. Let $f, A, g, B, \tilde{T}, A + f$ and $(A + f) \wedge B : \Omega \times X \to X$ be randomly comparison mapping respectively and two of them can be compared each other. If a equation*

$$(A(g(x(t),t),t)+f(u(t),t))\wedge B(x(t),t) = \theta,\ \theta \in X,\ t \in \Omega,\ u(t) \in \tilde{T}(x(t),t) \quad (5)$$

has a random solution sets $(x^*(t), u^*(t))$. Then $(x^*(t), u^*(t))$ is a random solution sets of fuzzy general nonlinear ordered random variational inequalities in ordered Banach spaces.

Proof. This directly follows from the definition of the $\wedge$ and the condition that $A, g, B, f, \tilde{T}, A + f$ and $(A + f) \wedge B : \Omega \times X \to X$ be randomly comparison respectively and any two of them can compared each other. □

From Theorem 3.1 and Lemma 3.2, we have the following Theorem.

Theorem 3.3. *Assume that (Ω, Σ, μ) is a complete σ-finite measurable space and X is a separable real Banach space, P a normal cone with normal constant N in X, $\leq$ is an ordered relation defined by the cone P, Ω is a nonempty open subset of X in which the $t \in X$. Let $T : \Omega \times X \to \mathfrak{F}(X)$ be the random fuzzy mapping satisfying condition (C) and $\tilde{T} : \Omega \times X \to CB(X)$ be the continuous random set valued mapping induced by T, respectively. Let mapping $\tilde{T}$ be the randomly $\mathcal{H}$-continuous ordered comparison mapping with the measure $\eta : \Omega \to (0,1)$. Let $A, g, f, A + f, \tilde{T}, B$ and $(A + f) \wedge B : \Omega \times X \to X$ be the some comparison random mappings to each other and $A(t), B(t)$ be the random $\beta_i(t)$-ordered compression measurable mapping with measure $\beta_i(t) : \Omega \to (0,1)$ for $i = 1,2$. Let f be a random σ-ordered compression mapping with measure $\sigma : \Omega \to (0,1)$ and g be a randomly $\gamma(t)$-ordered compression mapping with measure $\gamma : \Omega \to (0,1)$. If $A + f$ is a randomly $B(t)$- restricted accretive mapping with respect to $t \in \Omega$, for two measurable mappings $\alpha, \alpha' : \Omega \to (0,1)$ and $\rho : \Omega \to (0,1)$ is a any measure*

$$\rho(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t)] < \frac{1 - \alpha'(t)}{\alpha(t)} \quad (6)$$

holds. Then the fuzzy general nonlinear ordered random variational inequalities

$$[A(g(x(t),t),t)+f(u(t),t)]\wedge B(x(t),t) = \theta \ \text{ for } x(t) \in X, u(t) \in \tilde{T}(x(t),t), t \in \Omega. \quad (7)$$

There exists $x^*(t) \in X, u^*(t) \in \tilde{T}(x^*(t), t)$ for $t \in \Omega$, which is a random solution sets of problem (1) and for any $x_0(t) \in X$,

$$\|x^*(t) - x_0(t)\| \leq \{1 + \frac{N(\alpha(t)\rho(t)((\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t)) + \alpha'(t))}{1 - (\alpha(t)\rho(t)((\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t)) + \alpha'(t))}\} \times \\ N\|A(g(x_0(t), t), t) + f(u_0(t), t)\| \wedge \|B(x_0(t), t)\|.$$

Moreover $x_n(t) \to x^*(t), u_n(t) \to u^*(t)$ where $\{x_n(t)\}$ and $\{u_n(t)\}$ are the random sequences obtained by random iterative algorithm for fuzzy mapping.

Proof. Let X be a real ordered Banach space, P be a normal cone with normal constant N in the X, $\leq$ be an ordered relation defined by the cone P, Ω be a nonempty open subset of X in which the t takes values. For any $t \in \Omega$ and $x_1(t), x_2(t) \in X$ and $\rho : \Omega \to (0, 1)$ is a measurable mapping. Let $x_0(t) \propto x_1(t)$. Then for $x_0(t) \in X$ and $u_0(t) \in \tilde{T}(x_0(t), t)$,

$$x_1(t) = \rho(t)[A(g(x_0(t), t), t) + f(u_0(t), t)] \wedge B(x_0(t), t) + I(x_0(t), t).$$

Since $A(t)$ and $B(t)$ are the randomly comparison to each other so that $x_0(t) \propto x_1(t)$. Further we can have a random iterative algorithm for *fuzzy general nonlinear ordered random variational inequalities* (7) in ordered Banach space X.

$$x_{n+1}(t) = \rho(t)[A(g(x_n(t), t), t) + f(u_n(t), t)] \wedge B(x_n(t), t) + I(x_n(t), t),$$

for a measurable mapping $\rho : \Omega \to (0, 1)$ where $n = 0, 1, 2, \cdots$. It follows from the condition $A, g, f, A + f, B, \tilde{T}$ and $(A + f) \wedge B : \Omega \times X \to X$ of the random comparison mappings and $t \in \Omega$, $x_0(t) \propto x_1(t)$ that is $x_n(t) \propto x_{n+1}(t)$. By using the random $B(t)$-restricted accretive mapping and the randomly $\beta(t)$-ordered compression of $A(t)$, the random $\gamma(t)$-ordered compression of a random mapping $g : \Omega \times X \to X$ and $\tilde{T}$ be the random $\mathcal{H}$-continuous ordered compression mapping with the measurable mapping $\eta : \Omega \to (0, 1)$ and Lemma 2.4(7), we have

$$\begin{aligned}
\theta &\leq x_{n+1}(t) \oplus x_n(t) \\
&\leq [\rho(t)(A(g(x_n(t),t),t) + f(u_n(t),t)) \wedge B(x_n(t),t) + I(x_n(t),t)] \\
&\quad \oplus [\rho(t)(A(g(x_{n-1}(t),t),t) + f(u_{n-1}(t),t)) \wedge B(x_{n-1}(t),t) + I(x_{n-1}(t),t)] \\
&\leq \alpha(t)[(\rho(t)(A(g(x_n(t),t),t) + f(u_n(t),t)) \wedge B(x_n(t),t)) \\
&\quad \oplus (\rho(t)(A(g(x_{n-1}(t),t),t) + f(u_{n-1}(t),t)) \wedge B(x_{n-1}(t),t))] \\
&\quad + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \alpha(t)\rho(t)[(A(g(x_n(t),t),t) + f(u_n(t),t)) \oplus (A(g(x_{n-1}(t),t),t) \\
&\quad + f(u_{n-1}(t),t)) \vee (B(x_n(t),t) \oplus B(x_{n-1}(t),t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \rho(t)\alpha(t)[((A(g(x_n(t),t),t) \oplus A(g(x_{n-1}(t),t),t)) \\
&\quad + (f(u_n(t),t) \oplus f(u_{n-1}(t),t))) \vee (B(x_n(t),t) \oplus B(x_{n-1}(t),t))] \\
&\quad + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \rho(t)\alpha(t)[\beta_1(t)(g(x_n(t),t) \oplus g(x_{n-1}(t),t)) + \sigma(t)(u_n(t) \oplus u_{n-1}(t)) \\
&\quad \vee \beta_2(t)(x_n(t) \oplus x_{n-1}(t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \rho(t)\alpha(t)[\beta_1(t)\gamma(t)(x_n(t) \oplus x_{n-1}(t)) + \sigma(t)\mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x_{n-1}(t),t)) \\
&\quad \vee \beta_2(t)(x_n(t) \oplus x_{n-1}(t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq \rho(t)\alpha(t)[\beta_1(t)\gamma(t)(x_n(t) \oplus x_{n-1}(t)) + \sigma(t)\eta(t)(1 + \frac{1}{n})(x_n(t) \oplus x_{n-1}(t)) \\
&\quad \vee \beta_2(t)(x_n(t) \oplus x_{n-1}(t))] + \alpha^{'}(t)(x_n(t) \oplus x_{n-1}(t)) \\
&\leq (\rho(t)\alpha(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) \vee \beta_2(t)] + \alpha^{'}(t))(x_n(t) \oplus x_{n-1}(t)).
\end{aligned}$$

Since $\tilde{T}$ is randomly $\mathcal{H}$-continuous ordered compression mapping with measurable mapping $\eta : \Omega \to (0,1)$, we have

$$\begin{aligned}
(u_n(t) \oplus u_{n-1}(t)) &\leq \mathcal{H}(\tilde{T}(x_n(t),t), \tilde{T}(x_{n-1}(t),t)) \\
&\leq \eta(t)(1 + \frac{1}{n})(x_n(t) \oplus x_{n-1}(t)).
\end{aligned}$$

Now continuing these process, we have

$$\begin{aligned}
0 &\leq x_n(t) \oplus x_{n-1}(t) \\
&\leq (\rho(t)\alpha(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) \vee \beta_2(t)] + \alpha^{'}(t))(x_n(t) \oplus x_{n-1}(t)) \\
&\leq (\rho(t)\alpha(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) \vee \beta_2(t)] + \alpha^{'}(t))^n N(x_1(t) \oplus x_0(t)).
\end{aligned}$$

By Lemma 2.2 and Definition 9, we obtain

$$\|x_n(t) - x_{n-1}(t)\| \leq \triangle^n N \|x_1(t) - x_0(t)\| \tag{8}$$

where $\triangle^n = (\alpha(t)\rho(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)(1 + \frac{1}{n})) \vee \beta_2(t)] + \alpha^{'}(t))$.
Let $\triangle = (\alpha(t)\rho(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t)] + \alpha^{'}(t))$ with

$$\rho(t)[(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t)] < \frac{1 - \alpha^{'}(t)}{\alpha(t)}.$$

From the assumption (8) and Lemma 2.2. Hence for any $m > n > 0$ we have

$$\|x_m(t) - x_n(t)\| \leq N \sum_{i=n}^{m-1} \|x_{i+1}(t) - x_i(t)\| \leq N\|x_1(t) - x_0(t)\| \sum_{i=n}^{m-1} \triangle^i.$$

It follows from the condition (6) that $0 \leq \triangle \leq 1$ and

$$\|x_m(t) - x_n(t)\| \to 0, \text{ as } n \to \infty \text{ for } t \in \Omega.$$

So $\{x_n(t)\}$ is a random Cauchy sequence in complete space X. Let $x_n(t) \to x^*(t)$ as $n \to \infty (x^*(t) \in X, t \in \Omega)$. From the condition that $A, g, \tilde{T}, f, A + f$ and $(A + f) \wedge B : \Omega \times X \to X$ are randomly continuous and $\rho : \Omega \to (0, 1)$, we can have

$$\begin{aligned}
x^*(t) &= \lim_{n\to\infty} x_{n+1}(t) \\
&= \lim_{n\to\infty} (\rho(t)(A(g(x_n(t), t), t) + f(u_n(t), t)) \wedge B(x_n(t), t) + I(x_n(t), t)) \\
&= (\rho(t) \lim_{n\to\infty} A(g(x_n(t), t), t) + \rho(t) \lim_{n\to\infty} f(u_n(t), t)) \wedge \lim_{n\to\infty} B(x_n(t), t) \\
&\quad + \lim_{n\to\infty} I(x_n(t), t) \\
&= (\rho(t)A(g(\lim_{n\to\infty} x_n(t), t), t) + \rho(t)f(\lim_{n\to\infty} u_n(t), t)) \wedge B(\lim_{n\to\infty} x_n(t), t) \\
&\quad + I(\lim_{n\to\infty} x_n(t), t) \\
&= (\rho(t)A(g(\lim_{n\to\infty} x_n(t), t), t) + \rho(t)f(\tilde{T}(\lim_{n\to\infty} x_n(t), t))) \wedge B(\lim_{n\to\infty} x_n(t), t) \\
&\quad + I(\lim_{n\to\infty} x_n(t), t) \\
&= (\rho(t)A(g(x^*(t), t), t) + \rho(t)f(u^*(t), t)) \wedge B(x^*(t), t) + x^*(t),
\end{aligned}$$

where

$$\lim_{n\to\infty} u_n(t) = \tilde{T}(\lim_{n\to\infty} x_n(t), t) = \tilde{T}(x^*(t), t) = u^*(t).$$

Hence $x^*(t)$ is a solution of equation (1), *i.e.*,

$$(A(g(x(t), t), t) + f(u(t), t)) \wedge B(x(t), t) = 0.$$

By random $\mathcal{H}$-continuous order compression of $\tilde{T}$ we have

$$\begin{aligned}
u_{n+1}(t) \oplus u_n(t) &\leq \mathcal{H}(\tilde{T}(x_{n+1}(t), t), \tilde{T}(x_n(t), t)) \\
&\leq (1 + \frac{1}{n})\eta(t)(x_{n+1}(t) \oplus x_n(t)).
\end{aligned}$$

It follows that $\{u_n(t)\}$ is also a random Cauchy sequence in X and completeness of X, $u_n(t) \to u^*(t)$ as $n \to \infty$.

Note that $u_n(t) \in \tilde{T}(x_n(t), t)$, we have

$$\begin{aligned}\|u^*(t) - \tilde{T}(x^*(t), t)\| &\le \|u^*(t) - u_n(t)\| + \mathcal{H}(\tilde{T}(x_n(t), t), \tilde{T}(x^*(t), t)) \\ &\le \|u^*(t) - u_n(t)\| + (1 + \frac{1}{n})\eta(t)\|x_n(t) - x^*(t)\| \\ &\quad \to 0 \text{ as } n \to \infty.\end{aligned}$$

Hence

$$\|u^*(t) - \tilde{T}(x^*(t), t)\| = 0$$

and therefore $u^*(t) \in \tilde{T}(x^*(t), t)$.
We know that $(x^*(t), u^*(t))$ is a random solution sets of equation (7). It follows that

$$(A(g(x_n(t), t), t) + f(u_n(t), t)) \wedge B(x_n(t), t) \propto x^*(t),\ n = 0, 1, 2, \cdots, t \in \Omega,$$

from Lemma 2.3 and (8)

$$\|x^*(t) - x_0(t)\| = \lim_{n\to\infty} \|x_n(t) - x_0(t)\| \le \lim_{n\to\infty} \sum_{i=1}^{n} \|x_{i+1}(t) - x_i(t)\|$$

$$\le \lim_{n\to\infty} N \sum_{i=2}^{n} \triangle^{n-1}\|x_1(t) - x_0(t)\| + \|x_1(t) - x_0(t)\|$$

$$\begin{aligned}&\le \big(1 + \frac{N(\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}{1 - (\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}\big)\|(A(g(x_0(t), t), t) \\ &\qquad + f(u_0(t), t)) \wedge B(x_0(t), t)\| \\ &\le \big(1 + \frac{N(\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}{1 - (\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}\big)N\{\|A(g(x_0(t), t), t) \\ &\qquad + f(u_0(t), t)\|\} \wedge \|B(x_0(t), t)\| \\ &\le \big(1 + \frac{N(\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}{1 - (\alpha(t)\rho(t)(\beta_1(t)\gamma(t) + \sigma(t)\eta(t)) \vee \beta_2(t) + \alpha^{'}(t))}\big)N\{\|A(g(x_0(t), t), t)\| \\ &\qquad + \|f(u_0(t), t)\|\} \wedge \|B(x_0(t), t)\|\end{aligned}$$

holds. This complete the proof. □

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