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Finite Element Analysis of Functionally Graded Plates using Inverse Hyperbolic Shear Deformation Theory

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Abstract: Functionally graded materials (FGMs) are becoming very popular in various industries due to their effectiveness of the utilization of their constituent elements. However, the modelling of these materials is difficult due to the complex nature of variation of material properties across the thickness. Many shear deformation theories have been developed and employed for the analysis of such functionally graded plates (FGPs). A recently developed inverse hyperbolic shear deformation theory has been successfully employed by Grover et al. [1] for the analysis of laminated composites and sandwich plates. The objective of the study is to obtain finite element solution for the structural analysis of functionally graded plates using inverse hyperbolic shear deformation theory. Finite element analysis facilitates the analysis of complex problems such as functionally graded plates with different boundary conditions and different loadings.

Key Words : Functionally Graded Materials, Functionally Graded Plates, Inverse Hyperbolic Shear Deformation Theory, Numerical Solution, Finite Element Method

1. Introduction

Advances in engineering applications lead to higher expectations from the structural elements to perform under different challenging conditions. This leads to the development of advanced structural materials such as laminated composites. Laminated composites are layerwise structures made up by stacking laminae. They have better strength to weight ratio and can withstand greater loads. But the disadvantage of using laminated composites is that they tend to delaminate due to their discontinuous strength along the thickness. This can be eliminated by using a new class of materials – functionally graded materials (FGMs). The functionally graded materials exhibit continuous gradation of the two phases in the direction of thickness by continuous change in the volume fraction of the two phases. Since the material is varying continuously, the elastic and thermal properties also vary continuously and reduce the chances of delamination. The modelling of such materials is a challenge as they exhibit different properties at different layers. The analytical solution for these functionally graded plates (FGPs) can be obtained, but only for simple cases. Since the elasticity solution is not possible for complex problems, various shear deformation theories have been developed by the research community. The classical plate theory (CPT) and first order shear deformation theory (FSDT) provide fairly accurate

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results only for thin and moderately thick plates. Reddy's third order shear deformation theory (TSDT) satisfies the condition of zero transverse shear strain at the top and bottom surfaces of the plate and it can be applied for thick plates. Reddy's TSDT considers the polynomial shear strain function. Various shear deformation theories with polynomial shear strain functions have been reported in the literature. More advanced shear deformation theories with non-polynomial shear strain functions have also been developed. The shear strain functions may be trigonometric [2-4], hyperbolic [5], logarithmic [6] or exponential [7]. Grover et al. [1] has recently developed shear deformation theory with inverse trigonometric function and employed it for the analysis of laminated composite and sandwich plates. They have reported that the theory performs very well and can predict more accurate results than other equivalent single layer shear deformation theories. Complex problems such as various types of loadings, boundary conditions and geometry, can't be solved analytically. To solve such problems a numerical solution is suitable. Finite element method is a kind of numerical solution which facilitates solution of such complex problems. The present work focuses on deriving governing equations for the analysis of functionally graded plates using the inverse hyperbolic shear deformation theory developed by Grover et al. [1] and solving them numerically by applying finite element method. An eight-noded isoparametric serendipity bi-quadrilateral rectangular element has been considered for the finite element formulation. Seven field variables that is seven generalized displacements are considered at each node. Convergence of the results is obtained by increasing the number of elements in X and Y directions. A good convergence of the results is obtained, which ensures the correctness of the finite element solution. The obtained results are compared with the analytical and numerical results available in literature and good agreement has been achieved between them. Development of finite element formulation facilitates the analysis for different boundary conditions and loading conditions.

2. Mathematical Formulation

A typical functionally graded plate with dimensions a×b×h (Fig. 1) has been considered for the analysis. The variation of material properties has been observed along the thickness direction (along Z-axis) according to either exponential law or power law. The variation of the material properties include elastic properties such as elastic modulus, bulk modulus shear modulus, density etc.

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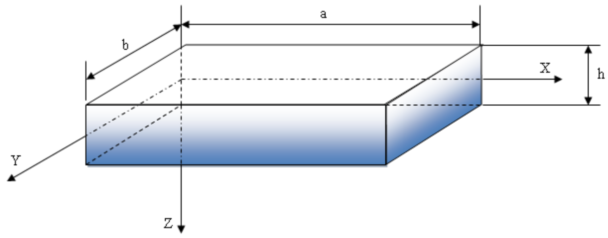

Fig. 1 Geometry of FGP

The displacements according to the inverse hyperbolic shear deformation theory (IHSDT) is given as below:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (1)$$

Where, $f(z) = g(z) + \Omega z$ such that $g(z) = \sinh^{-1}(rz/h)$ and $\Omega = -2r / [\, h(r^2+4)^{0.5}],\ r=3.$

The displacement field given in the Eq. 1 has C^1 continuity which is more difficult to solve. To convert it into C^0 continuity, new variables have been defined such that, $\partial w_0/\partial x = \phi_x$ and $\partial w_0/\partial y = \phi_y$. Hence the displacement field can be rewritten as:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\phi_x + f(z)\theta_x \\ v(x,y,z) &= v_0(x,y) - z\phi_y + f(z)\theta_y \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (2)$$

Linear strain-displacement relations and linear stress-strain relations have been considered for the analysis. The governing equations are obtained by applying Hamilton's principle (Eq. 3) which considers the variation of strain energy (Eq. 4) and potential energy (Eq. 5).

$$\int_T (\delta U + \delta W)dt = 0$$

(3)

$$\delta U = \int_v \left[\delta\varepsilon^T \sigma \right] dv$$

(4) $$\delta W = -\int_A Q_0 \delta u . dA$$

(5)

Where, $u = \{u_0\ v_0\ w_0\ \phi_x\ \phi_y\ \theta_x\ \theta_y\}^T$. Using strain-displacement relations and constitutive relations, above equation can be written in terms of generalized displacements given in Eq. 6.

$$\int_v \{\delta u\}[D]\{u\}\,dv - \int_A Q_0 \delta u . dA = 0 \qquad (6)$$

Where, $[D] = [T]^T [\bar{Q}][T]$ Discretization of the equation is done by considering eight noded bi-quadrilateral serendipity element with seven degrees of freedom at each node. $[\bar{Q}]$ is the matrix of elastic constants which relates stresses to the strains. [T] is the matrix of functions which relates strains with generalized strains. The shape functions at the node are given in Eq. 7.

$$N_i = \begin{cases} \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1) & for \quad i=1,2,3,4 \\ \frac{1}{2}(1-\xi^2)(1+\eta\eta_i) & for \quad i=5,7 \\ \frac{1}{2}(1-\eta^2)(1+\xi\xi_i) & for \quad i=6,8 \end{cases}$$

(7)

The discretized field variables can be represented as $\{u\} = [N]\{u^e\}$.

The final set of equations can be obtained as:

$$[K]\{u^e\} = \{F^e\}$$

(8)

where, stiffness matrix is given as, $[K] = \int_v B^T DBdv;\ \ and\ \ [B] = [L][N]$.

and [L] is operator matrix in the strain-displacement relations, [N] is the matrix of shape functions. The solution of the above simultaneous equations provides the displacements of the plate and hence stresses.

3. Results and discussion

Functionally graded plate with constituent elements as aluminum (E_m=70GPa) and zirconia (E_c=380GPa) have been considered for the analysis. For validation, thick FGPs (a/h=5) with simply supported edges (SSSS) and clamped edges (CCCC) have been considered and compared with quasi-3D theory by Gilhooley et al. [8] and a modified 2D theory by Thai and Choi [9], shown in Table. 1. The present theory produces reasonable results and can be adopted for the analysis of functionally graded plates. The effect of aspect ratio on the non-dimensional central deflection has been shown in Fig. 2; the plate considered is SSSS and CCCC with power indices as N=2, 5.

Table 1 comparison study for the thick functionally graded plate

Boundary condition	Theory	Power law index (N)			
		0	0.5	1	2
SSSS	Quasi-3D (Gilhooley et al.[8])	0.1671	0.2505	0.2905	0.3280
	2D (Thai and Choi [9])	0.1725	0.2330	0.2730	0.3148
	Present	0.1713	0.2316	0.2714	0.3130
CCCC	Quasi-3D (Gilhooley et al.[8])	0.0731	0.1073	0.1253	0.1444
	2D (Thai and Choi [9])	0.0755	0.1002	0.1171	0.1372
	Present	0.0751	0.0997	0.1169	0.1371

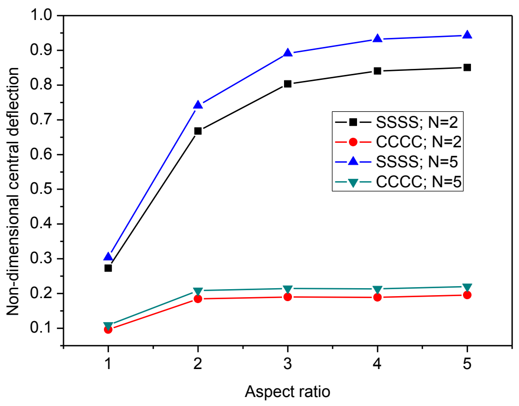

Fig. 2 Effect of aspect ratio on non-dimensional central deflection

4. Conclusions

Static analysis of the functionally graded plate by inverse hyperbolic shear deformation theory has been done numerically. The theory performs well even for the thick plates. The finite element solution facilitates the applicability of the solution for different boundary conditions and loading conditions. Some of the results of the study are presented in the present work. The effect of boundary condition, aspect ratio, power law index, and span-to-thickness ratio can also be studied using the present method. The results produced by IHSDT are fairly accurate as compared to the other two dimensional theories.

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