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ERRATUM: "A FIXED POINT METHOD FOR PERTURBATION OF BIMULTIPLIERS AND JORDAN BIMULTIPLIERS IN C^*-TERNARY ALGEBRAS" [J. MATH. PHYS. 51, 103508 (2010)]

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Abstract. Ebadian et al.[1] proved the Hyers-Ulam stability of bimultipliers and Jordan bimultipliers in C^*-ternary algebras by using the fixed point method.

Under the conditions in the main theorems for bimultipliers, we can show that the related mappings must be zero. Moreover, there are some mathematical errors in the statements and the proofs of the results. In this paper, we correct the statements and the proofs of the results, and prove the corrected theorems by using the direct method.

1. Introduction and Preliminaries

A C^*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x, y, z) \to [xyz]$ of A^3 into A, which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linear in the middle variable, and associative in the sense that $[xy[zwv]] = [x[wzy]v] = [[xyz]wv]$, and satisfies $\|[xyz]\| \le \|x\| \cdot \|y\| \cdot \|z\|$ and $\|[xxx]\| = \|x\|^3$.

Definition 1.1 ([1]). Let A be a C^*-ternary algebra. A $\mathbb{C}$-bilinear mapping $T : A \times A \to A$ is called a *C^*-ternary bimultiplier* if it satisfies

$$\begin{aligned} T([xyz], w) &= [T(x, y)zw], \\ T(x, [yzw]) &= [xyT(z, w)] \end{aligned}$$

for all $x, y, z \in A$.

The third variable of the left side in the first equality is $\mathbb{C}$-linear and the second variable of the left side in the first equality is conjugate $\mathbb{C}$-linear. But the third

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variable of the right side in the first equality is conjugate $\mathbb{C}$-linear and the second variable of the right side in the first equality is $\mathbb{C}$-linear. The third variable of the left side in the second equality is conjugate $\mathbb{C}$-linear and the second variable of the left side in the second equality is $\mathbb{C}$-linear. But the third variable of the right side in the second equality is $\mathbb{C}$-linear and the second variable of the right side in the second equality is conjugate $\mathbb{C}$-linear. So T must be zero. Hence all the mappings T, related to bimultiplers, must be zero. So the results on bimultipliers are meaningless.

Thus we correct the definition of C^*-ternary bimultiplier as follows.

Definition 1.2. Let A be a C^*-ternary algebra. A $\mathbb{C}$-bilinear mapping $T : A \times A \to A$ is called a *C^*-ternary bimultiplier* if it satisfies

$$\begin{aligned} T([xy^*z], w) &= [T(x,y)z^*w], \\ T(x, [yz^*w]) &= [xy^*T(z,w)] \end{aligned}$$

for all $x, y, z \in A$.

Definition 1.3 ([1])**.** Let A be a C^*-ternary algebra. A $\mathbb{C}$-bilinear mapping $T : A \times A \to A$ is called a *C^*-ternary Jordan bimultiplier* if it satisfies

$$\begin{aligned} T([xxx], x) &= [T(x,x)xx], \\ T(x, [xxx]) &= [xxT(x,x)] \end{aligned}$$

for all $x \in A$.

With respect to the definition of bimultipler, we can correct the definition of C^*-ternary Jordan bimultiplier as follows.

Definition 1.4. Let A be a C^*-ternary algebra. A $\mathbb{C}$-bilinear mapping $T : A \times A \to A$ is called a *C^*-ternary Jordan bimultiplier* if it satisfies

$$\begin{aligned} T([xx^*x], x) &= [T(x,x)x^*x], \\ T(x, [xx^*x]) &= [xx^*T(x,x)] \end{aligned}$$

for all $x \in A$.

The stability problem of functional equations originated from a question of Ulam[2] concerning the stability of group homomorphisms. Hyers[3] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki[4] for additive mappings and by Th.M. Rassias[5] for linear mappings by considering an unbounded Cauchy difference. The stability problems of various functional equations have been extensively investigated by a number of authors (see Refs. 6–11).

2. Hyers-Ulam Stability of Bimultipliers and Jordan Bimultipliers

Throughout this section, assume that A is a C^*-ternary algebra.

For a given mapping $f : A \times A \to A$, we define

$$\begin{aligned} E_{\lambda,\mu} f(x,y,z,w) &= f(\lambda x + \lambda y, \mu z - \mu w) + f(\lambda x - \lambda y, \mu z + \mu w) \\ &\quad -2\lambda\mu f(x,z) + 2\lambda\mu f(y,w) \end{aligned}$$

for all $x, y, z, w \in A$ and all $\lambda, \mu \in \mathbb{T}^1 := \{\nu \in \mathbb{C} : |\nu| = 1\}$.

From now on, assume that $f(0,0) = 0$.

We need the following lemma to obtain the main results.

Lemma 2.1 ([12]). *Let $f : A \times A \to A$ be a mapping satisfying $E_{\lambda,\mu} f(x,y,z,w) = 0$ for all $x, y, z, w \in A$ and all $\lambda, \mu \in \mathbb{T}^1$. Then the mapping $f : A \times A \to A$ is $\mathbb{C}$-bilinear.*

Theorem 2.2. *Let $f : A \times A \to A$ be a uniformly continuous mapping for which there exists a function $\varphi : A^4 \to [0, \infty)$ such that*

$$\|E_{\lambda,\mu} f(x,y,z,w)\| \leq \varphi(x,y,z,w), \tag{1}$$

$$\|f([xy^*z], w) - [f(x,y)z^*w]\| + \|f(x, [yz^*w]) - [xy^*f(z,w)]\| \leq \varphi(x,y,z,w), \tag{2}$$

$$\Phi(x,y,z,w) := \sum_{n=0}^{\infty} \frac{1}{4^n} \varphi(2^n x, 2^n y, 2^n z, 2^n w) < \infty \tag{3}$$

for all $\lambda, \mu \in \mathbb{T}^1$ and all $x, y, z, w \in A$. Then there exists a unique C^*-ternary bimultiplier $T : A \times A \to A$ such that

$$\|f(x,z) - T(x,z)\| \leq \frac{3}{8}\Phi(x,x,z,-z) + \frac{1}{8}\Phi(x,-x,z,z) + \frac{1}{4}\Phi(0,x,0,z) \tag{4}$$

for all $x, z \in A$.

Proof. Letting $\lambda = \mu = 1$, $y = -x$ and $w = z$ in (1), we get

$$\|f(2x, 2z) - 2f(x,z) + f(-x,z)\| \leq \varphi(x,-x,z,z) \tag{5}$$

for all $x, z \in A$. Letting $\lambda = \mu = 1$ and $x = z = 0$ in (1), we get

$$\|f(y,-w) + f(-y,w) + 2f(y,w)\| \leq \varphi(0,y,0,w) \tag{6}$$

for all $y, w \in A$. Replacing y by x and w by z in (6), we get

$$\|f(x,-z)+f(-x,z)+2f(x,x)\| \le \varphi(0,x,0,x) \tag{7}$$

for all $x, z \in A$. Letting $\lambda = \mu = 1$, $y = x$ and $w = -z$ in (1), we get

$$\|f(2x,2z)-2f(x,z)+f(x,-z)\| \le \varphi(x,x,z,-z) \tag{8}$$

for all $x, z \in A$. By (5) and (8), we obtain

$$\|2f(x,-z)-2f(-x,z)\| \le \varphi(x,x,z,-z)+\varphi(x,-x,z,z) \tag{9}$$

for all $x, z \in A$. By (7) and (8), we obtain

$$\begin{aligned} &\|f(2x,2z)-4f(x,z)+f(x,-z)-f(-x,z)\| \\ &\le \varphi(x,x,z,-z)+\varphi(0,x,0,z) \end{aligned} \tag{10}$$

for all $x, z \in A$. By (9) and (10), we have

$$\|f(2x,2z)-4f(x,z)\| \le \frac{3}{2}\varphi(x,x,z,-z)+\frac{1}{2}\varphi(x,-x,z,z)+\varphi(0,x,0,z) = M(x,z)$$

and so

$$\left\|f(x,z)-\frac{1}{4}f(2x,2z)\right\| \le \frac{1}{4}M\left(x,z\right) \tag{11}$$

for all $x, z \in A$. Here $M(x,z) := \frac{3}{2}\varphi(x,x,z,-z)+\frac{1}{2}\varphi(x,-x,z,z)+\varphi(0,x,0,z)$ for all $x, z \in A$.

It follows from (11) that

$$\left\|\frac{1}{4^l}f(2^l x,2^l z)-\frac{1}{4^m}f(2^m x,2^m z)\right\| \le \sum_{j=l}^{m-1}\frac{1}{4^{j+1}}M\left(2^j x,2^j z\right) \tag{12}$$

for all $x, z \in A$ and all nonnegative integers m, l with $m > l$. This implies that the sequence $\left\{\frac{1}{4^n}f(2^n x,2^n z)\right\}$ is a Cauchy sequence for all $x, z \in A$. Since A is complete, the sequence $\left\{\frac{1}{4^n}f(2^n x,2^n z)\right\}$ converges. Thus one can define the mapping $T : A \times A \to A$ by

$$T(x,z) := \lim_{n\to\infty}\frac{1}{4^n}f(2^n x,2^n z)$$

for all $x, z \in A$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (12), we get (4).

By the definition of the mapping T, we have

$$\begin{aligned} \|E_{\lambda,\mu}T(x,y,z,w)\| &= \lim_{n\to\infty}\frac{1}{4^n}\|E_{\lambda,\mu}f(2^n x,2^n y,2^n z,2^n w)\| \\ &\le \lim_{n\to\infty}\frac{1}{4^n}\varphi(2^n x,2^n y,2^n z,2^n w) = 0 \end{aligned}$$

for all $\lambda, \mu \in \mathbb{T}^1$ and all $x, y, z, w \in A$. By Lemma 2.1, the mapping $T : A \times A \to A$ is $\mathbb{C}$-bilinear.

Let $T' : A \times A \to A$ be another $\mathbb{C}$-bilinear mapping satisfying (4). Then we have

$$\begin{aligned}
&\|T(x,z) - T'(x,z)\| = \frac{1}{4^n}\|T(2^n x, 2^n z) - T'(2^n x, 2^n z)\| \\
&\le \frac{1}{4^n}\|T(2^n x, 2^n z) - f(2^n x, 2^n z)\| + \frac{1}{4^n}\|f(2^n x, 2^n z) - T'(2^n x, 2^n z)\| \\
&\le \frac{2}{4^n}\left(\frac{3}{8}\Phi(2^n x, 2^n x, 2^n z, -2^n z) + \frac{1}{8}\Phi(2^n x, -2^n x, 2^n z, 2^n z) + \frac{1}{4}\Phi(0, 2^n x, 0, 2^n z)\right),
\end{aligned}$$

which tends to zero as $n \to \infty$ for all $x, z \in A$. This proves the uniqueness of T.

It is easy to show that $T(x,z) = \lim_{n\to\infty} \frac{1}{16^n} f(8^n x, 2^n z) = \lim_{n\to\infty} \frac{1}{16^n} f(2^n x, 8^n z)$ for all $x, z \in A$, since T is bi-additive and f is uniformly continuous.

It follows from (2) and (3) that

$$\begin{aligned}
&\|T([xy^*z], w) - [T(x,y)z^*w]\| + \|T(x, [yz^*w]) - [xy^*T(z,w)]\| \\
&= \lim_{n\to\infty} \frac{1}{16^n}(\|f(8^n[xy^*z], 2^n w) - [f(2^n x, 2^n y)(2^n z)^*(2^n w)]\| \\
&\quad + \|f(2^n x, 8^n[yz^*w]) - [(2^n x)(2^n y)^* f(2^n z, 2^n w)]\|) \\
&\le \lim_{n\to\infty} \frac{1}{16^n}\varphi(2^n x, 2^n y, 2^n z, 2^n w) \le \lim_{n\to\infty} \frac{1}{4^n}\varphi(2^n x, 2^n y, 2^n z, 2^n w) = 0
\end{aligned}$$

for all $x, y, z, w \in A$. So

$$T([xy^*z], w) = [T(x,y)z^*w]$$

and

$$T(x, [yz^*w]) = [xy^*T(z,w)]$$

for all $x, y, z, w \in A$.

Therefore, the mapping $T : A \times A \to A$ is a unique C^*-ternary bimultiplier satisfying (4). $\square$

Corollary 2.3. *Let θ and p be positive real numbers with $p < 2$ and let $f : A \times A \to A$ be a uniformly continuous mapping such that*

$$\|E_{\lambda,\mu} f(x,y,z,w)\| \le \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p), \tag{13}$$

$$\begin{aligned}\|f([xy^*z], w) - [f(x,y)z^*w]\| + \|f(x, [yz^*w]) - [xy^*f(z,w)]\| \\ \le \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)\end{aligned} \tag{14}$$

for all $\lambda, \mu \in \mathbb{T}^1$ *and all* $x, y, z, w \in A$. *Then there exists a unique* C^**-ternary bimultiplier* $T : A \times A \to A$ *such that*

$$\|f(x,z) - T(x,z)\| \leq \frac{5\theta}{4-2^p}(\|x\|^p + \|z\|^p) \tag{15}$$

for all $x, z \in A$.

Proof. Letting $\varphi(x,y,z,w) := \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.2, we get the desired result. □

Theorem 2.4. *Let* $f : A \times A \to A$ *be a uniformly continuous mapping for which there exists a function* $\varphi : A^4 \to [0, \infty)$ *satisfying* (1), (2) *and*

$$\sum_{n=1}^{\infty} 16^n \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}, \frac{w}{2^n}\right) < \infty \tag{16}$$

for all $\lambda, \mu \in \mathbb{T}^1$ *and all* $x, y, z, w \in A$. *Then there exists a unique* C^**-ternary bimultiplier* $T : A \times A \to A$ *such that*

$$\|f(x,z) - T(x,z)\| \leq \frac{3}{8}\Phi(x,x,z,-z) + \frac{1}{8}\Phi(x,-x,z,z) + \frac{1}{4}\Phi(0,x,0,z) \tag{17}$$

for all $x, z \in A$. *Here*

$$\Phi(x,y,z,w) := \sum_{n=1}^{\infty} 4^n \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}, \frac{z}{2^n}, \frac{w}{2^n}\right)$$

for all $x, y, z, w \in A$.

Proof. It follows from (11) that

$$\left\| f(x,z) - 4f\left(\frac{x}{2}, \frac{z}{2}\right) \right\| \leq M\left(\frac{x}{2}, \frac{x}{2}, \frac{z}{2}, \frac{z}{2}\right) \tag{18}$$

for all $x, z \in A$.

It follows from (18) that

$$\left\| 4^l f\left(\frac{x}{2^l}, \frac{z}{2^l}\right) - 4^m f\left(\frac{x}{2^m}, \frac{z}{2^m}\right) \right\| \leq \sum_{j=l+1}^{m} 4^{j-1} M\left(\frac{x}{2^j}, \frac{z}{2^j} z\right) \tag{19}$$

for all $x, z \in A$ and all nonnegative integers m, l with $m > l$. This implies that the sequence $\left\{4^n f\left(\frac{x}{2^n}, \frac{z}{2^n}\right)\right\}$ is a Cauchy sequence for all $x, z \in A$. Since A is complete, the sequence $\left\{4^n f\left(\frac{x}{2^n}, \frac{z}{2^n}\right)\right\}$ converges. Thus one can define the mapping $T : A \times A \to A$ by

$$T(x,z) := \lim_{n\to\infty} 4^n f\left(\frac{x}{2^n}, \frac{z}{2^n}\right)$$

for all $x, z \in A$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (19), we get (17).

By the definition of the mapping T, we have

$$\begin{aligned}\|E_{\lambda,\mu}T(x,y,z,w)\| &= \lim_{n\to\infty} 4^n \left\|E_{\lambda,\mu}f\left(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n},\frac{w}{2^n}\right)\right\| \\ &\le \lim_{n\to\infty} 4^n\varphi\left(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n},\frac{w}{2^n}\right) \\ &\le \lim_{n\to\infty} 16^n\varphi\left(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n},\frac{w}{2^n}\right) = 0\end{aligned}$$

for all $\lambda,\mu \in \mathbb{T}^1$ and all $x,y,z,w \in A$. By Lemma 2.1, the mapping $T : A\times A \to A$ is $\mathbb{C}$-bilinear.

Let $T' : A\times A\to A$ be another $\mathbb{C}$-bilinear mapping satisfying (4). Then we have

$$\begin{aligned}&\|T(x,z)-T'(x,z)\| = 4^n\left\|T\left(\frac{x}{2^n},\frac{z}{2^n}\right)-T'\left(\frac{x}{2^n},\frac{z}{2^n}\right)\right\| \\ &\le 4^n\left\|T\left(\frac{x}{2^n},\frac{z}{2^n}\right)-f\left(\frac{x}{2^n},\frac{z}{2^n}\right)\right\|+4^n\left\|f\left(\frac{x}{2^n},\frac{z}{2^n}\right)-T'\left(\frac{x}{2^n},\frac{z}{2^n}\right)\right\| \\ &\le 2\cdot 4^n\left(\frac{3}{8}\Phi\left(\frac{x}{2^n},\frac{x}{2^n},\frac{z}{2^n},-\frac{z}{2^n}\right)+\frac{1}{8}\Phi\left(\frac{x}{2^n},-\frac{x}{2^n},\frac{z}{2^n},\frac{z}{2^n}\right)+\frac{1}{4}\Phi\left(0,\frac{x}{2^n},0,\frac{z}{2^n}\right)\right),\end{aligned}$$

which tends to zero as $n\to\infty$ for all $x,z\in A$. This proves the uniqueness of T.

It is easy to show that $T(x,z) = \lim_{n\to\infty} 16^n f\left(\frac{x}{8^n},\frac{z}{2^n}\right) = \lim_{n\to\infty} 16^n f\left(\frac{x}{2^n},\frac{z}{8^n}\right)$ for all $x,z\in A$, since T is bi-additive and f is uniformly continuous.

It follows from (2) and (16) that

$$\begin{aligned}&\|T([xy^*z],w)-[T(x,y)z^*w]\|+\|T(x,[yz^*w])-[xy^*T(z,w)]\| \\ &= \lim_{n\to\infty} 16^n\left(\left\|f\left(\frac{[xy^*z]}{8^n},\frac{w}{2^n}\right)-\left[f\left(\frac{x}{2^n},\frac{y}{2^n}\right)\frac{z^*}{2^n}\frac{w}{2^n}\right]\right\|\right. \\ &\quad \left.+\left\|f\left(\frac{x}{2^n},\frac{[yz^*w]}{8^n}\right)-\left[\frac{x}{2^n x}\frac{y^*}{2^n}f\left(\frac{z}{2^n},\frac{w}{2^n}\right)\right]\right\|\right) \\ &\le \lim_{n\to\infty} 16^n\varphi\left(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n},\frac{w}{2^n}\right) = 0\end{aligned}$$

for all $x,y,z,w\in A$. So

$$T([xy^*z],w) = [T(x,y)z^*w]$$

and

$$T(x,[yz^*w]) = [xy^*T(z,w)]$$

for all $x,y,z,w\in A$.

Therefore, the mapping $T : A\times A\to A$ is a unique C^*-ternary bimultiplier satisfying (17). $\square$

Corollary 2.5. *Let θ and p be positive real numbers with $p > 4$ and let $f : A\times A \to A$ be a uniformly continuous mapping satisfying* (13) *and* (14). *Then there exists a unique C^*-ternary bimultiplier $T : A \times A \to A$ such that*

$$\|f(x,z) - T(x,z)\| \le \frac{5\theta}{2^p - 4}(\|x\|^p + \|z\|^p) \tag{20}$$

for all $x, z \in A$.

Proof. Letting $\varphi(x,y,z,w) := \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.4, we get the desired result. $\square$

Now, we prove the Hyers-Ulam stability of Jordan bimultipliers in C^*-ternary algebras by using the direct method.

Theorem 2.6. *Let $f : A \times A \to A$ be a uniformly continuous mapping for which there exists a function $\varphi : A^4 \to [0,\infty)$ satisfying* (1), (3) *and*

$$\begin{aligned} &\|f([xx^*x], x) - [f(x,x)x^*x]\| + \|f(x,[xx^*x]) - [xx^*f(x,x)]\| \\ &\le \varphi(x,x,x,x) \end{aligned} \tag{21}$$

for all $x \in A$. Then there exists a unique C^*-ternary Jordan bimultiplier $T : A\times A \to A$ satisfying (4).

Proof. By the same reasoning as in the proof of Theorem 2.2, there exists a unique $\mathbb{C}$-bilinear mapping $T : A \times A \to A$ satisfying (4). The mapping $T : A \times A \to A$ is given by

$$T(x,z) := \lim_{n\to\infty} \frac{1}{4^n} f(2^n x, 2^n z)$$

for all $x, z \in A$.

It follows from (2) and (21) that

$$\begin{aligned} &\|T([xx^*x], x) - [T(x,x)x^*x]\| + \|T(x,[xx^*x]) - [xx^*T(x,x)]\| \\ &= \lim_{n\to\infty} \frac{1}{16^n}(\|f(8^n[xx^*x], 2^n x) - [f(2^n x, 2^n x)(2^n x)^*(2^n x)]\| \\ &\quad + \|f(2^n x, 8^n[xx^*x]) - [(2^n x)(2^n x)^* f(2^n x, 2^n x)]\|) \\ &\le \lim_{n\to\infty} \frac{1}{16^n}\varphi(2^n x, 2^n x, 2^n x, 2^n x) \le \lim_{n\to\infty} \frac{1}{4^n}\varphi(2^n x, 2^n x, 2^n x, 2^n x) = 0 \end{aligned}$$

for all $x \in A$. So

$$T([xx^*x], x) = [T(x,x)x^*x]$$

and

$$T(x, [xx^*x]) = [xx^*T(x,x)]$$

for all $x \in A$.

Therefore, the mapping $T : A \times A \to A$ is a unique C^*-ternary Jordan bimultiplier satisfying (4). $\square$

Corollary 2.7. *Let θ and p be positive real numbers with $p < 2$ and let $f : A \times A \to A$ be a uniformly continuous mapping satisfying* (13) *and*

$$(22)\quad \|f([xx^*x], x) - [f(x,x)x^*x]\| + \|f(x, [xx^*x]) - [xx^*f(x,x)]\| \leq 4\theta\|x\|^p$$

for all $x \in A$. Then there exists a unique C^*-ternary Jordan bimultiplier $T : A \times A \to A$ satisfying (15).

Proof. Letting $\varphi(x,y,z,w) := \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.6, we get the desired result. $\square$

Theorem 2.8. *Let $f : A \times A \to A$ be a uniformly continuous mapping for which there exists a function $\varphi : A^4 \to [0, \infty)$ satisfying* (1), (16) *and* (21). *Then there exists a unique C^*-ternary Jordan bimultiplier $T : A \times A \to A$ satisfying* (17).

Proof. The proof is similar to the proofs of Theorems 2.4 and 2.6. $\square$

Corollary 2.9. *Let θ and p be positive real numbers with $p > 4$ and let $f : A \times A \to A$ be a uniformly continuous mapping satisfying* (13) *and* (22). *Then there exists a unique C^*-ternary Jordan bimultiplier $T : A \times A \to A$ satisfying* (20).

Proof. Letting $\varphi(x,y,z,w) := \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.8, we get the desired result. $\square$

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