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COMPARISON OF NUMERICAL METHODS FOR OPTION PRICING UNDER THE CGMY MODEL

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Abstract. We propose a number of finite difference methods for the prices of a European option under the CGMY model. These numerical methods to solve a partial integro-differential equation (PIDE) are based on three time levels in order to avoid fixed point iterations arising from an integral operator. Numerical simulations are carried out to compare these methods with each other for pricing the European option under the CGMY model.

1. Introduction

Since a geometric Brownian motion in [1] was suggested to price derivatives in financial markets, a variety of researchers have been interested in stochastic processes to capture financial phenomena that are not accounted for by the Black-Scholes model. Lévy models can be widely used instead of the geometric Brownian motion in order to explain the stylized facts in the financial markets.

There are two kinds of the Lévy processes. The first one is called jump-diffusion processes in which sample paths have a finite number of jumps in a finite time interval. The other type is infinite activity processes with an infinite number of jumps in the sample paths. The CGMY process introduced by Carr, Geman, Madan, and Yor [2] is well known as one of the infinite activity processes.

We focus on a comparison of numerical methods to solve a PIDE for the prices of a European option when an underlying asset follows the CGMY model. Salmi and Toivanen [6] proposed implicit-explicit

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(IMEX) methods to solve the PIDE under a jump-diffusion model. The three IMEX methods are based on a framework using three time levels to avoid fixed point iterations at each time step and then the derived linear systems can be resolved with the inverses of trdiagonal matrices. In this paper we apply the three IMEX methods to option pricing problems under the infinite activity models. It is remarkable for us to use them since the PIDE with an infinite Lévy measure is transformed into the PIDE with a finite Lévy measure. A number of numerical simulations are performed to compare these IMEX methods and we can figure out the second-order convergence rate in the time variable.

This paper is organized as follows. In section 2 we introduce succinctly the CGMY process and the PIDE for pricing the European option in the financial markets. The three IMEX methods are proposed to solve the PIDE numerically in section 3. A variety of numerical simulations are carried out to compare these methods with each other in section 4. Finally, this paper ends with conclusions in section 5.

2. The CGMY option pricing model

In a risk-neutral world, we assume that an underlying asset S_t follows an exponential Lévy model given by $S_t = S_0 \exp((r-d)t + X_t)$ on a probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ with filtration $\mathcal{F}_t$, where r is the continuous risk-free interest rate, d is the continuous dividend rate, S_0 is an initial price at $t = 0$. The stochastic process X_t is considered by the CGMY process in [2] with the Lévy measure ν_X

$$\nu_X(dx) = C\left(\frac{e^{-G|x|}}{|x|^{1+Y}}1_{x<0} + \frac{e^{-Mx}}{x^{1+Y}}1_{x>0}\right)dx, \tag{2.1}$$

where $C > 0, G \geq 0, M \geq 0$, and $0 \leq Y < 2$. In the Lévy triplet $(\sigma_X^2, \gamma_X, \nu_X)$ of the process X_t, the value γ_X in the risk-neutral world is required to satisfy the martingale condition

$$\gamma_X = -\frac{1}{2}\sigma_X^2 - \int_{\mathbb{R}} (e^x - 1 - x1_{\{|x|\leq 1\}})\nu_X(dx)$$

with the indicator function 1_A of a set A.

In the option pricing problem, it is not easy to deal with small jumps in the CGMY process X_t. One of the possibility is in [4] that we can transform the infinite activity process X_t into the following jump-diffusion model Y_t with the Lévy triplet $(\sigma_Y^2, \gamma_Y, \nu_Y)$

$$\sigma_Y^2 = \sigma_X^2 + \int_{-\epsilon}^{\epsilon} x^2 \nu_X(dx),$$
$$\gamma_Y = -\frac{1}{2}\sigma_Y^2 - \int_{\mathbb{R}} (e^x - 1 - x1_{\{|x|\le 1\}})\nu_Y(dx),$$
$$\nu_Y = \nu_X 1_{|x|\ge\epsilon}.$$

Then the prices of the European option under the CGMY process X_t can be evaluated approximately by solving the initial and boundary valued PIDE problem

$$u_\tau(\tau, x) = (\mathcal{D} + \mathcal{I} - (r+\lambda_\epsilon))u(\tau, x), \quad (\tau, x) \in (0, T] \times \Omega, \tag{2.2}$$
$$u(\tau, x) = g(\tau, x), \quad (\tau, x) \in (0, T] \times \mathbb{R} \setminus \Omega, \tag{2.3}$$
$$u(0, x) = h(x), \quad x \in \Omega, \tag{2.4}$$

where $\Omega = (-X, X)$, $h(x)$ is the initial payoff function, $g(\tau, x)$ is the boundary function, and the differential and integral operator $\mathcal{D}$ and $\mathcal{I}$ are respectively given by

$$\mathcal{D}u = \frac{\sigma_Y^2}{2} u_{xx} + \left(r - d - \frac{\sigma_Y^2}{2} - \lambda_\epsilon \zeta_\epsilon \right) u_x, \tag{2.5}$$

$$\mathcal{I}u = \int_{|y|\ge\epsilon} u(\tau, x+y)\nu_Y(dy) \tag{2.6}$$

with $\lambda_\epsilon = \int_{|y|\ge\epsilon} \nu_Y(dy)$ and $\zeta_\epsilon = \int_{|y|\ge\epsilon} (e^y - 1)\nu_Y(dy)/\lambda_\epsilon$.

3. Numerical schemes for option pricing

We apply three numerical methods similar to those in [6] to option pricing problems under the infinite activity Lévy models. It is possible to use them since the CGMY process is transformed into the jump-diffusion model with the PIDE problem (2.2)–(2.4). The implicit-explicit (IMEX) method with Crank-Nicolson and leapfrog (CNLF) scheme is given by

$$\begin{aligned} \frac{U_m^{n+1} - U_m^{n-1}}{2\Delta\tau} &= (\mathcal{D}_\Delta - (1-c)(r+\lambda_\epsilon)) \left(\frac{U_m^{n+1} + U_m^{n-1}}{2} \right) \\ &\quad + (\mathcal{I}_\Delta - c(r+\lambda_\epsilon))U_m^n, \end{aligned} \tag{3.1}$$

where $\mathcal{D}_\Delta$ and $\mathcal{I}_\Delta$ are discretizations of the differential and integral operators $\mathcal{D}$ and $\mathcal{I}$ respectively, U_m^n is an approximate solution of $u(\tau_n, x_m)$, and c is an extra parameter to control the zeroth-order term $(r+\lambda_\epsilon)u$.

We refer to [5] for more details of the discrete operators $\mathcal{D}_\Delta$ and $\mathcal{I}_\Delta$. The following two schemes are concerned with the extrapolation to approximate the integral term $\mathcal{I}u$. The IMEX method with Crank-Nicolson and Adams-Bashforth (CNAB) scheme is

$$\frac{U_m^{n+1}-U_m^n}{\Delta\tau} = (\mathcal{D}_\Delta - (1-c)(r+\lambda_\epsilon))\left(\frac{U_m^{n+1}+U_m^n}{2}\right) + (\mathcal{I}_\Delta - c(r+\lambda_\epsilon))\left(\frac{3}{2}U_m^n - \frac{1}{2}U_m^{n-1}\right) \tag{3.2}$$

and the IMEX method based on the two-step backward differentiation formula (BDF2) is

$$\frac{3U_m^{n+1}-4U_m^n+U_m^{n-1}}{2\Delta\tau} = (\mathcal{D}_\Delta - (1-c)(r+\lambda_\epsilon))U_m^{n+1} + (\mathcal{I}_\Delta - c(r+\lambda_\epsilon))(2U_m^n - U_m^{n-1}). \tag{3.3}$$

These numerical methods are constructed to avoid any fixed point iteration techniques arising from the integral term $\mathcal{I}u$ at each time step. The IMEX-CNLF method with $c = 1$ formulated in [5] has the second-order accuracy when the underlying asset follows the CGMY model. Therefore we expect that the above three IMEX methods converge with the second-order in the time and spatial variables.

4. Numerical simulations

In this section we performed numerical simulations with MATLAB on a computer with Intel(R) Core(TM) i7-5820K CPU 3.30GHz to price a European call option when the underlying asset follows the CGMY model. The parameters used in the simulation are

$$\sigma = 0, \quad r = 0.1, \quad d = 0, \quad C = 16.97, \quad G = 7.08,$$

$$M = 29.97, \quad Y = 0.6442, \quad T = 0.25, \quad K = 98, \quad \epsilon = 0.01171875,$$

which are also given in [2, 7], and $X = 3$ in the truncated boundary domain Ω. We used the number of spatial steps $M = 8192$ to obtain more accurately the rate of convergence. The prices of the European call option on the first four time levels are computed by applying the explicit-implicit method in [4]. The reference values evaluated by using the FFT method in [3] are approximately 16.564028 at $S_1 = 90$, 21.438990 at $S_2 = 98$, and 26.781630 at $S_3 = 106$.

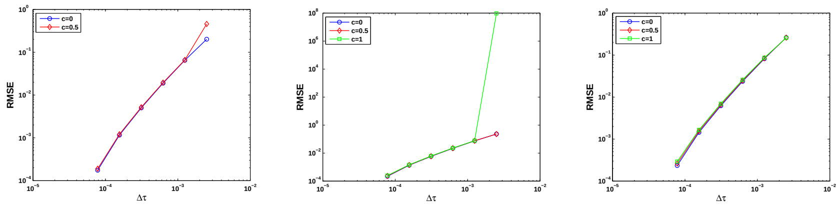

FIGURE 1. RMSEs for the European call option with the IMEX-CNLF scheme (left), IMEX-CNAB scheme (center), and IMEX-BDF2 scheme (right).

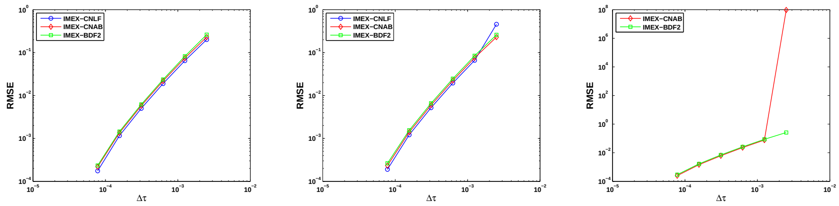

FIGURE 2. RMSEs for the European call option with the c=0 (left), c=0.5 (center), and c=1 (right).

In Figures 1 and 2, we present the root mean square errors (RMSEs) for the IMEX-CNLF, IMEX-CNAB, and IMEX-BDF2 schemes with $c = 0$, $c = 0.5$, and $c = 1$, where the RMSE is given by

$$\text{RMSE} = \sqrt{\left((U_1 - u_1^*)^2 + (U_2 - u_2^*)^2 + (U_3 - u_3^*)^2\right)/3} \tag{4.1}$$

for the computed price U_i and the reference value u_i^* at the stock price S_i with $i = 1$, 2, and 3. We observe that the described IMEX methods have the second-order convergence rate with respect to the time variable except for the IMEX-CNLF scheme with $c = 1$. It is unstable in the case of the IMEX-CNLF scheme with $c = 1$. The RMSEs for the IMEX-CNLF scheme tend to be smaller than those for the IMEX-CNAB and IMEX-BDF2 schemes provided that they are stable.

5. Conclusion

In this paper we considered the stability for the three IMEX methods when the underlying asset follows the CGMY model. These numerical methods for pricing the European option are designed to avoid any fixed point iteration techniques at each time step. A variety of simulations are carried out to study the stability and to price the European option under the CGMY model. The numerical results show that the IMEX-CNLF scheme has the smaller RMSEs than those obtained by the IMEX-CNAB and IMEX-BDF2 schemes except for $c = 1$. Moreover, we can observe that these schemes have the second-order convergence rate since the slopes of the lines in Figures 1 and 2 are almost equal to 2.

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