J. Appl. Math. & Informatics Vol. **34**(2016), No. 5 - 6, pp. 397 - 404
http://dx.doi.org/10.14317/jami.2016.397

QUANTUM CODES FROM CYCLIC CODES OVER $F_4 + vF_4$

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Abstract. In this work, a method is given to construct quantum codes from cyclic codes over $F_4 + vF_4$ which will be denoted as R throughout the paper, where $v^2 = v$ and a Gray map is defined between R and F_4^2, where F_4 is the field with 4 elements. Some optimal quantum code parameters and others will be presented at the end of the paper.

AMS Mathematics Subject Classification : 94B05, 94B65.
Key words and phrases : Cyclic Codes, Quantum Codes, Gray Map.

1. Introduction

Error correcting quantum codes are being used to deal with quantum noises such as decoherence which arises after by inventing quantum computers. The first discovery of error correcting quantum codes is made by Shor in [1]. After that discovery,a method is given for constructing quantum codes from widely known classical error correcting codes by Calderbank et al. in [2]. Recently, another widely known class of error correcting codes, cyclic codes over the field $\mathbb{F}_q$, are being used in the purpose of obtaining quantum error correcting codes, where q is a power of prime. In [6], a technique is given to construct for quantum error correcting codes over the finite ring $\mathbb{F}_2+u\mathbb{F}_2$, $u^2 = 0$ by Qian. Kai and Zhu in [7] gave a technique to construct quantum error correcting codes from cyclic codes with length n , where n is an odd integer, over finite chain ring $\mathbb{F}_4 + u\mathbb{F}_4$ with $u^2 = 0$. Moreover, Qian [5] gave an original method which uses cyclic codes over the finite ring $\mathbb{F}_2 + v\mathbb{F}_2$ with $v^2 = v$, to construct quantum error correcting codes. Motivated by this study, M.Ashraf [8] describes a similar construction method for quantum codes which is obtained from cyclic codes over $\mathbb{F}_3 + v\mathbb{F}_3$, where $v^2 = 1$. Making use of the cyclic codes over finite ring $R = \mathbb{F}_4+v\mathbb{F}_4$,where $v^2 = v$, we obtain quantum codes over $\mathbb{F}_4$. The structure of cyclic codes over $\mathbb{F}_4+v\mathbb{F}_4$ was given by A. Bayram in [9]. It was shown that this ring is isomorphic to $\mathbb{F}_4 \times \mathbb{F}_4$.

Received February 11, 2016. Revised April 11, 2016. Accepted April 23, 2016. *Corresponding author.

In this paper, a method is described for obtaining self-orthogonal codes over $\mathbb{F}_4$ as Gray map images of linear and cyclic codes over the ring $R = \mathbb{F}_4 + v\mathbb{F}_4$, $v^2 = v$. A sufficient and necessary condition for cyclic codes over R which contains its dual is given. At the end, parameters of associated quantum codes will be presented, some of them are optimal based on the table in [10].

2. Preliminaries

Let $R = \mathbb{F}_4 + v\mathbb{F}_4 = \{0, 1, w, w^2, v, 1+v, w+v, w^2+v, wv, 1+wv, w+wv, w^2+wv, w^2v, 1+w^2v, w+w^2v, w^2+w^2v\}$ where $v^2 = v$, $\mathbb{F}_4 = \{0,1,w,w^2\}$ where $w^2 = w+1$. R is a finite non-chain ring with 16 elements. The ring R has two maximal ideals $(v) = \{0, v, vw, v(w+1)\}$ and $(1+v) = \{0, v+1, (v+1)w, (v+1)(w+1)\}$. If we apply the Chinese Remainder Theorem, it is obtained that $R \cong \mathbb{F}_4[v]/(v-1) \oplus \mathbb{F}_4[v]/(v) \cong \mathbb{F}_4 \oplus \mathbb{F}_4 \cong (v) \oplus (1+v)$, which means that, every element of R can be expressed uniquely as $x + vy = v(x+y) + (v+1)x$, for some $x, y \in \mathbb{F}_4$. If $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n) \in R^n$, then the Hamming weight of γ is the number of nonzero coordinates in γ and is represented by $hw(\gamma)$. $d(\gamma, \delta) = hw(\gamma - \delta)$ gives the Hamming distance between γ and δ in R.

A linear code C over R of length n is an R submodule of R^n. If $\gamma \in C$, then we say that γ is a codeword. C is a cyclic code, if $\gamma = (\gamma_0, \gamma_1, \ldots, \gamma_{n-1}) \in C$, then $(\gamma_{n-1}, \gamma_0, \gamma_1, \ldots, \gamma_{n-2}) \in C$. Let $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ and $\delta = (\delta_1, \delta_2, \ldots, \delta_n)$ be two elements of R^n. Then the Euclidean inner product of γ and δ in R^n is defined as

$$\gamma.\delta = \gamma_1\delta_1 + \gamma_2\delta_2 + \ldots + \gamma_n\delta_n.$$

If C is a linear code of length n over R, then its dual code is defined as

$$C^\perp = \{\delta \in R^n \mid \langle \gamma, \delta \rangle = 0 \text{ for all } \gamma \in C\}.$$

We say that a code C is a self-orthogonal code, if it satisfies the condition $C \subseteq C^\perp$ and a self dual code if $C = C^\perp$.

If A_1 and A_2 are two linear codes then their direct sum and cartesian product is denoted by $A_1 \otimes A_2 = \{(a_1, a_2) | a_1 \in A_1, a_2 \in A_2\}$ and $A_1 \oplus A_2 = \{a_1 + a_2 | a_1 \in A_1, a_2 \in A_2\}$ respectively.

It can be seen directly that by suitable permutation of coordinates, the generator matrix of a nonzero linear code C over R can be considered in the form:

$$G = \begin{pmatrix} I_{k_1} & (1+v)B_1 & vA_1 & (1+v)A_2 + vB_2 & (1+v)A_3 + vB_3 \\ 0 & vI_{k_2} & 0 & vA_4 & 0 \\ 0 & 0 & (1+v)I_{k_3} & 0 & (1+v)B_4 \end{pmatrix},$$

where A_k and B_ℓ are matrices over $\mathbb{F}_4$ for $k = 1, 2, 3, 4$ and $\ell = 1, 2, 3, 4$. The next result is defined in [9]. Let

$$C_1 = \{a + b \in \mathbb{F}_4^n \mid (a+b)v + a(v+1) \in C, \text{ for some } a, b \in \mathbb{F}_4^n\}$$
$$C_2 = \{a \in \mathbb{F}_4^n \mid (a+b)v + a(v+1) \in C, \text{ for some } b \in \mathbb{F}_4^n\}.$$

Clearly C_1 and C_2 are linear codes over $\mathbb{F}_4$. Thus, $C = vC_1 \oplus (1+v)C_2$ and $|C| = 16^{k_1} 4^{k_2} 4^{k_3}$.

Theorem 2.1 (**CSS Construction**). *Let C has the parameter $[n, k, d]$ and $\bar{C}$ has the parameter $[n, \bar{k}, \bar{d}]$. If $\bar{C}^{\perp} \subseteq C$, then an $[n, k+\bar{k}-n, min\{d, \bar{d}\}]$ quantum code can be obtained. In particular, if we take $C^{\perp} \subseteq C$, then a quantum code that has the parameters $[n, 2k-n, d]$ can be obtained.*

3. Gray Images of Linear Codes over $\mathbb{F}_4 + v\mathbb{F}_4$

We know that every element of $R = \mathbb{F}_4 + v\mathbb{F}_4$ can be expressed as $a + vb$, where $a, b \in \mathbb{F}_4$.The Gray map ψ from R to $\mathbb{F}_4^2$ is defined as $\psi(a+vb) = (a+b, a)$. It is easy to show that ψ is a linear map. An extension of the Gray map ψ can be made in obvious way from R^n to $\mathbb{F}_4^{2n}$.

Proposition 3.1. *The Gray map ψ is a map that preserves the distance from (R^n, Lee distance) to ($\mathbb{F}_4^{2n}$, Hamming distance).*

Proposition 3.2. *Let $C = vC_1 \oplus (1+v)C_2$ be a linear code of length n over R and C_i be $[n, k_i, d(C_i)]$ linear codes for $i = 1, 2$. Then $\psi(C)$ is a $[2n, k_1 + k_2, min\{d(C_1), d(C_2)\}]$ code over $\mathbb{F}_4$.*

Proposition 3.3. *Let C be a code of length n over R. Assume C is a self-orthogonal code. Then $\psi(C)$ is self-orthogonal.*

Proof. Let $c_1 = \gamma_1 + v\delta_1$ and $c_2 = \gamma_2 + v\delta_2 \in C$, where $\gamma_1, \gamma_2, \delta_1, \delta_2 \in \mathbb{F}_4^n$. Then Euclidean inner product of c_1 and c_2, is

$$c_1 c_2 = \gamma_1\gamma_2 + (\gamma_1\delta_2 + \gamma_2\delta_1 + \delta_1\delta_2)v.$$

We know that C is self-orthogonal,then we have $\gamma_1\gamma_2 = \gamma_1\delta_2 + \gamma_2\delta_1 + \delta_1\delta_2 = 0$. On the other hand, $\psi(c_1)\psi(c_2) = \gamma_1\gamma_2 + \gamma_1\delta 2 + \gamma_2\delta_1 + \delta_1\delta_2 + \gamma_1\gamma_2 = 0$. Hence $\psi(C)$ is self-orthogonal. □

4. Quantum Codes from Cyclic Codes over R

In this section, we use cyclic codes over R of arbitrary length n to obtain self-orthogonal codes over $\mathbb{F}_4$. By using these self-orthogonal codes, the corresponding quantum code parameters will be determined.

The following lemmas are given in [4] and [9].

Lemma 4.1 ([9]). *Assume that $C = vC_1 \oplus (1+v)C_2$ be a linear code over $R = \mathbb{F}_4 + v\mathbb{F}_4$. So C is a cyclic code over $R = \mathbb{F}_4 + v\mathbb{F}_4$ if and only if C_1 and C_2 are both cyclic codes over $\mathbb{F}_4$.*

Lemma 4.2 ([9]). *If $C = vC_1 \oplus (1+v)C_2$ is a cyclic code of length n over $R = \mathbb{F}_4 + v\mathbb{F}_4$. Then $C = \langle vg_1(x), (1+v)g_2(x)\rangle$ and $|C| = 4^{2n-deg(g_1(x))-deg(g_2(x))}$, where $g_1(x), g_2(x)$ are the generator polynomials of C_1, C_2, respectively.*

Lemma 4.3 ([4]). *Let C be a cyclic code of length n over R. Then there exists unique polynomial $g(x)$ such that $C = \langle g(x)\rangle$,and $g(x)|x^n - 1$, where $g(x) = vg_1(x) + (1+v)g_2(x)$.*

Lemma 4.4 ([4]). *Let $C = vC_1 \oplus (1+v)C_2$ is a cyclic code of length n over R. Then $C^{\perp} = \langle (vh_1^*(x) + (1+v)h_2^*(x)\rangle$, and $|C^{\perp}| = 4^{deg(g_1(x))+deg(g_2(x))}$, where $h_i^*(x)$ are the reciprocal polynomials of $h_i(x)$, that is, $h_i(x) = x^n - 1/g_i(x), h_i^*(x) = x^{deg(h_i(x))}h_i(x)^{-1}$ for $i = 1, 2$.*

Now, we give a lemma that describes the necessary and sufficient condition for a cyclic code to be self-orthogonal.

Lemma 4.5. *Let C be a cyclic code with generator polynomial $g(x)$, then C contains its dual code if and only if*

$$x^n - 1 \equiv 0 \mod (g(x)g^*(x)),$$

where $g^*(x)$ is the reciprocal polynomial of $g(x)$.

Now, a sufficient and necessary condition for cyclic code over R that contains its dual is given.

Theorem 4.6. *Let $C = \langle g(x)\rangle$ is a cyclic code of length n over R, where $g(x) = vg_1(x) + (1+v)g_2(x)$. Then $C^{\perp} \subseteq C$ if and only if*

$$x^n - 1 \equiv 0 \mod (g_i(x)g_i^*(x)) \quad for \quad i = 1, 2.$$

Proof. Let $C = \langle g(x)\rangle = vC_1 \oplus (1+v)C_2$ be a cyclic code of length n over R, then $C = \langle vg_1(x), (1+v)g_2(x)\rangle$, $C_1 = \langle g_1(x)\rangle$ and $C_2 = \langle g_2(x)\rangle$. If

$$x^n - 1 \equiv 0 \mod (g_i(x)g_i^*(x))) \quad for \quad i = 1, 2.$$

Then

$$C_1^{\perp} \subseteq C_1, C_2^{\perp} \subseteq C_2.$$

This implies that

$$vC_1^{\perp} \subseteq vC_1, (1+v)C_2^{\perp} \subseteq (1+v)C_2.$$

Therefore

$$vC_1^{\perp} \oplus (1+v)C_2^{\perp} \subseteq vC_1 \oplus (1+v)C_2.$$

Hence

$$\langle vh_1^*(x), (1+v)h_2^*(x)\rangle \subseteq \langle vg_1(x), (1+v)g_2(x)\rangle,$$

that is, $C^{\perp} \subseteq C$.

On the other hand if $C^{\perp} \subseteq C$, then $vC_1^{\perp} \oplus (1+v)C_2^{\perp} \subseteq vC_1 \oplus (1+v)C_2$. Since C_1 (resp.C_2) be the code over $\mathbb{F}_4$ such that vC_1 (resp $(1+v)C_2$) is equal to C *modv* (resp. C *mod*$(1+v)$), $C_1^{\perp} \subseteq C_1, C_2^{\perp} \subseteq C_2$. Therefore,

$$x^n - 1 \equiv 0 \mod (g_i(x)g_i^*(x)), \quad for \quad i = 1, 2.$$

□

Corollary 4.7. *Let $C = vC_1 \oplus (1+v)C_2$ be a cyclic code of length n over $R = \mathbb{F}_4 + v\mathbb{F}_4$. Then we have $C^{\perp} \subseteq C$ if and only if*

$$C_1^{\perp} \subseteq C_1, C_2^{\perp} \subseteq C_2.$$

By Theorem 2.1 and Corollary 4.7, we can obtain quantum codes.

Theorem 4.8. *Let $C = vC_1 \oplus (1+v)C_2$ is a cyclic code of arbitrary length n over R and let the parameters of $\psi(C)$ be $[2n, k, d_L]$ where d_L is defined as the minimum Lee weight of C. If*

$$C_1^{\perp} \subseteq C_1, C_2^{\perp} \subseteq C_2$$

Then we have $C^{\perp} \subseteq C$ and thus we get a quantum error-correcting code which has the parameters $[[2n, 2k-2n, d_L]]$ where d_L.

5. Examples

Example 5.1. Let $R = \mathbb{F}_4 + v\mathbb{F}_4$ and $n = 10$. Then $x^{10} - 1 = (x+1)^2(x^2+wx+1)^2(x^2+w^2x+1)^2$ in $\mathbb{F}_4$. Let $g(x) = vg_1(x) + (1+v)g_2(x)$ with

$$g_1(x) = g_2(x) = x+1$$
$$g_1^*(x) = g_2^*(x) = x+1$$

and $C =< g(x) >$ be a cyclic code over R. Clearly $x^{10} - 1$ is divisible by $g_i g_i^*$ for $i = 1, 2$. Hence by Corollary 4.7 we have $C^{\perp} \subseteq C$. Then a quantum code with parameters $[[20, 16, 2]]$ is obtained, which is optimal based on [10]. Other optimal codes are presented in Table 1.

Example 5.2. Let $R = \mathbb{F}_4 + v\mathbb{F}_4$ and $n = 36$. Then $x^{36} - 1 = (x+1)^4(x+w)^4(x+w^2)^4(x^3+w)^4(x^3+w^2)^4$ in $\mathbb{F}_4$. Let $g(x) = vg_1(x) + (1+v)g_2(x)$ with

$$\begin{aligned}
g_1(x) &= x^9 + w^2x^8 + wx^7 + w^2x^5 + wx^4 + w^2x^2 + wx + 1\\
g_2(x) &= x^{10} + x^7 + w^2x^6 + x^4 + w^2x^3 + w^2\\
g_1^*(x) &= x^9 + wx^8 + w^2x^7 + wx^5 + w^2x^4 + wx^2 + w^2x + 1\\
g_2^*(x) &= w^2x^{10} + w^2x^7 + x^6 + w^2x^4 + x^3 + 1
\end{aligned}$$

and $C =< g(x) >$ be a cyclic code over R. Clearly $x^{36} - 1$ is divisible by $g_i g_i^*$ for $i = 1, 2$. Hence by Corollary 4.7 we have $C^{\perp} \subseteq C$. Then we obtain a quantum code with parameters $[[72, 34, 4]]$.

Example 5.3. Let $R = \mathbb{F}_4 + v\mathbb{F}_4$ and $n = 43$. Then $x^{43} - 1 = (x+1)(x^7+wx^5+x^4+x^3+w^2x^2+1)(x^7+w^2x^5+x^4+x^3+wx^2+1)(x^7+x^6+wx^5+w^2x^2+x+1)(x^7+x^6+w^2x^5+wx^2+x+1)(x^7+wx^6+wx^5+wx^4+w^2x^3+w^2x^2+w^2x+1)(x^7+w^2x^6+w^2x^5+w^2x^4+wx^3+wx^2+wx+1)$ in $\mathbb{F}_4$. Let $g(x) = vg_1(x) + (1+v)g_2(x)$ with

$$\begin{aligned}
g_1(x) &= g_2(x)\\
&= x^{14} + wx^{13} + w^2x^{12} + wx^{11} + w^2x^{10} + x^8 + x^7 + x^6 + wx^4 + w^2x^3 + wx^2 + w^2x + 1\\
g_1^*(x) &= g_2^*(x)\\
&= x^{14} + w^2x^{13} + wx^{12} + w^2x^{11} + wx^{10} + x^8 + x^7 + x^6 + w^2x^4 + wx^3 + w^2x^2 + wx + 1
\end{aligned}$$

and $C =< g(x) >$ be a cyclic code over R. Clearly $x^{43} - 1$ is divisible by $g_i g_i^*$ for $i = 1, 2$. Hence by Corollary 4.7 we have $C^{\perp} \subseteq C$. Then a quantum code with parameters $[[86, 30, 8]]$ is obtained.

Example 5.4. Let $R = \mathbb{F}_4 + v\mathbb{F}_4$ and $n = 57$. Then $x^{57} - 1 = (x+1)(x+w)(x+w^2)(x^9 + x^8 + wx^6 + x^5 + x^4 + w^2x^3 + x + 1)(x^9 + x^8 + w^2x^6 + x^5 + x^4 + x^3 + x + 1)(x^9 + x^8 + wx^6 + wx^5 + w^2x^4 + w^2x^3 + w^2x + 1)(x^9 + wx^8 + w^2x^6 + wx^5 + w^2x^4 + wx^3 + w^2x + 1)(x^9 + w^2x^8 + wx^6 + w^2x^5 + wx^4 + w^2x^3 + wx + 1)(x^9 + w^2x^8 + w^2x^6 + w^2x^5 + wx^4 + wx^3 + wx + 1)$ in $\mathbb{F}_4$. Let $g(x) = vg_1(x) + (1+v)g_2(x)$ with

$$\begin{aligned} g_1(x) &= g_2(x) \\ &= x^{19} + w^2x^{18} + wx^{17} + wx^{14} + x^{10} + w^2x^9 + wx^5 + wx^2 + x + w^2 \\ g_1^*(x) &= g_2^*(x) \\ &= w^2x^{19} + x^{18} + wx^{17} + wx^{14} + w^2x^{10} + x^9 + wx^5 + wx^2 + w^2x + 1 \end{aligned}$$

and $C =< g(x) >$ be a cyclic code over R. Clearly $x^{57} - 1$ is divisible by $g_i g_i^*$ for $i = 1, 2$. Hence by Corollary 4.7 we have $C^{\perp} \subseteq C$. Then a quantum code with parameters $[[114, 38, 6]]$ is obtained.

References

1. P.W. Shor, *Scheme for reducing decoherence in quantum memory*, Phys. Rev. A **52** (1995), 2493-2496.
2. A.R. Calderbank, E.M. Rains, P.M. Shor and N.J.A. Sloane, *Quantum error correction via codes over $GF(4)$*, IEEE Trans. Inf. Theory **44** (1998), 1369-1387.
3. A.M. Steane, *Simple quantum error correcting-codes*, Phys. Rev. A **54** (1996), 4741-4751.
4. S. Zhu, Y. Wang and M. Shi, *Some results on cyclic codes over $F_2 + vF_2$*, IEEE Trans. Inf. Theory **56** (2010), 1680-1684.
5. J. Qian, *Quantum codes from cyclic codes over F_2+vF_2*, Journal of Inform. & computational Science **10** (2013), 1715-1722.
6. J. Qian, W. Ma and W. Gou, *Quantum codes from cyclic codes over finite ring*, Int. J. Quantum Inform. **7** (2009), 1277-1283.
7. X. Kai and S. Zhu, *Quaternary construction of quantum codes from cyclic codes over $F_4 + uF_4$*, Int. J. Quantum Inform. **9** (2011), 92-119.
8. M. Ashraf and G. Mohammad, *Quantum codes from cyclic codes F_3+vF_3*, Int. J. Quantum Inform. **12** (2014), 1450042.
9. A. Bayram, E.S. Oztas and I. Siap, *Codes over $F_4 + vF_4$ and some DNA applications*, Des. Codes. Cryptogr. DOI 10.1007, 2015.
10. M. Grassl, *Online Linear Code Bounds*, Available Online at http://www.codetables.de, 2011.

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TABLE 1. The Parameters of Optimal Quantum Codes

n	**Generator Polynomials**	**Parameters**
4	$g_1 = g_2 = x + 1$	$[[8, 4, 2]]$
6	$g_1 = g_2 = x + w^2$	$[[12, 8, 2]]$
8	$g_1 = g_2 = x + 1$	$[[16, 12, 2]]$
9	$g_1 = g_2 = x + w^2$	$[[18, 14, 2]]$
12	$g_1 = g_2 = x + w^2$	$[[24, 20, 2]]$
14	$g_1 = g_2 = x + 1$	$[[28, 24, 2]]$
15	$g_1 = g_2 = x + w^2$	$[[30, 26, 2]]$
16	$g_1 = g_2 = x + 1$	$[[32, 28, 2]]$
18	$g_1 = g_2 = x + w^2$	$[[36, 32, 2]]$
20	$g_1 = g_2 = x + 1$	$[[40, 36, 2]]$
21	$g_1 = g_2 = x + w^2$	$[[42, 38, 2]]$
22	$g_1 = g_2 = x + 1$	$[[44, 40, 2]]$
24	$g_1 = g_2 = x + w^2$	$[[48, 44, 2]]$
26	$g_1 = g_2 = x + 1$	$[[52, 48, 2]]$
27	$g_1 = g_2 = x + w^2$	$[[54, 50, 2]]$
28	$g_1 = g_2 = x + 1$	$[[56, 52, 2]]$
33	$g_1 = g_2 = w^2x + 1$	$[[66, 62, 2]]$
34	$g_1 = g_2 = x + 1$	$[[68, 64, 2]]$
36	$g_1 = g_2 = w^2x + 1$	$[[72, 68, 2]]$
39	$g_1 = g_2 = w^2x + 1$	$[[78, 74, 2]]$
40	$g_1 = x^2 + w^2x + 1$ $g_2 = x + 1$	$[[80, 74, 2]]$
44	$g_1 = g_2 = x + 1$	$[[88, 84, 2]]$
48	$g_1 = g_2 = x + w^2$	$[[96, 92, 2]]$
50	$g_1 = g_2 = x + 1$	$[[100, 96, 2]]$
52	$g_1 = g_2 = x + 1$	$[[104, 100, 2]]$
54	$g_1 = g_2 = x + w^2$	$[[108, 104, 2]]$
56	$g_1 = g_2 = x + 1$	$[[112, 108, 2]]$
58	$g_1 = g_2 = x + 1$	$[[116, 112, 2]]$

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Table 2. The Parameters of Quantum Codes

n	**Generator Polynomials** $g_1 = g_2$	**Parameters**
11	$x^5 + w^2x^4 + x^3 + x^2 + wx + 1$	$[[22, 2, 5]]$
12	$x^4 + x^3 + w^2x^2 + wx + w$	$[[24, 8, 3]]$
15	$x^3 + x + w$	$[[30, 18, 3]]$
18	$x^5 + wx^3 + w^2x^2 + 1$	$[[36, 16, 3]]$
19	$x^9 + w^2x^8 + w^2x^6 + w^2x^5 + wx^4 + wx^3 + wx + 1$	$[[38, 2, 7]]$
21	$x^7 + wx^6 + x^4 + w^2x^3 + w^2x + w^2$	$[[42, 14, 5]]$
23	$x^{11} + x^{10} + x^6 + x^5 + x^4 + x^2 + 1$	$[[46, 2, 7]]$
28	$x^{10} + x^7 + x^5 + x^2 + x + 1$	$[[56, 16, 4]]$
31	$x^5 + x^4 + x^3 + x^2 + 1$	$[[62, 42, 3]]$
33	$x^{16} + wx^{15} + wx^{13} + wx^{12} + wx^{11} + wx^9$ $+wx^7 + wx^5 + wx^4 + wx^3 + wx + w^2$	$[[66, 2, 10]]$
33	$x^{11} + w^2x^{10} + x^8 + w^2x^7 + wx^6 + x^5$ $+w^2x^4 + wx^3 + w^2x + w$	$[[66, 22, 6]]$
35	$x^{12} + w^2x^{11} + wx^{10} + w^2x^9 + wx^8 +$ $w^2x^7 + wx^5 + w^2x^4 + wx^3 + w^2x^2 + wx + 1$	$[[70, 22, 5]]$
35	$x^6 + w^2x^5 + x^4 + wx^3 + wx^2 + 1$	$[[70, 46, 3]]$
39	$x^7 + wx^5 + w^2x^4 + wx^3 + x^2 + wx + w^2$	$[[78, 50, 3]]$
42	$x^5 + wx^4 + wx^3 + wx^2 + w$	$[[84, 64, 3]]$
42	$x^8 + w^2x^7 + x^5 + w$	$[[84, 52, 4]]$
42	$x^{11} + x^8 + w^2x^6 + wx^4 + x^3 + x^2 + x + w$	$[[84, 40, 5]]$
42	$x^{14} + x^{13} + wx^{12} + wx^{11} + w^2x^{10} + x^9$ $+wx^8 + x^7 + x^6 + wx^5 + x^4 + wx^3 + w^2x^2 + x + w$	$[[84, 28, 6]]$
43	$x^7 + w^2x^6 + w^2x^5 + w^2x^4 + wx^3 + wx^2 + wx + 1$	$[[86, 58, 5]]$
47	$x^{23} + x^{22} + x^{21} + x^{20} + x^{18} + x^{17}$ $+x^{16} + x^{14} + x^{13} + x^{11} + x^{10} + x^9 + x^5 + x^4 + 1$	$[[94, 2, 11]]$
48	$x^{11} + x^8 + wx^3 + w$	$[[96, 52, 4]]$
56	$x^{16} + x^{14} + x^{13} + x^{10} + x^9 + x^8 + x^4 + x^2 + x + 1$	$[[112, 48, 4]]$
57	$x^{27} + x^{26} + w^2x^{25} + x^{24} + wx^{23} + x^{22} + wx^{21}$ $+w^2x^{20} + w^2x^{19} + x^{17} + wx^{16} + w^2x^{14} + wx^{13} + w^2x^{11}+$ $x^{10} + wx^8 + wx^7 + w^2x^6 + x^5 + w^2x^4 + x^3 + wx^2 + x + 1$	$[[114, 6, 12]]$
59	$x^{29} + w^2x^{28} + x^{27} + x^{26} + w^2x^{25} + wx^{24}+$ $x^{23} + w^2x^{21} + x^{20} + w^2x^{19} + wx^{18} + x^{17} + x^{16} + w^2x^{15}$ $+wx^{14} + x^{13} + x^{12} + w^2x^{11} + wx^{10} + x^9 + wx^8 + x^6+$ $w^2x^5 + wx^4 + x^3 + x^2 + wx + 1$	$[[118, 2, 14]]$