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MAPPINGS OF CUBIC SETS

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Abstract. Images and inverse images of (almost) stable cubic sets are discussed. We show that the image and inverse image of stable cubic sets are also stable. Conditions for the image of almost cubic sets to be an almost cubic set are provided. The complement, the P-union and the P-intersection of (inverse) images of (almost) stable cubic sets are considered.

1. Introduction

Using a fuzzy set and an interval-valued fuzzy set, Jun et al. [5] introduced a new notion, called a (internal, external) cubic set, and investigated several properties. They dealt with P-union, P-intersection, R-union and R-intersection of cubic sets, and investigated several related properties. Cubic set theory is applied to CI-algebras (see [1]), B-algebras (see [8]), BCK/BCI-algebras (see [6, 7]), KU-Algebras (see [2, 9]), and semigroups (see [3]). In [4], Jun et al., introduced the notions of (almost) stable cubic set, stable element, evaluative set and stable degree. Regarding internal (external) cubic sets and the complement of cubic set, they investigated their (almost) stableness and unstableness. Regarding the P-union, R-union, P-intersection and R-intersection of cubic sets, they dealt with their (almost) stableness and unstableness.

In this paper, we discuss images and inverse images of (almost) stable cubic sets. We show that the image and inverse image of stable cubic sets are also stable. We provide conditions for the image of almost cubic sets to be an almost cubic set. We consider the complement, the P-union and P-intersection of (inverse) images of (almost) stable cubic sets.

2. Preliminaries

A *fuzzy set* in a set X is defined to be a function $\lambda : X \to [0,1]$. Denote by I^X the collection of all fuzzy sets in a set X. Define a relation $\leq$ on I^X as

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follows:

$$(\forall \lambda, \mu \in I^X)\,(\lambda \leq \mu \iff (\forall x \in X)(\lambda(x) \leq \mu(x))).$$

The join ($\vee$) and meet ($\wedge$) of λ and μ are defined by

$$(\lambda \vee \mu)(x) = \max\{\lambda(x), \mu(x)\},$$
$$(\lambda \wedge \mu)(x) = \min\{\lambda(x), \mu(x)\},$$

respectively, for all $x \in X$. The complement of λ, denoted by λ^c, is defined by

$$(\forall x \in X)\,(\lambda^c(x) = 1 - \lambda(x)).$$

For a family $\{\lambda_i \mid i \in \Lambda\}$ of fuzzy sets in X, we define the join ($\vee$) and meet ($\wedge$) operations as follows:

$$\left(\bigvee_{i\in\Lambda} \lambda_i\right)(x) = \sup\{\lambda_i(x) \mid i \in \Lambda\},$$
$$\left(\bigwedge_{i\in\Lambda} \lambda_i\right)(x) = \inf\{\lambda_i(x) \mid i \in \Lambda\},$$

respectively, for all $x \in X$.

Let $f : X \to Y$ be a mapping and let λ be a fuzzy set in X. Then the *image* of λ under f, denoted by $f(\lambda)$, is defined as follows:

$$f(\lambda)(y) = \begin{cases} \sup\limits_{y=f(x)} \lambda(x), & f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise} \end{cases}$$

for all $y \in Y$. Let μ be a fuzzy set in Y. Then the *inverse image* of μ under f, denoted by $f^{-1}(\mu)$, is defined as follows:

$$(\forall x \in X)\,(f^{-1}(\mu)(x) = \mu(f(x))).$$

Let $D[0,1]$ be the set of all closed subintervals of the unit interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \ldots$, and note that $M = [M^-, M^+]$, where M^- and M^+ are the lower and the upper end points, respectively. Especially, we denote $\mathbf{0} = [0,0]$, $\mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for every $a \in (0,1)$. We also note that

(i) $(\forall M, N \in D[0,1])\;(M = N \Leftrightarrow M^- = N^-,\, M^+ = N^+)$.
(ii) $(\forall M, N \in D[0,1])\;(M \leq N \Leftrightarrow M^- \leq N^-,\, M^+ \leq N^+)$.

For every $M \in D[0,1]$, the *complement* of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^+, 1 - M^-]$.

Let X be a nonempty set. A function $A : X \to D[0,1]$ is called an *interval-valued fuzzy set* (briefly, an *IVF set*) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^-$ and $A(x)^+$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. Denote by D^X the collection of all interval-valued fuzzy sets in a set X. In particular, for any $a \in [0,1]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all $x \in X$ is denoted by simply $\tilde{a}$.

For every $A, B \in D^X$, we define

$$A = B \Leftrightarrow (\forall x \in X)\,(A(x)^- = B(x)^-,\ A(x)^+ = B(x)^+),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)\,(A(x)^- \leq B(x)^-,\ A(x)^+ \leq B(x)^+).$$

The *complement* A^c of A is defined by

$$(\forall x \in X)\left(A^c(x)^- = 1 - A(x)^+,\ A^c(x)^+ = 1 - A(x)^-\right).$$

For a family $\{A_i \mid i \in \Lambda\}$ of IVF sets where Λ is an index set, the *union* $G = \bigcup_{i \in \Lambda} A_i$ and the *intersection* $F = \bigcap_{i \in \Lambda} A_i$ are defined by

$$(\forall x \in X)\left(G(x)^- = \sup_{i \in \Lambda} A_i(x)^-,\ G(x)^+ = \sup_{i \in \Lambda} A_i(x)^+\right),$$

$$(\forall x \in X)\left(F(x)^- = \inf_{i \in \Lambda} A_i(x)^-,\ F(x)^+ = \inf_{i \in \Lambda} A_i(x)^+\right),$$

respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the *image* of A under f, denoted by $f(A)$, is defined as follows:

$$f(A)(y)^- = \begin{cases} \sup\limits_{y=f(x)} A(x)^- & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

$$f(A)(y)^+ = \begin{cases} \sup\limits_{y=f(x)} A(x)^+ & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in Y$. Let B be an IVF set in Y. Then the *inverse image* of B under f, denoted by $f^{-1}(B)$, is defined as follows:

$$(\forall x \in X)\,(f^{-1}(B)(x)^- = B(f(x))^-,\ f^{-1}(B)(x)^+ = B(f(x))^+).$$

Definition 2.1 ([5])**.** Let X be a nonempty set. By a *cubic set* in X we mean a structure

$$\mathscr{A} = \{\langle x, A(x), \lambda(x)\rangle \mid x \in X\}$$

in which A is an IVF set in X and λ is a fuzzy set in X.

A cubic set $\mathscr{A} = \{\langle x, A(x), \lambda(x)\rangle \mid x \in X\}$ is simply denoted by $\mathscr{A} = \langle A, \lambda\rangle$. Note that a cubic set is a generalization of an intuitionistic fuzzy set.

Definition 2.2 ([5])**.** Let X be a nonempty set. A cubic set $\mathscr{A} = \langle A, \lambda\rangle$ in X is said to be an *internal cubic set* (briefly, ICS) if $A(x)^- \leq \lambda(x) \leq A(x)^+$ for all $x \in X$.

Definition 2.3 ([5])**.** Let X be a nonempty set. A cubic set $\mathscr{A} = \langle A, \lambda\rangle$ in X is said to be an *external cubic set* (briefly, ECS) if $\lambda(x) \notin (A(x)^-, A(x)^+)$ for all $x \in X$.

Theorem 2.4 ([5])**.** *Let $\mathscr{A} = \langle A, \lambda\rangle$ be a cubic set in X. If $\mathscr{A}$ is both an ICS and an ECS, then*

$$(\forall x \in X)\ (\lambda(x) \in U(A) \cup L(A)),$$

where $U(A) = \{A(x)^+ \mid x \in X\}$ and $L(A) = \{A(x)^- \mid x \in X\}$.

Definition 2.5 ([5])**.** Let $\mathscr{A} = \langle A, \lambda\rangle$ and $\mathscr{B} = \langle B, \mu\rangle$ be cubic sets in X. Then we define

(a) (Equality) $\mathscr{A} = \mathscr{B} \Leftrightarrow A = B$ and $\lambda = \nu$.
(b) (P-order) $\mathscr{A} \sqsubseteq \mathscr{B} \Leftrightarrow A \subseteq B$ and $\lambda \leq \nu$.
(c) (R-order) $\mathscr{A} \Subset \mathscr{B} \Leftrightarrow A \subseteq B$ and $\lambda \geq \nu$.

Definition 2.6 ([5])**.** Let $\mathscr{A} = \langle A, \lambda\rangle$, $\mathscr{B} = \langle B, \mu\rangle$ and $\mathscr{A}_i = \{\langle x, A_i(x), \lambda_i(x)\rangle \mid x \in X\}$ be cubic sets in X for $i \in \Lambda$. The *complement, P-union, P-intersection, R-union and R-intersection* are defined as follows:

(a) (Complement) $\mathscr{A}^c = \{\langle x, A^c(x), 1 - \lambda(x)\rangle \mid x \in X\}$.
(b) (P-union) $\mathscr{A} \sqcup \mathscr{B} = \{\langle x, (A \cup B)(x), (\lambda \vee \nu)(x)\rangle \mid x \in X\}$ and $\sqcup\mathscr{A}_i = \{\langle x, (\bigcup A_i)(x), (\bigvee \lambda_i)(x)\rangle \mid x \in X\}$ for $i \in \Lambda$.
(c) (P-intersection) $\mathscr{A} \sqcap \mathscr{B} = \{\langle x, (A \cap B)(x), (\lambda \wedge \nu)(x)\rangle \mid x \in X\}$ and $\sqcap\mathscr{A}_i = \{\langle x, (\bigcap A_i)(x), (\bigwedge \lambda_i)(x)\rangle \mid x \in X\}$ for $i \in \Lambda$.
(d) (R-union) $\mathscr{A} \Cup \mathscr{B} = \{\langle x, (A \cup B)(x), (\lambda \wedge \nu)(x)\rangle \mid x \in X\}$ and $\Cup\mathscr{A}_i = \{\langle x, (\bigcup A_i)(x), (\bigwedge \lambda_i)(x)\rangle \mid x \in X\}$ for $i \in \Lambda$.
(e) (R-intersection) $\mathscr{A} \Cap \mathscr{B} = \{\langle x, (A \cap B)(x), (\lambda \vee \nu)(x)\rangle \mid x \in X\}$ and $\Cap\mathscr{A}_i = \{\langle x, (\bigcap A_i)(x), (\bigvee \lambda_i)(x)\rangle \mid x \in X\}$ for $i \in \Lambda$.

3. Mappings of cubic sets

In what follows, let X and Y denote nonempty sets unless otherwise specified.

Definition 3.1 ([4])**.** Let $\mathscr{A} = \langle A, \lambda\rangle$ be a cubic set in X with the evaluative set

$$\mathbf{E}_{\mathscr{A}} = \{(x, E_{\mathscr{A}}(x)) \mid x \in X\}.$$

An element $a \in X$ is called a *stable element* of $\mathscr{A} = \langle A, \lambda\rangle$ in X if it satisfies:

$$l(E_{\mathscr{A}}(a)) = \lambda(a) - A(a)^- \geq 0,\ r(E_{\mathscr{A}}(a)) = A(a)^+ - \lambda(a) \geq 0.$$

Otherwise, we say that a is an *unstable element* of $\mathscr{A} = \langle A, \lambda\rangle$ in X. The set of all stable elements of $\mathscr{A} = \langle A, \lambda\rangle$ in X is called the *stable cut* of $\mathscr{A} = \langle A, \lambda\rangle$ in X and is denoted by $S_{\mathscr{A}}$. The set of all unstable elements of $\mathscr{A} = \langle A, \lambda\rangle$ in X is called the *unstable cut* of $\mathscr{A} = \langle A, \lambda\rangle$ in X and is denoted by $U_{\mathscr{A}}$. We say that $\mathscr{A} = \langle A, \lambda\rangle$ is a *stable* cubic set if $S_{\mathscr{A}} = X$. Otherwise, $\mathscr{A} = \langle A, \lambda\rangle$ is called an *unstable* cubic set.

It is clear that $X = S_{\mathscr{A}} \cup U_{\mathscr{A}}$,

$$S_{\mathscr{A}} = \{x \in X \mid l(E_{\mathscr{A}}(x)) \geq 0,\ r(E_{\mathscr{A}}(x)) \geq 0\}$$

and $U_{\mathscr{A}} = \{x \in X \mid l(E_{\mathscr{A}}(x)) < 0\} \cup \{x \in X \mid r(E_{\mathscr{A}}(x)) < 0\}$.

Definition 3.2. Let $f : X \to Y$ be a mapping and let $\mathscr{A} = \langle A, \lambda\rangle$ be a cubic set in X. Then the *image* of $\mathscr{A} = \langle A, \lambda\rangle$ under f is denoted by $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ and is defined by

$$(\forall y \in Y)\,(f(\mathscr{A})(y) = \langle f(A)(y), f(\lambda)(y)\rangle)\,.$$

Definition 3.3. Let $f : X \to Y$ be a mapping and let $\mathscr{B} = \langle B, \mu\rangle$ be a cubic set in Y. Then the *inverse image* of $\mathscr{B} = \langle B, \mu\rangle$ under f is denoted by $f^{-1}(\mathscr{B}) = \langle f^{-1}(B), f^{-1}(\mu)\rangle$ and is defined by

$$(\forall x \in X)\left(f^{-1}(\mathscr{B})(x) = \langle f^{-1}(B)(x), f^{-1}(\mu)(x)\rangle\right).$$

Example 3.4. (1) Let $\mathscr{A} = \langle A, \lambda\rangle$ be a cubic set in $I = [0, 1]$ defined by

$$A(x) = \begin{cases} \left[\frac{11}{20}, \frac{3}{5}\right] & \text{if } 0 \le x \le \frac{1}{2}, \\ \left[\frac{9}{10}, \frac{19}{20}\right] & \text{if } \frac{1}{2} < x \le 1, \end{cases}$$
$$\lambda(x) = -\tfrac{1}{2}x + \tfrac{3}{4}$$

for all $x \in I$. Consider a mapping $f : I \to I$ given by $f(x) = \frac{1}{2}x$ for all $x \in I$. Then $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is given as follows:

$$f(A)(y) = \begin{cases} \left[\frac{11}{20}, \frac{3}{5}\right] & \text{if } 0 \le y \le \frac{1}{4}, \\ \left[\frac{9}{10}, \frac{19}{20}\right] & \text{if } \frac{1}{4} < y \le \frac{1}{2}, \\ \mathbf{0} & \text{if } \frac{1}{2} < y \le 1, \end{cases}$$
$$f(\lambda)(y) = \begin{cases} -\frac{1}{2}y + \frac{3}{4} & \text{if } 0 \le y \le \frac{1}{2}, \\ 0 & \text{if } \frac{1}{2} < y \le 1 \end{cases}$$

for all $y \in Y$.

(2) Let $\mathscr{B} = \langle B, \mu\rangle$ be a cubic set in $Y = \{a, b, c\}$ defined as Table 1.

Table 1. Tabular representation of the cubic set $\mathscr{B}$

Y	$B(y)$	$\mu(y)$
a	$[1.0, 1.0]$	0.6
b	$[0.5, 1.0]$	0.7
c	$[0.6, 1.0]$	0.5

For a set $X = \{a, b, c\}$, let g be a function given as follows:

$$g : X \to Y,\ x \mapsto \begin{cases} b & \text{if } x = a, \\ c & \text{if } x = b, \\ a & \text{if } x = c. \end{cases}$$

Then $g^{-1}(\mathscr{B}) = \langle f^{-1}(B), f^{-1}(\mu)\rangle$ is given as Table 2.

Theorem 3.5. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ is a stable cubic set in X, then $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is a stable cubic set in Y.*

Table 2. Tabular representation of the cubic set $g^{-1}(\mathscr{B})$

X	$g^{-1}(B)(x)$	$g^{-1}(\mu)(x)$
a	$[0.5, 1.0]$	0.7
b	$[0.6, 1.0]$	0.5
c	$[1.0, 1.0]$	0.6

Proof. Let $\mathscr{A} = \langle A, \lambda\rangle$ be a stable cubic set in X. Then

$$S_{\mathscr{A}} = \{x \in X \mid l(E_{\mathscr{A}}(x)) \geq 0,\ r(E_{\mathscr{A}}(x)) \geq 0\} = X.$$

Hence $\lambda(x) - A(x)^- \geq 0$ and $A(x)^+ - \lambda(x) \geq 0$ for all $x \in X$. For each $y \in Y$, if $f^{-1}(y) \neq \emptyset$, then

$$f(\lambda)(y) = \sup_{y=f(x)} \lambda(x) \geq \sup_{y=f(x)} A(x)^- = f(A)(y)^-$$

and

$$f(A)(y)^+ = \sup_{y=f(x)} A(x)^+ \geq \sup_{y=f(x)} \lambda(x) = f(\lambda)(y).$$

Obviously, $f(\lambda)(y) \geq f(A)(y)^-$ and $f(A)(y)^+ \geq f(\lambda)(y)$ if $f^{-1}(y) = \emptyset$. This shows that $S_{f(\mathscr{A})} = Y$, and therefore $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is a stable cubic set in Y. □

Lemma 3.6 ([4])**.** *The complement of a stable cubic set is also stable.*

Theorem 3.7. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ is a stable cubic set in X, then the complement of $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is a stable cubic set in Y.*

Proof. It follows from Theorem 3.5 and Lemma 3.6. □

Lemma 3.8 ([4])**.** *The P-union and P-intersection of two stable cubic sets in X are stable cubic sets in X.*

Theorem 3.9. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ and $\mathscr{B} = \langle B, \mu\rangle$ are stable cubic sets in X, then the P-union and the P-intersection of $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ and $f(\mathscr{B}) = \langle f(B), f(\mu)\rangle$ is a stable cubic set in Y.*

Proof. By Theorem 3.5 and Lemma 3.8, it is straightforward. □

Theorem 3.10. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ and $\mathscr{B} = \langle B, \mu\rangle$ are stable cubic sets in X, then $f(\mathscr{A} \sqcup \mathscr{B})$ and $f(\mathscr{A} \sqcap \mathscr{B})$ are stable cubic sets in Y.*

Proof. Lemma 3.8 and Theorem 3.5 induces the result. □

Definition 3.11 ([4])**.** Let $\mathscr{A} = \langle A, \lambda\rangle$ be a cubic set with the evaluative set

$$\mathbf{E}_{\mathscr{A}} = \{(x, E_{\mathscr{A}}(x)) \mid x \in X\}$$

in X. Then the *stable degree* of $\mathscr{A}$ in X is denoted by $SD_{\mathscr{A}}$ and is defined by

$$SD_{\mathscr{A}} = \left(\sum_{x\in X} l(E_{\mathscr{A}}(x)),\ \sum_{x\in X} r(E_{\mathscr{A}}(x)) \right). \tag{3.1}$$

Definition 3.12 ([4])**.** A cubic set $\mathscr{A} = \langle A, \lambda\rangle$ with the evaluative set

$$\mathbf{E}_{\mathscr{A}} = \{(x, E_{\mathscr{A}}(x)) \mid x \in X\}$$

in X is said to be *almost stable* if there exists the stable degree $SD_{\mathscr{A}}$ in which $\sum_{x\in X} l(E_{\mathscr{A}}(x)) \geq 0$ and $\sum_{x\in X} r(E_{\mathscr{A}}(x)) \geq 0$.

Theorem 3.13. *Let $f : X \to Y$ be an injective mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ is an almost stable cubic set in X, then $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is an almost stable cubic set in Y.*

Proof. Let $\mathscr{A} = \langle A, \lambda\rangle$ be an almost stable cubic set in X. Then there exists a stable degree $SD_{\mathscr{A}}$ such that

$$\sum_{x\in X} l(E_{\mathscr{A}}(x)) = \sum_{x\in X} (\lambda(x) - A(x)^-) \geq 0,$$

and

$$\sum_{x\in X} r(E_{\mathscr{A}}(x)) = \sum_{x\in X} (A(x)^+ - \lambda(x)) \geq 0.$$

We have to show that $\sum_{x\in X} l(E_{f(\mathscr{A})}(x)) \geq 0$ and $\sum_{x\in X} r(E_{f(\mathscr{A})}(x)) \geq 0$ in the stable degree $SD_{f(\mathscr{A})}$. For each $y \in Y$, let $Y^* = \{y \in Y \mid f^{-1}(y) \neq \emptyset\}$ and $Y^{**} = Y \setminus Y^*$. Then Y is the disjoint union of Y^* and Y^{**}. If $y \in Y^{**}$, then $f(A)(y) = \mathbf{0}$ and $f(\lambda)(y) = 0$ which imply that

$$\sum_{y\in Y^{**}} (f(A)(y)^+ - f(\lambda)(y)) = 0 \text{ and } \sum_{y\in Y^{**}} (f(\lambda)(y) - f(A)(y)^-) = 0.$$

Since f is injective, $f^{-1}(y)$ is a singleton set for all $y \in Y^*$ and $\{f^{-1}(y) \mid y \in Y^*\} = X$. Hence

$$\begin{aligned}
\sum_{x\in X} l(E_{f(\mathscr{A})}(x)) &= \sum_{y\in Y} \big(f(\lambda)(y) - f(A)(y)^-\big) \\
&= \sum_{y\in Y^*} \big(f(\lambda)(y) - f(A)(y)^-\big) + \sum_{y\in Y^{**}} \big(f(\lambda)(y) - f(A)(y)^-\big) \\
&= \sum_{y\in Y^*} \big(f(\lambda)(y) - f(A)(y)^-\big) \\
&= \sum_{x\in X} (\lambda(x) - A(x)^-) \geq 0
\end{aligned}$$

and

$$\begin{aligned}\sum_{x\in X} r(E_{f(\mathscr{A})}(x)) &= \sum_{y\in Y}\big(f(A)(y)^+ - f(\lambda)(y)\big)\\ &= \sum_{y\in Y^*}\big(f(A)(y)^+ - f(\lambda)(y)\big) + \sum_{y\in Y^{**}}\big(f(A)(y)^+ - f(\lambda)(y)\big)\\ &= \sum_{y\in Y^*}\big(f(A)(y)^+ - f(\lambda)(y)\big)\\ &= \sum_{x\in X}(A(x)^+ - \lambda(x)) \geq 0.\end{aligned}$$

Therefore $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is an almost stable cubic set in Y. □

Lemma 3.14 ([4]). *The complement of an almost stable cubic set is also almost stable.*

Combining Theorem 3.13 and Lemma 3.14, we have the following theorem.

Theorem 3.15. *Let $f : X \to Y$ be an injective mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ is an almost stable cubic set in X, then the complement of $f(\mathscr{A}) = \langle f(A), f(\lambda)\rangle$ is an almost stable cubic set in Y.*

Proof. It is by Theorem 3.13 and Lemma 3.14. □

Theorem 3.16. *Let $f : X \to Y$ be a mapping. If $\mathscr{B} = \langle B, \mu\rangle$ is a stable cubic set in Y, then $f^{-1}(\mathscr{B}) = \langle f^{-1}(B), f^{-1}(\mu)\rangle$ is a stable cubic set in X.*

Proof. Let $\mathscr{B} = \langle B, \mu\rangle$ be a stable cubic set in X. Then

$$S_{\mathscr{B}} = \{y \in Y \mid l(E_{\mathscr{B}}(y)) \geq 0,\ r(E_{\mathscr{B}}(y)) \geq 0\} = Y.$$

Hence $\mu(y) - B(y)^- \geq 0$ and $B(y)^+ - \mu(y) \geq 0$ for all $y \in Y$. It follows that

$$l(E_{f^{-1}(\mathscr{B})}(x)) = f^{-1}(\mu)(x) - f^{-1}(B)(x)^- = \mu(f(x)) - B(f(x))^- \geq 0$$

and

$$r(E_{f^{-1}(\mathscr{B})}(x)) = f^{-1}(B)(x)^+ - f^{-1}(\mu)(x) = B(f(x))^+ - \mu(f(x)) \geq 0$$

for all $x \in X$. Thus

$$S_{f^{-1}(\mathscr{B})} = \{y \in Y \mid l(E_{f^{-1}(\mathscr{B})}(x)) \geq 0,\ r(E_{f^{-1}(\mathscr{B})}(x)) \geq 0\} = X,$$

and therefore $f^{-1}(\mathscr{B}) = \langle f^{-1}(B), f^{-1}(\mu)\rangle$ is a stable cubic set in X. □

Theorem 3.17. *Let $f : X \to Y$ be a mapping. If $\mathscr{B} = \langle B, \mu\rangle$ is a stable cubic set in Y, then $f^{-1}(\mathscr{B})^c$ is a stable cubic set in X.*

Proof. It is by Theorem 3.16 and Lemma 3.6. □

Theorem 3.18. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ and $\mathscr{B} = \langle B, \mu\rangle$ are stable cubic sets in Y, then the P-union and the P-intersection of $f^{-1}(\mathscr{A})$ and $f^{-1}(\mathscr{B})$ are stable cubic sets in X.*

Proof. By Theorem 3.16 and Lemma 3.8, the result is valid. □

Theorem 3.19. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ and $\mathscr{B} = \langle B, \mu\rangle$ are stable cubic sets in Y, then $f^{-1}(\mathscr{A} \sqcup \mathscr{B})$ and $f^{-1}(\mathscr{A} \sqcap \mathscr{B})$ are stable cubic sets in X.*

Proof. By Lemma 3.8 and Theorem 3.16, we have the desired result. □

Theorem 3.20. *Let $f : X \to Y$ be a mapping. If $\mathscr{A} = \langle A, \lambda\rangle$ is a stable cubic set in X, then $f^{-1}(f(\mathscr{A}))$ is a stable cubic set in X.*

Proof. It is by Theorems 3.5 and 3.16. □

Theorem 3.21. *Let $f : X \to Y$ be a mapping. If $\mathscr{B} = \langle B, \mu\rangle$ is a stable cubic set in Y, then $f(f^{-1}(\mathscr{B}))$ is a stable cubic set in Y.*

Proof. By Theorems 3.16 and 3.5, it is straightforward. □

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