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On the Starvation Period of CDF-Based Scheduling over Markov Time-Varying Channels

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ABSTRACT

In this paper, we consider a cumulative distribution function (CDF)-based opportunistic scheduling for downlink transmission in a cellular network consisting of a base station and multiple mobile stations. We present a closed-form formula for the average starvation period of each mobile station (i.e., the length of the time interval between two successive scheduling points of a mobile station) over Markov time-varying channels. Based on our formula, we investigate the starvation period of the CDF-based scheduling for various system parameters.

Key Words : CDF-based scheduling, starvation period, multiuser diversity, Markov channel

Ⅰ. Introduction

Opportunistic scheduling has been studied extensively in the last decade; it can maximize the sum throughput in wireless networks by selecting the user who has the largest channel gain at each time-slot[1]. As a result, a user having higher signal-to-noise ratio (SNR) on average is scheduled more frequently, which leads to *unfairness*[2]. A cumulative distribution function (CDF)-based opportunistic scheduling[3] was proposed to resolve the fairness problem while achieving high throughput by utilizing a set of weight parameters, and has been extended to various networks in recent work[1].

Along with the fairness, another important feature inherent in the opportunistic scheduling is *starvation*: a user may wait for a long time until it experiences peaks in its channel gain, which contrasts with non-opportunistic policies such as the round-robin scheduling. Such starvation can become severe in wireless networks with time-correlated channels. Note that the starvation period (i.e., the length of the time interval between two successive scheduling points of a user) determines the head-of-line packet delay, and thus understanding the starvation period helps to control delay performance.

In this paper, we present a closed-form expression for the average starvation period of the CDF-based scheduling over Markov time-varying channels. Through numerical studies, we investigate the starvation period for various system parameters.

Ⅱ. System Model

We consider downlink in a cellular network consisting of one base station (BS) and N mobile stations (MSs). We assume that the BS always has traffic to send to each MS.

2.1 Wireless channel model

Let $\gamma_n(t)$ be the SNR of the wireless channel between the BS and MS n at time-slot t. We assume that $\gamma_n(t)$ is a random variable having a general CDF $F_n(x) := \mathrm{P}(\gamma_n(t) \le x)$. To describe the time-correlated wireless channel, we use a two-state Markov model (also known as Gilbert-Elliott channel) as follows. Let $C_n(t)$ be the channel state of MS n at time-slot t. Then, $C_n(t) := 0$ if $\gamma_n(t) < l$ and $C_n(t) := 1$ if $\gamma_n(t) \ge l$ for a threshold $l > 0$. When $C_n(t) = k$, $k \in \{0,1\}$, and MS n is scheduled at time-slot t, the BS transmits packets to MS n with the transmission rate r_k where $r_0 < r_1$. The process $\{C_n(t)\}_{t=0}^{\infty}$ is assumed to be stationary and forms a Markov chain with the transition probability matrix

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$$\Lambda_n = \begin{bmatrix} p_{0,0} & p_{0,1} \\ p_{1,0} & p_{1,1} \end{bmatrix},$$

where $p_{k,k'} := \mathrm{P}(C_n(t+1) = k' | C_n(t) = k)$ $(k, k' = 0, 1)$. The steady-state distribution is

$$\pi_n^0 := \lim_{t \to \infty} \mathrm{P}(C_n(t) = 0) = \frac{p_{1,0}}{p_{0,1} + p_{1,0}},$$
$$\pi_n^1 := \lim_{t \to \infty} \mathrm{P}(C_n(t) = 1) = \frac{p_{0,1}}{p_{0,1} + p_{1,0}}.$$

2.2 CDF-based scheduling

A CDF-based scheduling operates slot-by-slot by selecting MS $n^*(t)$ at time-slot t as follows:

$$n^*(t) = \mathrm{argmax}_{1 \le n \le N} \left[F_n(\gamma_n(t))\right]^{1/w_n},$$

where $w_n (>0)$ denotes the weight of MS n and satisfies $\sum_{n=1}^{N} w_n = 1$. Under the condition $C_n(t) = k$, the scheduling probability of MS n at time-slot t is given by[4]

$$s_n^0 := \mathrm{P}(n^*(t) = n | C_n(t) = 0) = w_n (\pi_n^0)^{1/w_n - 1},$$
$$s_n^1 := \mathrm{P}(n^*(t) = n | C_n(t) = 1) = w_n \frac{1 - (\pi_n^0)^{1/w_n}}{\pi_n^1}.$$

The channel access probability of MS n in steady state is then

$$\lim_{t \to \infty} \mathrm{P}(n^*(t) = n) = \pi_n^0 s_n^0 + \pi_n^1 s_n^1 = w_n. \quad (1)$$

Ⅲ. Starvation Period Analysis

Let T_n be a random variable denoting the starvation period of MS n (see Fig. 1). By conditioning on the channel state at the initial slot of the starvation (denoted by h), we have

$$\begin{aligned} \mathrm{E}[T_n] &= \sum_{k=0}^{1} \mathrm{E}[T_n | C_n(h) = k] \cdot \mathrm{P}(C_n(h) = k) \\ &= \alpha_n^0 \mathrm{E}[T_n^0] + \alpha_n^1 \mathrm{E}[T_n^1], \end{aligned}$$

where $\alpha_n^k := \mathrm{P}(C_n(h) = k)$, and T_n^k is the starvation

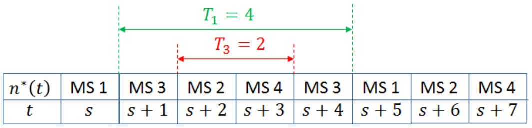

Fig. 1. Example of starvation period: $T_1 = 4$, $T_3 = 2$

period provided that its initial slot has the channel state $C_n(h) = k$. To find $\mathrm{E}[T_n^k]$, we introduce three events $\varepsilon_i (i = 0, 1, 2)$:

$$\varepsilon_0 := \{n^*(h+1) = n\},$$
$$\varepsilon_1 := \{n^*(h+1) \neq n\} \cap \{C_n(h+1) = 0\},$$
$$\varepsilon_2 := \{n^*(h+1) \neq n\} \cap \{C_n(h+1) = 1\}.$$

By the law of total probability, we have

$$\begin{aligned} \mathrm{E}[T_n^k] &= \mathrm{E}[T_n | C_n(h) = k] \\ &= \sum_{i=0}^{2} \mathrm{E}[T_n | \varepsilon_i, C_n(h) = k] \cdot \mathrm{P}(\varepsilon_i | C_n(h) = k). \end{aligned}$$

Applying first-step analysis, we then obtain

$$\mathrm{E}[T_n | \varepsilon_i, C_n(h) = k] = \begin{cases} 1 & i = 0, \\ 1 + \mathrm{E}[T_n^0] & i = 1, \\ 1 + \mathrm{E}[T_n^1] & i = 2, \end{cases}$$

$$\mathrm{P}(\varepsilon_i | C_n(h) = k) = \begin{cases} p_{k,0} s_n^0 + p_{k,1} s_n^1 & i = 0, \\ p_{k,0}(1 - s_n^0) & i = 1, \\ p_{k,1}(1 - s_n^1) & i = 2. \end{cases}$$

In matrix form, above equations are expressed as

$$\begin{bmatrix} \mathrm{E}[T_n^0] \\ \mathrm{E}[T_n^1] \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \Lambda_n \begin{bmatrix} 1 - s_n^0 & 0 \\ 0 & 1 - s_n^1 \end{bmatrix} \begin{bmatrix} \mathrm{E}[T_n^0] \\ \mathrm{E}[T_n^1] \end{bmatrix},$$

from which we obtain

$$\begin{bmatrix} \mathrm{E}[T_n^0] \\ \mathrm{E}[T_n^1] \end{bmatrix} = \left(I - \Lambda_n \begin{bmatrix} 1 - s_n^0 & 0 \\ 0 & 1 - s_n^1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad (2)$$

where I is the 2×2 identity matrix. To find α_n^k, we introduce

$$B_t := \{n^*(t-1) = n\} \cap \{n^*(t) \neq n\},$$

i.e., the event that time-slot t is the initial slot of the starvation. Due to Markov property and the stationarity, we have $\alpha_n^k = \mathrm{P}(C_n(t)=k|B_t)$ for any t. Hence,

$$\alpha_n^k = \frac{\mathrm{P}(C_n(t)=k, B_t)}{\mathrm{P}(C_n(t)=0, B_t)+\mathrm{P}(C_n(t)=1, B_t)} = \frac{\beta_n^k}{\beta_n^0+\beta_n^1},$$

where $\beta_n^k := \mathrm{P}(C_n(t)=k, B_t)$. By conditioning on the state pair $\{C_n(t), C_n(t-1)\}$ and then applying $\mathrm{P}(C_n(t)=k, C_n(t-1)=k') = \pi_n^{k'} \cdot p_{k',k}$, we obtain

$$\beta_n^k = \sum_{k'=0}^{1} \mathrm{P}(C_n(t)=k, C_n(t-1)=k', B_t)$$
$$= \sum_{k'=0}^{1} \mathrm{P}(B_t | C_n(t)=k, C_n(t-1)=k') \cdot \pi_n^{k'} \cdot p_{k',k}.$$

Note that $\mathrm{P}(B_t|C_n(t)=k, C_n(t-1)=k') = s_n^{k'}(1-s_n^k)$. Hence, β_n^k is given by

$$\beta_n^k = s_n^0(1-s_n^k)\pi_n^0 p_{0,k} + s_n^1(1-s_n^k)\pi_n^1 p_{1,k}.$$

In matrix form, above equation is expressed as

$$\begin{bmatrix}\beta_n^0 & \beta_n^1\end{bmatrix} = \begin{bmatrix}\pi_n^0 s_n^0 & \pi_n^1 s_n^1\end{bmatrix} \Lambda_n \begin{bmatrix}1-s_n^0 & 0 \\ 0 & 1-s_n^1\end{bmatrix}. \quad (3)$$

We summarize the main result of our analysis in the following theorem.

Theorem 1. Under the CDF-based scheduling, the average starvation period of MS $n \ (n=1,2,...,N)$ is

$$\mathrm{E}[T_n] = \frac{1}{\beta_n^0+\beta_n^1} \sum_{k=0}^{1} \beta_n^k \cdot \mathrm{E}[T_n^k],$$

where $\mathrm{E}[T_n^k]$ and β_n^k are given by (2) and (3), respectively.

Ⅳ. Numerical Study

Based on Theorem 1, we investigate the starvation period of the CDF-based scheduling.

In this section, we assume that $N=10$ and the transition probability matrix Λ_n is

$$\Lambda_n = \begin{bmatrix}\rho_n & 1-\rho_n \\ 1-\rho_n & \rho_n\end{bmatrix}.$$

First, we consider the scenario of equal weight $(w_1,...,w_{10}) = \frac{1}{10}(1,...,1); \quad (\rho_1,..., \rho_{10}) = (0.05,..., 0.95)$. Fig. 2 shows that the average starvation period is approximately linear with ρ_n (i.e., the probability of retaining the previous channel state). This is in contrast to the round-robin scheduling policy that always provides a constant starvation period regardless of channel statistics.

Next, we consider the scenario of linear weight

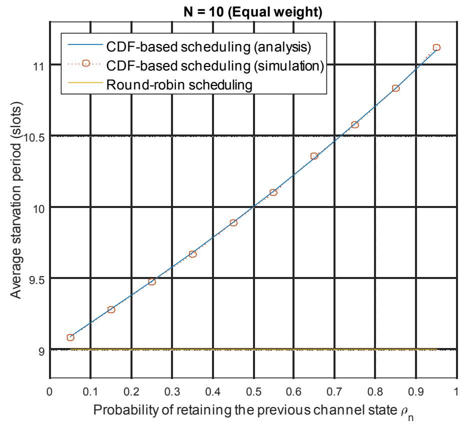

Fig. 2. The impact of the channel correlation on the starvation period

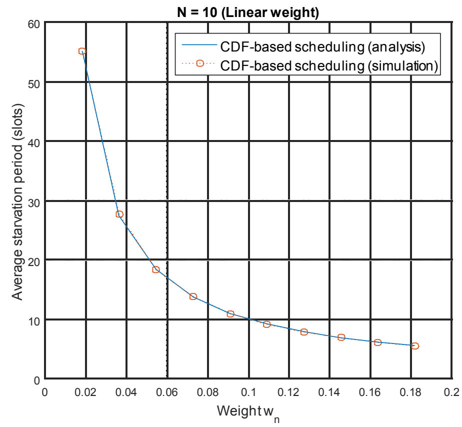

Fig. 3. The impact of the weight on the starvation period

$(w_1, ..., w_{10}) = \frac{1}{55}(1, ..., 10)$ and $\rho_n = 0.5$ for all n. Fig. 3 shows that the average starvation period decreases convexly with w_n. Note that the weight w_n represents the channel access probability of MS n by (1). Hence, Fig. 3 also indicates that decreasing the channel access probability lower than $1/(2N)$ significantly increases the starvation period. Our analytic results match the simulation results well, verifying the correctness of our analysis.

Ⅴ. Conclusion

In this paper, we present a formula for the average starvation period of the CDF-based scheduling. Based on the formula, we investigate the impact of the channel correlation and the weight on the starvation period. Extension of our analysis to a general finite-state Markov channel is our future work.

References

[1] H. Jin, B. C. Jung, and V. C. Leung, "Fundamental limits of CDF-based scheduling: Throughput, fairness, and feedback overhead," *IEEE/ACM Trans. Netw.*, vol. 23, no. 3, pp. 894-907, Jun. 2015.

[2] K. Lee, D.-H. Cho, and B. C. Chung, "Resource allocation method for improving energy efficiency and receiver fairness in wireless networks," *J. KICS*, vol. 40, no. 5, pp. 826-832, May 2015.

[3] D. Park, H. Seo, H. Kwon, and B. G. Lee, "Wireless packet scheduling based on the cumulative distribution function of user transmission rates," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1919-1929, Nov. 2005.

[4] D. Park and B. G. Lee, "QoS support by using CDF-based wireless packet scheduling in fading channels," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2051-2061, Nov. 2006.