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Small-World 망과 Scale-Free 망을 위한 일반적인 망 생성 방법

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Generalized Network Generation Method for Small-World Network and Scale-Free Network

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요 약

최근 들어 다양한 SNS(Social Network Service)에 대한 이해와 분석을 위해 가장 중요한 두 종류의 망인 small-world와 scale-free망에 대한 많은 연구가 수행되고 있다. 본 연구에서는 두 개의 입력 파라미터를 적절히 조정함으로서 small-world 망, scale-free 망 혹은 두 개의 성질을 동시에 모두 갖는 망을 생성 할 수 있는 보다 일반화된 망 생성 방법을 제안하였다. 두개의 입력 파라미터중 하나는 small-world 성질을 나태내주는 파라미터고 다른 하나는 scale-free와 small-world 성질 모두를 나타내주는 파라미터다. Small-world와 scale-free를 나타내주는 망의 성질로 군집계수, 평균 최단거리 그리고 power-law 상수를 이용하였다. 본 연구에서 제안한 방법을 사용하면 small-world 망과 scale-free 망의 성질과 관계에 대한 보다 명확한 이해를 할 수 있다. 다양한 여러 예제들을 통하여 두 개의 입력 파라미터들이 군집계수, 평균 최단거리 그리고 power-law 상수에 미치는 영향을 검증하였다. 이를 통해 어떠한 입력 파라미터들의 조합이 small-world 망, scale-free 망 혹은 두 개의 성질을 모두 갖는 망을 생성 할 수 있는지를 조사하였다.

Key Words : Small-world network, Scale-free network, Network generation model, Clustering coefficient, Power-law

ABSTRACT

To understand and analyze SNS(Social Network Service) two important classes of networks, small-world and scale-free networks have gained a lot of research interests. In this study, a generalized network generation method is developed, which can produce small-world network, scale-free network, or network with the properties of both small-world and scale-free by controlling two input parameters. By tuning one parameter we can represent the small-world property and by tuning the other one we can represent both scale-free and small-world properties. For the network measures to represent small-world and scale-free properties clustering coefficient, average shortest path distance and power-law property are used. Using the model proposed in this study we can have more clear understanding about relationships between small-world network and scale-free network. Using

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numerical examples we have verified the effects of two parameters on clustering coefficient, average shortest path distance and power-law property. Through this investigation it can be shown that small-world network, scale-free network or both can be generated by tuning two input parameters properly.

Ⅰ. Introduction

Many complex systems can be modeled as a network, where vertices are the elements of the system and edges represent the interactions between them. Internet, neural network, social network and transportation network are just some of numerous examples. Each network has complex dynamic activities between its own system elements(nodes). For example, routers of Internet must communicate with each other to deliver packets with least cost. And diseases are transmitted through the dynamic interaction of social network nodes. There have been several and important studies[1,2,3] concerning how a network structure characterizes the network dynamic behavior. Characterizing the structural properties of networks is then of fundamental importance to understand the complex dynamics of these systems. Although many measures and quantities are proposed, three measures including the average shortest path length, clustering coefficient, and degree distribution are the most widely used to understand the complex dynamics of network.

There have been efforts to develop a mathematical model, which can generate networks with a topology of similar structural properties. An accurate network generation model can have significant effect on network researches and provide a platform on which mathematical analysis is possible. For example, as Internet continues to expand exponentially, an Internet topology model is required which can yield insight into future behavior and suggest novel strategies for planning and long term network design as well. And, Magoni and Pansiot[4] found that network models played an important role in assessing network algorithm. That is, the effectiveness or performance of proposed algorithm and protocol is highly sensitive to the underlying Internet AS(Autonomous System) connectivity structure.

There are two extremes of network models: regular coupled networks and random networks. A regular network is a lattice in which every node is joined only by a few of its neighbors. It has a high clustering coefficient and long average shortest path length. That is, it is locally reliable and robust, but cannot provide synchronization that requires global coordination. Other extreme of network models is random network with a completely random graph, which was suggested by Erdos-Renyi[5]. In this model every two node is connected by a given probability P. Generally, it has a low clustering coefficient and small average path length.

Most of the a real world network is neither regular coupled network nor random network. So, in a past few decades researchers have tried to develop a network model with the properties of many real-world networks, which exists somewhere between regular coupled and random networks. There are two notable outcomes: small-world network and scale-free network. Watts and Strogatz[6] showed that some biological and social networks were not completely regular coupled nor random. They introduced small-world network model, which is highly clustered like regular lattice, yet has small average shortest path length like random network. It is known small-world network characterizes many real world networks such as the social network[7], the biochemical network[8], the Internet[2,9], VoD service over IP network[10], and etc. The other finding is scale-free network. Faloutsos et al.[11] examined the AS topology of the Internet and found that node degrees are well described by power-laws of the form $y = x^{-\gamma}$. This type of network is characterized by a highly heterogeneous degree distribution, There is a few, but significant number of nodes with a lot of connections. Several scale-free network generators have been proposed, which are based on incremental growth and preferential connectivity. Many real world networks such as the WWW, metabolic system, and the Internet AS system can be characterized by scale-free network.

All the current network models can generate small-world network or scale-free network, just one of them. However, there is no network model, which can produce a small-world network, a scale-free network, or a network with the properties of both small-world and scale-free. The objective of this study is to develop a more generalized network model, which can be done by controlling two input parameters. Using the model proposed in this study we can have more clear understanding about relationships between small-world network and scale-free network.

Following the Introduction we have reviewed some network generators in Chapter 2. And we have proposed our network model in Chapter 3. Using the model proposed in Chapter 3 we have shown several numerical examples in Chapter 4, which can give us valuable insight about relationship between small-world network and scale-free network. Final conclusion and summary are in Chapter 5.

Ⅱ. Network Generator Models

Starting from regularly coupled network and random network many network generator models have been proposed. In this study only small-world and scale-free networks are discussed. We will start this chapter by defining some network measures of interests.

2.1 Network Topology Measures

Tangmunarunkit et al.[12] insist that the goal of the network generator is not to produce exact replicas of the real network, but to produce network whose properties are similar to the real network. The question is what properties are relevant to this comparison. There seems to be no single answer to this question, as the relevant properties may well depend on how the generated networks are used. Although many measures and quantities are proposed, three measures including average shortest path length, clustering coefficient, and degree distribution are the most widely used to understand complex dynamics of network. These are defined as follows.

1) Clustering coefficient(C): It is a measure of how close a node's neighbors are to form a clique. It is based on triplets of nodes, where a triplet consists of three nodes that are connected by either two (open triplet) or three(closed triplet) undirected ties. Clustering coefficient is the percentage of closed triplets among all connected node triplets in the entire network.

2) Average shortest path length(L): It is the average of shortest lengths between all nodes of network. Let L_{ij} represents the shortest length between node i and j. Then, average shortest path length L is defined as

$$L = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} L_{ij}}{{}_N C_2} \tag{1}$$

3) Degree distribution: It is the probability distribution of node degree, which is the most frequently used properties. it contains more information about connectivity in a graph than average degree. If node degrees are well described by power-laws of form y = $x^{-\gamma}$, γ is called degree exponent.

2.2 Small-World Network Model

Small-world network is defined to be a network where the typical distance L between two randomly chosen nodes (the number of steps required) grows proportionally to the logarithm of number of nodes N in the network[6]. That is,

$$L \propto LogN \tag{2}$$

Watts and Strogatz[6] showed that small world network has the characteristics of small average shortest path length and higher clustering coefficient than that of random network. They proposed an interesting small-word network model. Watts and Strogatz summarized it in following two steps.

1) Begin with a nearest neighbor coupled network

consisting of N nodes arranged in a ring, where each node i is adjacent to its neighbor nodes, i = 1, 2, ⋯, K/2, with K being even.

2) Randomly rewire each link of the network with probability P;. varying P in such a way that the transition between order (P = 0) and randomness (P = 1) can be closely monitored.

Through the first step we can construct a network of regularly coupled type, which has a high clustering coefficient and long average shortest path length. The P in the second step plays a crucial role in determining average shortest path length and clustering coefficient. If P is 0, it is just a regularly coupled network. If P goes to 1, the average shortest path length becomes small and the clustering coefficient becomes big.

Another method to develop a small-world network was proposed by Barmpoutis et al[13], where a network with very small average path length and very high clustering coefficient is constructed.

2.3 Scale-Free Network Model

Scale-free network is a network whose degree distribution follows a power-law, y = x-ɣ, where ɣ is a parameter whose value is typically in the range of 2≤ɣ≤3. Barabasi and Albert[14] suggested two mechanisms as the origin of a scale-free power-law distribution and developed model as follows.

1) Increment growth: It places nodes gradually at a time as nodes join the network. In this case, a new node considers as candidate neighbors only those nodes that have already joined the network.

2) preferential connectivity: A newly considered node v connects to a candidate neighbor node i with the following probability

$$q_i = \frac{b_i}{\sum_{j \in B} b_j} \tag{3}$$

where bi is the current degree of node i, and B is the set of candidate neighbor nodes.

There have been several attempts for modeling scale-free network. Barabasi and Albert model has been extended and revised by several variations, which include two-level network model[15], non-linear preferential attachment[16], hierarchical network model[17], fitness model[18], and hyperbolic geometric graphs[19].

2.4 Relationship between Small-World Network and Scale-Free Network

Relationship between small-world network and scale-free network is not quite clear. Small-world network and random network can be viewed as a homogeneous network, in which all nodes have approximately same node degree. It has a peak at an average value and decay exponentially. However, some small-world networks can be also scale-free[1]. Scale-free network is basically a heterogeneous network, which has a few hub nodes whose node degrees are significantly higher than those of other nodes. The average shortest path length for scale-free network is somewhat smaller and the clustering coefficient is much higher compared to a random network of same node size. However, the characteristics of average shortest path length and clustering coefficient for scale-free network are not quite clearly known.

Ⅲ. Generalized Network Generation Method

In this Chapter a more generalized network model is developed, which can produce small-world network, scale-free network, or network with the properties of both small-world and scale-free by tuning two input parameters properly. Basically, two network models of small-world and scale-free, which are discussed in Chapter 2, are revised and combined properly. In secttion 3.1 and 3.2 the basis and validity for selecting two input parameters are discussed.

3.1 Input Parameter K_0

Instead of using constant K in the first step of constructing small-world network we assume K follows an exponential distribution with mean K_0. Through this revision we can make some rich nodes, which have large node degrees. With larger K_0 we can have higher clustering coefficient due to more connections with neighbors and vice versa. It is also expected that high clustering coefficient can decrease average shortest path and vice versa. Since high clustering coefficient and small average shortest path are characteristics of small-world network, we can represent the properties of small-world network by tunimg K_0 properly.

3.2 Input Parameter α

Instead of using constant P for wiring probability in the second step of constructing small-world network, the probability connecting two different nodes(P_{ij}) is determined based on the preferential connectivity used in building scale-free network.

P_{ij} is obtained using sigmoid function $1/(1 + e^{-x})$ by which we can get almost 1 for large input value and almost 0 for small input value as shown in Fig. 1.

x of sigmoid function is defined as $((K_i/\langle K\rangle)-\alpha)$, where K_i is the node degree of node i and $\langle K\rangle$ is mean node degree after completing the first step, random assignment of links to nodes. So, K_i is different from the initial number of links assigned randomly and K_0 is also different from $\langle K\rangle$. We can assign high probability if K_i is larger than $\langle K\rangle$ and vice versa. α plays a role of threshold. That is, to get an enough probability of connecting two nodes $K_i/\langle K\rangle$ should be big enough to exceed α. By controlling α we can have different degree of scale-free network. If α is large, only the nodes with high node degree can have a high probability of being connected to each other. So, rich nodes become richer, and we can have high degree of scale-free network.

$$P_{ij}=\frac{1}{1+e^{-(\frac{K_i}{\langle K\rangle}-\alpha)}}\times\frac{1}{1+e^{-(\frac{K_j}{\langle K\rangle}-\alpha)}} \tag{4}$$

Connecting of two randomly selected nodes with probability P_{ij} shifts one end of connection to a new node chosen at random from the whole network. And using P_{ij} defined above also gives high chance of connecting two different hub nodes, which have high node degree. Through this process average path length drops rapidly.

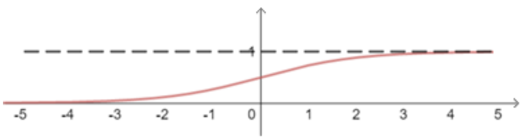


Fig. 1. Sigmoid Function

From above discussion we can say the probability of connecting two different nodes becomes smaller with larger α, which makes average shortest path length larger and vice versa. With larger α, rich nodes become richer and we can have higher degree of scale-free network and vice versa. So, we can get scale-free network by tuning α properly. The effect of α on clustering coefficient is very complicated since there is an interaction between α and K_0, which will be discussed in Chapter 4. Examples.

3.3 Generalized Network Generation Method

Now we can construct a generalized network using following steps. We assume that number of nodes is given as N, which is arranged in a ring.

Step1:

We start from node 1.

1) Generate random numbers from exponential distribution with mean K_0.

2) By rounding up the random numbers, an integer value n_i is assigned for node i(i = 1, 2, ⋯, N). If n_i is even, node i is connected to its neighbor nodes, i = i ± 1, i ± 2, ⋯, i ± $n_i/2$. If n_i is odd, node i is connected to its neighbor nodes, i = i ± 1, i ± 2, ⋯, i ± $(n_i-1)/2$. And it is also connected to node $(i + (n_i+1)/2)$.

Above procedure is continued until node N. Node i has a node degree K_i, which is different from n_i. It is because each node might have additional links, which come from other neighbor node j(j ≠ i).

Step 2:

In this step two randomly selected nodes are connected. There are ${}_NC_2$ candidates for connection.

1) After step 1 degree of node i is determined as K_i and average node degree $\langle K\rangle$ is also determined.

2) Select candidates k (k =1, 2, ⋯, NC2) randomly. Suppose candidates k consists of node i and node j.

3) Suppose candidate k consists of node i and node j. Connect node i and node j with probability P_{ij}, which is defined in Equation (4).

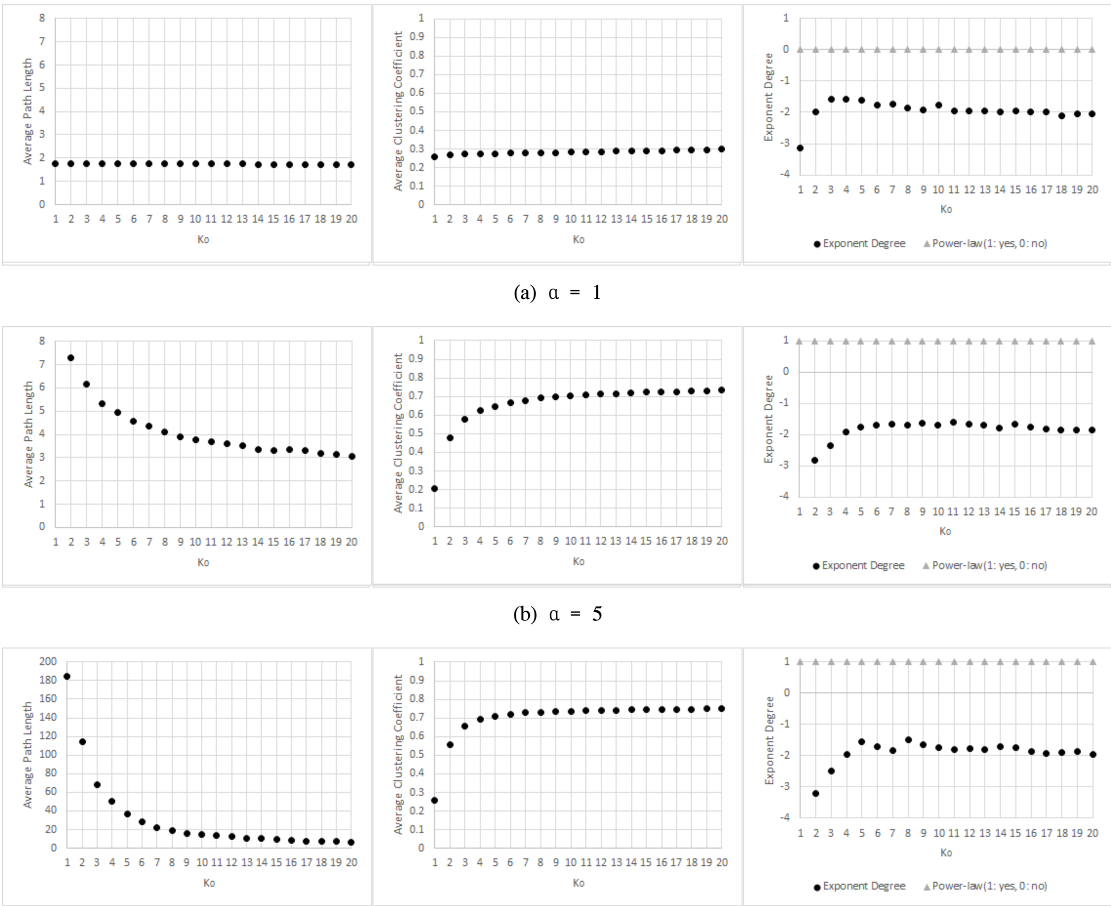
4) Step 2) and 3) are repeated until all the candidates are selected.

The nodes with high node degree after step 1 have a high chance of becoming hub nodes after completion of step 2.

Ⅳ. Examples

In previous Chapter we have developed generalized network generation method with two parameters K_0 and α. We have shown that K_0 affects clustering coefficient and average shortest path length. So, we can represent the property of small-world network by tuning K_0 properly. We have also shown that α determines power-law property. It also affects average shortest path length and clustering coefficients. So, we can represent the properties of both scale-free and small-world networks by tuning α properly.

Using several examples we will verify the arguments mentioned above. In section 4.1 effects of K_0 are investigated. For given α (1,5 and 8) and varying K_0 we generated networks and calculated the topological measures such as average shortest path length, clustering coefficient and node degree distribution. In section 4.2 effects of α are investigated. For given K_0 (1,5 and 10) and varying α we generated networks and calculated the



(a) α = 1

(b) α = 5

(c) α = 8

Fig. 2. Topological Measures with a Fixed α and Varied K_0

topological measures such as average shortest path length, clustering coefficient and node degree distribution.

Number of nodes(N) is given as 1,000. Each measure is the average of 10 networks. We also analyzed the change of topological measures when the size of network (number of nodes) grows from 1,000 to 10,000.

4.1 Effects of K_0

With a fixed α and varied K_0 the effects of K_0 on average shortest path length, clustering coefficient and node degree distribution are investigated. The results are shown in Fig. 2. In the figure of power-law, whether the degree distribution follows power-law or not is represented by 1(follow) and 0(not follow). The negative value represents the exponent of power-law.

As can be seen in Fig. 2 average shortest path length becomes smaller as K_0 increases. With larger K_0 node can have more number of neighbors, which leads to smaller average shortest path length. Since larger K_0 brings more connection with neighbor nodes, clustering coefficient becomes larger as K_0 increases as can be seen in Fig. 2.

From our discussion in '3. Generalized Network Generation Method' we can see that K_0 does not affect the power-law, which can be verified from power-law figure of Figure 2, The value of 0 or 1 which determines power-law remains almost same irrespective of K_0. From Figure 2 we can see that α is a determining parameter for power-law, which will be discussed in next section.

From above discussion we could verify our arguments proposed in Section 3.1. That is, K_0 affects clustering coefficient and average shortest path length. So, small-world network can be generated by tuning K_0 properly.

4.2 Effect of α

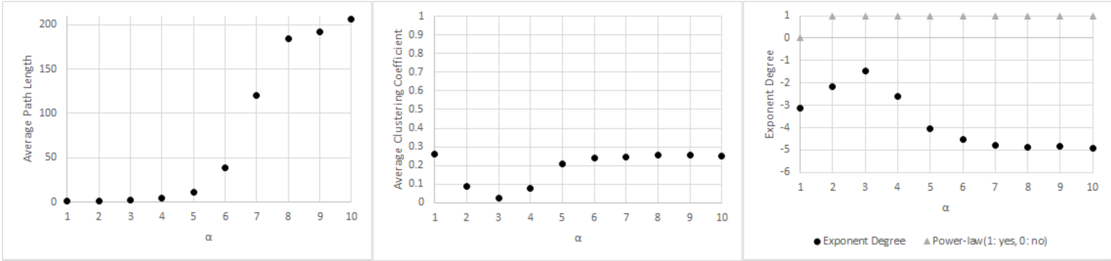
With a fixed K_0 and varying α the effects of α on average shortest path length, clustering coefficient and node degree distribution are investigated. The number of node is given as 1,000. The results are shown in Fig. 3.

As can be seen in Fig. 3 average shortest path length becomes larger as α increases. With larger α the probability of connecting two different nodes becomes smaller, which makes average shortest path length larger.

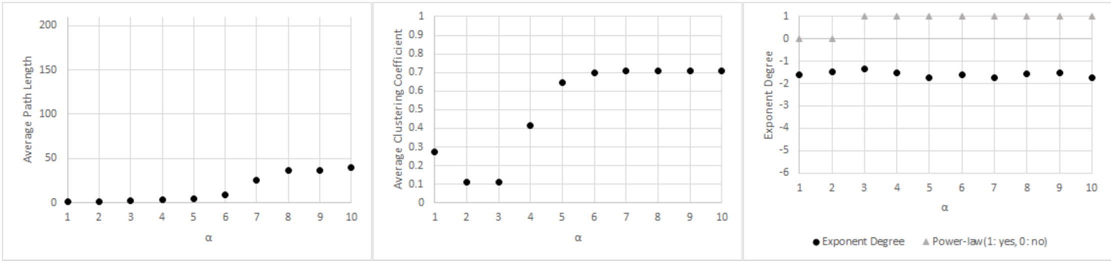
The effect of α on clustering coefficient seems to be complicated. When α is 10, the probability of connecting two different nodes is almost negligible. So, there is no step 2 in generalized network model. The clustering coefficient is mainly determined by K_0. When K_0 = 1, it is around 0.25. When K_0 = 10, it is above 0.7. As seen in previous section, C becomes larger as K_0 increases. When α goes down below 10, the probability of connecting two different nodes increases and we can have more number of links. It is expected that more number of links makes clustering coefficient larger. On the contrary, however, we can see clustering coefficient decreases as α goes down below 10 as shown in Figure 3. It is because additional link makes clustering coefficient smaller when it connects distant nodes, not neighboring nodes. Also, it is more probable that additional links increase the number of open triplets than they make the open triplets into closed ones. If α goes down further, below around 3, we can see that clustering coefficient increases, as shown in Figure 3. If α becomes small, we can have large number of additional links. Now, it is more probable that additional links make open triplets into closed ones than they increase the number of open triplets, which makes clustering coefficient larger.

To generate scale-free network rich nodes get richer as network grows. If α is large, only rich nodes, which have large value of K_i / $\langle K \rangle$, can have a relatively high probability of connecting to other nodes. So, we can see power-law property of generated network. If α is small, there is no significant difference of connecting to other nodes between poor and rich nodes. As can be seen in Fig. 3, if α is roughly less than 2, we can't see any power-law property.

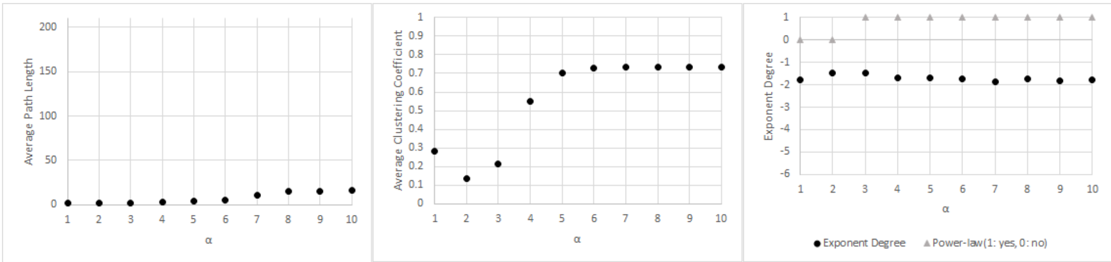
From above discussion we could verify the arguments proposed in Section 3.2. That is, α is a determining parameter for scale-free network. It also affects average shortest path length and clustering



(a) k_0 = 1



(b) k_0 = 5



(c) k_0 = 10

Fig. 3. Topological Measures with a Fixed K_0 and Varied α

coefficient. So, properties of both scale-free and small-world networks can be represented by tuning α properly.

4.3 Small-World Network and Scale-Free Network

In this section we try to answer which combinations of α and K_0 produce scale-free network, small-world network, both or none. For this purpose we have defined small-world network as the network whose average shortest path length is less than 6 and clustering coefficient is greater than 0.4. And scale-free network is defined as a network whose node distribution follows a power-law. Results are summarized in Fig. 4. For example, α = 4 and K_0 = 1 can produces a scale-free network and α = 5 and K_0 = 10 can produce both small-world and scale-free networks.

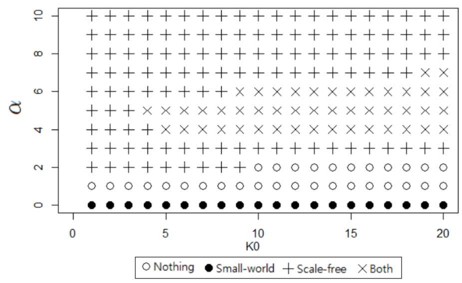


Fig. 4. Small-world Network and Scale-free Network according to α and K_0

4.4 Effects of N

Now, we want to see how topological measures change according to N. For this purpose we select three combinations of α and K_0.

- α = 0 and K_0 = 5 (can produce small-world network only when N = 1,000)
- α = 4 and K_0 = 1 (can produce scale-free network only when N = 1,000)
- α = 5 and K_0 = 10 (can produce both small-world and scale-free networks when N = 1,000)

1) α = 0 and K_0 = 5

As can be seen in Fig. 5 the generated networks still show the properties of small-world network as N increases from 1,000 to 10,000. Average shortest path length remains under 2 and average clustering coefficient stays above 0.5. We can also see that the generated network does not show the property of power-law.

2) α = 4 and K_0 = 1

As can be seen in Fig. 6 the generated networks still show the properties of power-law as N increases from 1,000 to 10,000. Average clustering coefficient decreases to almost 0. And average shortest path length stays above 3.

3) α = 5 and K_0 = 10

As can be seen in Fig. 7 the generated networks still show the properties of both small-world and scale-free networks as N increases from 1,000 to 10,000. Average shortest path length stays at about 3.5 and average clustering coefficient remains above 0.45. We can also see that the generated networks

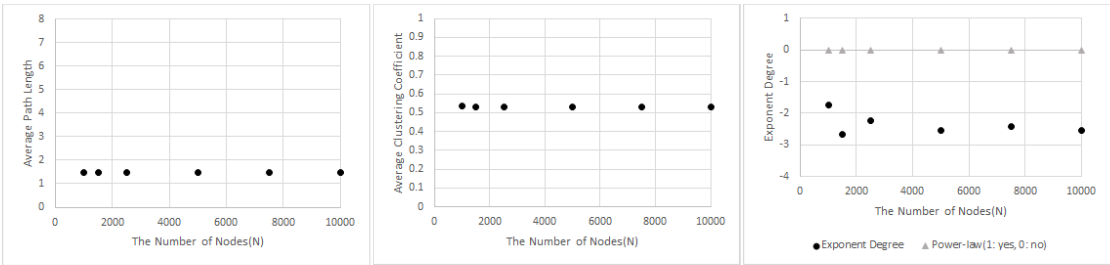


Fig. 5. α = 0 and K_0 = 5

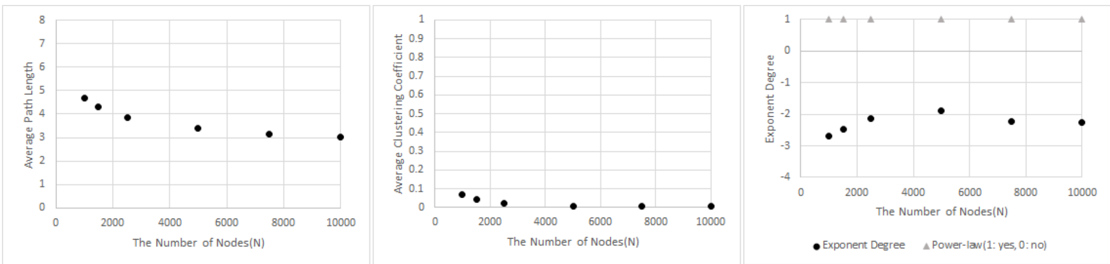


Fig. 6. α = 4 and K_0 = 1

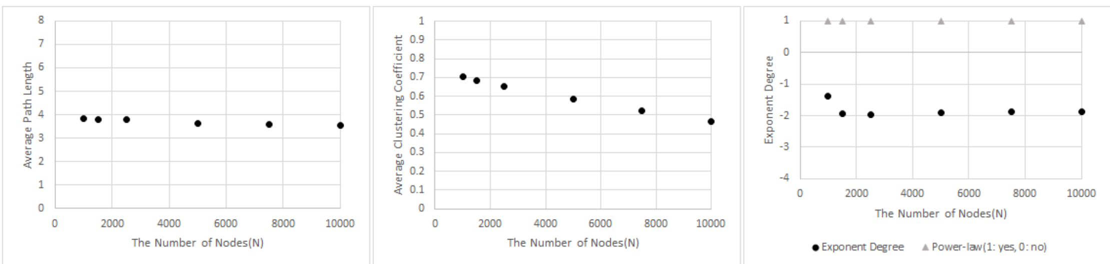


Fig. 7. α = 5 and K_0 = 10

shows the properties of power-law irrespective of size of N.

From above three cases we can say that the networks generated by using specific α and K_0 of three examples show the same network properties(small-world, scale-free or both) between the size of N =1,000 and size of N = 10,000. It will be a good future research topic whether network properties remains same irrespective of any size of N under specific α and K_0.

Ⅴ. Conclusion

In this study we have developed a generalized network generation model, which can produce small-world network, scale-free network, or network with the properties of both small-world and scale-free by controlling two input parameter, α and K_0. We have investigated the effects of α and K_0 on properties of the generated network. We have shown that K_0 affects clustering coefficient and average shortest path length. So, small-world network can be generated by tuning K_0 properly. And α is a determining parameter for scale-free network. It also affects average shortest path length and clustering coefficient. So, properties of both scale-free and small-world networks can be represented by tuning α properly.

It has been shown which combination of α and K_0. produces small-world network, scale-free network or both. We have also investigated the effect of N on clustering coefficient, average shortest path length and power-law, whish shows that network classes(scale-free, small-world or both) remain same between N = 1,000 and N = 10,000. Whether network classes remain same irrespective of node size will be a good future research topic.

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