J. Appl. Math. & Informatics Vol. **34**(2016), No. 5 - 6, pp. 519 - 526
http://dx.doi.org/10.14317/jami.2016.519

SOMEWHAT PAIRWISE FUZZY PRE-IRRESOLUTE CONTINUOUS MAPPINGS

A. SWAMINATHAN

Abstract. The concept of somewhat pairwise fuzzy pre-irresolute continuous mapping and somewhat pairwise fuzzy irresolute preopen mappings have been introduced and studied. Besides, some interesting properties of those mappings are given.

AMS Mathematics Subject Classification : 54A40, 03E72.
Key words and phrases : Somewhat pairwise fuzzy pre-irresolute continuous mapping, somewhat pairwise fuzzy irresolute preopen mapping.

1. Introduction and Preliminaries

The fundamental concept of fuzzy sets was introduced by L.A. Zadeh [13] provided a natural foundation for building new branches. In 1968 C.L. Chang [3] introduced the concept of fuzzy topological spaces as a generalization of topological spaces.

The class of somewhat continuous mappings was first introduced by karl.R. Gentry and others in [5]. Later, the concept of "somewhat" in classical topology has been extented to fuzzy topological spaces. In fact, somewhat fuzzy continuous mappings and somewhat fuzzy semicontinuous mappings were introduced and studied by G. Thangaraj and G. Balasubramanian in [9] and [10] respectively. In 1989, A. Kandil [4] introduced the concept of fuzzy bitopological spaces. The product related spaces and the graph of a function were found in Azad [1]. The concept of somewhat pairwise fuzzy continuous mappings was introduced and devoloped by M.K. Uma and others in [11].

Meanwhile, the concept of fuzzy irresolute continuous mappings on a fuzzy topological space was introduced and studied by M.N. Mukherjee and S. P.Shina

Received October 31, 2015. Revised December 5, 2015. Accepted December 6, 2015.

in [6] and fuzzy precontinuous mappings on a fuzzy topological space was introduced and studied by A.S. Bin Shahna in [2]. Also, fuzzy pre-irresolute continuous mappings on a fuzzy topological space were introduced and studiedby J.H. Park and B.H. Park in [7].

The concept of somewhat fuzzy pre-irresolute continuous mappings was introduced and studied by Young Bin Im and others in [12].

Recently, somewhat pairwise fuzzy precontinuous mappings on fuzzy bitopological spaces was introduced and studied by A. Swaminathan and others in [8].

In this paper, the concepts of somewhat pairwise fuzzy pre-irresolute continuous mappings and somewhat pairwise fuzzy irresolute preopen mappings on a fuzzy bitopological space are introduced and studied their properties.

Definition 1.1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy precontinuous [8] if $f^{-1}(\nu)$ is a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2) for any η_1-fuzzy open or η_2-fuzzy open set ν on (Y, η_1, η_2).

Definition 1.2. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy precontinuous [8] if there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1-fuzzy open or η_2-fuzzy open set ν on (Y, η_1, η_2) .

Definition 1.3. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy preopen [8] if $f(\mu)$ is an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2) for any τ_1-fuzzy open or τ_2-fuzzy open set μ on (X, τ_1, τ_2).

Definition 1.4. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy preopen [8] if there exists an η_1-fuzzy preopen or η_2-fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1-fuzzy open or τ_2-fuzzy open set μ on (X, τ_1, τ_2).

2. Somewhat pairwise fuzzy pre-irresolute continuous mappings

In this section, I introduce a somewhat pairwise fuzzy pre-irresolute continuous mapping which are stronger than a somewhat pairwise fuzzy precontinuous mapping. And we characterize a somewhat pairwise fuzzy pre-irresolute continuous mapping.

Definition 2.1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy pre-irresolute continuous if $f^{-1}(\nu)$ is a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2) for any η_1-fuzzy preopen or η_2-fuzzy preopen set ν on (Y, η_1, η_2).

Definition 2.2. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy pre-irresolute continuous if there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any η_1-fuzzy preopen or η_2-fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) .

From the definitions, it is clear that every pairwise fuzzy pre-irresolute continuous mapping is a somewhat pairwise fuzzy pre-irresolute continuous mapping.

And every somewhat pairwise fuzzy pre-irresolute continuous mapping is a pairwise fuzzy precontinuous mapping. Also, every pairwise fuzzy precontinuous mapping is a somewhat pairwise fuzzy precontinuous mapping. But the converses are not true in general as the following examples show.

Example 2.3. Let λ_1,λ_2,λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2,σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then $\lambda_1 = \frac{0.1}{a} + \frac{0.1}{b} + \frac{0.1}{c}$, $\lambda_2 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$, $\lambda_3 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\sigma_1 = \frac{0.3}{x} + \frac{0.0}{y} + \frac{0.3}{z}$, $\sigma_2 = \frac{0.5}{x} + \frac{0.0}{y} + \frac{0.5}{z}$, $\sigma_3 = \frac{0.5}{x} + \frac{0.2}{y} + \frac{0.5}{z}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_1\}$,$\tau_2 = \{0_X, 1_X, \lambda_2^c\}$, $\eta_1 = \{0_Y, 1_Y, \sigma_1\}$,$\eta_2 = \{0_Y, 1_Y, \sigma_3\}$. Then (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$,$f(b) = y$,$f(c) = y$. Then we have $f^{-1}(\sigma_1) = 0_X$, $f^{-1}(\sigma_2) == 0_X$ and $\lambda_1 < f^{-1}(\sigma_3) = \lambda_2$. Since λ_1 is a τ_1-fuzzy semiopen set on (X, τ_1, τ_2), f is somewhat pairwise fuzzy pre-irresolute continuous. But $f^{-1}(\sigma_3) = \lambda_2$ is not a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2). Hence f is not a pairwise fuzzy pre-irresolute continuous mapping.

Example 2.4. Let λ_1,λ_2,λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2,σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then $\lambda_1 = \frac{0.4}{a} + \frac{0.4}{b} + \frac{0.4}{c}$, $\lambda_2 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\lambda_3 = \frac{0.6}{a} + \frac{0.6}{b} + \frac{0.6}{c}$, $\sigma_1 = \frac{0.4}{x} + \frac{0.0}{y} + \frac{0.4}{z}$, $\sigma_2 = \frac{0.4}{x} + \frac{0.5}{y} + \frac{0.4}{z}$, $\sigma_3 = \frac{0.4}{x} + \frac{0.4}{y} + \frac{0.4}{z}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_2\}$,$\tau_2 = \{0_X, 1_X, \lambda_3\}$, $\eta_1 = \{0_Y, 1_Y, \sigma_1\}$,$\eta_2 = \{0_Y, 1_Y, \sigma_2\}$. Then (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$,$f(b) = y$,$f(c) = y$. Then we have $f^{-1}(\sigma_1) = 0_X$ and $f^{-1}(\sigma_2) = \lambda_2$ are τ_2-fuzzy preopen sets on (X, τ_1, τ_2), f is pairwise fuzzy precontinuous. But $f^{-1}(\sigma_3) = \lambda_1$ of an η_1-fuzzy preopen set σ_3 on (Y, η_1, η_2) is not τ_1-fuzzy preopen or τ_2-fuzzy preopen on (X, τ_1, τ_2). Hence f is not a somewhat pairwise fuzzy pre-irresolute continuous mapping.

Example 2.5. Let λ_1 and λ_2 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1 and σ_2 be fuzzy sets on $Y = \{x, y, z\}$. Then $\lambda_1 = \frac{0.1}{a} + \frac{0.1}{b} + \frac{0.1}{c}$, $\lambda_2 = \frac{0.3}{a} + \frac{0.3}{b} + \frac{0.3}{c}$,$\lambda_3 = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}$, $\lambda_4 = \frac{0.9}{a} + \frac{0.9}{b} + \frac{0.9}{c}$. Consider $\tau_1 = \{0_Y, 1_Y, \lambda_1\}$,$\tau_2 = \{0_Y, 1_Y, \lambda_2\}$, $\eta_1 = \{0_X, 1_X, \lambda_3\}$,$\eta_2 = \{0_X, 1_X, \lambda_4\}$. Then (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and consider an identity mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$. Then we have $\lambda_1 < f^{-1}(\lambda_3) = \lambda_3$ and $\lambda_1 < f^{-1}(\lambda_4) = \lambda_4$.Since λ_1 is τ_1-fuzzy preopen set on (X, τ_1, τ_2), f is somewhat pairwise fuzzy precontinuous. But $f^{-1}(\lambda_3) = \lambda_3$ and $f^{-1}(\lambda_4) = \lambda_4$ are not τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2). Hence f is not a pairwise fuzzy precontinuous mapping.

Definition 2.6. A fuzzy set μ on a fuzzy bitopological space (X, τ_1, τ_2) is called pairwise predense fuzzy set if there exists no τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set ν in (X, τ_1, τ_2) such that $\mu < \nu < 1$.

Theorem 2.7. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping. Then the following are equivalent:*
(1) f is somewhat pairwise fuzzy pre-irresolute continuous.

(2) If ν is an η_1-fuzzy preclosed or η_2-fuzzy preclosed set of (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$, then there exists a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set $\mu \neq 1_X$ of (X, τ_1, τ_2) such that $f^{-1}(\nu) \leq \mu$.
(3) If μ is a pairwise predense fuzzy set on (X, τ_1, τ_2), then $f(\mu)$ is a pairwise predense fuzzy set on (Y, η_1, η_2).

Proof. (1) $\Rightarrow$ (2): Let ν be an η_1-fuzzy preclosed or η_2-fuzzy preclosed set on (Y, η_1, η_2) such that $f^{-1}(\nu) \neq 1_X$. Then ν^c is an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2) and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$. Since f is somewhat pairwise fuzzy pre-irresolute continuous, there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\lambda \neq 0_X$ on (X, τ_1, τ_2) such that $\lambda \leq f^{-1}(\nu^c)$. Let $\mu = \lambda^c$. Then $\mu \neq 1_X$ is a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set such that $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \lambda = \lambda^c = \mu$.

(2) $\Rightarrow$ (3): Let μ be a pairwise predense fuzzy set on (X, τ_1, τ_2) and suppose $f(\mu)$ is not pairwise predense fuzzy set on (Y, η_1, η_2). Then there exists an η_1-fuzzy preclosed or η_2-fuzzy preclosed set ν on (Y, η_1, η_2) such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and $f^{-1}(\nu) \neq 1_X$, there exists a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set $\delta \neq 1_X$ such that $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$. This contradicts to the assumption that μ is a pairwise predense fuzzy set on (X, τ_1, τ_2). Hence $f(\mu)$ is a pairwise predense fuzzy set on (Y, η_1, η_2).

(3) $\Rightarrow$ (1): Let $\nu \neq 0_Y$ be an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2) and let $f^{-1}(\nu) \neq 0_X$. Suppose that there exists no τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu))^c$ is a τ_1-fuzzy set or τ_2-fuzzy set on (X, τ_1, τ_2) such that there is no τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set δ on (X, τ_1, τ_2) with $(f^{-1}(\nu))^c < \delta < 1$. In fact, if there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set δ^c such that $\delta^c \leq f^{-1}(\nu)$, then it is a contradiction. So $(f^{-1}(\nu))^c$ is a pairwise predense fuzzy set on (X, τ_1, τ_2). Then $f((f^{-1}(\nu))^c)$ is a pairwise predense fuzzy set on (Y, η_1, η_2) . But $f((f^{-1}(\nu))^c) = f(f^{-1}(\nu^c)) \neq \nu^c < 1$. This is a contradiction to the fact that $f((f^{-1}(\nu))^c)$ is pairwise predense fuzzy set on (Y, η_1, η_2) . Hence there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq f^{-1}(\nu)$. Consequently, f is somewhat pairwise fuzzy pre-irresolute continuous. □

Theorem 2.8. *Let $(X_1, \tau_1, \tau_2), (X_2, \omega_1, \omega_2), (Y_1, \eta_1, \eta_2), (Y_2, \sigma_1, \sigma_2)$ be fuzzy bitopological spaces. Let (X_1, τ_1, τ_2) be product related to $(X_2, \omega_1, \omega_2)$ and let (Y_1, η_1, η_2) be product related to $(Y_2, \sigma_1, \sigma_2)$. If $f_1 : (X_1, \tau_1, \tau_2) \to ?Y_1, \eta_1, \eta_2)$ and $f_2 : (X_2, \omega_1, \omega_2) \to (Y_2, \sigma_1, \sigma_2)$ is a somewhat pairwise fuzzy pre-irresolute continuous mappings, then the product $f_1 \times f_2 : (X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2) \to (Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2)$ is also somewhat pairwise fuzzy pre-irresolute continuous.*

Proof. Let $\lambda = \bigvee_{i,j} (\mu_i \times \nu_j)$ be η_i-fuzzy preopen or σ_j-fuzzy preopen set on $(Y_1, \eta_1, \eta_2) \times (Y_2, \sigma_1, \sigma_2)$ where $\mu_i \neq 0_{Y_1}$ is η_i-fuzzy preopen set and $\nu_j \neq 0_{Y_2}$ is σ_j-fuzzy preopen set on (Y_1, η_1, η_2) and $(Y_2, \sigma_1, \sigma_2)$ respectively. Then $(f_1 \times$

$f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is somewhat pairwise fuzzy pre-irresolute continuous, there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. And, since f_2 is somewhat pairwise fuzzy pre-irresolute continuous, there exists a ω_1-fuzzy preopen or ω_2-fuzzy preopen set $\gamma_j \neq 0_{X_2}$ such that $\gamma_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now $\delta_i \times \gamma_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \gamma_j \neq 0_{X_1 \times X_2}$ is a δ_i-fuzzy preopen or ν_j-fuzzy preopen set on $(X_1 \times X_2)$. Hence $\bigvee_{i,j}(\delta_i \times \gamma_j) \neq 0_{X_1 \times X_2}$ is a τ_i-fuzzy preopen or ω_j-fuzzy preopen set on $(X_1, \tau_1, \tau_2) \times (X_2, \omega_1, \omega_2)$ such that $\bigvee_{i,j}(\delta_i \times \gamma_j) \leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat pairwise fuzzy pre-irresolute continuous. □

Theorem 2.9. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping. If the graph $g : (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$ of f is a somewhat pairwise fuzzy pre-irresolute continuous mapping, then f is also somewhat pairwise fuzzy pre-irresolute continuous.*

Proof. Let ν be an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2). Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is somewhat pairwise fuzzy pre-irresolute continuous and $1 \times \nu$ is a τ_i-fuzzy preopen or η_j-fuzzy preopen set on $(X, \tau_1, \tau_2) \times (Y, \eta_1, \eta_2)$, there exists a τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2) such that $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X$. Therefore, f is somewhat pairwise fuzzy pre-irresolute continuous. □

3. Somewhat pairwise fuzzy irresolute preopen mappings

In this section, I introduce a somewhat pairwise fuzzy irresolute preopen mapping which are stronger than a somewhat pairwise fuzzy preopen mapping. And we characterize a somewhat pairwise fuzzy irreolute preopen mapping.

Definition 3.1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy irresolute preopen if $f(\mu)$ is an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2) for any τ_1-fuzzy preopen or τ_2-fuzzy preopen set μ on (X, τ_1, τ_2).

Definition 3.2. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy irresolute preopen if there exists an η_1-fuzzy preopen or η_2-fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu) \neq 0_Y$ for any τ_1-fuzzy preopen or τ_2-fuzzy preopen set $\mu \neq 0_X$ on (X, τ_1, τ_2).

From the definitions, it is clear that every pairwise fuzzy irresolute preopen mapping is a somewhat pairwise fuzzy irresolute preopen mapping. And every somewhat pairwise fuzzy irresolute preopen mapping is a pairwise fuzzy preopen mapping. Also, every pairwise fuzzy preopen mapping is a somewhat pairwise fuzzy preopen mapping. But the converses are not true in general as the following examples show.

Example 3.3. Let λ_1,λ_2,λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2,σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then $\lambda_1 = \frac{0.1}{a} + \frac{0.1}{b} + \frac{0.1}{c}$, $\lambda_2 = \frac{0.2}{a} + \frac{0.2}{b} + \frac{0.2}{c}$, $\sigma_1 = \frac{0.0}{x} + \frac{0.1}{y} + \frac{0.0}{z}$, $\sigma_2 = \frac{0.0}{x} + \frac{0.2}{y} + \frac{0.0}{z}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_1\}$,$\tau_2 = \{0_X, 1_X, \lambda_2\}$, $\eta_1 = \{0_Y, 1_Y, \sigma_1\}$,$\eta_2 = \{0_Y, 1_Y, \sigma_2^c\}$. Then (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. Then we have $f(\lambda_1) = \sigma_1$, $\sigma_1 < f(\lambda_2) = \sigma_2$. Since f is somewhat pairwise fuzzy irresolute preopen mapping. But $f(\lambda_2) = \sigma_2$ is not η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2). Hence f is not pairwise fuzzy irresolute preopen mapping.

Example 3.4. Let λ_1,λ_2,λ_3 be fuzzy sets on $X = \{a, b, c\}$ and let σ_1,σ_2,σ_3 be fuzzy sets on $Y = \{x, y, z\}$. Then $\lambda_1 = \frac{0.4}{a} + \frac{0.1}{b} + \frac{0.4}{c}$, $\lambda_2 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\lambda_3 = \frac{1.0}{a} + \frac{0.0}{b} + \frac{1.0}{c}$, $\sigma_1 = \frac{0.0}{x} + \frac{0.1}{y} + \frac{0.0}{z}$, $\sigma_2 = \frac{0.0}{x} + \frac{0.5}{y} + \frac{0.0}{z}$ are defined as follows: Consider $\tau_1 = \{0_X, 1_X, \lambda_1\}$,$\tau_2 = \{0_X, 1_X, \lambda_2\}$, $\eta_1 = \{0_Y, 1_Y, \sigma_1\}$,$\eta_2 = \{0_Y, 1_Y, \sigma_2\}$. Then (X, τ_1, τ_2) and (Y, η_1, η_2) are fuzzy bitopologies and $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ defined by $f(a) = y$, $f(b) = y$, $f(c) = y$. Then we have $f(\lambda_1) = \sigma_1$, $f(\lambda_2) = \sigma_2$ and $f(\lambda_2) = 0_Y$ are η_2-fuzzy preopen set on (Y, η_1, η_2). Since f is pairwise fuzzy preopen mapping. But λ_3 is a τ_1-fuzzy preopen set on (X, τ_1, τ_2) and $f(\lambda_3) = 0_Y$. Hence f is not somewhat pairwise fuzzy irresolute preopen mapping.

Example 3.5. Let λ_1,λ_2,λ_3,λ_4,λ_5 be fuzzy sets on $I = [0, 1]$ with

$$\lambda_1 = 0.1, 0 \le x \le 1,$$
$$\lambda_2 = 0.3, 0 \le x \le 1,$$
$$\lambda_3 = 0.7, 0 \le x \le 1,$$
$$\lambda_4 = 0.8, 0 \le x \le 1,$$

Let $\tau_1 = \{0_I, 1_I, \lambda_3\}$, $\tau_2 = \{0_I, 1_I, \lambda_4\}$, $\eta_1 = \{0_I, 1_I, \lambda_1\}$ and $\eta_2 = \{0_I, 1_I, \lambda_2\}$. Then (I, τ_1, τ_2) and (I, η_1, η_2) be fuzzy bitopologies on I. Consider an identity mapping $f : (I, \tau_1, \tau_2) \to (I, \eta_1, \eta_2)$ defined by $f(x) = x, 0 \le x \le 1$. We have $\lambda_2 < f(\lambda_3) = \lambda_3$, $\lambda_2 < f(\lambda_4) = \lambda_4$.Since λ_2 is an η_2-fuzzy preopen set on (I, η_1, η_2) ,f is somewhat pairwise fuzzy preopen. But $f(\lambda_3) = \lambda_3$ is not an η_1-fuzzy preopen or η_2-fuzzy preopen set on (I, τ_1, τ_2).Hence f is not a pairwise fuzzy preopen mapping.

Theorem 3.6. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a bijection. Then the following are equivalent:*
(1) f is somewhat pairwise fuzzy irresolute preopen.
(2) If μ is a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set on (X, τ_1, τ_2) such that $f(\mu) \neq 1_Y$, then there exists an η_1-fuzzy preclosed or η_2-fuzzy preclosed set $\nu \neq 1_Y$ on (Y, η_1, η_2) such that $f(\mu) < \nu$.

Proof. (1) $\Rightarrow$ (2): Let μ be a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set on (X, τ_1, τ_2) such that $f(\mu) \neq 1_Y$. Since f is bijective and μ^c is a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2), $f(\mu^c) = (f(\mu))^c \neq 0_Y$. And, since f is somewhat pairwise fuzzy irresolute preopen mapping, there exists an η_1-fuzzy preopen or η_2-fuzzy preopen set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \neq 1_Y$ and ν is an η_1-fuzzy preclosed or η_2-fuzzy preclosed set on (Y, η_1, η_2).

(2) $\Rightarrow$ (1): Let μ be a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2) such that $f(\mu) \neq 0_Y$. Then μ^c is a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set on (X, τ_1, τ_2) and $f(\mu^c) \neq 1_Y$. Hence there exists an η_1-fuzzy preclosed or η_2-fuzzy preclosed set $\nu \neq 1_Y$ on (Y, η_1, η_2) such that $f(\mu^c) < \nu$. Since f is bijective, $f(\mu^c) = (f(\mu))^c < \nu$. Hence $\nu^c < f(\mu)$ and $\nu^c \neq 0_X$ is an η_1-fuzzy preopen or η_2-fuzzy preopen set on (Y, η_1, η_2). Therefore, f is somewhat pairwise fuzzy irresolute preopen. □

Theorem 3.7. *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a surjection. Then the following are equivalent:*
(1) f is somewhat pairwise fuzzy irresolute preopen.
(2) If ν is a pairwise predense fuzzy set on (Y, η_1, η_2), then $f^{-1}(\nu)$ is a pairwise predense fuzzy set on (X, τ_1, τ_2).

Proof. (1) $\Rightarrow$ (2): Let ν be a pairwise predense fuzzy set on (Y, η_1, η_2). Suppose $f^{-1}(\nu)$ is not pairwise predense fuzzy set on (X, τ_1, τ_2). Then there exists a τ_1-fuzzy preclosed or τ_2-fuzzy preclosed set μ on (X, τ_1, τ_2) such that $f^{-1}(\nu) < \mu < 1$. Since f is somewhat pairwise fuzzy irreolute preopen and μ^c is a τ_1-fuzzy preopen or τ_2-fuzzy preopen set on (X, τ_1, τ_2), there exists an η_1-fuzzy preopen or η_2-fuzzy preopen set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta \leq f(Int\mu^c) \leq f(\mu^c)$. Since f is surjective, $\delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c$. Thus there exists an η_1-fuzzy preclosed or η_2-fuzzy preclosed set δ^c on (Y, η_1, η_2) such that $\nu < \delta^c < 1$. This is a contradiction. Hence $f^{-1}(\nu)$ is pairwise predense fuzzy set on (X, τ_1, τ_2).

(2) $\Rightarrow$ (1): Let μ be a τ_1-fuzzy open or τ_2-fuzzy open set on (X, τ_1, τ_2) and $f(\mu) \neq 0_Y$. Suppose there exists no η_1-fuzzy preopen or η_2-fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu)$. Then $(f(\mu))^c$ is an η_1-fuzzy set or η_2-fuzzy set δ on (Y, η_1, η_2) such that there exists no η_1-fuzzy preclosed or η_2-fuzzy preclosed set δ on (Y, η_1, η_2) with $(f(\mu))^c < \delta < 1$. This means that $(f(\mu))^c$ is pairwise predense fuzzy set on (Y, η_1, η_2) . Thus $f^{-1}((f(\mu))^c)$ is pairwise predense fuzzy set on (X, τ_1, τ_2). But $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$. This is a contradiction to the fact that $f^{-1}(f(\nu))^c$ is pairwise predense fuzzy set on (X, τ_1, τ_2). Hence there exists an η_1-fuzzy preopen or η_1-fuzzy preopen set $\nu \neq 0_Y$ on (Y, η_1, η_2) such that $\nu \leq f(\mu)$. Therefore, f is somewhat pairwise fuzzy irresolute preopen. □

References

1. K.K. Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. **82** (1981), 14-32.
2. A.S. Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems **44** (1991), 303-308.
3. C.L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182-190.

4. A. Kandil and M.E. El-Shafee, *Biproximities and fuzzy bitopological spaces*, Simon Stevin **63** (1989), 45-66.
5. Karl R. Gentry and Hughes B. Hoyle, III, *Somewhat continuous functions*, Czech. Math. Journal **21** (1971), 5-12.
6. M.N. Mukherjee and S.P. Sinha, *Irresolute and almost open functions between fuzzy topological spaces*, Fuzzy Sets and Systems **29** (1989), 381-388.
7. J.H. Park and B.H. Park, *Fuzzy pre-irresolute mappings*, Pusan Kyongnam Math. J. **10** (1994), 303-312.
8. A. Swaminathan and S. Sudhakar, *Somewhat pairwise fuzzy precontinuous mappings*, accepted in Thai Journal of Mathematics.
9. G. Thangaraj and G. Balasubramanian, *On somewhat fuzzy continuous functions*, J. Fuzzy Math. **11** (2003), 725-736.
10. G. Thangaraj and G. Balasubramanian, *On somewhat fuzzy semi-continuous functions*, Kybernetika **137** (2001), 165-170.
11. M.K. Uma, E. Roja and G. Balasubramanian, *On somewhat pairwise fuzzy continuous functions*, East Asian Math. J. **23** (2007), 83-101.
12. Young Bin Im, Joo Sung Lee and Yung Duk Cho, *Somewhat fuzzy pre-irresolute continuous mappings*, J. Appl. Math. & Informatics **30** (2012), 1077-1084.
13. L.A. Zadeh, *Fuzzy sets*, Inform. And Control **8** (1965), 338-353.

A. Swaminathan received M.Sc., from Bharathidasan University, Tiruchirapalli and Ph.D., at Annamalai University, Chidambaram. Since 2007 he has been at Annamalai University. His research interests include Topology and Fuzzy Topology.

Department of Mathematics(FEAT), Annamalai University, Chidambaram, Tamilnadu, India.
e-mail: asnathanway@gmail.com