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Dynamic Analysis of Laminated Composite and Sandwich Plates Using Trigonometric Layer-wise Higher Order Shear Deformation Theory

Suganyadevi S, B. N. Singh[†] Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, INDIA [†]E-mail: bnsingh@aero.iitkgp.ernet.in

Abstract : A trigonometric Layerwise higher order shear deformation theory (TLHSDT) is developed and implemented for free vibration and buckling analysis of laminated composite and sandwich plates by analytical and finite element formulation. The present model assumes parabolic variation of out-plane stresses through the depth of the plate and also accomplish the zero transverse shear stresses over the surface of the plate. Thus a need of shear correction factor is obviated. The present zigzag model able to meet the transverse shear stress continuity and zigzag form of in-plane displacement continuity at the plate interfaces. Hence, botheration of shear correction coefficient is neglected. In the case of analytical method, the governing differential equation and boundary conditions are obtained from the principle of virtual work. For the finite element formulation, an efficient eight noded C^0 continuous isoparametric serendipity element is established and employed to examine the dynamic analysis. Like FSDT, the considered mathematical model possesses similar number of variables and which decides the present models computationally more effective. Several numerical predictions are carried out and results are compared with those of other existing numerical approaches.

Key Words : Dynamic analysis, Navier solution, Finite element formulation, inter laminar-continuity, composite plates, sandwich plates.

1. Introduction

In the past few decades several shear deformation theories have developed and established by various authors. Pagano [1] presented given an exact three dimensional (3D) elasticity solution for laminated plates. Herein, each lamina considered as 3D solid and hence the computational effort becomes more. To bypass the above confines of computational cost, several single layer theories have been developed. Generally single layer theories can be grouped as

Received: March 23, 2016 Revised: April 26, 2016 Accepted: May 29, 2016 †Corresponding Author Tel:+91-3222-283026, E-mail: bnsingh@aero.iitkgp.ernet.in Copyright © The Society for Aerospace System Engineering classical laminated plate theory, first order shear deformation theory and higher order shear deformation theory (HSDT). The classical laminated theory completely neglects the shear effects. The first order theory interprets the shear effects. However, it fails to fulfill the traction free boundary condition at the plate surfaces. In order to satisfy the traction free boundary condition, an artificial shear correction factor must be considered. The above discussed limitations can be overcome by HSDTs.

Based on the expressions of mathematical filed, the HSDT classified as polynomial and non-polynomial higher order theory. The polynomial higher order theories represent Taylor series expansion of thickness coordinates. It varies approximately parabolic variations of transverse shear stress across plate thickness and also satisfies the traction

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free boundary condition over the plate surfaces. Henceforth, the shear correction factor is evaded. Levinson [2] proposed a plate theory in terms of cubic order of thickness coordinates. Kant and Pandya [3] presented a higher order theory with 7 unknown field variables for ant symmetric laminated composite plates. Talha and Singh [4] developed a plate theory with transverse normal strain effect using 13 unknowns for functionally graded plate. However, developing the displacement field in terms of Taylor's series expansions will introduces additional unknown variables and also these unknown variables are physically hard to interpret.

The non-polynomial shear deformation theory leads to more accuracy and easy formulation. Touratier [5] proposed HSDT in terms of sinusoidal shear strain shape function. Karama et al. [6], Soldataos [7], Aydogdu [8], Mantari et al. [9], Neves et al. [10] and Suganyadevi and Singh [11] have paid much attention for developing the displacement field in terms of shear strain function. Though, the above discussed theories give nonlinear variations of displacement across the plate thickness, they fails to meet the interlaminar shear stress continuity across the plate thickness.

Toledano and Murakami [12] used a mixed variational principal to account the zigzag requirement and transverse shear stress continuity at each layer interfaces. Cho et al. [13] studied dynamic response of laminated plates using a Layerwise theory. Carrera developed [14] a mixed theory by interpolating Layerwise legendre polynomials. Ferreira [15] given a Layerwise theory in which the equilibrium equations and boundary conditions are achieved through mesh free methodology. Though above discussed theories, evaluate the structural responses of laminated plates with sufficient precision. However, these models required high computational efforts. Because the unknowns are strongly depends on each layer. Hence forth, several authors focused towards zigzag theory in which the unknowns are taken at each layer interfaces in terms of those at the reference plane. Cho and Parmeter [16] presented model where the zigzag requirement and transverse shear stress continuity are obtained by implementing Heaviside step functions. Icardi [17] used a zigzag model for quadratic plate element using 8 noded element with 56 field variables per element. Carrera et al. [18] employed a zigzag model and they obtained cubic variation of in-plane displacement through the plate thickness. Though, the above noted polynomial zigzag theories are layer independent, interpreting the higher order terms in the formulation is quite difficult.

In order to avoid the above shortcomings, nonpolynomial zigzag theories have been developed. It represents non-linear variation of in-plane displacement, parabolic variation of transverse share stress continuity, zigzag requirement, adequate accuracy and the traction free boundary condition is guaranteed. Shimpi and Ghugal [19] have introduced a Layerwise shear deformation theory with involving trigonometric shear strain function for laminated beams. Mantari et al. [9] studied a zigzag theory using a trigonometric shear strain function for laminated composite plates. Notable works based on various shear deformation theories can be seen in [20][21].

To the best of authors knowledge, the present work presents buckling and vibration analysis of laminated composite and sandwich plates using trigonometric shear deformation theory. To handle the dynamic analysis analytical and finite element formulation is implemented. The current theory estimates the zigzag requirement and interlaminar shear stress continuity with easy formulation and augmented results. Moreover, the non-polynomial shear strain function makes the shear stress free conditions at the top and bottom surfaces of the plate a priori. In the case of analytical approach, the governing differential equations and boundary conditions are obtained from principle of virtual work. For the finite element formulation, the governing equations are obtained minimizing the total potential energies. By employing eight noded C^0 continuous isoparametric serendipity element the dynamic analysis are carried out. Various numerical examples are carried out and they are validated with the available results.

2. Theoretical formulation

A rectangular laminated composite plate is taken as shown in Fig [1]. The plate is composed of equally thickened four layer with theta angle orientation. It has plate thickness h, length a and width b. The present work is the redefined work of Arya et al. [5] to the multilayered laminated composites and sandwich plates.



Fig. 1 Schematic diagram of a rectangular laminated plate.

Further, it is the combination of Layerwise parameters and a trigonometric shear strain function [22] which can be represented as follows

$$U^{k}(x, y, z) = u_{0}(x, y) - z \left(\frac{\partial w_{0(x, y)}}{\partial x}\right) + \left[A^{k} + zB^{k} + f(z)\right]\phi_{x}(x, y)$$
$$V^{k}(x, y, z) = v_{0}(x, y) - z \left(\frac{\partial w_{0(x, y)}}{\partial y}\right) + \left[C^{k} + zD^{k} + f(z)\right]\phi_{y}(x, y)$$
$$W(x, y, z) = w_{0}(x, y)$$
(1)

Where

$$f(z) = \left[g(z) + \aleph z\right]$$

$$g(z) = \frac{h}{m} \tan h^{-1} \left(\frac{mz}{h}\right); \quad \aleph = -\left[\frac{1}{(m^2 z^2/h^2) + 1}\right]_{z=h/2}; \quad m = 2.5$$
(2)

Throughout this work superscript k represents the layer number. Here $U^k_{and} V^k$ are the in-plane displacement at (x^k, y^k, z^k) whereas u_0 and are v_0 the in-plane displacement at (x, y, 0). The transverse displacement (w_0) is the function of x and y. Midplane rotations ϕ_x and ϕ_y with respect to y and x axis. Here A^k, B^k, C^k, D^k are the Layerwise parameters which dependent on each layer geometry and material property and consequently, they varies in each layer. The generalized expressions of A^k, B^k, C^k, D^k are given in Appendix.

2.2. Formulation for analytical methodology

The governing differential equation is obtained from the dynamic version of principal of virtual work as follows:

$$\int_{t_0}^t \left(\delta T - \delta U\right) dt + \int_{t_0}^t \delta W dt = 0 \quad (3)$$

$$\delta U = \int_{\Omega} \left\{ \int_{\frac{2}{n}}^{\frac{h}{2}} \left[\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{yy} \delta \varepsilon_{yy} + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \right] dz \right\} dxdy$$

$$\delta T = \int_{\Omega} \left\{ \int_{\frac{2}{n}}^{\frac{h}{2}} \rho \left[u \delta u_0 + v \, \delta v_0 + v \delta w_0 \right] dz \right\} dxdy; \\ \delta V = \int_{\Omega} (q + \overline{N}) \delta w dxdy \quad (4)$$

Where

 $\delta W = -\delta V; \overline{N} = \overline{N}_{xx} w_{0,xx} + 2\overline{N}_{xy} w_{0,xy} + \overline{N}_{yy} w_{0,yy}$ Employing the above equations [4] in equation [3] the governing differential equation is obtained as represented in Equation [5].

$$\begin{split} \delta u_{0} &: N_{xx,x}^{k} + N_{xy,}^{k} = I_{0}^{k} u_{0} - I_{1}^{k} w_{0,x} + I_{3}^{k} \phi_{x}; \\ \delta v_{0} &: N_{xy,}^{k} + N_{yy,y}^{k} = I_{0}^{k} \ddot{v_{0}} - I_{1}^{k} w_{0,y} + I_{5}^{k} \ddot{\phi_{y}}; \\ \delta w_{0} &: M_{xx,xx}^{k} + 2M_{xy,xy}^{k} + M_{yy,yy}^{k} + \bar{N}^{k} + q = I_{1}^{k} \left(\ddot{u}_{0,x} + \ddot{v}_{0,y} \right) \\ &- I_{2}^{k} \left(\ddot{w}_{0,xx} + \ddot{w}_{0,yy} \right) + I_{4}^{k} \ddot{\phi}_{x,x} + I_{6}^{k} \ddot{\phi}_{y,y} + I_{0}^{k} \ddot{w_{0}}; \\ \delta \phi_{x} &: A^{k} \left(N_{xx,x}^{k} + N_{xy,y}^{k} \right) + (B^{k} + \aleph) \left(M_{xx,x}^{k} + M_{xy,y}^{k} - Q_{1}^{k} \right) \\ &+ P_{xx,x}^{k} + P_{xy,y}^{k} - L_{1}^{k} = I_{3}^{k} \ddot{u}_{0} - I_{4}^{k} w_{0,x} + I_{7}^{k} \ddot{\phi}_{x}; \\ \delta \phi_{y} &: C^{k} \left(N_{yy,y}^{k} + N_{xy,x}^{k} \right) + (D^{k} + \aleph) \left(M_{yy,y}^{k} + M_{xy,x}^{k} - Q_{2}^{k} \right) \\ &+ P_{yy,y}^{k} + P_{xy,x}^{k} - L_{2}^{k} = I_{5}^{k} \ddot{v}_{0} - I_{6}^{k} w_{0,y} + I_{8}^{k} \ddot{\phi}_{y}; \end{split}$$
(5)

The following Navier solution equation [7] is considered for the five unknowns which satisfy the simply supported boundary conditions given in equation [6] and differential equation [5].

$$v_0 = w_0 = \phi_y = N_{xx} = M_{xx} = 0 \text{ at } x = 0, a;$$

$$u_0 = w_0 = \phi_x = N_{yy} = M_{yy} = 0 \text{ at } y = 0, b;$$
 (6)

$$\begin{cases} u_{0}(x, y, t) \\ v_{0}(x, y, t) \\ w_{0}(x, y, t) \\ \phi_{x}(x, y, t) \\ \phi_{y}(x, y, t) \end{cases} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{cases} U_{mn} e^{i o t} cos(\alpha x) sin(\beta y) \\ V_{mn} e^{i o t} sin(\alpha x) cos(\beta y) \\ W_{mn} e^{i o t} sin(\alpha x) sin(\beta y) \\ X_{mn} e^{i o t} cos(\alpha x) sin(\beta y) \\ Y_{mn} e^{i o t} sin(\alpha x) cos(\beta y) \end{cases}$$
(7)

2.2. Formulation for finite element method

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The total potential energy of the system can be written as

$$\Pi_{e} = T - U_{s} - U_{in} - U_{ac} - W_{ext}$$
(8)
Here

T -represents the kinetic energy of the system

Us - represents the strain energy due to deformation

Uin - represents the potential energy due to inplane loads

Uac - represents the strain energy due to artificial constraints

Wext - represents the work done due to external loading

The elemental potential energy can be represented as:

$$\prod_{e} = \frac{1}{2} \left\{ \delta \right\}^{T} \left[M_{e} \right] \left\{ \delta \right\}^{T} \left[K_{e} \right] \left\{ \delta \right\}^{T} \left[K_{e} \right] \left\{ \delta \right\}^{T} \left[K_{Ge} \right] \left\{ \delta \right\}^{T} \left[K_{Ge} \right] \left\{ \delta \right\}^{T} \left[R_{Ge} \right] \left\{ \delta \right\}^{T} \left[P_{e} \right] - \frac{1}{2} \left\{ \delta \right\}^{T} \left[K_{Fe} \right] \left\{ \delta \right\}^{T} \left\{ K_{Fe} \right\}^{T} \left\{ \delta \right\}^{T} \left\{ K_{Fe} \right\}^{T} \left\{ \delta \right\}^{T} \left\{ K_{Fe} \right\}^{T} \left\{ K_{Fe} \right\}^{T} \left\{ \delta \right\}^{T} \left\{ K_{Fe} \right\}^{T} \left\{ K_{FF} \left\{ K_{FF} \right\}^{T} \left\{ K_{FF} \left\{ K_{FF} \right\}^{T} \left$$

Where

 $\begin{bmatrix} K_e \end{bmatrix} = \iint \begin{bmatrix} B \end{bmatrix}^T \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} B \end{bmatrix} dxdy;$ $\begin{bmatrix} K_{Ge} \end{bmatrix} = \iint \begin{bmatrix} B_G \end{bmatrix}^T \begin{bmatrix} G \end{bmatrix} \begin{bmatrix} B_G \end{bmatrix} dxdy;$ $\begin{bmatrix} K_{Pe} \end{bmatrix} = \gamma \iint \left(P_x^T P_x + P_y^T P_y \right) dxdy$

where $[\kappa_e]$, $[\kappa_{pe}]$ are elemental stiffness matrix and penalty matrix whereas $[\kappa_{Ge}]$ are the elemental geometric matrix.

The following eigen value equations are obtained [9,10] for the buckling and free vibration analysis respectively.

 $\begin{bmatrix} \begin{bmatrix} \bar{K} \end{bmatrix} - \lambda^2 [G] \end{bmatrix} \{ \chi \} = \{ 0 \}$ (9) $\begin{bmatrix} \begin{bmatrix} \bar{K} \end{bmatrix} - \omega^2 [M] \end{bmatrix} \{ \chi \} = \{ 0 \}$ (10)

Here, [G] denotes the geometric matrix due to the uniaxial load and [M] denotes mass matrix, λ denotes the buckling parameter and ω denotes the frequency parameter.

The following [11,12] non-dimensionalised equations and material models (MM) are used for the buckling and free vibration analysis.

$$\overline{\lambda} = \lambda b^2 / E_2 h^3; \qquad (11)$$
$$\overline{\omega} = \omega b^2 / h \sqrt{\rho / E_2} \qquad (12)$$

$$\begin{split} MP1: E_1 / E_2 &= 25, G_{12} / E_2 = 0.5, G_{23} / E_2 = 0.2, \\ G_{13} / E_2 &= 0.5, \upsilon_{12} = 0.25; \\ MP2: E_1 / E_2 &= open, G_{12} / E_2 = 0.6, G_{23} / E_2 = 0.5, \\ G_{13} / E_2 &= 0.6, \upsilon_{12} = 0.25 \end{split}$$

3. Numerical results and discussion

3.1. Free vibration Analysis for a four layered composite plate

A four layered laminated [0/90/90/0] square plate is considered. The plate is simply supported at its four edges. The fundamental frequencies equation [12] are evaluated by varying the modular ratio from 3 to 40 (MP2). The span to thickness ratio of the plate is assumed as a/h=5. From the Figure [1.a] it can be seen that, the present analytical and finite element formulation are in good agreement, also better than the available higher order theory [23]. It is also noticed that the higher order theory overestimate the vibration especially for thick plate. Further, it is also noticed from the Figure [2] that the fundamental frequencies increases when the modular ratio increase.



3.1. Buckling Analysis of laminated composite plate

A simply supported three layered symmetric cross ply [0/90/0] laminated plate subjected to uniaxial load. The non-dimensional buckling load parameter is obtained using MP2 model with modular ratio 40. By varying the side to thickness ratio from thick to thin plate (a/h=2-100) the buckling load parameters are calculated and are compared with higher order theory [24]. From the Figure [3] it is clearly confirms that the compared higher order theory [24] overestimates the buckling loads as compared to the present zigzag model. It is also observed that the buckling load parameter increases with the side to thickness ratio (a/h) increment.

Table 1: Influence of BCs & a/h on buckling load

BC	a/h					
	2	4	5	10	50	100
SSS	2.18	7.82	11.43	30.2	58.921	60.211
S	21	41	61	164		0
SSS	2.19	7.85	11.46	31.3	62.764	64.627
С	24	91	12	81	1	1
SCS	2.19	7.93	11.67	33.1	73.745	76.789
С	36	61	23	767	9	1
CCC	2.19	7.94	11.69	33.9	97.499	104.11
С	48	73	69	158	4	62

Further, Figure [4] shows the first six buckling mode shapes for the same four layered cross-ply plate under uniaxial load along with respective eigen values given.

Besides, a four layered anti symmetric [45/-45/45/-45] angle ply laminated plate under uniaxially loaded is taken for this example. The material properties specified in MP2 is considered. By varying the boundary condition from SSS to CCCC the buckling analysis is carried out and tabulated in Table [1].





Fig. 4 Six buckling mode shapes of four layered plate

4. Conclusions

This paper presented an effective model with the combination of Layerwise parameters and trigonometric shear strain function using Navier closed type solution technique and finite element formulation for the buckling and free vibration analysis of laminated composite and sandwich plates. Like FSDT the present model utilizes same unknowns. which reduce the complexity of computational efforts and formulation. This theory represents a non-linear representation of transverse shear stress and satisfies the axial displacement and transverse shear stress continuity at the layer interfaces. Through constitutive relation the interlaminar shear stress continuity effect is achieved. The requirement of shear correction coefficient is evaded. Because, it vanishes the transverse shear stresses at the upper and lower surfaces of the plate. Several numerical predictions are carried out independently for the laminated plates under considerations of number of layers, layer orientation, side to thickness ratio and different loading conditions. In all the cases, the present results are well matching with the 3D elasticity solutions and provide adequate accuracy than the existing shear deformation theories. The present analytical approach doesn't carry numerical and computational error, however concern to classical boundary condition. Henceforth, various boundary conditions and loading conditions are comprehensively analyzed using finite element formulation. Hence, from the above results and discussions it can be concluded that the present model have the capability to analyze the static

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behavior of any multilayered cross-ply plate with adequate accuracy. Also, it can be suggested as the most favorable and simplest one to examine the laminated plates.

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Appendix

$$\begin{split} A^{k} &= -\sum_{n=2}^{k} z^{n} \Bigg[\left\{ f'(z) \Big|_{z^{n}} \left(\frac{Q_{55}^{n-1}}{Q_{55}^{n}} - 1 \right) + \sum_{p=2}^{n-1} f'(z) \Big|_{z^{p}} \frac{Q_{55}^{p}}{Q_{55}^{n}} \left(\frac{Q_{55}^{p-1}}{Q_{55}^{p}} - 1 \right) \right\} \\ &- \left\{ f'(z) \Big|_{z^{n-1}} \left(\frac{Q_{55}^{n-2}}{Q_{55}^{s-1}} - 1 \right) + \sum_{p=2}^{n-2} f'(z) \Big|_{z^{p}} \frac{Q_{55}^{m}}{Q_{55}^{n-1}} \left(\frac{Q_{55}^{p-1}}{Q_{55}^{p}} - 1 \right) \right\} \Bigg] \\ B^{k} &= f'(z) \underbrace{=}_{z^{k}} \left\{ \frac{Q_{55}^{k-1}}{Q_{55}^{k}} - 1 \right\} + \sum_{m=2}^{k-1} f'(z) \Big|_{z^{m}} \frac{Q_{55}^{m}}{Q_{55}^{k}} \left(\frac{Q_{55}^{m-1}}{Q_{55}^{m}} - 1 \right) \right\} \\ C^{k} &= -\sum_{n=2}^{k} z^{n} \Bigg[\left\{ f'(z) \Big|_{z^{n}} \left(\frac{Q_{44}^{n-1}}{Q_{44}^{n}} - 1 \right) + \sum_{p=2}^{n-1} f'(z) \Big|_{z^{p}} \frac{Q_{44}^{p}}{Q_{44}^{n}} \left(\frac{Q_{44}^{p-1}}{Q_{44}^{p}} - 1 \right) \right\} \\ - \left\{ f'(z) \Big|_{z^{n-1}} \left(\frac{Q_{44}^{n-1}}{Q_{44}^{n-1}} - 1 \right) + \sum_{m=2}^{n-2} f'(z) \Big|_{z^{p}} \frac{Q_{44}^{p}}{Q_{44}^{m}} \left(\frac{Q_{44}^{p-1}}{Q_{44}^{p}} - 1 \right) \right\} \Bigg] \\ D^{k} &= f'(z) \underbrace{=}_{z^{k}} \left\{ \frac{Q_{44}^{k-1}}{Q_{44}^{k}} - 1 \right\} + \sum_{m=2}^{k-1} f'(z) \Big|_{z^{m}} \frac{Q_{44}^{m}}{Q_{44}^{k}} \left(\frac{Q_{44}^{m-1}}{Q_{44}^{p}} - 1 \right) \right\} \end{split}$$

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Authors

Suganya devi S



Suganyadevi S graduated in Aeronautical Engineering with first class from V.S.B. Engineering College, under Anna University, Karur, Tamilnadu, in 2009. She studied Master programme in Aeronautical Engineering specialization Anna University Thirunelveli, at Thirunelveli, Tamilnadu. In 2012, she joined the doctoral programme in the Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur. Her research interests lie in development and analysis of shear deformation theories in deterministic and non-deterministic environment for composite and functionally graded plates and shells by analytically and Numerically.

B. N. Singh

Dr. Bhrigu Nath Singh, is currently professor and Head of the Department of Aerospace Engineering, Indian Institute of Technology Kharagpur. Presently, he is Chair of the ICTACEM. He has discovered the enormous impact of material uncertainty on Aerospace structures response, vibratory loads and stability. He is one of the delegates of Aerospace Industry-Academia Conclave, HAL and also had adorned for taking Lead in Establishing First Aerospace University in India. He is best known for his pioneering research and works in Smart and Composite Structures, Uncertainty Quantification in Aircraft & Design, Multi-scale Analysis Modelling, FGM Plates and Shells, Adaptive Nonlinear FEM, Aerospace Structures and Solid Mechanics. He has publications in repute journals like AIAA J,ASME J,ASCE J,…etc.



