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ON FUZZY $k-$IDEALS, $k-$FUZZY IDEALS AND FUZZY $2-$PRIME IDEALS IN $\Gamma-$SEMIRINGS

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ABSTRACT. The notion of $\Gamma-$semiring was introduced by M. Murali Krishna Rao [8] as a generalization of $\Gamma-$ring as well as of semiring. In this paper fuzzy $k-$ideals, $k-$fuzzy ideals and fuzzy-2$-$prime ideals in $\Gamma-$semirings have been introduced and study the properties related to them. Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M with $|Im(\mu)| = 2$ and $\mu(0) = 1$. Then we establish that M_μ is a $2-$prime ideal of $\Gamma-$semiring M if and only if μ is a fuzzy prime ideal of $\Gamma-$semiring M.

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1. Introduction

Semiring is one of the universal algebras which is a generalization not only of ring but also of distributive lattice. As an universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, *i.e.*, $a(b + c) = ab + ac,\ (a + b)c = ac + bc,$ for all $a, b, c \in S$. Semiring was first introduced by H. S. Vandiver [14] in 1934. Though semiring is a generalization of ring, ideals of semiring do not coincide with ring ideals. For example an ideal of semiring need not be the kernel of some semiring homomorphism. To solve this problem Henriksen [4] defined $k-$ideals and Iijuka [5] defined $h-$ideals in semirings to obtain analogues of ring results for semirings. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. Semiring is very useful for solving problems in applied mathematics and information science because semiring provides an algebraic frame work for modeling. Semirings play an important role in studying matrices and determinants.

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The notion of Γ-ring was introduced by Nobusawa [10] as a generalization of ring in 1964. Sen [12] introduced the notion of Γ- semigroup in 1981. The notion of ternary algebraic system was introduced by Lehmer [6] in 1932, Lister [7] introduced ternary ring. Dutta & Kar [3] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. In 1995, Murali Krishna Rao [8] introduced the notion of Γ- semiring which is a generalization of Γ- ring, ternary semiring and semiring.

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was first introduced by Zadeh [15]. The concept of fuzzy subgroup was introduced by Rosenfeld [11]. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc . Uncertain data in many important applications in the areas such as economics, engineering, environment, medical sciences and business management could be caused by data randomness, information incompleteness, limitations of measuring instrument, delayed data updates etc. Swamy and Swamy [13] studied fuzzy prime ideal of rings. Dheena et al. [2] studied fuzzy 2−prime ideal in semirings. Murali krishna Rao [9] studied fuzzy soft Γ−semirings and fuzzy soft $k-$ideals over Γ−semirings. In this paper fuzzy $k-$ideals, $k-$fuzzy ideals and fuzzy-2−prime ideals in Γ−semirings have been introduced and study the properties related to them. We study the homomorphic images and pre-images of fuzzy $k-$ideals and $k-$fuzzy ideals of Γ−semirings. Let μ be a fuzzy $k-$ideal of Γ−semiring M with $|Im(\mu)| = 2$ and $\mu(0) = 1$. Then we establish that M_μ is a 2−prime ideal of Γ−semiring M if and only if μ is a fuzzy prime ideal of Γ−semiring M.

2. Preliminary Results

In this section we will recall some of the fundamental concepts and definitions, these are necessary for this paper.

Definition 2.1 ([1])**.** A set R together with two associative binary operations called addition and multiplication (denoted by + and · respectively) will be called a semiring provided

(i). Addition is a commutative operation.
(ii). There exists $0 \in R$ such that $x+0 = x$ and $x \cdot 0 = 0\cdot = 0$ for each $x \in R$.
(iii). Multiplication distributes over addition both from the left and from the right.

Definition 2.2 ([8])**.** Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a Γ−semiring, if there exists a mapping $M \times \Gamma \times M \to M$ written as (x, α, y) as $x\alpha y$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

(i). $x\alpha(y + z) = x\alpha y + x\alpha z$ and $(x + y)\alpha z = x\alpha z + y\alpha z$
(ii). $x(\alpha + \beta)y = x\alpha y + x\beta y$
(iii). $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ as the usual semiring multiplication. A $\Gamma-$semiring M is said to have zero element if there exists element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$ for all $x \in M$. A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. An additive subsemigroup I of $\Gamma-$semiring M is said to be a left (right) ideal of $\Gamma-$semiring M if $M\Gamma I \subseteq I$ ($I\Gamma M \subseteq I$). If I is both left and right ideal then I is called an ideal of $\Gamma-$semiring M. An ideal I of $\Gamma-$semiring M is called a $k-$ideal, if $b \in M, a + b \in I$ and $a \in I$ then $b \in I$. A function $f : R \to S$ where R and S are $\Gamma-$semirings is said to be a $\Gamma-$semiring homomorphism if $f(a + b) = f(a) + f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in R, \alpha \in \Gamma$.

Let S be a nonempty set. A mapping $f : S \to [0, 1]$ is called a fuzzy subset of S. Let A be a subset of S. The characteristic function χ_A of A is a fuzzy subset of S is defined by $\chi_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$. Let f be a fuzzy subset of a nonempty set S for $t \in [0, 1]$, the set $f_t = \{x \in S \mid f(x) \geq t\}$ is called level subset of S with respect to f. For any $x \in M$ and $t \in [0, 1]$, we define the fuzzy point x_t as $x_t(y) = \begin{cases} t, & \text{if } y = x \\ 0, & \text{if } y \neq x \end{cases}$. If x_t is a fuzzy point and μ is any fuzzy subset of $\Gamma-$semiring M and $x_t \subseteq \mu$ then we write $x_t \in \mu$. $x_t \in \mu$ if and only if $x \in \mu_t$. Let μ be a fuzzy subset of R. Then the image of μ denoted by $Im(\mu) = \{\mu(r) \mid r \in R\}$ and $|Im(\mu)|$ denotes the cardinality of $Im(\mu)$. A fuzzy subset $\mu : S \to [0, 1]$ is a nonempty, if μ is not the constant function. For any two fuzzy subsets λ and μ of $S, \lambda \subseteq \mu$ means $\lambda(a) \leq \mu(a)$ for all $a \in S$. Let S and R be two nonempty sets and $\psi : S \to R$ be any function. A fuzzy subset f of S is called a ψ invariant if $\psi(x) = \psi(y) \Rightarrow f(x) = f(y)$ for all $x, y \in S$. Let f and g be two fuzzy subsets of $\Gamma-$semiring M. the product fg is defined by

$$fg(x) = \begin{cases} \sup\limits_{x = y\alpha z} \min\{f(y), g(z)\}, \\ 0, \quad \text{otherwise.} \end{cases} ; \text{ for } x, y, z \in M, \alpha \in \Gamma.$$

3. MAIN RESULTS

In this section fuzzy-2$-$prime ideals, fuzzy $k-$ideals and $k-$fuzzy ideals in $\Gamma-$semirings have been introduced and study the properties related to them. Throughout this paper M is a commutative $\Gamma-$semiring with zero element.

Definition 3.1. A fuzzy subset f of $\Gamma-$semiring M is called a fuzzy ideal of $\Gamma-$semiring M, if for all $x, y \in M, \alpha \in \Gamma$

(i). $f(x + y) \geq \min\{f(x), f(y)\}$ (ii). $f(x\alpha y) \geq \max\{f(x), f(y)\}$.

Definition 3.2. A fuzzy ideal f of $\Gamma-$semiring M is said to be fuzzy $k-$ideal of $\Gamma-$semiring M, if $f(x) \geq \min\{f(x + y), f(y)\}$ for all $x, y \in M$.

This definition can also be written as a fuzzy ideal f of $\Gamma-$semiring M is said to be fuzzy $k-$ideal of $\Gamma-$semiring M if $f(x+y) \geq \lambda,\ f(y) \geq \lambda \Rightarrow f(x) \geq \lambda$ for all $x, y \in M, \lambda \in [0,1]$.

Definition 3.3. A fuzzy ideal f of $\Gamma-$semiring M is said to be $k-$fuzzy ideal of $\Gamma-$semiring M if $f(x+y) = f(0)$ and $f(y) = f(0) \Rightarrow f(x) = f(0)$ for all $x, y \in M$.

Example 3.4. Let M be the additive commutative semigroup of all non negative integers and Γ be the additive commutative semigroup of all natural numbers. Ternary operation is defined by $a\alpha b =$ product of a, α, b, for all $a, b \in M$ and $\alpha \in \Gamma$. Then M is a $\Gamma-$semiring.

Let μ be a fuzzy subset of $\Gamma-$semiring M, defined by

$$\mu(x) = \begin{cases} 0.3, & \text{if } x \text{ is odd} \\ 0.5, & \text{if } x \text{ is even} \\ 0, & \text{if } x = 0 \end{cases} ;\ \lambda(x) = \begin{cases} 1, & \text{if } x > 7 \\ 0.5, & \text{if } 5 < x < 7 \\ 0, & \text{if } 0 \leq x < 5\ . \end{cases}$$

Then μ is a fuzzy $k-$ideal of $\Gamma-$semiring M and μ is also $k-$fuzzy ideal of $\Gamma-$semiring M. λ is a fuzzy ideal but not a fuzzy $k-$ideal.

Definition 3.5. The ideal generated by $a, a \in M$ is defined as the smallest ideal of $\Gamma-$semiring M which contains a and it is denoted by (a). The $k-$ideal generated by a, $a \in M$ is defined as the smallest $k-$ideal of $\Gamma-$semiring M which contains a and it is denoted by $(a)_k$

Definition 3.6. If A is an ideal of $\Gamma-$semiring M then $\bar{A} = \{a \in M \mid a + x \in A,$ for some $x \in A\}$ is called a $k-$closure of A.

Definition 3.7. Let M be a $\Gamma-$semiring and f be a fuzzy ideal of $\Gamma-$semiring M. The $k-$fuzzy closure $\overline{f}$ of f is defined by

$$\overline{f}(x) = \begin{cases} f(x) & \text{if , } x \notin \overline{f}_{f(0)} \\ f(0) & \text{if, } x \in \overline{f}_{f(0)}. \end{cases}$$

Definition 3.8. An ideal P of $\Gamma-$semiring M is called a prime ideal if for any ideals A, B of $\Gamma-$semiring M, $A\Gamma B \subseteq P$ then $A \subseteq P$ or $B \subseteq P$

Definition 3.9. An ideal P of $\Gamma-$semiring M is called a $2-$prime ideal if for any $k-$ideals A, B of $\Gamma-$semiring M, $A\Gamma B \subseteq P$ then $A \subseteq P$ or $B \subseteq P$

Definition 3.10. A fuzzy ideal μ of $\Gamma-$semiring M is called a fuzzy prime ideal if for any fuzzy ideals f, g of $\Gamma-$semiring M, $fg \subseteq \mu$ then $f \subseteq \mu$, $g \subseteq \mu$.

Definition 3.11. A fuzzy ideal μ of $\Gamma-$semiring M is called a fuzzy $2-$prime ideal if for any fuzzy $k-$ideals f, g of $\Gamma-$semiring M, $fg \subseteq \mu$ then $f \subseteq \mu$ or $g \subseteq \mu$.

Definition 3.12. Let M and N be $\Gamma-$semirings, $\phi : M \to N$ be a homomorphism of $\Gamma-$semiring M and f be a subset of $\Gamma-$semiring M. We define a fuzzy subset $\phi(f)$ of $\Gamma-$semiring N by

$$\phi(f)(y) = \begin{cases} \sup\limits_{x\in\phi^{-1}(y)} f(x), & \text{if } \phi^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

We call $\phi(f)$ is the image of f under ϕ.

Definition 3.13. Let $\phi : M \to N$ be a homomorphism of $\Gamma-$semiring and f be a fuzzy subset of $\Gamma-$semiring N. We define a fuzzy subset $\phi^{-1}(f)$ of $\Gamma-$semiring M by $\phi^{-1}(f)(x) = f(\phi(x))$ for all $x \in M$, we call $\phi^{-1}(f)$ is a pre-image of f.

We now state the following lemmas, proofs are which are analogous to the corresponding lemmas in semirings [1] similar, so we omit the proofs.

Lemma 3.14. *If A is an ideal of $\Gamma-$semiring M then $\overline{A}$ is a $k-$ideal of $\Gamma-$semiring M.*

Lemma 3.15. *If A is an ideal of $\Gamma-$semiring M then A is a $k-$ideal if and only if $A = \overline{A}$*

Lemma 3.16. *Let f_1 and f_2 be any two fuzzy subsets of $\Gamma-$semiring M. If $f_1 \subseteq f, f_2 \subseteq g$ then $f_1f_2 \subseteq fg$ for any fuzzy subsets f and g of $\Gamma-$semiring M.*

Lemma 3.17. *If μ is a fuzzy prime ideal of $\Gamma-$semiring M then μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M.*

Lemma 3.18. *If μ is a fuzzy ideal of $\Gamma-$semiring M and $a \in M$ then $\mu(x) > \mu(a)$ for all $x \in (a)$.*

Lemma 3.19. *Let f, g be any fuzzy ideals of $\Gamma-$semiring M and μ be a fuzzy $k-$ideal of $\Gamma-$semiring M. If $fg \subseteq \mu$ then $\overline{f}\ \overline{g} \subseteq \mu$.*

Lemma 3.20. *Let μ be a fuzzy subset of $\Gamma-$semiring M. Then μ is a fuzzy prime ideal if and only if $M_\mu = \{x \in M \mid \mu(x) = \mu(0)\}$ is a prime ideal of $\Gamma-$semiring M.*

Lemma 3.21. *Let I be an ideal of $\Gamma-$semiring M and $\alpha < \beta \neq 0$. If the fuzzy subset μ of $\Gamma-$semiring M is defined by $\mu(x) = \begin{cases} \beta, & \text{if } x \in I \\ \alpha, & \text{otherwise} \end{cases}$. Then μ is a fuzzy ideal of $\Gamma-$semiring M.*

Theorem 3.22. *Let f be a fuzzy ideal of $\Gamma-$semiring M. Then $f(x) \leq f(0)$ for all $x \in M$.*

Proof. Let $x \in M, \alpha \in \Gamma, f(0) = f(0\alpha x) \geq f(x)$. Therefore $f(x) \leq f(0)$, for all $x \in M$. □

Theorem 3.23. *Let f and g be fuzzy ideals of $\Gamma-$semiring M. If f and g are fuzzy ideals of $\Gamma-$semiring M then $f \cap g$ is a fuzzy ideal of $\Gamma-$semiring M.*

Proof. Let f and g be fuzzy ideals of $\Gamma-$semiring M, $x, y \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} f \cap g(x\alpha y) &= \min\{f(x\alpha y), g(x\alpha y)\} \\ &\geq \min\{\max\{f(x), f(y)\},\ \max\{g(x), g(y)\}\} \\ &\geq \max\{\min\{f(x), g(x)\}, \min\{f(y), g(y)\}\} \\ &= \max\{f \cap g(x), f \cap g(y)\}. \\ f \cap g(x+y) &= \min\{f(x+y), g(x+y)\} \\ &\geq \min\{\min\{f(x), f(y)\}, \min\{g(x), g(y)\}\} \\ &= \min\{\min\{f(x), g(x)\}, \min\{f(y), g(y)\}\} \\ &= \min\{f \cap g(x), f \cap g(y)\}. \end{aligned}$$

Hence $f \cap g$ is a fuzzy ideal of $\Gamma-$semiring M. □

Theorem 3.24. *Let f and g be fuzzy $k-$ideals of $\Gamma-$semiring M. Then $f \cap g$ is a fuzzy $k-$ideal of $\Gamma-$semiring M.*

Proof. Let f and g are fuzzy $k-$ideals of $\Gamma-$semiring M. By Theorem [3.23], $f \cap g$ is a fuzzy ideal of $\Gamma-$semiring M. Let $x, y \in M$. We have

$$\begin{aligned} f \cap g(x) &= \min\{f(x), g(x)\} \\ &\geq \min\{\min\{f(x+y), f(y)\},\ \min\{g(x+y), g(y)\}\} \\ &= \min\{\min\{f(x+y), g(x+y\},\ \min\{f(y), g(y)\}\} \\ &= \min\{f \cap g(x+y), f \cap g(y)\}. \end{aligned}$$

Hence $f \cap g$ is a fuzzy $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.25. *Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M and $a \in M$. Then $\mu(x) > \mu(a)$ for all $x \in (a)_k$.*

Proof. Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M and $a \in M$. If $x \in (a)_k$ then $x + y \in (a)$ for some $y \in (a)$, by Lemma 3.18, $\mu(x+y) > \mu(a)$ and $\mu(y) > \mu(a)$.

$$\begin{aligned} &\text{We have } \mu(x) \geq \min\{\mu(x+y), \mu(y)\},\ \text{since } \mu \text{ is a fuzzy } k-\text{ideal} \\ &\Rightarrow \mu(x) > \min\{\mu(a), \mu(a)\} \\ &\Rightarrow \mu(x) > \mu(a). \end{aligned}$$

Hence the theorem. □

Theorem 3.26. *Let μ be a fuzzy ideal of $\Gamma-$semiring M. If $x_r y_t \subseteq \mu \Rightarrow x_r \subseteq \mu$ or $y_t \subseteq \mu$ then μ is a fuzzy prime ideal of $\Gamma-$semiring M.*

Proof. Let σ and θ be fuzzy ideals of $\Gamma-$semiring M and $\sigma\theta \subseteq \mu$.
Suppose $\sigma \nsubseteq \mu$. Then there exists $x \in M \ni \sigma(x) \geq \mu(x)$.
Let $\sigma(x) = a,\ y \in M$ and $\theta(y) = b$. If $z = x\gamma y$, for some $\gamma \in \Gamma$ then $(x_a y_b)(z) = \min\{a, b\}$. Therefore

$$\mu(z) = \mu(x\gamma y) \geq \sigma\theta(x\gamma y)$$

$$\geq \min\{\sigma(x), \theta(y)\}$$
$$= \min\{a, b\} = (x_a y_b)(z).$$

Therefore

$$\begin{aligned} x_a y_b \subseteq \mu \Rightarrow & x_a \subseteq \mu \ \text{ or } y_b \subseteq \mu \\ \Rightarrow & a \leq \mu(x) \ \text{ or } b \leq \mu(y), \\ \Rightarrow & \theta(y) = b \leq \mu(y), \ \text{ since } a \nleq \mu(x), \\ \Rightarrow & \theta \subseteq \mu. \end{aligned}$$

Hence μ is a fuzzy prime ideal of $\Gamma-$semiring M. □

Theorem 3.27. *Let I be an ideal of $\Gamma-$semiring $M, \alpha \in [0,1)$. If μ be a fuzzy subset of $\Gamma-$semiring M defined by $\mu(x) = \begin{cases} 1, & if x \in I \\ \alpha, & otherwise \end{cases}$. If I is a $2-$prime ideal then μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M.*

Proof. Let I be a $2-$prime ideal of $\Gamma-$semiring M. Clearly μ is a non constant fuzzy ideal of $\Gamma-$semiring M. Let σ, θ be fuzzy ideals of $\Gamma-$semiring M such that $\sigma\theta \subseteq \mu, \sigma \nsubseteq \mu$ and $\theta \nsubseteq \mu$.
Then there exist $x, y \in M$ such that

$$\begin{aligned} \sigma(x) > \mu(x), \ \theta(y) > \mu(y) & \Rightarrow \mu(x) = \mu(y) = \alpha \\ & \Rightarrow x, y \notin I. \\ & \Rightarrow (x)_k \gamma (y)_k \nsubseteq I, \end{aligned}$$

since I is a $2-$prime ideal of $\Gamma-$semiring $M, \gamma \in \Gamma$. Therefore there exist $c \in (x)_k, d \in (y)_k$ such that $c\gamma d \notin I$.
Let $a = c\gamma d, \mu(a) = \mu(c\gamma d) = \alpha, \ \sigma\theta(a) \leq \mu(a) = \alpha$. Now

$$\begin{aligned} \sigma\theta(a) & \geq \min\{\theta(c), \theta(d)\} \\ & \geq \min\{\theta(x), \theta(y)\} \\ & \geq \min\{\mu(x), \mu(y)\} \\ & = \alpha = \mu(a). \end{aligned}$$

Which is a contradiction to the fact that $\sigma\theta \subseteq \mu$.
Hence μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M. □

Corollary 3.28. *Let I be an ideal of $\Gamma-$semiring M. If I is a $2-$prime ideal of $\Gamma-$semiring M then the characteristic function χ_1 is a fuzzy $2-$prime ideal of $\Gamma-$semiring M.*

Lemma 3.29. *If I is an ideal of $\Gamma-$semiring M then χ_I is a fuzzy ideal of $\Gamma-$semiring M.*

Proof. Suppose I is an ideal of $\Gamma-$semiring M and $a, b \in I$. Then $a+b \in I, a\alpha b \in I$ for all $\alpha \in \Gamma$.

$$\chi_I(a+b) = 1 \geq \min\{\chi_I(a), \chi_I(b)\}, \chi_I(a\alpha b) = 1 \geq \max\{\chi_I(a), \chi_I(b)\}.$$

Suppose $a \in I, b \notin I, \alpha \in \Gamma$ and $a+b \notin I$. Then

$$\chi_I(a+b) = 0 \geq \min\{\chi_I(a), \chi_I(b)\}, \chi_I(a\alpha b) \ \geq \max\{\chi_I(a), \chi_I(b)\} = 1.$$

Hence χ_I is a fuzzy ideal of $\Gamma-$semiring M. □

Lemma 3.30. *If $t \in [0,1]$ such that $f_t \neq \phi, f_t$ is a $k-$ideal of $\Gamma-$semiring M then f is a $k-$fuzzy ideal of $\Gamma-$semiring M.*

Proof. Let M be a $\Gamma-$semiring, $f(x+y) = f(0)$ and $f(y) = f(0)$. Then

$$\begin{aligned} &x + y \in f_{f(0)} \text{ and } y \in f_{f(0)} \\ \Rightarrow &x \in f_{f(0)}, \text{ since } f_{f(0)} \text{ is a } k-\text{ideal.} \\ \Rightarrow &f(x) \geq f(0). \end{aligned}$$

We have $f(x) \leq f(0)$. Therefore $f(x) = f(0)$. Hence f is a $k-$fuzzy ideal of $\Gamma-$semiring M. □

Lemma 3.31. *Let w be a fixed element of $\Gamma-$semiring M. If μ is a fuzzy $k-$ideal of $\Gamma-$semiring M then $\mu^w = \{x \in \mu \mid \mu(x) \geq \mu(w)\}$ is a $k-$ideal of $\Gamma-$semiring M.*

Proof. Let w be a fixed element of $\Gamma-$semiring M. and $x, y \in \mu^w$. Then

$$\mu(x) \geq \mu(w) \text{ and } \mu(y) \geq \mu(w), \ \mu(x+y) \geq \min\{\mu(x), \mu(y)\} \geq \mu(w).$$

Let $x \in \mu^w, \ r \in M$ and $\alpha \in \Gamma$. Then

$$\mu(x\alpha r) \geq \max\{\mu(x), \mu(r)\} \geq \mu(w) \Rightarrow x\alpha r \in \mu^w.$$

Therefore μ^w is an ideal of M.
Let $x, x+y \in \mu^w$. Then

$$\begin{aligned} &\mu(x) \geq \mu(w), \mu(x+y) \geq \mu(w) \\ &\Rightarrow \min\{\mu(x), \mu(x+y)\} \geq \mu(w) \\ &\Rightarrow \mu(y) \geq \mu(w) \end{aligned}$$

Therefore $y \in \mu^w$. Hence μ^w is a $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.32. *Let M be a $\Gamma-$semiring and $I \subseteq M$. Then I is a $k-$ideal of $\Gamma-$semiring M if and only if χ_I is a $k-$fuzzy ideal of $\Gamma-$semiring M.*

Proof. Let I be a $k-$ideal of $\Gamma-$semiring M. Then χ_I is a fuzzy ideal by Lemma 3.29. Let $x, y \in M$.

$$\begin{aligned} &\chi_I(x+y) = \chi_I(0), \ \chi_I(y) = \chi_I(0) \\ &\Rightarrow \chi_I(0) = 1 \Rightarrow \chi_I(x+y) = 1. \\ &\Rightarrow x + y \in I. \end{aligned}$$

Now $\chi_I(y) = \chi_I(0) = 1 \Rightarrow y \in I$.
Now $x + y \in I$ and $y \in I$. Therefore $x \in I$, since I is a $k-$ideal. Then $\chi_I(x) = 1 = \chi_I(0)$ and hence χ_I is a $k-$fuzzy ideal of $\Gamma-$semiring M.

Conversely, suppose that χ_I is a $k-$fuzzy ideal of $\Gamma-$semiring $M \Rightarrow I$ is an ideal of $\Gamma-$semiring $M \Rightarrow \chi_I(0) = 1$, since $0 \in I$. Suppose $x + y$ and $y \in I$. Then $\chi_I(x + y) = \chi_I(y) = \chi_I(0) \Rightarrow \chi_I(x) = 0$, since χ_I is a $k-$fuzzy ideal of $\Gamma-$semiring M. Therefore $x \in I$. Hence I is a $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.33. *f is a fuzzy ideal of $\Gamma-$semiring M if and only if for any $t \in [0, 1]$ such that $f_t \neq \varphi$, f_t is an ideal of $\Gamma-$semiring M.*

Proof. Let f_t be an ideal of $\Gamma-$semiring M, $x, y \in M$ and $t = \min\{f(x), f(y)\}$. Then

$$\begin{aligned} f(x) \geq t, f(y) \geq t &\Rightarrow x, y \in f_t \\ &\Rightarrow x + y \in f_t, \\ &\Rightarrow f(x + y) \geq t = \min\{f(x), f(y)\}. \end{aligned}$$

Let $s = \max\{f(x), f(y)\}$, $\alpha \in \Gamma$. Then

$$\begin{aligned} f(x) = s \text{ or } f(y) = s &\Rightarrow x \in f_s \text{ or } y \in f_s \\ &\Rightarrow x\alpha y \in f_s \\ &\Rightarrow f(x\alpha y) \geq s = \max\{f(x), f(y)\}. \end{aligned}$$

Hence f is a fuzzy ideal of $\Gamma-$semiring M.

Conversely suppose that f is a fuzzy ideal of $\Gamma-$semiring M and $t \in [0, 1]$ so that

$$\begin{aligned} x, y \in f_t &\Rightarrow f(x) \geq t,\ f(y) \geq t, \\ &\Rightarrow f(x + y) \geq \min\{f(x), f(y)\} \geq t \\ &\Rightarrow x + y \in f_t. \end{aligned}$$

Let $x \in f_t, y \in M \mid f_t$ then $f(x) \geq t$, $\alpha \in \Gamma$. Then

$$f(x\alpha y) \geq \max\{f(x), f(y)\} \geq f(x) \geq t \Rightarrow x\alpha y \in f_t.$$

Hence f_t is an ideal of $\Gamma-$semiring M. □

Theorem 3.34. *A fuzzy subset μ of $\Gamma-$semiring M is a fuzzy $k-$ideal of $\Gamma-$semiring M if and only if μ_t is a $k-$ideal of $\Gamma-$semiring M for any $t \in [0, 1], \mu_t \neq \phi$.*

Proof. Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M. Clearly μ_t is an ideal of $\Gamma-$semiring M, by Theorem 3.33. Suppose $a, a + x \in \mu_t$, $x \in M \Rightarrow \mu(a) \geq t, \mu(a + x) \geq t$. Since μ is a fuzzy $k-$ideal, we have

$$\mu(x) \geq \min\{\mu(a + x), \mu(a)\} \Rightarrow \mu(x) \geq t \Rightarrow x \in \mu_t.$$

Hence μ_t is a $k-$ideal of $\Gamma-$semiring M.

Conversely, assume that μ_t is a $k-$ideal of $\Gamma-$semiring M for any $t \in [0,1]$ with $\mu_t \neq \phi$. Let $\mu(a) = t_1, \mu(x+a) = t_2$ and $t = \min\{t_1, t_2\}$.
Then $a \in \mu_t$ and $a + x \in \mu_t$ for some $x \in M$, since μ_t is a $k-$ideal, we have $x \in \mu_t, \mu(x) \geq \min\{\mu(x+a), \mu(a)\}$. Therefore μ is a fuzzy $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.35. *Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M. Then two $k-$ideals μ_s, μ_t of $\Gamma-$semiring M (with $s < t, s, t \in [0,1]$) are equal if and only if there is no $x \in M$ such that $s \leq \mu(x) < t$.*

Proof. Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M and two $k-$ideals μ_s, μ_t of $\Gamma-$semiring M. Suppose $s < t \in [0,1]$ and $\mu_s = \mu_t$. If there exists $x \in M$ such that $s < \mu(x) < t$. Then μ_t is a proper subset of μ_s, which is a contradiction. Therefore they are equal.

Conversely, suppose that there is no $x \in M$ such that $s < \mu(x) < t$. Then we have $s < t \Rightarrow \mu_s \subset \mu_t$. If $x \in \mu_s$ then $\mu(x) \geq s$ and no $\mu(x) \geq t$. Since $\mu(x) \nless t, \Rightarrow x \in \mu_t$. Hence $\mu_s = \mu_t$. □

Theorem 3.36. *Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M. If $Im(\mu) = \{t_1, t_2, \cdots, t_n\}$ where $t_1 < t_2 < t_3 \cdots < t_n$ then family of $k-$ideals $\mu_{t_i}, i = 1, 2, 3, \cdots, n$, is a collection of all level ideals of $\Gamma-$semiring M.*

Proof. If $t \in [0,1]$ with $t < t_1$ then $\mu_{t_1} \subset \mu_t$. Since $\mu_{t_1} = M, \mu_t = M$. If $t \in [0,1]$ with $t_i < t < t_{i+1}$ then there is no $x \in M$ such that $t_i < \mu(x) < t_{i+1}$. It follows $\mu_{t_i} \subseteq \mu_{t_{i+2}}$. Hence family of k-ideals $\mu_{t_i}, i = 1, 2, \cdots, n$, is collection of all level ideals of $\Gamma-$semiring M. □

Theorem 3.37. *Let M be a $\Gamma-$semiring, $t \in [0,1]$ and f_t be a $k-$ideal of $\Gamma-$semiring M. Then f is a $k-$fuzzy ideal of $\Gamma-$semiring M.*

Proof. Let f_t be a $k-$ideal of $\Gamma-$semiring M, $x, y \in M$. Suppose $f(x+y) = f(0), f(y) = f(0)$ and $t = f(0)$. Then $x + y \in f_t, y \in f_t$.
Since f_t is a $k-$ideal of $\Gamma-$semiring M, we have $x \in f_t \Rightarrow f(x) \geq t = f(0)$, we have $f(x) \leq f(0)$ for all $x \in M$. Therefore $f(x) = f(0)$. Hence f_t is a $k-$fuzzy ideal of $\Gamma-$semiring M. □

Theorem 3.38. *If I be a $k-$ideal of $\Gamma-$semiring M then there exists a fuzzy $k-$ideal μ of $\Gamma-$semiring M such that $\mu_t = I$ for some $t \in (0,1]$.*

Proof. We define a fuzzy subset μ of $\Gamma-$semiring M by

$$\mu(x) = \begin{cases} t, & \text{if } x \in I \\ 0, & \text{if } x \notin I, \text{ for some } t \in (0,1]. \end{cases}$$

Clearly $\mu_t = I$. If $s \in (0,1]$ then μ_s is a $k-$ideal of $\Gamma-$semiring M. Hence, by Theorem 3.34, fuzzy subset μ is a fuzzy $k-$ideal. □

Theorem 3.39. *Let M and N be $\Gamma-$semirings, $\phi : M \to N$ be a homomorphism and f be a ϕ invariant fuzzy ideal of $\Gamma-$semiring M. If $x = \phi(a)$ then $\phi(f)(x) = f(a), a \in M$.*

Proof. Let M and N be $\Gamma-$semirings, $a \in M, x \in N$, $x = \phi(a)$. Then $a \in \phi^{-1}(x)$ and $t \in \phi^{-1}(x)$. Therefore $\phi(t) = x = \phi(a)$, since f is ϕ invariant.

$$f(t) = f(a) \Rightarrow \phi(f)(x) = \sup_{t\in\phi^{-1}(x)} \{f(t)\} = f(a).$$

Hence $\phi(f)(x) = f(a)$. □

Theorem 3.40. *Let M and N be $\Gamma-$semirings and $\phi : M \to N$ be an onto homomorphism. If f is a ϕ invariant fuzzy ideal of $\Gamma-$semiring M then $\phi(f)$ is a fuzzy ideal of $\Gamma-$semiring N.*

Proof. Let M and N be $\Gamma-$semirings, $\phi : M \to N$ be an onto homomorphism and $x, y \in N$. Then there exist $a, b \in M$ such that $\phi(a) = x, \phi(b) = y$.

$$\begin{aligned} &\Rightarrow \phi(a + b) = x + y \\ &\Rightarrow \phi(f)(x + y) = f(a + b) \geq \min\{f(a), f(b)\} = \min\{\phi(f)(x), \phi(f)(y)\}, \\ \phi(f)(x\alpha y) &= f(a)\alpha f(b) \geq \max\{f(a), f(b)\} = \max\{\phi(f)(x), \phi(f)(y)\}. \end{aligned}$$

Hence $\phi(f)$ is a fuzzy ideal of $\Gamma-$semiring N. □

Theorem 3.41. *Let M and N be $\Gamma-$semirings, $\phi : M \to N$ be an onto homomorphism and f be a ϕ invariant fuzzy ideal of $\Gamma-$semiring M. Then f is a $k-$fuzzy ideal if and only if $\phi(f)$ is a $k-$fuzzy ideal of $\Gamma-$semiring N.*

Proof. Let M and N be $\Gamma-$semirings, $\phi : M \to N$ be an onto homomorphism and f be a ϕ invariant fuzzy ideal of $\Gamma-$semiring M. By Theorem 3.40, $\phi(f)$ is a fuzzy ideal of $\Gamma-$semiring N. Let $x, y \in N$. Then there exist $a, b \in M$ such that $\phi(a) = x, \phi(b) = y$.

$$\begin{aligned} &\Rightarrow \phi(a + b) = x + y \Rightarrow \phi(f)(x + y) = f(a + b) \\ &\Rightarrow \phi(f)(x + y) = \phi(f)(0) \Rightarrow \phi(f)(y) = \phi(f)(0). \end{aligned}$$

Since $\phi(a + b) = \phi(a) + \phi(b) = x + y$,

$$\phi(f)(y) = f(b) \Rightarrow f(a + b) = f(0) \text{ and } f(b) = f(0).$$

Since f is a $k-$ideal of $\Gamma-$semiring M,

$$\begin{aligned} f(a) = f(0) &\Rightarrow \phi(f)(x) = f(a) = f(0) = \phi(f)(0) \\ &\Rightarrow \phi(f) \text{ is a } k - \text{fuzzy ideal of } \Gamma - \text{semiring } M. \end{aligned}$$

Suppose $\phi(f)$ is a $k-$fuzzy ideal of $\Gamma-$semiring N. Then $\phi^{-1}(\phi(f))$ is a $k-$fuzzy ideal of $\Gamma-$semiring M. Therefore f is a $k-$fuzzy ideal of $\Gamma-$semiring M. □

Theorem 3.42. *Let $\phi : M \to N$ be an onto homomorphism of $\Gamma-$semirings and f be the fuzzy $k-$ideal of of $\Gamma-$semiring N. Then $\phi^{-1}(f)$ is a fuzzy $k-$ideal of $\Gamma-$semiring M.*

Proof. Let $\phi : M \to N$ be an onto homomorphism of $\Gamma-$semirings and f be the fuzzy $k-$ideal of of $\Gamma-$semiring N. By Definition 3.13, $\phi^{-1}f(x+y) = f\{\phi(x+y)\}$ for all $x, y \in M$

$$\begin{aligned}\phi^{-1}(f)(x+y) &= f\{\phi(x)+\phi(y)\}\\ &\geq \min\{f(\phi(x)), f(\phi(y))\}\\ &= \min\{\phi^{-1}(f)(x), \phi^{-1}(f)(y)\}.\\ \phi^{-1}(f)(x\alpha y) &= f(\phi(x\alpha y))\\ &= f(\phi(x)\alpha\phi(y))\\ &\geq \max\{f(\phi(x)), f(\phi(y))\}\\ &= \max\{\phi^{-1}(f)(x), \phi^{-1}(f)(y)\}.\end{aligned}$$

Let $a, b \in N$. Then there exist $x, y \in M$ such that $\phi(x) = a, \phi(y) = b$.

$$\begin{aligned}\phi^{-1}(f)(x) = f(\phi(x)) &= f(a)\\ &\min\{f(a+b), f(b)\}\\ &= \min\{f(\phi(x+y)), f(\phi(y))\}\\ &= \min\{\phi^{-1}f(x+y), \phi^{-1}(f(x))\}.\end{aligned}$$

Hence $\phi^{-1}(f)$ is a fuzzy $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.43. *Let $\phi : M \to N$ be an onto homomorphism of $\Gamma-$semiring and f be the fuzzy ideal of $\Gamma-$semiring N. If $\phi^{-1}(f)$ is a $k-$fuzzy ideal of $\Gamma-$semiring M then f is a $k-$fuzzy ideal of $\Gamma-$semiring N.*

Proof. Let $\phi : M \to N$ be an onto homomorphism of $\Gamma-$semiring, f be the fuzzy ideal of N and $\phi^{-1}(f)$ be a $k-$fuzzy ideal of $\Gamma-$semiring M. Suppose $x, y \in N$, $f(x+y) = f(0)$ and $f(y) = f(0)$. Since ϕ is onto, there exist $a, b \in M$ such that $\phi(a) = x, \phi(b) = y$,

$$\begin{aligned}f[\phi(a)+\phi(b)] = f[\phi(0)] &\Rightarrow f[\phi(a+b)] = f[\phi(0)]\\ &\Rightarrow \phi^{-1}(f)[a+b] = \phi^{-1}(f)(0) \quad \text{and}\\ \phi^{-1}(f)(b) = \phi^{-1}(f)(0) &\Rightarrow \phi^{-1}(f)(0)\\ &\Rightarrow f(\phi(a)) = f(\phi(0))\\ &\Rightarrow f(x) = f(0).\end{aligned}$$

Therefore f is a $k-$fuzzy ideal of $\Gamma-$semiring N. □

Theorem 3.44. *Let M be a $\Gamma-$semiring . Then f is a $k-$fuzzy ideal of $\Gamma-$semiring M if and only if $f = \overline{f}$.*

Proof. Let M be a $\Gamma-$semiring, f be a $k-$fuzzy ideal of$M$ and $x \in \overline{f}_{f(0)}$. Then $\overline{f}(x) = f(0)$, since $x \in \overline{f}_{f(0)}$ then there exist $a, b \in f_{f(0)}$ such that $a + x = b$ $\Rightarrow f(a) = f(0), f(a+x) = f(b) = f(0)$ then $f(x) = f(0)$. Therefore $\overline{f}(x) = f(x)$.

Conversely suppose that $f = \overline{f}$ and $f_{f(0)} = \overline{f}_{f(0)}$. Then $f_{f(0)}$ is a $k-$ideal of $\Gamma-$semiring M. Let $x, y \in M$. Then

$$\begin{aligned} f(x+y) = f(0) = f(y) &\Rightarrow x+y \in f_{f(0)},\, y \in f_{f(0)} \\ &\Rightarrow x \in f_{f(0)} \\ &\Rightarrow f(x) = f(0). \end{aligned}$$

Hence f is a $k-$fuzzy ideal of $\Gamma-$semiring M. □

Let M be a $\Gamma-$semiring and μ be a fuzzy subset of $\Gamma-$semiring M. The set $\{x \mid \mu(x) = \mu(0)\}$ is denoted by M_μ.

Theorem 3.45. *Let μ be a fuzzy subset of $\Gamma-$semiring M, $|Im(\mu)| = 2$ and $\mu(0) = 1$. If M_μ is an ideal of $\Gamma-$semiring M then μ is a fuzzy $k-$ideal of $\Gamma-$semiring M.*

Proof. Let μ be a fuzzy subset of $\Gamma-$semiring M, $|Im(\mu)| = 2$, $\mu(0) = 1$, $Im(\mu) = \{t, 1\}$, $t < 1$ and $x, y \in M$. If $x, y \in M_\mu$ then

$$x + y \in M_\mu \Rightarrow \mu(x+y) = \mu(0), \mu(x+y) \geq \min\{\mu(x), \mu(y)\} = \mu(0).$$

Let $x \in M_\mu, y \notin M_\mu$ and $x+y \in M_\mu$. Then $\mu(x+y) \geq \min\{\mu(x), \mu(y)\} = \mu(0)$. Suppose $x+y \notin M_\mu$. Then $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$. If $x \notin M_\mu$ and $y \notin M_\mu$ then $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$. Hence $\mu(x+y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in M$.

Let $x \in M_\mu, y \in M, \alpha \in \Gamma$. Then $\mu(x) = \mu(0), \mu(x\alpha y) \geq \max\{\mu(x), \mu(y)\}$. Let $x, y \in M, x \notin M_\mu$ and $y \notin M_\mu$. Then

$$\begin{aligned} x + y \notin M_\mu &\Rightarrow \mu(x) = t,\ \mu(x+y) = t, \mu(y) = t \\ &\Rightarrow \mu(x) = \min\{\mu(x+y), \mu(y)\}. \end{aligned}$$

Let $x, y \in M, x \in M_\mu$ and $y \in M_\mu$. Then

$$x + y \in M_\mu, 1 = \mu(x) \geq \min\{\mu(x+y), \mu(y)\} = \min\{1, 1\} = 1.$$

Let $x, y \in M, x \notin M_\mu$ and $y \in M_\mu$. Then $x+y \notin M_\mu$, $\mu(x) \geq t$. Hence μ is a fuzzy $k-$ideal of $\Gamma-$semiring M. □

Theorem 3.46. *Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M. Then M_μ is a $k-$ideal of $\Gamma-$semiring M.*

Proof. Let μ be a fuzzy $k-$ideal of $\Gamma-$semiring M, $x, y \in M_\mu$ and $\alpha \in \Gamma$. Then

$$\mu(x) = \mu(0) = \mu(y),\ \mu(x+y) \geq \min\{\mu(0), \mu(0)\} = \mu(0) \Rightarrow \mu(x+y)\} \geq \mu(0).$$

We have $\mu(0) \geq \mu(x+y)$. Therefore $\mu(0) = \mu(x+y) \Rightarrow x+y \in M_\mu$.
$\mu(x\alpha y) \geq \max\{\mu(x), \mu(y)\} = \max\{\mu(0), \mu(y)\} = \mu(0)$, we have

$$\begin{aligned} \mu(0) \geq \mu(x\alpha y) &\Rightarrow \mu(x\alpha y) = \mu(0) \\ &\Rightarrow x\alpha y \in M_\mu. \end{aligned}$$

Therefore M_μ is an ideal of $\Gamma-$semiring M.
Let $x+y, x \in M_\mu$. Then $\mu(x+y) = \mu(0), \mu(x) = \mu(0)$ since μ is a fuzzy $k-$ideal of

$\Gamma-$semiring M. We have $\mu(y) \geq \min\{\mu(x+y), \mu(x)\} = \min\{\mu(0), \mu(0)\} = \mu(0)$, $\mu(y) \geq \mu(0)$ and $\mu(y) \leq \mu(0) \Rightarrow \mu(y) = \mu(0) \Rightarrow y \in M_\mu$.
Hence M_μ. is a $k-$ideal of $\Gamma-$semiring M. □

Proof of the following theorem follows from Theorems 3.45 and 3.46.

Theorem 3.47. *Let M be a $\Gamma-$semiring and μ be a fuzzy subset of $\Gamma-$semiring M with $|Im(\mu)| = 2$ and $\mu(0) = 1$. Then M_μ is a $k-$ideal of $\Gamma-$semiring M if and only if μ is a fuzzy $k-$ideal of $\Gamma-$semiring M.*

Theorem 3.48. *If μ is a fuzzy $2-$prime ideal and a fuzzy $k-$ideal of $\Gamma-$semiring M then μ is a fuzzy prime ideal of $\Gamma-$semiring M.*

Proof. Suppose μ is a fuzzy $2-$prime ideal and fuzzy $k-$ideal of $\Gamma-$semiring M, f and g be fuzzy ideals of $\Gamma-$semiring M such that $fg \subseteq \mu$. As μ is a fuzzy $k-$ideal, by Lemma [3.19], $\overline{f}\overline{g} \subseteq \mu \Rightarrow \overline{f} \subseteq \mu$ or $\overline{g} \subseteq \mu$ since μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M and $\overline{f}, \overline{g}$ are fuzzy $k-$ideals $\Rightarrow f \subseteq \overline{f} \subseteq \mu$ or $g \subseteq \overline{g} \subseteq \mu \Rightarrow f \subseteq \mu$ or $g \subseteq \mu$. Hence μ is a fuzzy prime ideal of $\Gamma-$semiring M. □

Theorem 3.49. *Let μ be any fuzzy subset of $\Gamma-$semiring M, $|Im(\mu)| = 2$ and $\mu(0) = 1$. If M_μ is a $2-$prime ideal of $\Gamma-$semiring M then μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M.*

Proof. Let μ be any fuzzy subset of $\Gamma-$semiring M, $|Im(\mu)| = 2, \mu(0) = 1$. Suppose M_μ is a $2-$prime ideal of $\Gamma-$semiring M and $Im(\mu) = \{t, 1\},\ t < 1$. Clearly μ is a $k-$fuzzy ideal of $\Gamma-$semiring M. Let f and g be fuzzy $k-$ideals of $\Gamma-$semiring M such that $fg \subseteq \mu$. Suppose that $f \nsubseteq \mu$ and $g \nsubseteq \mu$. Then there exist $x, y \in M$ $f(x) > \mu(x)$ and $g(y) > \mu(y)$. Clearly $\mu(x) = t = \mu(y) \Rightarrow x, y \notin M_\mu$. Since M_μ is a $2-$prime ideal of $\Gamma-$semiring M, there exist $x_1 \in (x)_k$ and $y_1 \in (y)_k$ such that $x_1\alpha y_1 \notin M_\mu$, $\alpha \in \Gamma$. By Lemma 3.25, we have

$$\begin{aligned} f(x_1) > f(x),\ g(y_1) > g(y) &\Rightarrow f(x_1) > \mu(x) \text{ and } g(y_1) > \mu(y) \\ &\Rightarrow \mu(x) = \mu(y) = \mu(x_1\alpha y_1) = t. \end{aligned}$$

Now

$$\begin{aligned} fg(x_1\alpha y_1) &= \sup_{x_1\alpha y_1 = a\gamma b}\{\min\{f(a), g(b)\}\} \\ &\geq \min\{f(x_1), g(y_1) \\ &> \min\{\mu(x), \mu(y)\} = t \\ &= \mu(x_1\alpha y_1). \end{aligned}$$

Therefore $fg \nsubseteq \mu$, which is a contradiction. Hence μ is a fuzzy $2-$prime ideal of $\Gamma-$semiring M. □

The proof of the following theorem follows from Theorems 3.48, 3.49 and Lemma 3.20.

Theorem 3.50. *Let M be a $\Gamma-$semiring, μ be a fuzzy $k-$ideal of $\Gamma-$semiring M with $|Im(\mu)| = 2$ and $\mu(0) = 1$. Then M_μ is a $2-$prime ideal of $\Gamma-$semiring M if and only if μ is a fuzzy prime ideal of $\Gamma-$semiring M.*

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