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Reliability computation technique for ball bearing under the stress-strength model

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Abstract: Stress function of ball bearing is function of multiple stochastic factors and this system is so complex that analytical expression for reliability is difficult to obtain. To address this pressing problem, in this article, we have made an attempt to approximate system reliability of this important item based on reliability bounds under the stress strength setup. This article also provides level of error of this item. Numerical analysis has been adopted to show the closeness between the upper and lower bounds of this item.

Key Words: *ball bearing, level of error, reliability bounds, reliability approximation, stress strength model*

1. INTRODUCTION

Designing of items in manufacturing industries is an important task. The designation of an item should be done in such a way that its reliability is high. Now to assess the same one has to study the behaviour of the response function of the item which is random in nature. The response is function of its random components values. Furthermore, there are other factors also which makes the response function a random variable. These factors are like change of operators, uncertainty in the environmental situations, measure mental errors etc. Therefore, one should study the system response function. But in most cases the task is unmanageable due to complexity of the response function. As a way out, we have some alternative techniques.

These are (i) Taylor-series method (ii) Monte-Carlo method (iii) Quadrature method (iv) Discretization Method and (v) Discrete approximation method and (vi) Reliability approximation technique. Discretization approach plays important role for computing reliability of different complex engineering items in the recent past. Several author's have

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proposed several discretization procedure for computing reliability of complex engineering items.

D`Errico and Zaino (1988) suggested a moment equalization approach for discretizing stress and strength distribution. English et al (1996), in the stress-strength setup, applied this moment equalization rule, and they continued the discrete variable up to six points. They considered different alternative of strength parameters under normality of the factor variables and presented that the errors from discretization were on the lower side under the normal situation. This moment based discretizing approach of English et al has attracted attention to the reliability analyst. This approach has been referred as moment equalization technique. Roy and Ghosh (2009) made an attempt to discretize a continuous random variable by equating the raw moments of the original distribution and they discretized distribution. They carried out discretization of Exponential distribution. Barbiero (2010) proposed a discretizing method for reliability computation in complex stress-strength model.

Krishna and Pundir (2007) considered the discrete Maxwell distribution. They defined it using the discrete concentration approach of Roy (1993). Roy and Dasgupta (2001) presented discretizing procedure using discrete concentration of Roy (1993) and they provided a new approximating technique by using survival function. This method is mentioned as method of discrete concentration. Their approach was based on survival function and is easy to understand and implement. Further they considered four engineering items of importance and demonstrated the admissibility of their approach to that of English et al (1996) in terms of mean absolute deviation. Roy and Dasgupta (2002) proposed a discretization approach by using a survival function of a continuous random variable. They numerically showed that their discrete concentration approach has greater applicability because discrete mass points are not dependent on the normal law and can be used for Weibull setup.

Recently Roy and Ghosh (2009) introduced a new discretization technique that retains the basic structure of the failure rate function of the original life distribution. They discretized Rayleigh and Lomax distribution based on failure rate consideration. Their proposed approach was used for approximating the reliability of complex systems where exact determination of survival probability is analytically intractable. Ghosh et al (2011) established this approach for the Weibull distribution also. Ghosh et al (2013) described the usefulness of discretization of a random variable using the reversed hazard rate function.

All the approaches, mentioned above suffer from effects that the approximated reliability values remain constant towards changes in the value of the strength parameter. The authors concentrated on simulation study. But further manipulation in terms of design parameter can`t be under taken under their approaches. There are cases where discrete approximations are extremely weak. For example, under the Exponential setup where lack of memory property holds, the discretization approach doesn't offer close approximate value. So, to bridge this gap of the literature, authors have addressed bound based reliability approximation. This reliability approximation represents alternative solution to numerical integration method, Taylor`s series expansion method, Monte Carlo simulation technique, and discretization method.

Reliability approximation technique is an alternative technique of the above discussed techniques to approximate the system reliability of complex engineering items. This technique helps in finding reliability of complex engineering items when analytical evaluation of reliability is not tractable. Nayak and Roy (2012) proposed bound based reliability approximation under the Weibull, Rayleigh and Exponential setups for simplified form of I-beam. Here they showed that reliability of the item can be increased or decreased according to requirement. Numerical studies were also worked out by them to show the sharpness of the reliability upper and lower bounds. Nayak and Roy (2012) also studied a new approach for approximating reliability of a complex system under the Weibull setup. Here they approximated reliability of I-beam. Nayak (2013) established reliability approximation of hollow rectangular tube under the Weibull and Rayleigh setups. Nayak et al (2014) determined reliability of solid shaft under the Gamma set up. Nayak and Roy (2015) also examined bound based reliability approximation of an engineering item, resistor under the stress-strength model.
This article deals with reliability approximation of ball bearing under the Weibull set up. Level of error is also established for this important engineering item. One can increase or decrease reliability of this item using our approach by varying the design parameters. Therefore, our method will help to the manufacturing industries for producing high reliable product.

Abbreviations

$F_x(X)$: Cumulative distribution (df) function
W (λ, α) : Weibull distribution with scale parameter λ and shape parameter α
E(X) : Expectation on random variable X
R : System reliability
U(λ_P, λ_E, β, μ, □, θ, ϕ, α) : Reliability upper bound of ball bearing under the Weibull setup
L(λ_P, λ_E, β, μ, □, θ, ϕ, α) : Reliability lower bound of ball bearing under the Weibull setup
R(λ_P, λ_E, β, μ, □, θ, ϕ, α) : Reliability approximation of ball bearing under the Weibull setup
E(λ_P, λ_E, β, μ, □, θ, ϕ, α) : Level of error of ball bearing under the Weibull setup

2. RELIABILITY BOUNDS

Ball bearing is a common type of item, used in almost mechanical system. It is a one type of complex engineering item. Analytical evaluation of reliability of this crucial engineering item is difficult to manage. The form of contact stress of ball bearing is given by the following relational model:

$$S_c = 0.616\sqrt[3]{PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2}$$

where, P is dynamic applied load, E is modulus of elasticity, d_1 is diameter of ball bearing, $d_2$is diameter of bearing race, S_c is contact stress.
We now first determine reliability bounds of ball bearing and on the basis of these bounds, we will find approximated reliability values of this important engineering item in terms of design parameters. For this purpose, we have assumed the following parametric setups.

We assume that P $\sim W(\lambda_P, \square)$, E$\sim W(\lambda_E, \theta)$, $d_1 \sim W(\beta, \phi)$ and $d_2 \sim W(\mu, \alpha)$. Let us assume that the built in contact strength X follows i.e. X$\sim W(\lambda, \psi)$. Another assumption is that P, E, d_1, d_2, and X are independent. Under this distributional setup, we are interested to determine reliability upper and lower bounds, reliability approximation and extent of error. If X is the strength and Y is the contact stress of this item then under the stress strength model reliability, R, for this item is given by

$$R = P(X > Y)$$
$$= P\{X > .616 \sqrt[3]{PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2}\}$$
$$= E_P E_E E_{d_1} E_{d_2}\{ X > .616 \sqrt[3]{PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2} \mid P = p, E = e, D_1 = d_1 and D_2 = d_2\}$$
$$= E_P E_E E_{d_1} E_{d_2}\{e^{-\lambda(.616)^{\psi}\left(PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\right)^{\frac{\psi}{3}}}\}$$

Hence, the unconditional reliability value is given by

$$R = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-\lambda(.616)^{\psi}\left(PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\right)^{\frac{\psi}{3}}} dF_P dF_E dF_{d_1} dF_{d_2}$$

It is very difficult to integrate R with the above expression due to mathematical complexity. So, for the sake of simplicity, we will integrate R with the choice of ψ =3. Therefore, R reduces to

$$R = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-(.616)^3 \lambda PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2} dF_P dF_E dF_{d_1} dF_{d_2} \quad (1)$$

Result 1: Reliability upper bound under the Weibull frame work for Ball bearing is given by

$$U(\lambda_P, \lambda_E, \beta, \mu, \square, \theta, \phi, \alpha) = 1 - a^3 \lambda \frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}} \frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}} [\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - 2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}$$
$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + \Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}] + a^6 \frac{\lambda^2}{2} \frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}} \frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}} [\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}} - 4\,\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}$$
$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + 6\,\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\,\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}} - 4\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{3}{\alpha}\right)\mu^{\frac{3}{\alpha}} +$$
$$\Gamma\left(1-\frac{4}{\alpha}\right)\mu^{\frac{4}{\alpha}}],$$

where, a =.616

Proof: Note that, for X > 0, $e^{-X} \leq 1 - X + \frac{X^2}{2}$ (a)

Using (a) in (1), we get

$$R \leq \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty [1 - a^3 \lambda PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2 + a^6 \frac{\lambda^2 P^2 E^4}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^4] dF_P dF_E dF_{d_1} dF_{d_2} (2)$$
$$= \int_0^\infty \int_0^\infty \int_0^\infty f(E, d_1, d_2)\, dF_E dF_{d_1} dF_{d_2} \quad (3)$$

where,

$$f(E, d_1, d_2) = \int_0^\infty [1 - \lambda PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2 + \frac{\lambda^2 P^2 E^4}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^4] dF_P$$
$$= 1 - \lambda E^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2 E(P) + \frac{\lambda^2 E^4}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^4 E(P^2)$$

$$=1-\lambda E^{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}+\frac{\lambda^{2}E^{4}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}} \quad (4)$$

Substituting (4) in (3) we get

$$R\leq \int_0^\infty\int_0^\infty\int_0^\infty\left[1-\lambda E^{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}+\frac{\lambda^{2}E^{4}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\right]dF_E\,dF_{d_1}dF_{d_2}$$

$$=\int_0^\infty\int_0^\infty w(d_1,d_2)\,dF_{d_1}dF_{d_2} \quad (5)$$

where,

$$w(d_1,d_2)=\int_0^\infty\left[1-\lambda E^{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}+\frac{\lambda^{2}E^{4}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\right]dF_E$$

$$=1-\lambda E(E^{2})\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}+\frac{\lambda^{2}E(E^{4})}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}$$

$$=1-\lambda\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}}+\frac{\lambda^{2}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma(1+\frac{4}{\theta})}{\lambda_E^{\frac{4}{\theta}}} \quad (6)$$

Substituting (6) in (5), we get

$$R\leq \int_0^\infty\int_0^\infty\left[1-\lambda\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}}+\frac{\lambda^{2}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma(1+\frac{4}{\theta})}{\lambda_E^{\frac{4}{\theta}}}\right]dF_{d_1}dF_{d_2}$$

$$=\int_0^\infty w^{*}(d_2)\,dF_{d_2} \quad (7)$$

$$w^{*}(d_2)=\int_0^\infty\left[\left[1-\lambda\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{2}\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}+\frac{\lambda^{2}}{2}\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^{4}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma(1+\frac{4}{\theta})}{\lambda_E^{\frac{4}{\theta}}}\right]\right]dF_{d_1}$$

$$=1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}\left\{E\left(\frac{1}{d_1^{2}}\right)-\frac{2}{d_2}E\left(\frac{1}{d_1}\right)+\frac{1}{d_2^{2}}\right\}+\frac{\lambda^{2}}{2}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}\left\{E\left(\frac{1}{d_1^{4}}\right)-\frac{4}{d_2}E\left(\frac{1}{d_1^{3}}\right)+\frac{6}{d_2^{2}}E\left(\frac{1}{d_1^{2}}\right)-\frac{4}{d_2^{3}}E\left(\frac{1}{d_1}\right)+\frac{1}{d_2^{4}}\right\}$$

$$=1-\lambda\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}}\left\{\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}-\frac{2}{d_2}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}+\frac{1}{d_2^{2}}\right\}+\frac{\lambda^{2}}{2}\frac{\Gamma(1+\frac{2}{\square})}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma(1+\frac{4}{\theta})}{\lambda_E^{\frac{4}{\theta}}}\left\{\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}}-\frac{4}{d_2}\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}+\frac{6}{d_2^{2}}\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}-\frac{4}{d_2^{3}}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}+\frac{1}{d_2^{4}}\right. \quad (8)$$

Substituting (8) in (7), we get

$$R\leq\int_0^\infty\left[1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}\left\{\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}-\frac{2}{d_2}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}+\frac{1}{d_2^{2}}\right\}+\frac{\lambda^{2}}{2}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}\left\{\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}}-\frac{4}{d_2}\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}+\frac{6}{d_2^{2}}\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}-\frac{4}{d_2^{3}}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}+\frac{1}{d_2^{4}}\right\}\right]dF_{d_2}$$

$$=1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}[\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\text{- }2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}}+\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}]+$$
$$\frac{\lambda^2}{2}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}[\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}}-4\,\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}}+6\,\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}$$
$$\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}-4\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{3}{\alpha}\right)\mu^{\frac{3}{\alpha}}+\Gamma\left(1-\frac{4}{\alpha}\right)\mu^{\frac{4}{\alpha}} \tag{9}$$

Therefore, under the Weibull setup, reliability upper bound for Ball bearing is given by

$$U(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha)=1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}[\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\text{- }2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}$$
$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}}+\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}]+\frac{\lambda^2}{2}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}[\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}}-4\,\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}$$
$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}}+6\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\,\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}-4\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{3}{\alpha}\right)\mu^{\frac{3}{\alpha}}+$$
$$\Gamma\left(1-\frac{4}{\alpha}\right)\mu^{\frac{4}{\alpha}}$$

Result2: Reliability lower bound under the Weibull frame work for ball bearing is given by

$$L(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha)=1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}[\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\text{- }2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}$$
$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}}+\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}]$$

Proof: We also note that, for X > 0, $e^{-X}\ \geq\ 1-X$ (b)

Using lemma (b) in (1), we get

$$R\geq\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty[1-\lambda PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2]dF_P dF_E dF_{d_1}dF_{d_2}$$
$$=\int_0^\infty\int_0^\infty\int_0^\infty f(E,d_1,d_2)\,dF_E dF_{d_1}dF_{d_2} \tag{10}$$

where,

$$f(E,d_1,d_2)=\int_0^\infty[1-\lambda PE^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2]dF_P$$
$$=1-\lambda E^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2E(P)$$
$$=1-\lambda E^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}} \tag{11}$$

Substituting (11) in (10) we get

$$R\geq\int_0^\infty\int_0^\infty\int_0^\infty[1-\lambda E^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}]\,dF_E dF_{d_1}dF_{d_2}$$
$$=\int_0^\infty\int_0^\infty w(d_1,d_2)dF_{d_1}dF_{d_2} \tag{12}$$

where,

$$w(d_1,d_2) = \int_0^{\infty}[1-\lambda E^2\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\;]\,dF_E$$

$$= 1-\lambda E(E^2)\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}$$

$$= 1-\lambda\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}} \tag{13}$$

Substituting (13) in (12), we get

$$\mathrm{R} \geq \int_0^{\infty}\int_0^{\infty}[1-\lambda\left(\frac{1}{d_1}-\frac{1}{d_2}\right)^2\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}}]\,dF_{d_1}dF_{d_2}$$

$$=\int_0^{\infty} w^*(d_2)\,dF_{d_2} \tag{14}$$

$$\mathrm{w}^*(\mathrm{d}_2) = \int_0^{\infty}[[1-\lambda\left(\frac{1}{\mathrm{d}_1}-\frac{1}{\mathrm{d}_2}\right)^2\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}]]\,\mathrm{dF}_{\mathrm{d}_1}$$

$$= 1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}\{E\left(\frac{1}{d_1^2}\right)-\frac{2}{d_2}E\left(\frac{1}{d_1}\right)+\frac{1}{d_2^2}\}$$

$$= 1 - \lambda\frac{\Gamma(1+\frac{1}{\square})}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma(1+\frac{2}{\theta})}{\lambda_E^{\frac{2}{\theta}}}\{\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - \frac{2}{d_2}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}} + \frac{1}{d_2^2}\} \tag{15}$$

Substituting (15) in (14), we get

$$\mathrm{R} \geq \int_0^{\infty}[1 - \lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}\{\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - \frac{2}{d_2}\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}} + \frac{1}{d_2^2}\}\,]\,dF_{d_2}$$

$$=1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}[\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - 2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + \Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}]$$

Therefore, reliability lower bound under the Weibull frame work for Ball bearing is given

$$\mathrm{L}(\lambda_P, \lambda_E, \beta, \mu, \square, \theta, \phi, \alpha) = 1-\lambda\frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}}\frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}}[\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - 2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}$$

$$\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + \Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}$$

3. RELIABILITY APPROXIMATION AND LEVEL OF ERROR

Now based on result1 and result 2, we now find reliability approximation and extent of error of this important engineering item. Reliability approximation and extent of error will be obtained as function of design parameters. One can increase or decrease reliability according to their requirements. Reliability approximation of this important engineering item is given by

$$\mathrm{R}(\lambda_P, \lambda_E, \beta, \mu, \square, \theta, \phi, \alpha) = \frac{[\mathrm{U}(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha)+\mathrm{L}(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha)\,]}{2}$$

$$= 1 - \lambda \frac{\Gamma\left(1+\frac{1}{\square}\right)}{\lambda_P^{\frac{1}{\square}}} \frac{\Gamma\left(1+\frac{2}{\theta}\right)}{\lambda_E^{\frac{2}{\theta}}} [\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}} - 2\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + \Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}}] + \frac{\lambda^2}{4}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}[\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}} - 4\,\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + 6\,\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\,\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}} - 4\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{3}{\alpha}\right)\mu^{\frac{3}{\alpha}} + \Gamma\left(1-\frac{4}{\alpha}\right)\mu^{\frac{4}{\alpha}}$$

Level of error of this item in terms of design parameter is given by

$$E(\lambda_P, \lambda_E, \beta, \mu, \square, \theta, \phi, \alpha) \leq \frac{[U(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha) - L(\lambda_P,\lambda_E,\beta,\mu,\square,\theta,\phi,\alpha)]}{2}$$

$$= \frac{\lambda^2}{4}\frac{\Gamma\left(1+\frac{2}{\square}\right)}{\lambda_P^{\frac{2}{\square}}}\frac{\Gamma\left(1+\frac{4}{\theta}\right)}{\lambda_E^{\frac{4}{\theta}}}[\Gamma\left(1-\frac{4}{\phi}\right)\beta^{\frac{4}{\phi}} - 4\,\Gamma\left(1-\frac{3}{\phi}\right)\beta^{\frac{3}{\phi}}\,\Gamma\left(1-\frac{1}{\alpha}\right)\mu^{\frac{1}{\alpha}} + 6\,\Gamma\left(1-\frac{2}{\phi}\right)\beta^{\frac{2}{\phi}}\,\Gamma\left(1-\frac{2}{\alpha}\right)\mu^{\frac{2}{\alpha}} - 4\,\Gamma\left(1-\frac{1}{\phi}\right)\beta^{\frac{1}{\phi}}\,\Gamma\left(1-\frac{3}{\alpha}\right)\mu^{\frac{3}{\alpha}} + \Gamma\left(1-\frac{4}{\alpha}\right)\mu^{\frac{4}{\alpha}}]$$

4. NUMERICAL STUDY

We now depict numerical study to find the numerical values of the reliability bounds, reliability approximation and extent of error of ball bearing.

For this purpose, we have chosen the stress parameters as □ = 5, **θ** = 6, **ϕ** = 9, α = 10, β = 4, μ = 8, $\lambda_P = 20$ and $\lambda_E = 30$. we have allowed the strength parameter, λ, to vary so that it can cover high reliability values. Reliability bounds, reliability approximation and extent of errors are reported in the following tables. We have also depicted approximated values of this item in the Table 2 and Table 3 for different values of λ_E *and* λ_P respectively. For Table 2, selection of other design parameters are □ = 5, **θ** = 6, **ϕ** = 9, α = 10, β = 4, μ = 8, λ = 30 and $\lambda_P = 20$. For Table 3, choice of other design parameters are □ = 5, **θ** = 6, **ϕ** = 9, α = 10, β = 4, μ = 8, λ = 20 and $\lambda_E = 30$.

Numerical calculations indicate that the proposed approach gives good approximation of reliability of ball bearing under the stress strength set up.

Table 1. Bound based reliability approximation and level of error of ball bearing under the Weibull set up for different values of strength parameter, λ

Sl. No.	Strength parameter(λ)	Lower bound	Upper bound	Reliability approximation	Error terms
1	80	0.776464	0.776465	0.776464	8.08E-07
2	75	0.790435	0.790436	0.790435	7.10E-07
3	70	0.804406	0.804407	0.804406	6.19E-07
4	65	0.818377	0.818378	0.818377	5.34E-07
5	60	0.832348	0.832349	0.832348	4.55E-07
6	55	0.846319	0.846319	0.846319	3.82E-07
7	50	0.860290	0.860290	0.86029	3.16E-07
8	45	0.874261	0.874261	0.874261	2.56E-07
9	40	0.888232	0.888232	0.888232	2.02E-07
10	35	0.902203	0.902203	0.902203	1.55E-07
11	30	0.916174	0.916174	0.916174	1.14E-07
12	25	0.930145	0.930145	0.930145	7.89E-08
13	20	0.944116	.944116	0.944116	5.05E-08
14	15	0.958087	0.958087	0.958087	2.84E-08
15	10	0.972058	0.972058	0.972058	1.26E-08
16	9	0.974852	0.974852	0.974852	1.02E-08
17	8	0.977646	0.977646	0.977646	8.08E-09
18	7	0.980441	0.980441	0.980441	6.19E-09
19	6	0.983235	0.983235	0.983235	4.55E-09
20	5	0.986029	0.986029	0.986029	3.16E-09
21	4	0.988823	0.988823	0.988823	2.02E-09
22	3	0.991617	0.991617	0.991617	1.14E-09
23	2	0.994412	0.994412	0.994412	5.05E-10

Table 2. Bound based reliability approximation and level of error of ball bearing under the Weibull set up for different values of stress parameter, λ_E

Sl. No.	Stress parameter (λ_E)	Lower bound	Upper bound	Reliability approximation	Error terms
1	20	0.811391	0.811392	0.811392	5.75E-07
2	21	0.828926	0.828927	0.828927	4.73E-07
3	21.5	0.836791	0.836791	0.836791	4.31E-07
4	22	0.844125	0.844126	0.844125	3.93E-07
5	23	0.857385	0.857385	0.857385	3.29E-07
6	24	0.869022	0.869022	0.869022	2.77E-07
7	24.5	0.874313	0.874314	0.874313	2.56E-07
8	26	0.888397	0.888398	0.888397	2.01E-07
9	27	0.896511	0.896511	0.896511	1.73E-07
10	28	0.903771	0.903771	0.903771	1.50E-07
11	29	0.910293	0.910293	0.910293	1.30E-07
12	32	0.926325	0.926325	0.926325	8.78E-08
13	34	0.934737	0.934738	0.934737	6.89E-08
14	36	0.941787	0.941788	0.941787	5.48E-08
15	40	0.952848	0.952848	0.952848	3.60E-08
16	45	0.962744	0.962744	0.962744	2.25E-08
17	50	0.969823	0.969823	0.969823	1.47E-08
18	55	0.97506	0.97506	0.97506	1.01E-08
19	60	0.979043	0.979043	0.979043	7.10E-09
20	65	0.982144	0.982144	0.982144	5.16E-09
21	70	0.984603	0.984603	0.984603	3.83E-09
22	90	0.990686	0.990686	0.990686	1.40E-09
23	100	0.992456	0.992456	0.992456	9.21E-10

Table 3. Bound based reliability approximation and level of error of ball bearing under the Weibull set up for different values of stress parameter, λ_p

Sl. No.	Stress parameter (λ_p)	Lower bound	Upper bound	Reliability approximation	Error terms
1	6	0.81372	0.813721	0.813720	5.61E-07
2	6.3	0.82259	0.822591	0.822591	5.09E-07
3	6.6	0.830654	0.830655	0.830655	4.64E-07
4	6.9	0.838017	0.838018	0.838018	4.24E-07
5	7	0.840331	0.840332	0.840332	4.12E-07
6	7.5	0.850976	0.850976	0.850976	3.59E-07
7	8	0.860290	0.860290	0.860290	3.16E-07
8	9	0.875813	0.875814	0.875813	2.49E-07
9	10	0.888232	0.888232	0.888232	2.02E-07
10	11	0.898393	0.898393	0.898393	1.67E-07
11	12	0.90686	0.906860	0.906860	1.40E-07
12	13	0.914024	0.914025	0.914025	1.20E-07
13	14	0.920166	0.920166	0.920166	1.03E-07
14	15	0.925488	0.925488	0.925488	8.98E-08
15	17	0.934254	0.934254	0.934254	6.99E-08
16	20	0.944116	0.944116	0.944116	5.05E-08
17	25	0.955293	0.955293	0.955293	3.23E-08
18	30	0.962744	0.962744	0.962744	2.25E-08
19	40	0.972058	0.972058	0.972058	1.26E-08
20	60	0.981372	0.981372	0.981372	5.61E-09
21	200	0.994412	0.994412	0.994412	5.05E-10
22	300	0.996274	0.996274	0.996274	2.25E-10
23	400	0.997206	0.997206	0.997206	1.26E-10

5. CONCLUSION

Analytical computation of exact reliability of ball bearing is very difficult task. It also becomes very difficult task if we want to compute reliability of it under the Weibull frame work if strength parameter, $\alpha, exceeds, 3$. so, to overcome this problem, we have addressed here bound based reliability approximation of this important engineering item for strength parameter α = 3 due to mathematical complexity. Numerical analysis has been adopted to show the closeness between the upper and lower bounds of this item. From the numerical studies, we can claim that mid value of the two bounds fairly approximate the reliability value. Reliability approximation and level of error for different strength and stress parameters have been provided in the above tables.

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