

## INT-SOFT MIGHTY FILTERS IN *BE*-ALGEBRAS

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ABSTRACT. In this paper, we introduce the notions about *int-soft mighty filters*, *int-soft  $n$ -fold mighty filters*, and *int-soft  $n$ -fold positive implicative filters* of *BE*-algebras. We investigate their properties and provide conditions which have connecting relationship among int-soft filters, int-soft mighty filters, and int-soft positive implicative filters. Also, characterizations of int-soft  $n$ -fold mighty filters and int-soft  $n$ -fold positive implicative filters are provided in *BE*-algebras.

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### 1. Introduction

Two classes of abstract algebras called *BCK*-algebra and *BCI*-algebra were introduced by K. Iséki and S. Tanaka ([5, 6, 7]). The class of *BCK*-algebra is a proper subclass of the class of *BCI*-algebras. The *BCK* and *BCI*-algebras were more investigated as a generalization of propositional logics ([13, 14]). Especially, H. S. Kim and Y. H. Kim established the concepts and properties of *BE*-algebras as a dualization for a generalization of *BCK*-algebras ([9]).

Sometimes it is not able to apply with classical methods successfully for many of complicated problems in engineering, economics, medical science, and environment because of various uncertainties. We investigate to approach their vagueness with wide extended ranges for theories of probability, fuzzy sets, vague sets, and other mathematical tools. However, most of these theories still have their own difficulties because of inadequate parametrization tools of the theories. To overcome these difficulties, the concept of *soft set* as a mathematical tool was suggested by D. Molodtsov ([15]). Since then the soft set which is a parameterized family of subsets of a universe has been based on algebraic structures by

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several authors. Acar et al.([1]) introduced initial concepts of soft rings and the notion of an *int-soft filter* in a *BE*-algebra was discussed by Ahn et al.([2]).

In this paper, we review some definitions and properties for positive implicative filters, mighty filters, and int-soft filters in *BE*-algebras. We define the notions for an int-soft mighty filter, an int-soft  $n$ -fold mighty filter, and an int-soft  $n$ -fold positive implicative filter of *BE*-algebras. We investigate their properties and provide several examples to clarify them. We introduce a condition for a mighty filter to be a positive implicative filter and then state some properties between int-soft mighty filters and int-soft positive implicative filters in a *BE*-algebra. We discuss characterizations of int-soft  $n$ -fold mighty filters and int-soft  $n$ -fold positive implicative filters of *BE*-algebras.

## 2. Preliminaries

We recall some definitions and results that will be useful in the process of our paper.

**Definition 2.1** ([9]). An algebra  $(X; *, 1)$  is called a *BE*-algebra if it satisfies:

- (BE1)  $x * x = 1$  for all  $x \in X$ ;
- (BE2)  $x * 1 = 1$  for all  $x \in X$ ;
- (BE3)  $1 * x = x$  for all  $x \in X$ ;
- (BE4)  $x * (y * z) = y * (x * z)$  for all  $x, y, z \in X$ .

We introduce a relation “ $\leq$ ” on  $X$  by  $x \leq y$  if and only if  $x * y = 1$ . A *BE*-algebra  $(X; *, 1)$  is said to be *self-distributive* if  $x * (y * z) = (x * y) * (x * z)$ , *commutative* if  $(x * y) * y = y * (y * x)$  ([16]), and *transitive* if  $y * z \leq (x * y) * (x * z)$  for all  $x, y, z \in X$  ([4]).

**Definition 2.2** ([9]). Let  $(X; *, 1)$  be a *BE*-algebra and  $F$  a non-empty subset of  $X$ . Then  $F$  is called a *filter of  $X$*  if it satisfies:

- (F1)  $1 \in F$ ;
- (F2)  $x * y \in F$  and  $x \in F$  imply  $y \in F$  for all  $x, y \in X$ .

**Proposition 2.3** ([9]). Let  $(X; *, 1)$  be a self-distributive *BE*-algebra, then the followings hold: for any  $x, y, z \in X$ ,

- (1) if  $x \leq y$ , then  $z * x \leq z * y$  and  $y * z \leq x * z$ ;
- (2)  $y * z \leq (z * x) * (y * z)$ ;
- (3)  $y * z \leq (z * x) * (y * x)$ ;
- (4)  $y * z \leq (x * y) * (x * z)$ .

A *BE*-algebra  $(X, *, 1)$  is commutative,  $X$  has the same properties of Proposition 2.3 ([3, 16]).

**Proposition 2.4** ([4]). Let  $(X; *, 1)$  be a *BE*-algebra and  $F$  a filter of  $X$ . If  $x \leq y$  and  $x \in F$ , then  $y \in F$  for any  $x, y \in X$ .

**Definition 2.5** ([10]). Let  $X$  be a *BE*-algebra. A nonempty subset  $F$  of  $X$  is called a *positive implicative filter of  $X$*  if it satisfies:

- (1)  $1 \in F$ ;
- (2)  $x * ((y * z) * y) \in F$  and  $x \in F$  imply  $y \in F$  for all  $x, y, z \in X$ .

Every positive implicative filter of a BE algebra  $X$  is a filter of  $X$ .

**Theorem 2.6** ([10]). *Let  $F$  be a filter of a BE-algebra  $X$ . Then  $F$  is a positive implicative filter of  $X$  if and only if*

$$(x * y) * x \in F \text{ implies } x \in F \text{ for all } x, y \in X.$$

**Definition 2.7** ([12]). A non-empty subset  $F$  of a BE-algebra  $X$  is called a *mighty filter of  $X$*  if it satisfies:

- (M1)  $1 \in F$ ;
- (M2)  $x * (y * z) \in F$  and  $x \in F$  imply  $((z * y) * y) * z \in F$  for all  $x, y, z \in X$ .

Every mighty filter of a BE-algebra  $X$  is a filter of  $X$ .

**Theorem 2.8** ([12]). *A filter  $F$  of a BE-algebra  $X$  is a mighty filter of  $X$  if and only if it satisfies :*

$$y * x \in F \text{ implies } ((x * y) * y) * x \in F \text{ for all } x, y \in X.$$

In what follows, we take a BE-algebra  $X$  as a set of parameters unless otherwise specified. Let  $U$  be an initial universe set,  $A, B, C \dots \subseteq X$  and  $\mathcal{P}(U)$  denote the power set of  $U$ .

A *soft set*  $(f, A)$  of  $X$  over  $U$  ([15]) is defined to be the set of ordered pairs  $(f, A) := \{(x, f(x)) : x \in X, f(x) \in \mathcal{P}(U)\}$ , where  $f : X \rightarrow \mathcal{P}(U)$  such that  $f(x) = \emptyset$  if  $x \notin A$ .

**Definition 2.9** ([2]). A soft set  $(f, X)$  of a BE-algebra  $X$  over  $U$  is called an *intersection-soft filter (briefly, int-soft filter) over  $U$*  if it satisfies: for any  $x, y \in X$ ,

- (IS1)  $f(x) \subseteq f(1)$ ;
- (IS2)  $f(x * y) \cap f(x) \subseteq f(y)$ .

**Proposition 2.10** ([2]). *Every int-soft filter  $(f, X)$  of a BE-algebra  $X$  over  $U$  satisfies the following properties:*

- (1) For all  $x, y \in X$ ,  $x \leq y \Rightarrow f(x) \subseteq f(y)$ ;
- (2) For all  $x, y, z \in X$ ,  $f(x * (y * z)) \cap f(y) \subseteq f(x * z)$ .

**Proposition 2.11.** *Every int-soft filter  $(f, X)$  of a BE-algebra  $X$  over  $U$  satisfies the followings:*

- (1) For all  $x, y \in X$ ,  $f(x) \subseteq f((x * y) * y)$ ;
- (2) For all  $x, y \in X$ ,  $f(x) \subseteq f((x * y) * x)$ ;
- (3) For all  $x, y \in X$ ,  $f(x) \subseteq f(y * x)$ .

*Proof.* In BE-algebra, it satisfies  $x \leq (x * y) * y$ . Thus it is implied  $f(x) \subseteq f((x * y) * y)$  by Proposition 2.10-(1). It has the same way in the cases of (2), (3).  $\square$

**Proposition 2.12** ([8]). *Let  $(f, X)$  be a soft set of a  $BE$ -algebra  $X$  over  $U$ , then  $(f, X)$  is an int-soft filter of  $X$  over  $U$  if and only if*

$$z \leq x * y \Rightarrow f(x) \cap f(z) \subseteq f(y) \quad \text{for all } x, y, z \in X.$$

**3. Int-soft mighty filters in  $BE$ -algebras**

In this section, we define an int-soft mighty filter of a  $BE$ -algebra. We investigate some relations among int-soft filter, int-soft mighty filter, and int-soft positive implicative filter in  $BE$ -algebras.

**Definition 3.1.** A soft  $(f, X)$  of a  $BE$ -algebra  $X$  over  $U$  is called an *int-soft mighty filter of  $X$  over  $U$*  if it satisfies:

- (IM1)  $f(x) \subseteq f(1)$ ;
- (IM2)  $f(x * (y * z)) \cap f(x) \subseteq f(((z * y) * y) * z)$  for all  $x, y, z \in X$ .

**Theorem 3.2.** *Every int-soft mighty filter of a  $BE$ -algebra  $X$  is an int-soft filter of  $X$ .*

*Proof.* Let  $(f, X)$  be an int-soft mighty filter of  $X$ . If we take  $y = 1$  in (IM2),  $f(x * (1 * z)) \cap f(x) \subseteq f(((z * 1) * 1) * z) \Rightarrow f(x * z) \cap f(x) \subseteq f(z)$  for any  $x, z \in X$ . Thus  $(f, X)$  is an *int-soft filter of  $X$* . □

**Example 3.3.** Let  $X$  be a set of parameters and  $U = X$  the initial universe set. Let  $X := \{1, a, b, c, d\}$  be a  $BE$ -algebra with the following Cayley table:

$*$	1	$a$	$b$	$c$	$d$
1	1	$a$	$b$	$c$	$d$
$a$	1	1	$b$	$c$	$d$
$b$	1	$a$	1	$c$	$d$
$c$	1	$a$	$b$	1	1
$d$	1	1	$b$	1	1

Let a soft set  $(f, X)$  be a soft set of  $X$  over  $U$  defined as follows:

$$f : X \rightarrow \mathcal{P}(U), \quad x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, a\} \\ \gamma_1 & \text{if } x \in \{c, d\} \\ \emptyset & \text{if } x \in \{b\}, \end{cases}$$

where  $\gamma_1$  and  $\gamma_2$  are subsets of  $U$  with  $\gamma_1 \subsetneq \gamma_2$ . It is easy to check that  $(f, X)$  is both an int-soft mighty filter of  $X$  and an int-soft filter of  $X$  over  $U$ .

The converse of Theorem 3.2 is not true in general as the following example.

**Example 3.4.** Let  $X := \{1, a, b, c, d\}$ , then  $X$  is a  $BE$ -algebra with the following Cayley table:

*	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	1
c	1	a	1	1	1
d	1	1	b	1	1

Let a soft set  $(f, X)$  be defined as follows:

$$f : X \rightarrow \mathcal{P}(U), x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, a\} \\ \gamma_1 & \text{if } x \in \{b, c, d\}, \end{cases}$$

where  $\gamma_1$  and  $\gamma_2$  are subsets of  $U$  with  $\gamma_1 \subsetneq \gamma_2$ . Then  $(f, X)$  is an int-soft filter, but  $(f, X)$  is not an int-soft mighty filter since  $f(1 * (c * b)) \cap f(1) = \gamma_2 \not\subseteq \gamma_1 = f(((b * c) * c) * b)$ .

**Theorem 3.5.** *An int-soft filter is an int-soft mighty filter of a BE-algebra  $X$  if and only if*

$$f(y * z) \subseteq f(((z * y) * y) * z) \text{ for all } y, z \in X.$$

*Proof.* Assume  $X$  is an int-soft mighty filter and  $y, z \in X$ .

$$f(y * z) \subseteq f(1 * (y * z)) \cap f(1) \subseteq f(((z * y) * y) * z).$$

Conversely, let  $X$  is an int-soft filter and  $x, y, z \in X$ .

$$f(x * (y * z)) \cap f(x) \subseteq f(y * z) \subseteq f(((z * y) * y) * z).$$

Thus it is an int-soft mighty filter of  $X$ . □

**Theorem 3.6.** *Let  $(f, X)$  and  $(g, X)$  be int-soft filters of a transitive BE-algebra  $X$  with  $f(x) \subseteq g(x)$  and  $f(1) = g(1)$ . If  $(f, X)$  is an int-soft mighty filter of  $X$ , then so is  $(g, X)$ .*

*Proof.* Suppose  $(f, X)$  is an int-soft mighty filter of a be-algebra  $X$ . Let  $x, y \in X$ , using Theorem 3.5, we have  $f(1) = f(y * ((y * x) * x)) \subseteq f((((y * x) * x) * y) * y * ((y * x) * x)) \subseteq g((((y * x) * x) * y) * y * ((y * x) * x)) = g((y * x) * (((y * x) * x) * y) * y * x) = g(1)$  since  $f(1) = g(1)$ . It follows from Definition 2.9 that

$$\begin{aligned} g(y * x) &= g(1) \cap g(y * x) \\ &= g((y * x) * (((y * x) * x) * y) * y * x) \cap g(y * x) \\ &\subseteq g((((y * x) * x) * y) * y * x). \end{aligned}$$

Since  $X$  is transitive,

$$\begin{aligned} &((((y * x) * x) * y) * y) * x * (((x * y) * y) * x) \\ &\geq ((x * y) * y) * (((y * x) * x) * y) * y) \\ &\geq ((y * x) * x) * y * (x * y) \\ &\geq x * ((y * x) * x) \\ &= 1. \end{aligned}$$

We get  $g(\(((y * x) * x) * y) * y) * x \cap g(1) \subseteq g((x * y) * y) * x$  by Proposition 2.12. Thus  $g(y * x) \subseteq g(\(((y * x) * x) * y) * y) * x \subseteq g((x * y) * y) * x$ . It satisfies Theorem 3.5. Thus  $(g, X)$  is an int-soft mighty filter of  $X$ .  $\square$

We introduce a condition for a mighty filter to be a positive implicative filter and then provide some properties of int-soft mighty filters and int-soft positive implicative filters in a  $BE$ -algebra.

**Theorem 3.7.** *Every positive implicative filter of a commutative  $BE$ -algebra  $X$  is a mighty filter of  $X$ .*

*Proof.* Suppose a subset  $F$  of a  $BE$ -algebra  $X$  is a positive implicative filter. Using the commutative property of Proposition 2.3-(3), let  $y * x \in F$  for  $x, y \in X$ .

$$\begin{aligned} y * x &\leq ((x * y) * y) * (y * x) \\ &\leq ((y * x) * x) * (((x * y) * y) * x) \\ &= (((x * y) * y) * ((x * y) * y) * x) \\ &\leq (((x * y) * y) * x) * x * (((x * y) * y) * x). \end{aligned}$$

We get  $\(((x * y) * y) * x) * x * (((x * y) * y) * x) \in F$  by Proposition 2.4, so that  $((x * y) * y) * x \in F$  by Theorem 2.6. Hence  $F$  satisfies Theorem 2.8.  $\square$

The converse of Theorem 3.7 is not true in general as seen the following example.

**Example 3.8.** Let  $X := \{1, a, b, c, d\}$ , then  $X$  is a commutative  $BE$ -algebra with the following Cayley table:

$*$	1	$a$	$b$	$c$	$d$
1	1	$a$	$b$	$c$	$d$
$a$	1	1	$a$	$c$	$d$
$b$	1	1	1	$c$	$d$
$c$	1	$a$	$b$	1	$c$
$d$	1	$a$	$b$	1	1

Let  $F := \{1, c, d\}$ , then  $F$  is a mighty filter  $X$ . But it is not a positive implicative filter of  $X$  since  $c * ((a * b) * a) = 1 \in F$  and  $c \in F$ , but  $a \notin F$ .

**Definition 3.9** ([11]). A soft set  $(f, X)$  of a  $BE$ -algebra  $X$  over  $U$  is called an *int-soft positive implicative filter of  $X$  over  $U$*  if it satisfies:

- (IS1)  $f(x) \subseteq f(1)$ ;
- (IS2)  $f(x * ((y * z) * y)) \cap f(x) \subseteq f(y)$  for all  $x, y, z \in X$ .

Every int-soft positive implicative filter of a  $BE$ -algebra  $X$  over  $U$  is an int-soft filter of  $X$  over  $U$  ([11]).

**Theorem 3.10** ([11]). *Let  $X$  be a  $BE$ -algebra. Then an int-soft filter  $(f, X)$  is an int-soft positive implicative filter of  $X$  if and only if*

$$f((x * y) * x) \subseteq f(x) \text{ for all } x, y \in X.$$

**Theorem 3.11.** *Every int-soft positive implicative filter of a commutative  $BE$ -algebra  $X$  is an int-soft mighty filter of  $X$ .*

*Proof.* Suppose  $(f, X)$  is an int-soft positive implicative filter. It follows from Theorem 3.7,  $y * x \leq (((x * y) * y) * x) * x * (((x * y) * y) * x)$ . We have  $f(y * x) \subseteq f((((x * y) * y) * x) * x * (((x * y) * y) * x)) \subseteq f(((x * y) * y) * x)$  from Theorem 3.10. It satisfies Theorem 3.5. Hence  $(f, X)$  is an int-soft mighty filter of  $X$ .  $\square$

The converse of Theorem 3.11 does not hold in general as the following example.

**Example 3.12.** Let  $X := \{1, a, b, c\}$ , then  $X$  is a commutative  $BE$ -algebra with the following Cayley table:

$*$	1	$a$	$b$	$c$
1	1	1	$a$	$c$
$a$	1	1	$a$	$c$
$b$	1	1	1	$b$
$c$	1	$a$	$a$	1

Let a soft set  $(f, X)$  be defined as follows:

$$f : X \rightarrow \mathcal{P}(U), x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1\} \\ \gamma_1 & \text{if } x \in \{a, b, c\}, \end{cases}$$

where  $\gamma_1$  and  $\gamma_2$  are subsets of  $U$  with  $\gamma_1 \subsetneq \gamma_2$ , then  $(f, X)$  is an int-soft mighty filter of  $X$ . But it is not an int-soft positive implicative filter since  $f(1 * ((b * c) * b)) \cap f(1) = \gamma_2 \not\subseteq \gamma_1 = f(b)$ .

**Theorem 3.13.** *Every int-soft mighty filter of a  $BE$ -algebra  $X$  is an int-soft positive implicative filter of  $X$  if it satisfies  $f((x * y) * x) \subseteq f(x)$  for all  $x, y \in X$ .*

*Proof.* It is obvious from Theorem 3.10.  $\square$

#### 4. Int-soft $n$ -fold mighty filter and Int-soft $n$ -fold positive implicative filter of a $BE$ -algebra

In this chapter, we recall the definition of  $n$ -fold mighty filter  $F$  of a  $BE$ -algebra  $X$  in ([12]) which is satisfied two conditions: for all  $x, y, z \in X$ ,

- (1)  $1 \in F$ ;
- (2)  $x * (y * z) \in F$  and  $x \in F \Rightarrow ((z^n * y) * y) * z \in F$ .

Let  $X$  be a  $BE$ -algebra and  $n$  denote a positive integer. For any elements  $x, y \in X$ , let  $f(x^n * y)$  denote  $f(x * (x * (\dots(x * y)))) \dots$ .

**Definition 4.1.** A soft set  $(f, X)$  of a  $BE$ -algebra  $X$  is called an *int-soft  $n$ -fold mighty filter of  $X$*  if it satisfies:

- (NM1)  $f(x) \subseteq f(1)$ ;
- (NM2)  $f(x * (y * z)) \cap f(x) \subseteq f(((z^n * y) * y) * z)$  for all  $x, y, z \in X$ .

**Example 4.2.** Let  $X$  be the set of parameters and  $U = X$  the universal set. Let  $X := \{1, a, b, c, d\}$ , then  $X$  is a  $BE$ -algebra with the following Cayley table:

$*$	1	$a$	$b$	$c$	$d$
1	1	$a$	$b$	$c$	$d$
$a$	1	1	$b$	$c$	$d$
$b$	1	$a$	1	$c$	$c$
$c$	1	$a$	$b$	1	$b$
$d$	1	1	1	1	1

Let a soft set  $(f, X)$  be defined as follows:

$$f : X \rightarrow \mathcal{P}(U), x \mapsto \begin{cases} \gamma_2 & \text{if } x \in \{1, a, b\} \\ \gamma_1 & \text{if } x \in \{c, d\}, \end{cases}$$

where  $\gamma_1$  and  $\gamma_2$  are subsets of  $U$  with  $\gamma_1 \subsetneq \gamma_2$ , then  $(f, X)$  is an int-soft  $n$ -fold mighty filter ( $n \geq 2$ ) of  $X$ .

**Definition 4.3.** A soft set  $(f, X)$  of a  $BE$ -algebra  $X$  over  $U$  is called an *int-soft  $n$ -fold positive implicative filter of  $X$*  if it satisfies:

- (NP1)  $f(x) \subseteq f(1)$ ;
- (NP2)  $f(x * ((y^n * z) * y)) \cap f(x) \subseteq f(y)$  for all  $x, y, z \in X$ .

In Example 4.2,  $(f, X)$  is an int-soft  $n$ -fold positive implicative filter of a  $BE$ -algebra  $X$ .

**Proposition 4.4.** For a soft set  $(f, X)$  of a  $BE$ -algebra  $X$ ,

- (1) Every int-soft  $n$ -fold mighty filter is an int-soft filter of  $X$ .
- (2) Every int-soft  $n$ -fold positive implicative filter of a  $BE$ -algebra  $X$  is an int-soft filter of  $X$ .

*Proof.* (1) If we take  $y = 1$  in (NM2),

$$\begin{aligned} f(x * (1 * z)) \cap f(x) &\subseteq f(((z^n * 1) * 1) * z) \\ f(x * z) \cap f(x) &\subseteq f(z), \quad \text{for all } x, z \in X. \end{aligned}$$

Thus  $(f, X)$  is an int-soft filter of  $X$ .

(2) has the same way if we take  $z = 1$  in (NP2). □

**Theorem 4.5.** Let  $(f, X)$  be an int-soft filter of a  $BE$ -algebra  $X$ . Then  $(f, X)$  is an int-soft  $n$ -fold mighty filter of  $X$  if and only if  $f(x * y) \subseteq f(((y^n * x) * x) * y)$  for all  $x, y, z \in X$ .

*Proof.* Suppose that  $(f, X)$  is an int-soft  $n$ -fold mighty filter of  $X$ . For any  $x, y \in X$ ,  $f(x * y) = f(1 * (x * y)) \subseteq f(1 * (x * y)) \cap f(1) \subseteq f(((y^n * x) * x) * y)$ . Conversely, assume  $(f, X)$  is an int-soft filter of  $X$  and it satisfies

$$f(x * y) \subseteq f(((y^n * x) * x) * y),$$

then

$$\begin{aligned} f(z * (x * y)) \cap f(z) &\subseteq f(x * y) \\ &\subseteq f(((y^n * x) * x) * y) \quad \text{for all } x, y, z \in X. \end{aligned}$$

Thus  $(f, X)$  is an int-soft  $n$ -fold mighty filter of  $X$ .  $\square$

**Theorem 4.6.** *An int-soft filter  $(f, X)$  of a BE-algebra  $X$  is an int-soft  $n$ -fold positive implicative filter of  $X$  if and only if  $f((y^n * x) * y) \subseteq f(y)$ .*

*Proof.* Suppose that  $(f, X)$  is an int-soft  $n$ -fold positive implicative filter of  $X$ . For any  $x, y \in X$ ,  $f((y^n * x) * y) \subseteq f(1 * ((y^n * x) * y)) \cap f(1) \subseteq f(y)$ , thus

$$f((y^n * x) * y) \subseteq f(y).$$

Conversely, If  $(f, X)$  is an int-soft filter of  $X$  satisfying  $f((y^n * x) * y) \subseteq f(y)$ , then

$$f(x * ((y^n * z) * y)) \cap f(x) \subseteq f((y^n * z) * y) \subseteq f(y) \quad \text{for all } x, y, z \in X.$$

Thus  $(f, X)$  is an int soft  $n$ -fold positive implicative filter of  $X$ .  $\square$

**Proposition 4.7.** *For soft set  $(f, X)$  of a BE-algebra  $X$ ,*

- (1) *Every int-soft  $n$ -fold positive implicative filter of a commutative BE-algebra  $X$  is an int-soft  $n$ -fold mighty filter of  $X$ .*
- (2) *Every int-soft  $n$ -fold mighty filter of a BE-algebra  $X$  is an int-soft  $n$ -fold positive implicative filter of  $X$  if it satisfies  $f((x^n * y) * x) \subseteq f(x)$  for all  $x, y \in X$ .*

*Proof.* It is obvious by Theorem 3.11, Theorem 3.13.  $\square$

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