Joint Lattice-Reduction-Aided Precoder Design for Multiuser MIMO Relay System

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Received February 11, 2016; revised May 20, 2016; accepted May 31, 2016; published July 31, 2016

Abstract

Lattice reduction (LR) has been used widely in conventional multiple-input multiple-output (MIMO) systems to enhance the performance. However, LR is hard to be applied to the relay systems which are important but more complicated in the wireless communication theory. This paper introduces a new viewpoint for utilizing LR in multiuser MIMO relay systems. The vector precoding (VP) is designed along with zero force (ZF) criterion and minimum mean square error (MMSE) criterion and enhanced by LR algorithm. This implementable precoder design combines nonlinear processing at the base station (BS) and linear processing at the relay. This precoder is capable of avoiding multiuser interference (MUI) at the mobile stations (MSs) and achieving excellent performance. Moreover, it is shown that the amount of feedback information is much less than that of the singular value decomposition (SVD) design. Simulation results show that the proposed scheme using the complex version of the Lenstra--Lenstra--Lovász (LLL) algorithm significantly improves system performance.

Keywords: Lattice reduction, multiuser, MIMO relay, precoder design, multiuser interference.

This paper is supported by The National Natural Science Foundation of China under Grant Nos. 61501224 and 61401236.

http://dx.doi.org/10.3837/tiis.2016.07.008 **ISSN : 1976-7277**

1. Introduction

Cooperative data transmission has attracted significant attention from the research community recently. By transmitting information via a relay instead of direct channels, cooperative communication can enhance coverage, reliability, power efficiency and the capacity of wireless networks [1-4]. Among different cooperative transmission protocols, amplify-and-forward (AF) relays require less signal processing and are transparent for source/destination coding and modulation schemes. Thus, research on precoder designs for multiuser relay system mainly focuses on cooperation under the AF protocol. In [1], the multiuser precoding strategy is implemented under the AF protocol by jointly designing Tomlison-Harashima precoding (THP) at the multiple-input multiple-output (MIMO) base station (BS) and linear signal processing at the MIMO relay station (RS). However, this method is of high complexity and requires full feedback of the channel state information (CSI) from mobile users (MSs) to BS, which is not always feasible in practice. In [5-7], various precoding schemes are designed for AF MIMO relay systems, combined with a Tomlinson-Harashima precoding. The channel estimation errors, which are inevitable in practical relay systems, are taken into account in the transceiver design.

It is known that a MIMO system can exhibit optimal performance if the channel matrix is orthogonal. However, the original channel matrices could be ill conditioned, resulting in significant performance degradation. Lattice reduction (LR) is known to be capable of transforming an ill-conditioned matrix closer to an orthogonal matrix by identifying a set of basis that is closer to orthogonality than the columns of a matrix that span the same lattice. It has been widely used in MIMO systems, not only to help conventional data detection collect a high diversity, but also help precoder reduce average transmitted energy [8-16].

In the paper, we consider the use of Lenstra-Lenstra-Lovász (LLL) algorithm [17] to implement a joint BS/RS precoding design for multiuser MIMO relay systems that combines vector perturbation precoding at the BS and linear signal processing at the relay. Furthermore, the minimum mean square error (MMSE) criterion is incorporated to achieve an improved performance. The precoder is capable of cancelling out the multiuser interference (MUI) at the MSs due to relay links. In addition, we reduce the amount of feedback information. Compared to full feedback of the CSI in the SVD design in [1], the partial information, which between the RS and the MSs and is fed back to the BS, is a Gaussian integer matrix bounded by a modulo operation. Besides, this LR-aided precoder significantly improves the bit error rate (BER) performance of multiuser MIMO relay networks with only a small increase in complexity. In simulation results, the BER performance shows a prospect of high diversity gain achieved. At last, since the unimodular matrix, which is derived by LR algorithm, may be dependent on the CSI quality, this paper also investigates the impact of imperfect CSI and channel asymmetry in the simulation.

The rest of this paper is organized as follows. The proposed system model and relay based transmission framework in a MIMO system are introduced in Section 2. Next, joint lattice-reduced-aided precoder design is proposed in Section 3. Selected simulation results are presented in Section 4 to demonstrate the performance of proposed scheme. Finally, conclusions are drawn in Section 5.

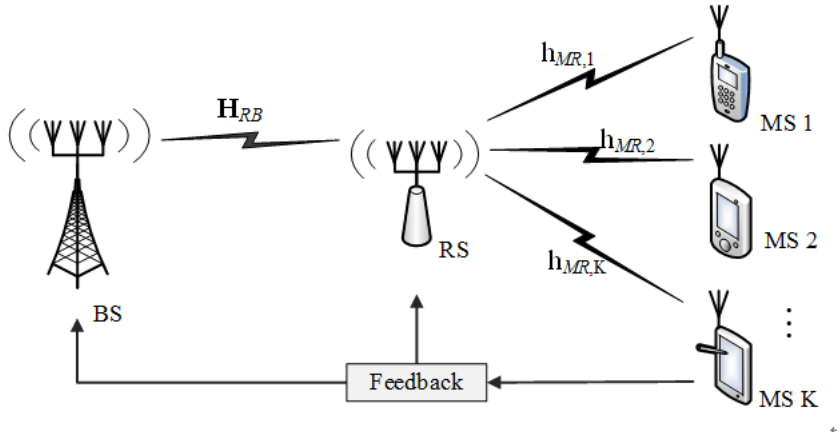

Fig. 1. Multiuser broadcast system with a fixed multi-antenna RS.

2. System Model

We consider an AF MIMO broadcast channel modeling the downlink of a multiuser relay system as depicted in **Fig. 1**, where the BS and RS are equipped with M and K antennas, respectively. Due to size and cost constraints, each MS is assumed to be equipped with a single antenna. There are K MSs being served via the fixed half-duplex RS. The channels remain constant within a block. To ensure that the compound channel can support K independent substreams we assume $K \le M$. The data vector to be transmitted is $\mathbf{s} = [s_1, s_2, \cdots, s_K]^T$. To apply the lattice viewpoint, the constellation should be of lattice type, such as BPSK, QPSK, QAM, or PAM. Specifically, the data symbols are chosen from an m -QAM constellation with the real and imaginary parts of s_k belonging to the set $\mathcal{A} = \{\pm 1, \pm 3, \cdots, \pm(\sqrt{m}-1)\}$. We neglect any direct connections between the BS and the MSs. Due to the half-duplex relay we use the typical two-phase transmission scheme where the BS and RS transmissions are synchronized. In detail, the BS transmits a symbol vector to a RS in the first phase. During the second phase, the RS processes and sends the received signal to the MSs. Both phases span equal time durations.

The $K \times 1$ received signal vector at the RS is given as

$$\mathbf{y}_{RB} = \mathbf{H}_{RB}\mathbf{x} + \mathbf{n}_{RB} \,, \tag{1}$$

where $\mathbf{x}$ is the precoded signal vector with the transmit power constraint $\mathbb{E}[\mathbf{x}^H\mathbf{x}] = P_B$; $\mathbf{H}_{RB}$ represents the $K \times M$ MIMO channel matrix between BS and RS where each element is assumed to be independent and identically distributed (i.i.d.) zero-mean unit-variance complex Gaussian random variable; $\mathbf{n}_{RB}$ is a Gaussian noise vector where each term is assumed to be i.i.d. zero-mean complex Gaussian random variable with unit variance.

During the second phase, we consider the AF relay processing, while a precoding matrix $\mathbf{F}_R$ is used at the RS. The signal transmitted by the RS is

$$\mathbf{x}_R = \mathbf{F}_R \mathbf{y}_{RB} \,. \tag{2}$$

The power constraint $\mathbb{E}\| \mathbf{x_R} \|^2 = P_R$ is imposed, where P_R is the transmit power at the RS. Also, P_B and P_R can be viewed as the transmit SNR of BS-RS link and RS-MSs link respectively.

Let the row vector $\mathbf{h}_{MR,k}$ denote the channel between the RS and the k th MS. $\mathbf{h}_{RM,k}$ is a row vector and the elements are zero-mean unit-variance i.i.d. variables. The received signal at the k th MS is therefore

$$y_{MR,k} = \mathbf{h}_{MR,k}\mathbf{x}_R + n_{MR,k}, \quad (3)$$

where $n_{MR,k}$ is noise at the k th MS receiver assumed to be an i.i.d. zero-mean complex Gaussian random variable with unit variance. We stack the vector $\mathbf{h}_{MR,k}$ row-wise to form the $K \times K$ matrix $\mathbf{H}_{MR}$. Therefore, the received signal at the MSs via RS transmission can be represented as

$$\mathbf{y}_{MR} = \mathbf{H}_{MR}\mathbf{F}_R\mathbf{H}_{RB}\mathbf{x} + \mathbf{H}_{MR}\mathbf{F}_R\mathbf{n}_{RB} + \mathbf{n}_{MR}, \quad (4)$$

where $\mathbf{n}_{MR}$ is a noise vector with elements $n_{MR,k}$. It is necessary to resolve any information collision at the MSs.

3. Joint Lattice-Reduction-Aided Precoder Design

3.1 Initialization

For a multiuser MIMO relay downlink, the interference due to signals transmitted to other users occurs at the transmitters, and in principle, a precoder can be used to essentially mitigate its effects. The well-known channel inversion (zero-forcing) precoding will amplify the transmit power, especially if the channel matrix is poorly conditioned. A more power-efficient precoding method is vector perturbation (VP) precoding [18-19] known to be capable of suppressing the power enhancement. Our focus in the joint precoding design is implementation of VP precoding at a BS along with LR-based precoder design at a RS.

It is known that the system can exhibit optimal performance if the original channel matrix is orthogonal. However, the channel matrices are not orthogonal and may be poorly conditioned. Fortunately, LR is capable of transforming an ill-conditioned matrix closer to an orthogonal matrix by identifying a set of basis that is closer to orthogonality than the columns of a matrix that span the same lattice. Two widely used used LR algorithms are the Korkine-Zolotareff (KZ) algorithm [20] and the LLL algorithm [17]. The KZ algorithm can identify the optimal basis for a lattice, at the cost of high complexity. In contrast, the LLL algorithm is a suboptimum solution of the shortest vector problem with polynomial complexity, and provides an acceptable tradeoff between performance and computational complexity. Specially, in [21] a complex LLL (CLLL) algorithm is introduced for direct application to a complex-valued lattice. Compared to the traditional LLL algorithm, the CLLL algorithm can reduce the complexity by half without degrading performance.

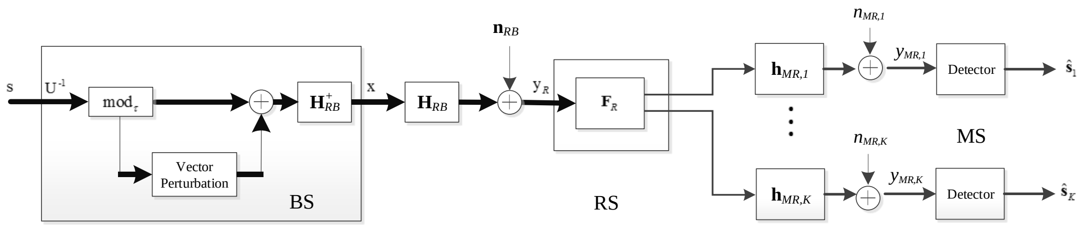

Fig. 2. Block diagram for the MIMO relay LR-aided precoding scheme.

Fig. 2 shows the precoding scheme for a cooperative MIMO relay system. Each MS collects its CSI and feeds back it to the RS. At the RS, we use the CLLL algorithm, which is a suboptimum solution of the shortest vector problem with a polynomial-time computational complexity [21], to reduce the lattice basis of $\mathbf{H}_{MR}^{\dagger}$ and obtain $\bar{\mathbf{H}} = \mathbf{H}_{MR}^{\dagger}\mathbf{U}$, where $\mathbf{H}_{MR}^{\dagger}$ is the right pseudoinverse of $\mathbf{H}_{MR}$ (i.e., $\mathbf{H}_{MR}^{\dagger} = \mathbf{H}_{MR}^{H}(\mathbf{H}_{MR}\mathbf{H}_{MR}^{H})^{-1}$; $\mathbf{U} = [u_{i,j}]_{1\le i\le K, 1\le j\le K}$ is a $K\times K$ unimodular matrix, i.e., a square matrix with Gaussian integer entries, such that $\det(\mathbf{U}) = \pm 1$). Then, the RS feeds back $\text{mod}_{\tau}(\mathbf{U}^{-1})$ to the BS and the original precoded data vector at the BS is

$$\mathbf{d} = \text{mod}_{\tau}(\text{mod}_{\tau}(\mathbf{U}^{-1})\mathbf{s}) = \text{mod}_{\tau}(\mathbf{U}^{-1}\mathbf{s}), \tag{5}$$

where τ is the length of the edges of the hypercube [22] (e.g., $\tau = 2\sqrt{m}$ for m-QAM). Note that the volume of $\mathbf{d}$ is preserved as that of $\mathbf{s}$. The modulo operator [9], which is originally derived for the THP, is denoted by:

$$\text{mod}_{\tau}(x) = x - \tau\left\lfloor \frac{\text{Real}(x)}{\tau} + \frac{1}{2}\right\rfloor - i\tau\left\lfloor \frac{\text{Imag}(x)}{\tau} + \frac{1}{2}\right\rfloor. \tag{6}$$

This nonlinear modulo operator is employed to bound the signals in a square region $[-\sqrt{m},\sqrt{m})\times[-\sqrt{m},\sqrt{m})$.

The feedback of $\text{mod}_{\tau}(\mathbf{U}^{-1})$ to the BS and equivalently multiplying $\mathbf{U}^{-1}$ to $\mathbf{s}$ at the BS is an essential processing to eliminate the effect of $\mathbf{U}$ at MSs by a simple modulo operation. It is worth highlighting that $\text{mod}_{\tau}(\mathbf{U}^{-1})$ instead of $\mathbf{H}_{MR}$ is required to be fed back to the BS from the RS. Since $\text{mod}_{\tau}(\mathbf{U}^{-1})$ is bounded in $[-\sqrt{m},\sqrt{m})\times[-\sqrt{m},\sqrt{m})$, the feedback signaling overhead is much lower. In contrast, the study in [1] assume that perfect channel information is available at the BS. That is, the channel information between the MSs and the RS and between the RS and the BS can be collected at the BS. Once the optimal relay processing coefficients are determined, the BS also need to configure the relay by determining the RS matrix and the precoding matrix via downlink signalling. The direct feedback channel information is required. Since the elements of the channel matrix is complex, finite feedback suffers from truncation error and quantization error.

A vector perturbation (VP) approach [8,18] is utilized to suppress the power enhancement. With VP, the transmit signal at the BS is formed by adding a (scaled) Gaussian integer perturbation $\mathbf{p}\in\Lambda^{K}$ ($\Lambda = \{a+bi \mid a,b\in\mathbb{Z}\}$ is a Gaussian integer set) to the data vector before pre-equalization, and can be expressed by

$$\mathbf{x} = \sqrt{\frac{P_B}{\Gamma_B}} \mathbf{H}_{RB}^{\dagger} (\mathbf{d} + \tau \mathbf{p}) . \tag{7}$$

where $\Gamma_B = \sqrt{\| \mathbf{H}_{RB}^{\dagger} (\mathbf{d} + \tau \mathbf{p}) \|^2}$ is the power normalization coefficient. $\mathbf{p}$ is designed so that the transmit signal requires minimum power, i.e.,

$$\mathbf{p} = \arg\min_{\mathbf{p}' \in \Lambda^K} \| \mathbf{H}_{RB}^{\dagger} (\mathbf{d} + \tau \mathbf{p}') \|^2 . \tag{8}$$

3.2 LR-aided Zero-Forcing Vector Perturbation (ZF-VP)

VP enables the data vector to be reoriented to a more favorable signal space direction. Finding the optimum perturbation vector is identical to searching the vector closest to $-\mathbf{H}_{RB}^{\dagger}\mathbf{d}$ in the lattice $\tau \mathbf{H}_{RB}^{\dagger} \Lambda^K$. This full search problem is very similar to the maximum likelihood (ML) detection problem, and can be effectively solved by sphere-encoding [18]. However, such lattice encoding results in high computational complexity, especially for complex lattices. In this case, we use the suboptimal LR-aided precoding technique. The original pre-equalization matrix $\mathbf{H}_{RB}^{\dagger}$ is CLLL-reduced and the corresponding reduced matrix is given by $\mathbf{W} = \mathbf{H}_{RB}^{\dagger}\mathbf{T}$, where $\mathbf{T}$ is a unimodular transformation matrix. Then, the LR-transformed cost function in Eq. (8) can be expressed in terms of the reduced matrix $\mathbf{W}$ as

$$\begin{aligned} \| \mathbf{H}_{RB}^{\dagger} (\mathbf{d} + \tau \mathbf{p}') \|^2 &= \| \mathbf{H}_{RB}^{\dagger} \mathbf{T}\mathbf{T}^{-1} (\mathbf{d} + \tau \mathbf{p}') \|^2 \\ &= \| \mathbf{W} (\tilde{\mathbf{d}} + \tau \tilde{\mathbf{p}}) \|^2, \end{aligned} \tag{9}$$

where $\tilde{\mathbf{d}} = \mathbf{T}^{-1}\mathbf{d}$ and $\tilde{\mathbf{p}} = \mathbf{T}^{-1}\mathbf{p}'$. This transformation does not incur any loss of performance and is more advantageous because $\mathbf{W}$ is nearly orthogonal compared to $\mathbf{H}_{RB}^{\dagger}$. Then, Eq. (9) can be solved approximately by using zero-forcing (ZF) criterion. As a result, the approximate perturbation vector is obtained by the rounding operation and is

$$\mathbf{p}_{\text{LR-ZF}} = -\mathbf{T} \left\lceil \frac{\mathbf{T}^{-1}\mathbf{d}}{\tau} \right\rfloor \tag{10}$$

3.3 LR-aided Tomlison-Harashima Precoding VP (THP-VP)

There is another variant of this algorithm that makes use of decision feedback approach similar to THP. In other words, THP can be interpreted as successive optimization of the elements of the perturbation vector. To achieve the performance better than that of ZF criterion, THP is implemented with aid of LR at the BS, while the RS just needs a simple modulo operation. By applying QR decomposition to $\mathbf{W}$, a lower triangular matrix $\mathbf{B}$ with unit diagonal is obtained as:

$$\mathbf{B} = \text{diag}(r_{11}^{-1}, \cdots, r_{KK}^{-1})\mathbf{R}, \qquad \mathbf{R} = \mathbf{F}\mathbf{W}, \tag{11}$$

where $\mathbf{F}$ is an orthogonal matrix, and $r_{11}, \cdots, r_{KK}$ are the diagonal elements of $\mathbf{R}$.

During successive quantization process, the previous quantized values are made use of in a feedback way. The first step of the algorithm is to set a vector $\mathbf{q} = [q_1, q_2, \cdots, q_K]^T$

$$\mathbf{q} = -\mathbf{F}\mathbf{H}_{RB}^{\dagger}\mathbf{d}. \tag{12}$$

Then, let $\tilde{q}_1 = q_1$, and the remaining components $\tilde{q}_2 \cdots, \tilde{q}_K$ are iteratively computed by correcting previous decisions as follows

$$\tilde{q}_k = \left\lceil \frac{1}{\tau}(q_k - \sum_{t=1}^{k-1} b_{kt}\tilde{q}_t) \right\rfloor, \tag{13}$$

where b_{kt} is the (k,t)th element of $\mathbf{B}$. Assuming $\tilde{\mathbf{q}} = [\tilde{q}_1, \cdots, \tilde{q}_K]^T$, the perturbation vector is calculated finally as

$$\mathbf{p}_{\text{LR-THP}} = \mathbf{T}\tilde{\mathbf{q}}. \tag{14}$$

The received signals at the RS contain some redundant information, which can be removed by subjecting the receive values to a modulo operation. Note that a normalization factor is required. The modulo operation leads to

$$\mathbf{y}_R = \text{mod}_\tau(\sqrt{\frac{\Gamma_B}{P_B}}\mathbf{y}_{RB}). \tag{15}$$

The transmitted signal at the RS can be written as

$$\mathbf{x}_R = \mathbf{F}_R \mathbf{y}_R. \tag{16}$$

As $\mathbf{H}_{MR}^\dagger$ is reduced at the RS, the joint LR-aided precoder at the RS can be achieved by choosing

$$\mathbf{F}_R = \sqrt{\frac{P_R}{\Gamma_R}}\bar{\mathbf{H}} = \sqrt{\frac{P_R}{\Gamma_R}}\mathbf{H}_{MR}^\dagger \mathbf{U}, \tag{17}$$

where $\Gamma_R = \sqrt{\| \bar{\mathbf{H}}\mathbf{y}_R \|^2}$ is the power normalization coefficient. This shorter and nearly orthogonal precoding matrix will suffer from less noise enhancement.

As a result, the received vector at the MSs is

$$\begin{aligned} \mathbf{y}_{MR} &= \sqrt{\frac{P_R}{\Gamma_R}}\mathbf{U}\text{mod}_\tau(\sqrt{\frac{\Gamma_B}{P_B}}\mathbf{H}_{RB}\mathbf{x} + \sqrt{\frac{\Gamma_B}{P_B}}\mathbf{n}_{RB}) + \mathbf{n}_{MR} \\ &= \sqrt{\frac{P_R}{\Gamma_R}}\mathbf{U}\text{mod}_\tau(\mathbf{d} + \tau\mathbf{p} + \sqrt{\frac{\Gamma_B}{P_B}}\mathbf{n}_{RB}) + \mathbf{n}_{MR} \\ &= \sqrt{\frac{P_R}{\Gamma_R}}\mathbf{U}\text{mod}_\tau(\mathbf{U}^{-1}\mathbf{s} + \sqrt{\frac{\Gamma_B}{P_B}}\mathbf{n}_{RB}) + \mathbf{n}_{MR}. \end{aligned} \tag{18}$$

To compensate for the effect of amplification by a factor of $\sqrt{P_R / \Gamma_R}$, the received signal must be divided by $\sqrt{P_R / \Gamma_R}$ via automatic gain control (AGC) circuit at the MSs.

In the practical system, the BS would compute the expected power $\mathbb{E}[\Gamma_B]$, where the expectation is taken over $\mathbf{s}$. That is, the expected power required to transmit each block is constant. Specially, if the packet is long enough, the empirical and expected values of both $\mathbb{E}[\Gamma_B]$ and $\mathbb{E}[\Gamma_R]$ will be close. Hence, with the knowledge of all the channels, the RS is possible to compute the expected power $\mathbb{E}[\Gamma_B]$ and $\mathbb{E}[\Gamma_R]$. What is more, the MSs do not need to know anything at all about the channels, only need to know $\mathbb{E}[\Gamma_R]$, which is a data independent quantity. In other words, the relay just needs to send the information of the product of $\mathbb{E}[\Gamma_R]$ at the beginning of the fading block. **Fig. 3** shows that the difference in performance is not significant.

Since $\mathbf{U}$ is a unimodular matrix, the effect of $\mathbf{U}^{-1}$ is removed and the data is recovered by the same modulo operator employed at the BS:

$$\hat{\mathbf{s}} = \mathrm{mod}_\tau(\mathbf{s} + \tilde{\mathbf{n}}), \tag{19}$$

where $\tilde{\mathbf{n}} = \sqrt{\Gamma_B / P_B}\mathbf{U}\mathbf{n}_{RB} + \sqrt{\Gamma_R / P_R}\mathbf{n}_{MR}$. Thus, each row of signal vector can be sliced as the ZF detector. The effective noise in Eq. (19) is an additive colored noise with variance

$$\begin{aligned} \sigma^2_{\tilde{\mathbf{n}}_i} &= \frac{1}{K}\mathbb{E}\| \mathrm{mod}_\tau(\tilde{\mathbf{n}}_i)\|^2 \\ &\le \frac{1}{K}\mathbb{E}\| \tilde{\mathbf{n}}_i\|^2 \\ &= \frac{\Gamma_B}{P_B}\sum_{j=1}^{K}|\mu_{i,j}|^2 + \frac{\Gamma_B\Gamma_R}{P_B P_R}. \end{aligned} \tag{20}$$

It is worthwhile to point out that the complexity of LR can be shared by symbols within the coherent time. Hence, the proposed precoder scheme is well suited for application in delay-constrained MIMO relay systems.

3.4 LR-aided MMSE Vector Perturbation Precoding (MMSE-VP)

Another alternative to ZF pre-equalization with improved performance is the MMSE pre-equalizer, which takes the noise term into account and thereby leads to an improved performance. In [23] the MMSE vector perturbation precoding (MMSE-VP) method was proposed for a MIMO system to minimize the mean-square error (MSE) of the received signal. We may incorporate the MMSE criterion in the LR-based precoding algorithm. This leads to LR-aided MMSE-VP with the perturbing vector given by

$$\mathbf{p} = \arg\min_{\mathbf{p}' \in \Lambda^K} \| \mathbf{L}(\mathbf{d} + \tau\mathbf{p}')\|^2. \tag{21}$$

where $\mathbf{L}$ is a $K \times K$ matrix obtained through Cholesky decomposition of the following matrix

$$(\mathbf{H}_{MR}\mathbf{H}_{MR}^H + \frac{K}{P_B}\mathbf{I}_K)^{-1} = \mathbf{L}^*\mathbf{L}, \tag{22}$$

In this case the precoded data vector is given as

$$\mathbf{x} = \sqrt{\frac{P_B}{\Gamma_B}}\mathbf{H}_{RB}^H(\mathbf{H}_{RB}\mathbf{H}_{RB}^H + \frac{K}{P_B}\mathbf{I}_K)^{-1}(\mathbf{d} + \tau\mathbf{p}), \tag{23}$$

where Γ_B is the power scaling factor enforcing power constraints.

The received signal at the RS is multiplied by the scaling factor and then fed to the modulo function. Similarly, we may employ a MMSE filter instead of the ZF method in order to get an improved estimate for transmit signal $\mathbf{x}_R$. By the MMSE method of the LR system we obtain

$$\mathbf{H}_{RMMSE} = \mathbf{H}_{MR}^{H}(\mathbf{H}_{MR}\mathbf{H}_{MR}^{H} + \frac{K}{P_R}\mathbf{I}_K)^{-1} . \tag{24}$$

It is necessary to mention that the LR algorithm is imposed on $\mathbf{H}_{RMMSE}$ instead of $\mathbf{H}_{MR}^{\dagger}$ and leads to $\bar{\mathbf{H}} = \mathbf{H}_{RMMSE}\mathbf{U}$ and

$$\mathbf{F}_{RMMSE} = \sqrt{\frac{P_R}{\Gamma_R}}\mathbf{H}_{RMMSE}\mathbf{U} \tag{25}$$

where Γ_R is the power scaling factor enforcing power constraints at the RS.

Furthermore, the MMSE method can be combined with THP scheme. The LR aided MMSE-THP-VP is capable of achieving the performance better than that of the MMSE and THP method. In [24], a precoding for MIMO-relay broadcast communication was proposed. Also, the performances were improved through LR algorithm. However, the RS has to send the information about translation vector to the MSs for compensation at the beginning of each block. This overload of these data is not necessary in our proposed scheme.

3.5 Complexity Analysis

The average complexity of the CLLL reduction algorithm is correlated to the distribution of the random basis matrix. There is no universal upper bound on the number of LLL iterations in the MIMO context, or equivalently that the worst-case complexity is unbounded. Fortunately, atypically large values of conditional number are very rare [8]. When the entries of a $m \times n$ matrix $\mathbf{H}$ is i.i.d. complex Gaussian $\mathcal{CN}(0,1)$, the overall average complexity is $\mathcal{O}(n^3 m \log n)$. The precoder design is implemented with an additional polynomial time preprocessing.

It is worth to note that when an LR method is used in a MIMO communication system, the complexity of an LR algorithm is only related to channel matrices and does not depend on signal to noise ratio (SNR) or constellation size [20]. So the computational complexity for LR algorithm corresponds to the effort for a 4-QAM-modulation system.

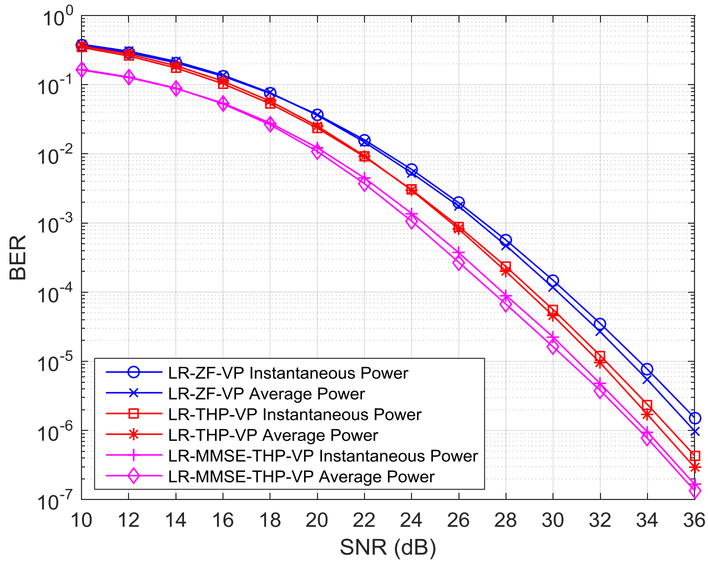

Fig. 3. BER performance of precoded MIMO relay systems using instantaneous power and average power with $M = K = 4$ ($\mathrm{SNR} = \mathrm{SNR}_{RS} = \mathrm{SNR}_{MR}$).

4. Simulation Results

In this section, we investigate the performance of the proposed precoder schemes. The transmitted symbols are modulated with 4-QAM. The CLLL algorithm is utilized and its reduction parameter is set to 0.75, which is a factor selected to yield a good quality-complexity tradeoff. The channel matrix is generated randomly from one burst to the next. The corresponding LR-aided precoding schemes are refered to as LR-aided ZF-VP and LR-aided THP-VP. The SNRs of the BS-RS link and RS-MSs links are defined as $\mathrm{SNR}_{RB} = P_B$ and $\mathrm{SNR}_{MR} = P_R$, respectively. Here, we make the quantities SNR_{RB} and SNR_{MR} equal to each other. This implies that the path losses for the first and the second link are the same. We assume $\mathrm{SNR} = \mathrm{SNR}_{RB} = \mathrm{SNR}_{MR}$. Further, we take into account the effect of channel estimation error which is inevitable in practice. The estimated channel $\hat{\mathbf{H}}$ can be modeled as $\hat{\mathbf{H}} = \sqrt{1-\rho^2}\mathbf{H} + \rho\Delta\mathbf{H}$, where $\Delta\mathbf{H}$ represents the channel estimation error with elements being complex Gaussian random variables with zero mean and same variance, and ρ ($0 \le \rho \le 1$) is a measure of the channel estimation quality. We assume the channel estimation quality is same for both $\mathbf{H}_{RB}$ and $\mathbf{H}_{MR}$ and set ρ^2 to 0.01, 0.001 and 0.0001.

Fig. 3 shows the performance of LR-aided precoding schemes with normalization by average power and normalization by instantaneous power. From the figure we see that using normalization by average power achieves slight improvement in BER performance. Apparently, it is a most practical manner to choose normalization by average power, since the MSs then do not require to know Γ_R instantaneously.

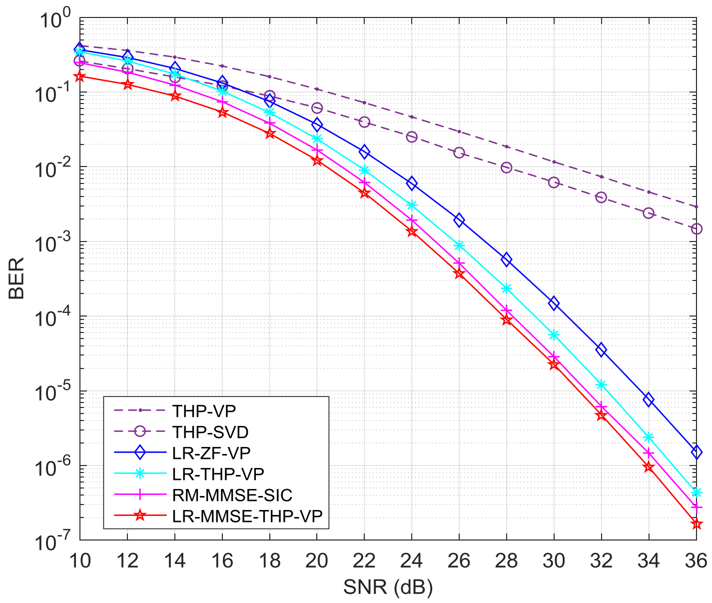

Fig. 4. BER performance comparison of precoded MIMO relay systems with $M = K = 4$

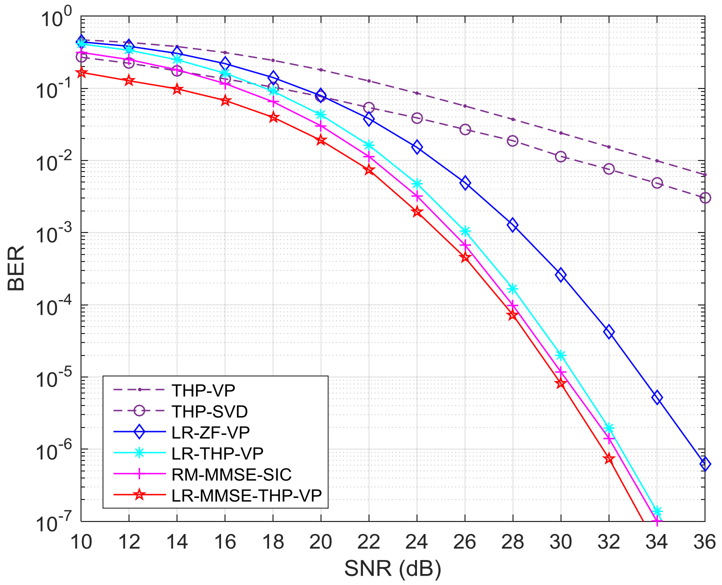

Fig. 5. BER performance comparison of precoded MIMO relay systems with $M = K = 8$

In **Fig. 4** and **Fig. 5**, the BER performances of THP SVD scheme [1], THP-VP scheme with and without LR, and LR-aided ZF-VP scheme are plotted with $M = K = 4$ and $M = K = 8$. Note that THP-VP scheme without LR means the channel matrices are not reduced, i.e., $\mathbf{U}$ and $\mathbf{T}$ are all identity matrices. From the figures, we observe that THP-VP with LR performs much better than that without LR. We conclude that LR is able to aid relay systems achieve more diversity, like it does in the MIMO scenarios. The LR-MMSE-THP-VP approach can obtain more code gain and thus performs the best. The THP SVD scheme outperforms two LR-aided schemes when SNR is small. However, the curves of LR-aided schemes decrease much faster. When SNR is large, performance of our two LR-aided schemes is much better than THP SVD scheme. This is because the distribution of singular value is highly unequal and the worst subchannel (smallest singular value) is the dominant contributor to the BER. For the sake of comparison, the performances of RS modulo MMSE successive interference cancellation (RM-MMSE-SIC) [24] is also shown. The proposed LR-MMSE-THP-VP approach shows a slight advantage over RM-MMSE-SIC approach. It is worth to note that LR method is helpful to achieve a higher diversity.

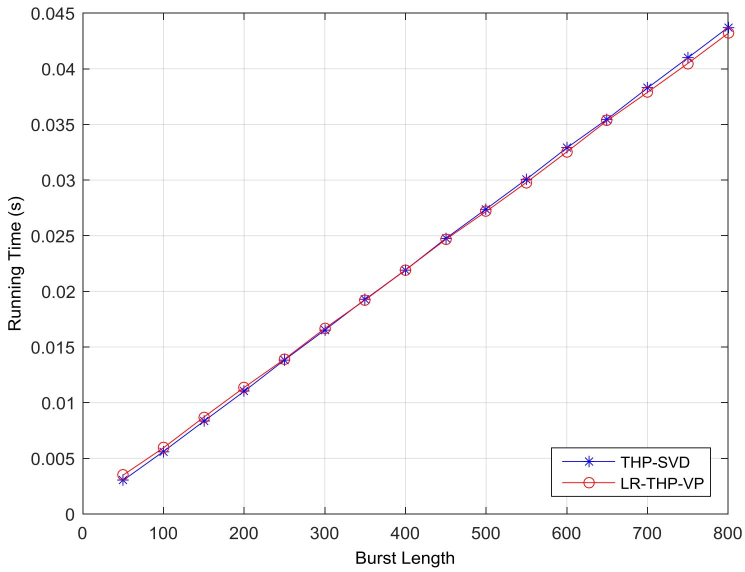

Fig. 6. Comparison of the average running times between LR-THP-VP scheme and THP-SVD scheme.

We then compare the LR-THP-VP scheme with the THP-SVD scheme in terms of complexity. **Fig. 6** depicts our results, where each point is given in average time (in seconds) of burst length, using a HP computer with a 3.6-GHz Core processor, with MATLAB running under Windows 7. The running times are averaged using the 1000 bursts. The channel matrix is generated randomly from one burst to the next and remains constant within a burst. From the figure we observe that the difference of running time between LR-THP-VP scheme and THP-SVD scheme is very slight. When the burst is long, the THP-VP scheme shows a little faster speed. This is because that the LR algorithm is just carried out once during one burst and the complexity is shared by symbols within the burst.

In **Fig. 7** and **Fig. 8**, we also observe that channel estimation error significantly reduces the BER performance at high SNR. At low SNR, the BER performance is seen to be insensitive to the channel estimation error. When ρ^2 is small, the proposed scheme is less susceptible and yields a better performance. Generally, the BER reaches a maximum when SNR increases to a certain point. Additionally, we explore the impact of channel asymmetry (CA). It is assumed that the average channel gains from different BSs to RS are different. Let the variance of channel between the RS and a MS is randomly drawn from [1,5]. That is, the differences of channel gains can be up to 5 dB between the MSs. We can observe that the existence of the channel asymmetry affects the system performance, but still yields a high diversity. It is worth noting that the BER performance of LR-aided ZF-VP scheme with $K = 8$ is not superior to the case when $K = 4$ in the low SNR regime. This is due to insufficient orthogonality yielded by the CLLL algorithm in the high dimension. In this case, the Korkine-Zolotareff algorithm [20] can be used as an alternative to LR since it is able to provide a better reduced basis.

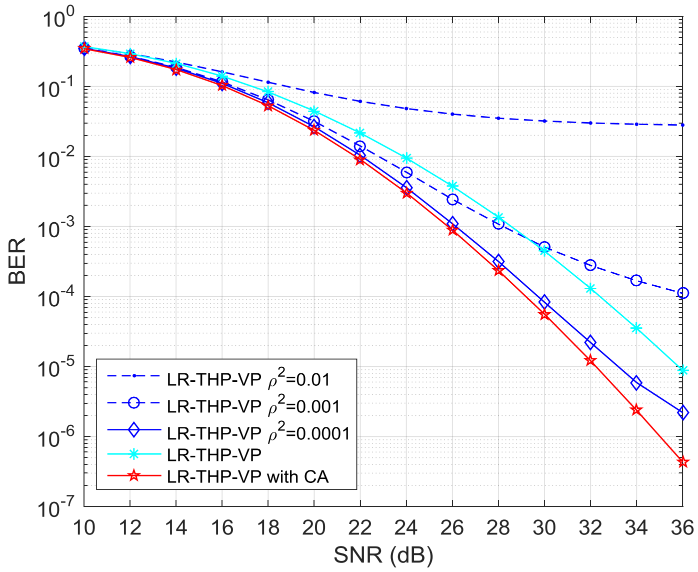

Fig. 7. BER performance of precoded MIMO relay systems with channel estimation error and channel asymmetry ($M = K = 4$, $\mathrm{SNR} = \mathrm{SNR}_{RB} = \mathrm{SNR}_{MR}$).

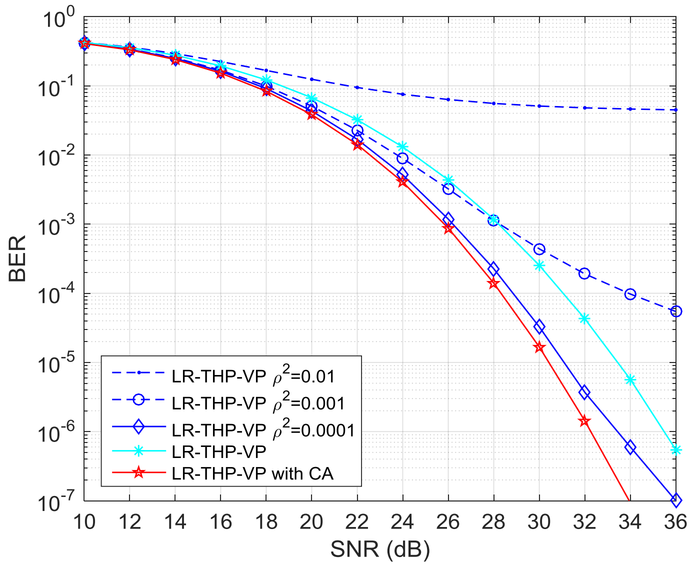

Fig. 8. BER performance of precoded MIMO relay systems with channel estimation error and channel asymmetry ($M = K = 8$, $\mathrm{SNR} = \mathrm{SNR}_{RB} = \mathrm{SNR}_{MR}$).

5. Conclusions

In this study, we develop a new and efficient joint precoder design for multiuser MIMO relay systems. An upside of this proposed scheme is that LR is exploited to provide a significant performance improvement at low computational complexity. With a reduction of the feedback for the LR-aided precoding, its practical implementation could be feasible. Simulation results have shown that the proposed precoding design with the application of LR is capable of achieving a superior BER performance, especially in high SNR.

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