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Bilevel-programming based failure-censored ramp-stress ALTSP for the log-logistic distribution with warranty cost

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Abstract: In this paper accelerated life testing is incorporated in quality control technique of acceptance sampling plan to induce early failures in high reliability products.Stress under accelerated condition can be applied in constant-stress, step-stress and progressive-stress or combination of such loadings. A ramp-stress results when stress is increased linearly (from zero) with time. In this paper optimum failure-censored ramp-stress accelerated life test sampling plan for log-logistic distribution has been formulated with cost considerations. The log-logistic distribution has been found appropriate for insulating materials. The optimal plans consist in finding optimum sample size, sample proportion allocated to each stress, and stress rate factor such that producer's and consumer's interests are safeguarded. Variance optimality criterion is used when expected cost per lot is not taken into consideration, and bilevel programming approach is used in cost optimization problems. The methods developed have been illustrated using some numerical examples, and sensitivity analyses carried out in the context of ramp-stress ALTSP based on variable SSP for proportion nonconforming.

Key Words: a*ccelerated life test, ramp-test, producer's risk, consumer's risk, cumulative exposure model, Type-II censoring*

1. INTRODUCTION

The lifetime of a product is an important quality characteristic for determining its acceptability with regard to its usefulness at the time it is put into the service. Sampling plans meant to determine the acceptability of the product with respect to its lifetime are called life test sampling plan or reliability sampling plans. Life test sampling coupled with Accelerated life testing help in testing the acceptability of high reliable products at lower cost and in shorter time. This has necessitated design of Accelerated Life Test Sampling Plans (ALTSPs).

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Accelerated test condition includes stresses in the form of temperature, voltage, pressure, vibration, cycling rate, humidity, etc. In life testing, a fixed number of items are often tested simultaneously and the test continues for a fixed period of time (time-censoring or Type-I censoring) or until some fixed number of items on test fail (failure-censoring or Type-II censoring). Stress under accelerated condition can be applied using constant-stress, step-stress, progressive- stress, cyclic- stress, random- stress, or combinations of such loadings. In a progressive-stress loading, the stress level is increased continually either until the censoring time or up to the maximum stress level, which is maintained until the censoring time. A ramp-stress test results when stress is increased linearly (from zero) with time. In particular, a ramp test with two linearly increasing stresses is a simple ramp test. Ramp tests are used for example in fatigue testing (Prot (1948)), capacitors (Endicott et al (1965), Starr and Endicott (1961)), insulation (Goba (1969), Solomon et. al. (1965)), and integrated circuits (Chan (1990)), Nelson (1990), Chapter 10) has given some practical situations in which ramp-stress ALT has been used. Bai et. al. (1992), Yin and Sheng (1987), Srivastava and Shukla (2009), Srivastava and Mittal (2012a), Srivastava and Mittal (2012b) have studied ramp-stress ALT test.

Life test sampling plans (LSPs) involving single sample under Type-I and Type-II censoring schemes have been studied extensively in the literature. A rich literature exists on designing LSPs in the case of exponential life distribution. Early work includes Epstein and Sobel (1953) in which LSPs are developed under Type-II censoring. In Spurrier and Wei (1980), Type-I censoring is assumed and LSPs are obtained considering the producer risk only. Later, Jeong and Yum (1995) extended the Spurrier and Wei (1980) work to the case where both the producer and consumer risks are considered. The design of LSPs under Type-I censoring and intermittent monitoring has been presented by Kim and Yum (2000). For the Weibull distribution, many authors have developed LSPs under Type-II censoring (e.g., see Soland (1968), Schneider (1985), Kwon (1996), Wu and Tsai (2000), McKane et. al. (2005)).

However these LSPs are designed under use conditions. (Wallace, (1985)) has stressed the need for introducing ALT to the future plan of MIL-STD-781. Yum and Kim (1990) have developed an accelerated life test sampling plan for the exponential distribution based on two stress levels and Type II censored data, and Hsiesh (1994) has extended their work and obtained sampling plans that minimize the total censoring number.Bai et. al. (1993) have designed failure-censored accelerated life test sampling plans for lognormal and Weibull distributions in which optimum total sample size, sample proportions allocated to each stress and lot acceptability constants are determined when the test uses two prescribed levels of stress higher than use condition with a given degree of censoring at each stress. Bai et. al. (1995) have devised failure-censored constant-stress accelerated life test sampling plans for Weibull distribution under expected test time constraint. A cost model for an ALTSP has been constructed by Lin and Chiu (1995). Seo et. al. (2009) have designed optimum time-censored and failure-censored constant-stress ALTSPs for Weibull distribution with non constant shape parameter. Optimum failure–censored step-stress ALTSPs for Weibull distribution have been formulated by Chung et. al. (2002).

However, no work seems to exist in the literature that incorporates ramp-stress accelerated life testing in acceptance sampling plan. In this paper, we have formulated failure-censored ramp-stress ALTSPs with warranty. We have used single sampling plan by variables for proportion nonconforming. The basic idea behind variable sampling for proportion nonconforming is to show that the sample results are sufficiently far within the specification limit(s) to assure the acceptability of lot with reasonable probability. The methods commonly used to estimate the proportion nonconforming of a lot are $\bar{X}$-Method, k-Method, and M-Method (Schilling and Neubauer (2009)). We have used Schneider's (1985) approach to k-Method with the sample average $\bar{X}$ replaced by MLE, $\hat{\mu}$, of location parameter, and σ replaced by MLE, $\hat{\sigma}$, of scale parameter of the log lifetime distribution. Warranty is a seller's assurance to the purchaser that the goods or property is as represented and, if not, will be replaced or repaired (Neufeldt and Guralnik (1953)). The seller's assurance to the buyer can be considered to be a contractual agreement between the two parties and becomes effective upon the sale of the product. The optimal plan consists in finding optimum sample size, sample proportions allocated to each stress, and stress rate factor by minimizing the expected total cost per lot. Finding the optimal plan by minimizing the asymptotic variance of test-statistic and expected total cost subject to the constraints that the probability of accepting a good lot is at least $100(1-\tilde{\alpha})$ % and probability of accepting a bad lot is at the most $100\tilde{\beta}$ % helps in safeguarding producer's interest as well as consumer's interest. Sensitivity analysis has been carried out, and the method developed has been illustrated using a numerical example.

Acronyms

ALT	accelerated life test
ALTSP	accelerated life test sampling plan
SSP	single sampling plan
Asvar	asymptotic variance
cdf	cumulative distribution function
pdf	probability density function
MLE	maximum likelihood estimate
FRW	free replacement warranty
PRW	pro-rata warranty
GRW	general rebate warranty

Notation

ψ_i	Stress rate at level i, $\psi_i > 0$, i=1,2
ξ_i	Stress-rate factor; $\xi_1 < \xi_2 = 1$; $\xi_i = \psi_i / \psi_2$, i=1,2
ϕ, $\bar{\phi}$	Proportion of sample allocated to stress rates ψ_1 and ψ_2, respectively; $\bar{\phi} = 1-\phi$
s(y)	Stress at failure time y
s_0	Stress level under normal operating conditions or design constant stress
ϕ^*	Optimum Optimum proportion of sample allocated to stress rate ψ_1
N	Total number of test items in a ALT
n_i	Total number of test items allocated to stress level i; i = 1, 2 ($n = n_1 + n_2$)

r_i	Number of items failed before censoring at stress rate ψ_i, i = 1, 2
q_i	Proportion of failures at stress rate ψ_i, i = 1, 2, $q_1 = r_1 / n_1$, $q_2 = r_2 / n_2$
τ	Censoring time
τ_1, τ_2	Time at which nq_1^{th} and nq_2^{th} failures occurs
τ_i'	$(\ln(\tau_i) - \mu(\xi_i))/\sigma$; Standardized log censoring time at stress rate factor ξ_i, i = 1, 2
α, λ	Shape and scale parameters of log-logistic life distribution
γ_0, γ_1	Parameters of the inverse power law, $\gamma_1 > 0$, $-\infty < \gamma_0 < \infty$
μ, σ	Location, scale parameter of the transformed distribution
T	Test-statistic used to decide the lot acceptability
$\tilde{\alpha}$, $\tilde{\beta}$	Producer's risk and consumer's risk; $0 < \tilde{\alpha} < 1$, $0 < \tilde{\beta} < 1$
p_1, p_2	Fraction non-conforming to be accepted with probability atleast $1-\tilde{\alpha}$, and rejected with probability $1-\tilde{\beta}$
$\Phi(\cdot)$	Cdf of standard normal distribution
$H(\cdot)$	Cdf of logistic distribution
w_p	Quantile p of standard normal distribution; $\Phi(w_p) = p$
z_p	Quantile p of standard logistic distribution; $z_p = -\ln[(1-p)/p]$
K	Lot acceptability constant
C_s	Cost of sampling and putting an item on test
C_τ	Cost per unit of test time
C_{re}	Cost of rejecting an item
C_w	Cost associated with an external failure
P_1	probability that a unit will fail by time τ at ψ_2 and is given by $1-\{1 + [\tau/((e^{\gamma_0}(s_0/(\psi_2))^{\gamma_1}(1+\gamma_1))^{1/(1+\gamma_1)})]^{\alpha(1+\gamma_1)}\}^{-1}$
P_2	probability that a unit will fail by time τ at ψ_2 and is given by $1-\{1 + [\tau/((e^{\gamma_0}(s_0/(\psi_2\xi_1))^{\gamma_1}(1+\gamma_1))^{1/(1+\gamma_1)})]^{\alpha(1+\gamma_1)}\}^{-1}$

2. THE MODEL

2.1 *Basic assumptions*

The stress rates ψ_1 and ψ_2 ($\psi_1 < \psi_2$) are used in a ramp-test.

1) At any constant stress, s, the product life, Y, has a log-logistic distribution.
2) For the effect of changing stress, the linear cumulative exposure model holds (see Nelson (1990); Yin and Sheng (1987); and Nilsson (1985)).
3) The inverse power law holds for $\lambda(s)$ where λ is linear function of a (possibly transformed) stress, i.e.,

$$\lambda(s) = e^{\gamma_0}\left(\frac{s_0}{s}\right)^{\gamma_1},$$

where parameters γ_0 and γ_1 are the characteristics of the product .

4) The test units are statistically independent and identically distributed.

5) The stress applied to test units is continuously increased with constant rate ψ from zero.

2.2 *Log-logistic distribution*

The cdf and pdf of log-logistic distribution is given by:

$$F(y;\alpha,\lambda) = 1-(1+(y/\lambda)^{\alpha})^{-1},\ y \geq 0,\ \alpha > 0,\ \lambda > 0, \tag{1}$$

$$f(y;\alpha,\lambda) = (\alpha/\lambda)(y/\lambda)^{\alpha-1}(1+(y/\lambda)^{\alpha})^{-2},\ y \geq 0,\ \alpha > 0,\ \lambda > 0, \tag{2}$$

where α and λ are shape and scale parameters, respectively.

2.3 *Life distribution under ramp-test*

The stress at failure time y is (from assumption #6) $s(y) = \psi y$.
From the linear cumulative exposure model and the inverse power law, the cdf of the lifetime Y of a unit tested under stress rate ψ is:

$$G(y) = F(\varepsilon(y)), \tag{3}$$

where $F(\cdot)$ is the assumed cdf (see (3)) with the scale parameter λ set equal to one,

$$\varepsilon(y) = \int_0^y 1/\lambda(s(u))du \tag{4}$$

is the cumulative exposure (damage) model. Hence, the cdf, and pdf, respectively, of log-logistic distribution reduces to

$$\begin{aligned} G(y;\alpha,\eta) &= \left(\int_0^y [du/\lambda(s(u))]\right)^{\alpha} \Big/ 1+\left(\int_0^y [du/\lambda(s(u))]\right)^{\alpha}, \\ &= (y/\eta)^{\alpha'} / 1+(y/\eta)^{\alpha'}, \end{aligned} \tag{5}$$

$$g(y;\alpha,\eta) = \alpha'/\eta(y/\eta)^{\alpha'-1} \Big/ \left(1+(y/\eta)^{\alpha'}\right)^2, \tag{6}$$

where $\alpha' = \alpha\ (1+\gamma_1)$, $\eta = \left(e^{\gamma_0}(\gamma_1+1)(s_0/\psi)^{\gamma_1}\right)^{1/(\gamma_1+1)}$, $G(\cdot)$ is the log-logistic distribution with scale parameter η and shape parameter α'.

2.4 *Life test procedure*

1) Out of total 'n' ($n = n\phi + n\bar{\phi}$) items, '$n\phi$' items randomly chosen are allocated to stress rate ψ_1 and the remaining '$n\bar{\phi}$' items are allocated to stress rate ψ_2.
2) The test is continued until: a) all failure times are observed, or b) at each stress rate ψ_i, the test run until r_i failures are observed, i = 1, 2.

2.5 *Lot acceptance sampling procedure*

Assume that one-sided lower specification limit, L, is assigned to the lifetime of a product. Instead of using the actual life time, Y, X = ln(Y) is used. Thus, X follows logistic distribution with location parameter $\mu = \ln(\eta)$ and scale parameter $\sigma = (1/\alpha')$. The lower specification limit on X is $L' = \ln(L)$.

The pdf and cdf for X, respectively are

$$h(x;\mu,\sigma) = e^{(x-\mu)/\sigma}\Big/\sigma\left(1+e^{(x-\mu)/\sigma}\right)^2, \tag{7}$$

$$H(x;\mu,\sigma) = 1-\left(e^{(x-\mu)/\sigma}/(1+e^{(x-\mu)/\sigma})\right). \tag{8}$$

2.6 *Log likelihood*

Let $X_1, X_2, ..., X_{n_i}$ be a random sample of size n_i from logistic distribution and let $X_{(1)}, X_{(2)}, ..., X_{(n_i)}$ be the order statistic based on $X_1, X_2, ..., X_{n_i}$. The stress rate factor ξ at i^{th} stress level is $\xi_i = \psi_i/\psi_2$, where ψ_i is the stress rate ψ at the i^{th} stress level ; i = 1, 2. For the high stress rate, ψ_2, $\xi_2 = 1$, and the low stress rate factor is $\xi_1 = \psi_1/\psi_2$. The location parameter of lifetime distribution of a unit tested under stress rate factor ξ_i is

$$\mu(\xi_i) = \ln(\eta(\xi_i)) = (1/(1+\gamma_1))(\gamma_0 + \gamma_1 \ln(s_0/\xi_i\psi_2) + \ln(1+\gamma_1));\ i = 1,\ 2.$$

Define the indicator function $I \equiv I(x)$ in terms of the censoring time τ at stress ξ_i by

$$I = \begin{cases} 1, & \text{if } x \le \ln(\tau), \text{ failure observed by time } \ln(\tau) \\ 0, & \text{if } x > \ln(\tau), \text{ censored at time } \ln(\tau) \end{cases}.$$

The log-likelihood function L from an observation x at stress ξ is

$$L = I(x)\left\{-\ln\sigma + \left(x-\mu(\xi)\right)/\sigma - 2\ln(1+\exp\left(\left(x-\mu(\xi)\right)/\sigma\right)\right\} - \bar{I}(x)\ \ln\left(1+\exp\left(\tau'\right)\right).$$

Since ramp-test is to be carried out in two test chambers, namely, one in which test units are tested at low stress rate and the other in which they are tested at higher stress rate, so τ' takes the form τ_1' and τ_2' accordingly, where τ_1' is taken as the nq_1^{th} order statistic of a random sample of size n from the standard logistic distribution under low stress rate factor ξ_1, and τ_2' as nq_2^{th} order statistic from the same distribution under high stress rate factor ξ_2.

The maximum likelihood estimates $\hat{\gamma}_0, \hat{\gamma}_1$, and $\hat{\alpha}$ are the parameter values that maximize the sample log-likelihood L summed over all the test units.

3. THEORITICAL DERIVATIONS

3.1 *Fisher information matrix*

The elements of the Fisher information matrix are the negative expectations of second order partial derivatives of the log-likelihood function of a test unit with respect to γ_0, γ_1, and α.

Since τ_1' and τ_2' are taken as the nq_1^{th} and nq_2^{th} order statistics of a random sample of size n from the standard logistic distribution, the expectations can be approximated by replacing τ_1' by $E[\tau_1'] \approx H^{-1}(q_1) = z_{q_1}$ and τ_2' by $E[\tau_2'] \approx H^{-1}(q_2) = z_{q_2}$ (Schneider(20)), where $H^{-1}(\cdot)$ is the inverse function of the standard logistic distribution, and q_i is proportion of failures at stress rate ψ_i, i = 1, 2. The Fisher information matrix with ramp rate factor, ξ_i, is obtained as:

$$F_{\xi_i}(\gamma_0,\gamma_1,\alpha)=\begin{bmatrix} E\{-\partial^2 L_i/\partial\gamma_0^2\} & E\{-\partial^2 L_i/\partial\gamma_0\partial\gamma_1\} & E\{-\partial^2 L_i/\partial\gamma_0\partial\alpha\} \\ E\{-\partial^2 L_i/\partial\gamma_0\partial\gamma_1\} & E\{-\partial^2 L_i/\partial\gamma_1^2\} & E\{-\partial^2 L_i/\partial\gamma_1\partial\alpha\} \\ E\{-\partial^2 L_i/\partial\gamma_0\partial\alpha\} & E\{-\partial^2 L_i/\partial\gamma_1\partial\alpha\} & E\{-\partial^2 L_i/\partial\alpha^2\} \end{bmatrix}$$

where the values of these elements are given in Appendix A.

Since n_1 units are tested under stress rate factor $\xi_1 = \psi_1/\psi_2$, and remaining, n_2, units are tested under stress rate factor $\xi_2 = 1$, the Fisher information matrix for the plan with a sample of n independent items at two stress levels is

$$F = n\phi F_{\xi_1}(\gamma_0,\gamma_1,\alpha) + n\bar{\phi} F_{\xi_2}(\gamma_0,\gamma_1,\alpha).$$

3.2 *Asymptotic variance of the test-statistic*

For any plan, the asymptotic variance-covariance matrix of the maximum likelihood estimates $\hat{\gamma}_0$, $\hat{\gamma}_1$ and $\hat{\alpha}$ is the inverse of the corresponding Fisher information matrix, i.e.,

$$\Sigma=\begin{bmatrix} Var[\hat{\gamma}_0] & Cov[\hat{\gamma}_0,\hat{\gamma}_1] & Cov[\hat{\gamma}_0,\hat{\alpha}] \\ Cov[\hat{\gamma}_1,\hat{\gamma}_0] & Var[\hat{\gamma}_1] & Cov[\hat{\gamma}_1,\hat{\alpha}] \\ Cov[\hat{\alpha},\hat{\gamma}_0] & Cov[\hat{\alpha},\hat{\gamma}_1] & Var[\hat{\alpha}] \end{bmatrix} = F^{-1}.$$

The asymptotic variance of the test-statistic T is given by

$$Asvar(T) = Asvar(\hat{\mu}_0) - 2k\, Ascov(\hat{\mu}_0,\hat{\sigma}_0) + k^2\, Asvar(\hat{\sigma}_0), \tag{9}$$

where $Asvar(\hat{\mu}_0)$, $Ascov(\hat{\mu}_0,\hat{\sigma}_0)$ and $Asvar(\hat{\sigma}_0)$ are obtained from the elements of inverse of the Fisher information matrix.

3.3 *Operating characteristic (OC) curve*

Based on the asymptotic distribution theory, the test-statistic T is asymptotically normally distributed with mean $\mu_0 - k\sigma_0$ and variance= V/n. The standardized variate,

$$U = \sqrt{n}\left[T-(\mu_0-k\sigma_0)\right]\big/\sqrt{V} \sim N(0, 1) \text{ as } n \to \infty, \text{ where } Asvar(T) = V/n \tag{10}$$

The OC curve is obtained by plotting L_p against the fraction nonconforming, p, where

$$L_p = P[T \geq L'] = 1-\Phi\left[U \geq \left(\sigma_0 \times \sqrt{n}\,(z_P + k)\right)\big/\sqrt{V}\right], \tag{11}$$

and $z_p = (L' - \mu_0)/\sigma_0$ is the quantile of the standard logistic distribution corresponding to the fraction nonconforming p, and Φ is the standard normal distribution function.

4. OPTIMAL PLANS

The optimal plans under ramp-stress have been explained in 4.1 and 4.2 without and with cost consideration respectively.

4.1 *Optimal plan without cost consideration*

The optimal plan is obtained using variance optimality criterion. It consists in finding out optimal sample size, lot acceptability constant, low stress rate, and sample proportion allocated to each stress such that producer's risk and consumer's risk are safeguarded by minimizing V = nAsvar(T) (see (9) and (10)). The high stress rate ψ_2 is known from technical considerations.

4.2 *Optimal plan with cost consideration*

The optimal plan is determined by finding out optimum sample size, lot acceptability constant, sample proportion allocated to each stress, and low stress rate by minimizing the expected total cost per lot comprising warranty costs with respect to acceptance or rejection of the lot, sampling cost, and testing cost such that producer's risk and consumer's risk are safeguarded. The high stress rate ψ_2 is known from technical considerations. Bilevel programming approach is used for this purpose.

5. COST STRUCTURE

In this section cost structure that takes into consideration testing cost, sampling cost, and warranty costs based on rejection and acceptance of a lot for adoption of an optimal test planning has been obtained.

A warranty policy is defined mainly by two elements, viz., the period of coverage, and the terms of payment or compensation to the buyer. The commonly used warranty policies are **free replacement warranty** (FRW), and **pro-rata warranty** (PRW). The FRW does not charge any fee from the consumer during the term of warranty; the cost of replacement or repair of the warranted item is covered completely by the manufacturer during the warranty period. The **pro-rata warranty** (PRW) involves a sharing of the repair or replacement cost by the manufacturer and customer based on some product age dependent formula, the most common being linear. Occasionally, FRW, and PRW can be combined as one policy, called a general rebate warranty (GRW), to provide consumers with more choices (see Murthy and Blischke (2000)).

The company takes responsibility for the product warranty once the batch has been accepted by the company; and the periods of FRW and PRW are predetermined as 'c_f', and 'c_p', respectively. However, the pro-rata warranty of the producer between c_f, and c_p is assumed to vary in time instead of being constant. As a result, the warranty costs of the company could be expressed by a piecewise function of life time X which is given as:

$$\mathrm{Cost}_w(X) = \begin{cases} C_w, & \text{if } x < c_f \\ C_w(c_p - x)/(c_p - c_f), & \text{if } c_f \le x < c_p, \\ 0, & \text{if } x \ge c_p \end{cases}$$

where X is the lifetime of the product .

Since X follows log-logistic distribution, therefore the expected warranty cost per product is given by

$$E(\gamma_0,\alpha)=C_w\int_0^{c_f} f(t)dt+C_w\int_{c_f}^{c_p}((c_p-t)/(c_p-c_f))f(t)dt\ .$$

Thus, the expected cost resulting from selling the acceptance lot is given by

$$W(\gamma_0,\alpha)=(N-n)E(\gamma_0,\alpha)P[T\geq L'\mid p_1]$$

and the expected rejection cost resulting from the rejecting the lot is given by

$$W(\gamma_0,\alpha)=(N-n)C_{re}P[T< L'\mid p_1]$$

Let C_s be the cost of sampling and putting an item on test and C_τ be the cost per unit of test time. Because the sampling cost C_s, and the testing cost C_τ can both be obtained by simply multiplying the unit cost of sampling with the sample size, and multiplying the unit cost of testing with the censoring time, i.e. 'nC_s', and 'τC_τ', respectively, therefore, the expected total cost per lot for the given sampling plan is given by

$$ECost(n,\gamma_0,\alpha)=nC_s+\tau C_\tau+W(\gamma_0,\alpha)+R(\gamma_0,\alpha)\ , \tag{15}$$

6. DESIGN OF OPTIMAL SAMPLING PLAN

In this section, the optimal sampling plans under ramp-stress based on variable SSP for proportion nonconforming has been devised.

6.1 Optimization problem without cost consideration

In determining an optimal sampling plan for two given points $(p_1,\tilde{\alpha})$ and $(p_2,\tilde{\beta})$ on the OC curve, the two equations have to be solved for n^* and k^*, where $w_{\tilde{\alpha}}$ and $w_{1-\tilde{\beta}}$ denotes the quantiles of the standard normal distribution.

$$w_{\tilde{\alpha}}-\left(\sigma_0\times\sqrt{n}\left(z_{P_1}+k\right)\right)\Big/\sqrt{V}=0, \tag{12}$$

$$w_{1-\tilde{\beta}}-\left(\sigma_0\times\sqrt{n}\left(z_{P_2}+k\right)\right)\Big/\sqrt{V}=0. \tag{13}$$

On solving (12) and (13) for n^* and k^*, we have

$$n^*=\left(\left(w_{\tilde{\alpha}}-w_{1-\tilde{\beta}}\right)\Big/\left(z_{P_1}-z_{P_2}\right)\right)V\Big/\sigma_0^2, \tag{14a}$$

and

$$k^*=\left(w_{1-\tilde{\beta}}z_{P_1}-w_{\tilde{\alpha}}z_{P_2}\Big/w_{\tilde{\alpha}}-w_{1-\tilde{\beta}}\right). \tag{14b}$$

The value of k^* is determined by the two points $(p_1,\tilde{\alpha})$ and $(p_2,\tilde{\beta})$ on the OC curve and n^* depends on these two points and V. See Schneider (1985) for reference.

To minimize total sample size n, it is reasonable to design the test plan so that V is minimized.

Thus, the optimal design problem for obtaining optimal sampling plan under ramp-test with time-censored data is formulated as:

Minimize V

$$\begin{aligned} \text{s.t.}\quad & 0 \le \phi \le 1, \\ & \phi + \bar{\phi} = 1, \\ & \phi P_1 \ge 0.1, \\ & \bar{\phi} P_2 \ge 0.1, \\ & 0 < \xi_1 < 1. \end{aligned} \tag{15}$$

Since the sample size is unknown while minimizing V, therefore instead of giving minimum mean number of failures (MMNF) a pre-specified value in advance, 0.1, say, is assigned to the ratio of MMNF and the sample size. Another suitable number for MMNF can be specified according to cost, time, and precision implications.

6.2 Optimization problem with cost consideration

Bilevel-programming problem is a hierarchical mathematical optimization problem containing an optimization problem in the constraints. In this paper, in the first stage, optimum stress rates and sample proportion allocated to each stress level are obtained by minimizing Asvar(T); followed by the second stage in which optimal sample size has been obtained by minimizing the expected total cost per lot of the sampling plan. To obtain an optimal total sample size, it is necessary to make sufficiently low size of risk that the producer and consumer are willing to accept, say at most $\tilde{\alpha}$ and $\tilde{\beta}$, respectively evaluated at the corresponding acceptable and rejectable non-conforming proportions p_1 and p_2. Thus, the optimum sample size is determined by minimizing the expected warranty cost per lot subject to the constraints $P[T \ge L' \mid p_1] \ge 1-\tilde{\alpha}$ and $P[T \ge L' \mid p_2] \le \tilde{\beta}$ are satisfied. *Mathematica 8.0* has been used to formulate the optimal plan.

The optimal design problem can be formulated as:

$$\left\{ \begin{aligned} & \min_{\phi,\xi_1} \ \text{Asvar(T)} \\ \text{s.t.}\quad & 0 \le \phi \le 1, \\ & 0 < \xi_1 < 1, \text{ where } (\phi, \xi_1) \text{ solves} \\ & \left\{ \begin{aligned} & \min_{n} \ \text{ECost}(n,\ \gamma_0,\ \alpha) \\ \text{s.t.}\quad & P[T \ge L' \mid p_1] \ \ge\ 1-\tilde{\alpha}, \\ & P[T \ge L' \mid p_2] \le \ \tilde{\beta},\ N \ge n, n \in Z^{+} \end{aligned} \right. \end{aligned} \right. \tag{16}$$

Let

$$\Psi_1 = \{(\phi, \xi_1) : 0 \le \phi \le 1,\ 0 < \xi_1 < 1\},$$

$$\Psi_2 = \{(\text{ECost}(n,\ \gamma_0,\ \alpha)) : P[T \ge L' \mid p_1] \ \ge\ 1-\tilde{\alpha},\ P[T \ge L' \mid p_2] \le \ \tilde{\beta},\ N \ge n, n \in Z^{+}\},$$

and $(\phi^*, \xi_1^*) = \underset{\phi,\xi_1}{\text{Argmin}}\{n/|F| : (\phi, \xi_1) \in \Psi_1\}$. Then, the optimization problem reduces to:

$$\underset{n}{\text{Min}} \left\{ ECost(n,\ \gamma_0, \alpha)\ :\ n \in \Omega_2,\ (\phi, \xi_1) = (\phi^*, \xi_1^*) \right\} \quad \text{(using (16))}.$$

Since, the optimum ramp-test depends on $q_1, q_2,\ s_0, \alpha, \gamma_0, \gamma_1$ and ψ_2; one must obtain their values from experience, similar data, or a preliminary test to achieve the minimum of Asvar(T) in the ALTSP for the optimum values of ϕ and ξ_1. The optimal value of sample size is obtained by minimizing expected warranty cost per lot.

7. AN ILLUSTRATIVE EXAMPLE

Consider an IC manufacturing company, XYZ. Since IC products are relatively expensive, it is worthwhile to conduct a more cost saving life testing program in which an appropriate sample size can be found out to determine the acceptance or rejection of a batch of IC products with the objective of cost minimization for the company. However, given that various relevant costs would influence the profits of XYZ Corporation in the process of the life test, it would be essential for the managers to select the appropriate decision parameters regarding the accelerated life test sampling plan.

7.1 Without cost consideration

The determination of failure-censored optimal sampling plan under ramp-stress based on SSP for proportion nonconforming depends on: $\alpha,\ \gamma_0,\ \gamma_1,\ s_0,\ \psi_2,\ q_1$, and q_2.
One must obtain their values from experience, similar data, or a preliminary test. $(p_1, 1-\tilde{\alpha})$ and $(p_2,\ \tilde{\beta})$ are chosen by taking in to consideration producer's and consumer's interests. The hypothetical data set used is:

$$\alpha = 1.5,\ \gamma_0 = 7.5,\ \gamma_1 = 0.1,\ s_0 = 20,\ \psi_2 = 1.1,\ q_1 = q_2 = 0.2.$$

Step 1. OC Curve
Select two points $(p_1,\ 1 ? \tilde{\alpha}$ and $(p_2,\ \tilde{\beta})$ on the OC curve as (0.01, 0.99) and (0.10, 0.10), respectively.

Step 2. Acceptability constant
Compute the acceptability constant k, using (14b).

Step 3. Optimal plan
Determine (ϕ_1^*, ξ_1^*) which minimize V evaluated at k^*. The value of k^* obtained is $k^* = 3.049$.
The optimal low stress rate factor, optimal proportion of test units allocated at low and high stress rates, optimal sample size, optimal number of samples at low stress rate and at

high stress rate, respectively, of the test plan are: $\xi_1^* = 0.129$, $\phi_1^* = 0.811$ and $\phi_2^* = 0.189$.

Thus, the optimum low stress rate is $\psi_1^* = \xi_1^* \psi_2 = 0.499 \approx 0.5$ KV/sec. Optimal sample size is 38 and optimal number of units to be tested at low stress rate ψ_1 is 31, and at high stress rate ψ_2 is 7.

Step 4. Simulated data

The simulated data set is given in Table 1 include 38 simulated observations for low and high stress rates.

Table 1. Simulated Data: Failure-censored sample on a simple ramp-test using log-logistic model ($\alpha = 1.5$, $\gamma_0 = 7.5$, $\gamma_1 = 0.1$, $s_0 = 20$, $\psi_2 = 1.1$, $\mathbf{q_1 = q_2 = 0.2,\ k^* = 3.049}$)

Stress rate	Failure times	Number of censored units
$\boldsymbol{\xi_1^* = 0.129}$ (Low Stress rate)	6.459, 7.095, 5.940, 7.112, 6.720, 6.931, 7.304, 6.367, 6.277, 5.973, 7.214, 6.820, 5.943, 6.493, 5.951, 7.243, 6.675, 6.963, 7.007, 7.135, 6.147, 5.627, 6.189, 6.105	7
$\boldsymbol{\xi_2 = 1}$ (High Stress rate)	6.585, 6.489, 6.850, 6.627, 6.623, 6.802	1

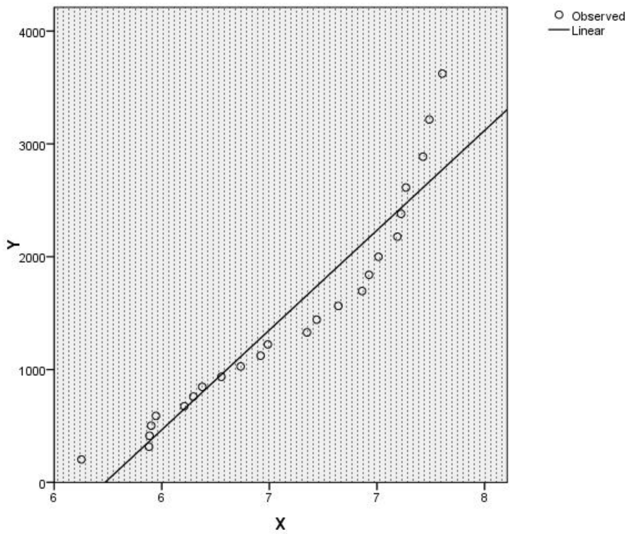

Figure 1. The log-logistic probability plot at low stress for the simulated data
$(\boldsymbol{\alpha = 1.5,\ \gamma_0 = 7.5,\ \gamma_1 = 0.1,\ s_0 = 20,\ \psi_2 = 1.1,\quad q_1 = q_2 = 0.2,\ k^* = 3.049})$

Step 5. Graphical goodness of fit
Figure 1 shows the associated log-logistic probability plot at low stress. The failure times in Table 1 at low stress are arranged in increasing order and are ranked from 1,2,…,i,…,n. The plotted points tend to follow straight line, which is substantiated by fitting straight line to these points resulting in high value of coefficient of determination $r^2 = 0.937$. The log-logistic distribution therefore appears to describe the data adequately.

Similarly, Figure 2 shows the associated log-logistic probability plot at high stress. The plotted points tend to follow straight line, which is substantiated by fitting straight line to these points resulting in high value of coefficient of determination $r^2 = 0.603$. The log-logistic distribution therefore appears to describe the data adequately.

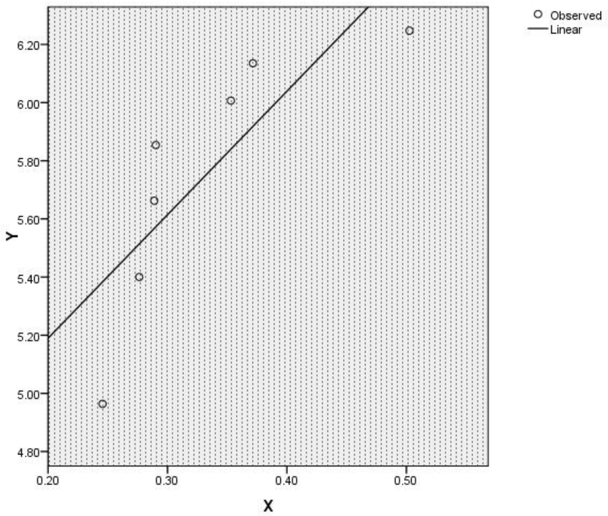

Figure 2. The log-logistic probability plot at high stress for the simulated data
$(\alpha = 1.5,\ \gamma_0 = 7.5,\ \gamma_1 = 0.1,\ s_0 = 20,\ \psi_2 = 1.1,\ q_1 = q_2 = 0.2,\ k^* = 3.049)$

Step 6. MLEs of the design parameters
The MLEs of the design parameters obtained using simulated data in Table 1 are $\hat{\alpha} = 1.278, \hat{\gamma}_0 = 1.770, \hat{\gamma}_1 = 1.089$.

These are obtained by using the NMaximize option of *Mathematica* 8.0.

Step 7. Decision Criterion
For given lower specification limit L′, lot is accepted if $\hat{\mu}_0 - k\hat{\sigma}_0 = \hat{\mu}_0 - 3.049\hat{\sigma}_0 > L'$, otherwise rejected.

Figure 3 shows changes in the OC curve by taking values of $(p_1, 1-\tilde{\alpha})$ and $(p_2, \tilde{\beta})$ as (0.01, 0.99), (0.10, 0.10).

Table 2 depicts optimum sampling plan for various values of p_1 and p_2 when $(1-\tilde{\alpha}, \tilde{\beta}) = (0.99, 0.10)$. It is observed that for given p_1 as p_2 increases n^* and V^* decrease.

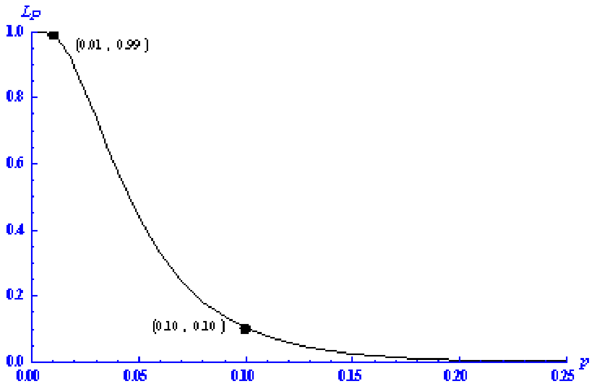

Figure 3. OC Curve under failure-censored variable SSP for proportion nonconforming without cost consideration ($\alpha = 1.5$, $\boldsymbol{\gamma_0 = 7.5, \gamma_1 = 0.1,}$ $\boldsymbol{s_0 = 20, \psi_2 = 1.1, q_1 = q_2 = 0.2, k^* = 3.049}$)

Table 2. Optimal ALTSP under failure-censored variable SSP for proportion nonconforming without cost consideration when $\mathbf{p_1}$ and $\mathbf{p_2}$ change

$$(\boldsymbol{\alpha = 1.5, \gamma_0 = 7.5, \gamma_1 = 0.1, s_0 = 20, \psi_2 = 1.1, q_1 = q_2 = 0.2})$$

$1-\tilde{\alpha}$	$\tilde{\beta}$	p_1	p_2	k^*	ξ_1^*	ϕ_1^*	ϕ_2^*	V^*	n^*	n_1^*	n_2^*
0.99	0.10	0.001	0.01	5.416	0.866	0.586	0.414	33.261	182	107	75
			0.05	4.352	0.121	0.375	0.625	18.873	35	13	22
			0.10	3.870	0.293	0.909	0.091	13.854	18	17	2
		0.01	0.05	3.531	0.183	0.789	0.211	10.879	117	92	25
			0.10	3.049	0.129	0.811	0.189	7.449	38	31	7
			0.15	2.751	0.050	0.622	0.378	5.792	21	13	8

7.2 With cost consideration

Let the product lot size N be 2000 and the manager assesses various cost (in appropriate monetary units) with regard to the lot of IC products as:

$$C_s = 250,\ C_{re} = 1200,\ C_w = 2500,\ C_\tau = 500.$$

The components are sold under the general rebate warranty policy with $c_f = 100$ and $c_p = 450$ unit times. An agreement between the producer and the consumer is considered to obtain an optimal failure censored acceptance sampling plan such that the respective

probabilities of rejecting a good item and accepting a bad item are at most $\tilde{\alpha}$ = 0.01 and $\tilde{\beta}$ = 0.05, respectively.
The following steps yield an optimum ramp-stress ALTSP.

Step1. Select two points $(p_1,\ 1?\tilde{\alpha}$ and $(p_2,\ \tilde{\beta})$ as (0.01, 0.99) and (0.05, 0.10) on the OC curve.

Step2. Compute the acceptability constant k^*, using formula in equation (14b).
Since $w_{\tilde{\alpha}}$ = –2.326, $w_{1-\tilde{\beta}}$ = 1.282, z_{p_1} = –4.595, and z_{p_2} = –2.944, we have k^* = 3.531.

Step3. Consider a hypothetical data set as $\alpha = 1.5,\ \gamma_0 = 7.5\ ,\ \gamma_1 = 0.1,\ s_0 = 20,\ \psi_2 = 1.1\ ,$ $q_1 = q_2 = 0.2$. Determine $\left(\phi^*,\ \xi^*\right)$ which minimize asymptotic variance V evaluated at k^*. The optimal stress rate factor is obtained as $\xi_1^* = 0.189$. The optimal proportion of test units allocated at low and high stress rates, respectively, are

$$\phi^* = 0.805 \Rightarrow \bar{\phi}^* = 0.195\ .$$

Step 4. The value of optimal sample size n^* is 189 which is precisely the minimal number of failures. The optimal number of samples at low stress rate n_1^* = 149 and at high stress rate n_2^* = 40. The optimal cost is 6,72,184 monetary units. The test is run until the number of failures at low stress rate r_1^* = 15 and at high stress rate r_2^* = 3 are reached. Let the lower specification limit of L hours (say) in the used condition, indicating that items with lifetime shorter than ln L are nonconforming. Using the data from the test, MLEs $\hat{\mu}$ and $\hat{\sigma}$ are computed. Thus, the lot is accepted if $\hat{\mu}_0 - k^*\hat{\sigma}_0 = \hat{\mu}_0 - 3.531\hat{\sigma}_0$ > ln L and reject it otherwise.

Table 3. Optimal ALTSP under failure-censored variable SSP for proportion nonconforming with cost consideration when $\mathbf{p_1}$ and $\mathbf{p_2}$ change

$(\alpha = 1.5,\ \gamma_0 = 7.5,\ \gamma_1 = 0.1,\ s_0 = 20,\ \psi_2 = 1.1,\ \ q_1 = q_2 = 0.2)$

$1-\tilde{\alpha}$	$\tilde{\beta}$	p_1	p_2	k^*	ξ_1^*	ϕ^*	V^*	n^*	ECost(n)
0.99	0.10	0.001	0.01	5.416	0.866	0.586	47.165	320	540,395
			0.05	4.352	0.121	0.375	23.513	81	508,242
			0.10	3.870	0.293	0.909	15.954	44	503,549
		0.01	0.05	3.531	0.189	0.805	11.807	189	522,361
			0.10	3.049	0.128	0.809	7.590	73	507,221
			0.15	2.751	0.050	0.622	5.962	45	503,609

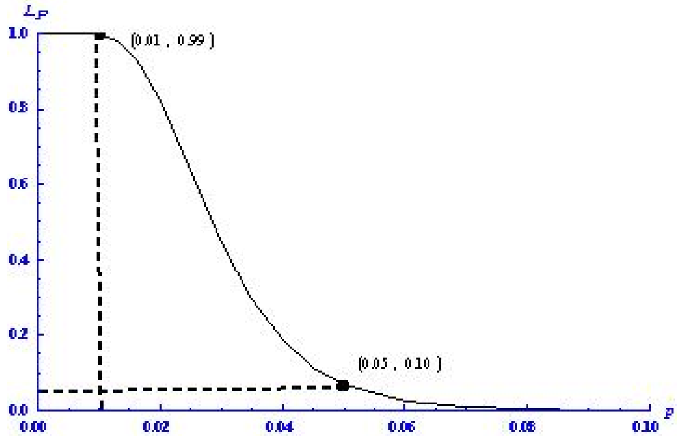

Figure 4. OC Curve under failure-censored variable SSP for proportion nonconforming with cost consideration ($\alpha = 1.5$, $\gamma_0 = 7.5$, $\gamma_1 = 0.1$, $s_0 = 20$, $\psi_2 = 1.1$, $q_1 = q_2 = 0.2$, $k^* = 3.531$)

Table 3 presents optimum sampling plans assuming, $\alpha = 1.5$, $\gamma_0 = 7.5$, $\gamma_1 = 0.1$, $s = 20$, $\psi_2 = 1.1$, $q_1 = q_2 = 0.2$ when $(p_1, 1-\tilde{\alpha})$ and $(p_1, \tilde{\beta})$ are (0.01, 0.99), (0.05, 0.10), respectively. It shows that for fixed p_1 as p_2 increases acceptability constant, optimal sample size, and expected total cost also decrease.

The Figure 4 shows changes in the OC curve by taking values of $(p_1, 1 ? \tilde{\alpha}$ and $(p_2, \tilde{\beta})$ as (0.01, 0.99), (0.05, 0.10).

8. SENSITIVITY ANALYSIS

The sensitivity analysis identifies the sensitive parameters which need to be estimated with special care for the purpose of minimizing the risk of obtaining an erroneous optimal solution. In this section, the effects of % change in the pre-estimated parameters γ_0, γ_1, α in terms of the relative increase of asymptotic variance of test-statistic, T, are presented for failure-censored (Type-II censored) data set in Table 2.

The analysis is performed by changing only one parameter at a time and keeping others unchanged. The percentage deviation, PD, of an optimal setting is measured by $PD = (|Z^{**} - Z^*|/Z^*) \times 100$, where Z^* is the optimal asymptotic variance of T obtained with the given design parameters, and Z^{**} is the one obtained when the parameter is misspecified. Let n^{**} be the optimal sample size Z resulting from the misspecified parameter. Table 2 shows that if the pre-estimates deviate from the true values by $\pm 1\%$ to $\pm 5\%$, then the absolute relative change in asymptotic variance is not significantly large. Thus, the optimal setting of parameters is robust to the deviations from baseline parameters.

8.1 *Without cost consideration*

Table 4 shows that irrespective of whether the incorrect variance is smaller or larger than the true variance, if the percentage deviation in variance is small then the proposed optimum plan is robust.

Table 4. Optimal ALTSP when design parameter estimates change $(\boldsymbol{\alpha = 1.5, \gamma_0 = 7.5, \gamma_1 = 0.1, s_0 = 20, \psi_2 = 1.1, q_1 = q_2 = 0.2, k = 3.049, Z^* = 7.449, n^* = 38})$

% Change	α	γ_0	γ_1	ψ_1^*	ξ_1^*	ϕ^*	$\bar{\phi}^*$	Z^{**}	n^*	$\mathbf{n_1^*}$	$\mathbf{n_2^*}$	PD
1%	1.616	7.575	5.050	0.163	0.148	0.881	0.119	7.302	38	33	5	1.971
-1%	1.584	7.425	4.950	0.156	0.142	0.841	0.159	7.600	38	32	6	2.030
2%	1.632	7.650	5.100	0.146	0.133	0.844	0.156	7.160	38	32	6	3.883
-2%	1.568	7.350	4.900	0.181	0.164	0.898	0.102	7.756	38	34	4	4.123
3%	1.648	7.725	5.150	0.136	0.123	0.822	0.178	7.021	38	31	7	5.740
-3%	1.552	7.275	4.850	0.188	0.171	0.906	0.094	7.917	38	34	4	6.281
4%	1.664	7.800	5.200	0.132	0.120	0.821	0.179	6.887	38	31	7	7.544
-4%	1.536	7.200	4.800	0.191	0.174	0.902	0.098	8.083	38	34	4	8.507
5%	1.680	7.875	5.250	0.112	0.102	0.771	0.229	6.756	38	29	9	9.297
-5%	1.520	7.125	4.750	0.138	0.126	0.750	0.250	8.254	38	29	10	10.803

8.2 With cost consideration

In Table 5, it is observed that if the pre-estimates deviate from the true values by $\pm 1\%$ to $\pm 5\%$, then the absolute relative change in asymptotic variance is not significantly large. Thus, the optimal setting of Z is robust to the deviations from baseline parameters.

Table 5. Optimal ALTSP when design parameter estimates change
$(\boldsymbol{\alpha = 1.5, \gamma_0 = 7.5, \gamma_1 = 0.1, s = 20, \psi_2 = 1.1, q_1 = q_2 = 0.2, Z^* = 11.807, n^* = 189})$

% Change	α	γ_0	γ_1	ψ_1^*	ξ_1^*	ϕ^*	$\bar{\phi}^*$	Z^{**}	n^*	$\mathbf{n_1^*}$	$\mathbf{n_2^*}$	Ecost(n)	PD
1%	1.515	7.575	0.101	0.156	0.142	0.698	0.302	10.665	174	122	52	670282	9.671
-1%	1.485	7.425	0.099	0.202	0.184	0.777	0.223	11.100	180	140	40	671013	5.985
2%	1.530	7.650	0.102	0.151	0.138	0.699	0.301	10.457	172	120	52	669930	11.433
-2%	1.470	7.350	0.098	0.179	0.163	0.712	0.288	11.328	183	130	53	671393	4.056
3%	1.545	7.725	0.103	0.234	0.212	0.910	0.090	10.255	169	154	15	669587	13.145
-3%	1.455	7.275	0.097	0.195	0.178	0.734	0.266	11.563	186	136	50	671782	2.067
4%	1.560	7.800	0.104	0.101	0.092	0.600	0.400	10.059	167	100	67	669252	14.807
-4%	1.440	7.200	0.096	0.149	0.136	0.622	0.378	11.805	189	118	71	672181	0.017
5%	1.575	7.875	0.105	0.104	0.095	0.617	0.383	9.868	164	101	63	668924	16.422
-5%	1.425	7.125	0.095	0.122	0.111	0.554	0.446	12.055	192	106	86	672591	2.100

9. CONCLUSION

In this paper, we have obtained an optimum ramp-stress accelerated life test sampling plan based on log-logistic distribution under Type-II censoring, assuming inverse power law and a cumulative exposure model. The optimal plans consist in finding optimum sample size, sample proportion allocated to each stress, and stress rate factor such that producer's and consumer's interests are safeguarded. Variance optimality criterion is used when cost is not taken into consideration, and bilevel programming approach is used in cost optimization problem. The methods developed have been illustrated using numerical examples, and results of sensitivity analyses carried out with respect to ramp-stress ALTSP.

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Appendix

The expectations of negative of second order derivatives given L_i are

$$E[-\partial^2 L_i/\partial\gamma_0^{\,2}] = \alpha^2/3\left(1-1/\left(1-q_i\right)^3\right),$$

$$E[-\partial^2 L_i/\partial\gamma_0\partial\gamma_1] = E[-\partial^2 L_i/\partial\gamma_1\partial\gamma_0] = 1/(1+\gamma_1)\left(\alpha E[-\partial^2 L_i/\partial\gamma_0\partial\alpha] + T_1 E[-\partial^2 L_i/\partial\gamma_0^{\,2}]\right),$$

$$E[-\partial^2 L_i/\partial\gamma_0\partial\alpha] = E[-\partial^2 L_i/\partial\alpha\partial\gamma_0] = -\left(\int_{-\infty}^{z_{q_i}} u_i e^{-2u_i}/(1+e^{-u_i})^4 du_i + z_{q_i} q_i\left(1-q_i\right)^2\right),$$

$$E[-\partial^2 L_i/\partial\gamma_1^{\,2}] = 1/(1+\gamma_1)\left(\alpha E[-\partial^2 L_i/\partial\gamma_1\partial\alpha] + T_1 E[-\partial^2 L_i/\partial\gamma_0\partial\gamma_1]\right),$$

$$E[-\partial^2 L_i/\partial\gamma_1\partial\alpha] = E[-\partial^2 L_i/\partial\alpha\partial\gamma_1] = 1/(1+\gamma_1)\left(\alpha E[-\partial^2 L_i/\partial\alpha^2] + T_1 E[-\partial^2 L_i/\partial\gamma_0\partial\alpha]\right),$$

$$E[-\partial^2 \ln L_i/\partial\alpha^2] = 1/\alpha^2\left(\int_{-\infty}^{z_{q_i}} 2u_i^{\,2} e^{-2u_i}/(1+e^{-u_i})^4 du_i + q_i + z_{q_i}^2 q_i\left(1-q_i\right)^2\right),$$

where $\tau_i' = z_{q_i}$ and $H(\tau_i') = q_i$, $T_1 = 1/(1+\gamma_1)\left(\log(1+\gamma_1) + \gamma_1 \log(s_0/\xi_i\zeta_2) + \gamma_0\right)$.

These expectations are calculated with the aid of, $E[\partial \ln L_i/\partial\gamma_{i'}] = 0$, for $i' = 0,1, E[\partial \ln L_i/\partial\alpha] = 0$