IJASS
International Journal of
Aeronautical and Space Sciences

On-Board Orbit Propagator and Orbit Data Compression for Lunar Explorer using B-spline

Junghyun Lee*

Gwangju Institute of Science and Technology (GIST), Gwangju 61005, Republic of Korea

Sujin Choi**

Korea Aerospace Research Institute (KARI), Daejeon 34133, Republic of Korea

Kwanghee Ko***

Gwangju Institute of Science and Technology (GIST), Gwangju 61005, Republic of Korea

Abstract

In this paper, an on-board orbit propagator and compressing trajectory method based on B-spline for a lunar explorer are proposed. An explorer should recognize its own orbit for a successful mission operation. Generally, orbit determination is periodically performed at the ground station, and the computed orbit information is subsequently uploaded to the explorer, which would generate a heavy workload for the ground station and the explorer. A high-performance computer at the ground station is employed to determine the orbit required for the explorer in the parking orbit of Earth. The method not only reduces the workload of the ground station and the explorer, but also increases the orbital prediction accuracy. Then, the data was compressed into coefficients within a given tolerance using B-spline. The compressed data is then transmitted to the explorer efficiently. The data compression is maximized using the proposed methods. The methods are compared with a fifth order polynomial regression method. The results show that the proposed method has the potential for expansion to various deep space probes.

Key words: B-spline, lunar exploration, orbit propagator, trajectory, data compression

1. Introduction

Once put into the earth's orbit by a launch vehicle, deep space probes such as a Mars probe, an asteroid probe, or a lunar explorer escape the earth orbit using engine injection to reach their target planets after staying in the earth orbit for a certain period of time. In the case of a lunar explorer, Trans-lunar injection (TLI) is performed in the parking orbit to enter the lunar transfer orbit. When entering the earth-moon transfer orbit, the explorer is tracked by ground antennae distributed around the world; then, the orbit determination is performed based on the tracking data from the explorer. This data is used for the initial orbit determination value, which

predicts the overall trajectory of the explorer. Basic research on a lunar explorer has been performed by Korea Aerospace Research Institute (KARI) [1-3]. However, development of an orbit propagator and a method for compact transmission of orbit data has not been addressed yet.

1.1 Previous Works

In general, orbital motion analysis is based on the integration of a non-linear orbital equation. Special perturbation and general perturbation methods are utilized for solving the equation. The special perturbation theory yields a small error in the numerical integration of orbit

(cc)   * PhD Student
     ** MS
   *** Professor, Corresponding author: khko@gist.ac.kr

propagation, but it requires a high performance computer to obtain the desired results; the general perturbation method does not require a high performance computer. Significant numerical integration errors are expected over time. Ephemeris compression is proposed to overcome such a shortcoming [4].

The on-board orbit propagator, i.e., the propagator mounting the explorer, has been mainly studied in low-orbit satellites. The residual, i.e., the difference between the reference orbit and true orbit, is generated. The residual orbit, which has a periodic characteristic, is transmitted to the satellite in the form of coefficients of Fourier functions along with the reference orbit [4]. The on-board computer enhances the accuracy of the orbit propagation and reduces the computational load by generating a reconstruction orbit using the coefficients and the reference orbit [5, 6]. Jung et al. [7] proposed a method which compresses and transmits the residuals between the real and the reference trajectory for a station keeping orbit of geostationary satellites. Lee et al. [8] studied about a multiple compression method for the residual. However, the methods cannot be applied to a deep space probe because the orbital characteristic for the deep space navigation is not periodic. Orbit determination within the satellite itself using GPS measurement data has also been studied [9-12]. The orbit estimation methods for satellite mounting include a batch filter, a sequential filter, and Kalman filter, etc. [13-16]. However, the accuracy of the propagated orbit using these methods is lower than that of the ground-determined orbit due to the measurement data noise and unexpected elements. Currently, an effective on-board orbit propagator of deep space crafts, including the lunar explorer, has not been established.

Satellite data compression has been an active research topic. Compression using polynomial coefficients for transmission of antenna data, i.e., azimuth and elevation, was studied for the Korea Multi-purpose Satellite (KOMPSAT-3) [17, 18]. Gwangju Institute of Science and Technology (GIST) suggested a B-spline based algorithm for compressing antenna data [19, 20]. However, the aim of their study was to send the mechanical movement data for a short time period. In the case of a lunar explorer, the quantity of orbital data is immense compared to the quantity of antenna data; moreover, communication is constrained for the deep space case. Therefore the existing polynomial based compression approach may not produce stable results for a deep space orbit determination; furthermore, the compression ratio should be increased remarkably to avoid various communication constraints. A B-spline based algorithm for accurate orbit determination was studied, as described in [21, 22]. However, a B-spline based compression algorithm

for orbital data transmission of a deep space probe has not been studied yet. Moreover, an on-board orbit propagator for deep space probes has not been addressed.

1.2 Motivation

In this study, an on-board orbit propagator and an orbit data compression methodology based on the B-spline for a lunar explorer is proposed, which has not been reported in the literature. Existing methods are orbit determination in the ground stations, not onboard orbit propagation or compression. The explorer must be able to obtain information regarding its current and future trajectory for a stable navigation because the communication between Earth and the probe is sometimes limited (such as in the case of eclipse by other planet). Additionally, it should be able to perform tasks (such as maneuvering attitude to increase power production efficiency).

Generally, precise orbit determination is mainly performed by the ground station due to the complexity of the orbital equation including perturbations. Orbit determination is periodically performed and the computed orbit data is uploaded to the probe, which would be the main work load to the ground station and the probe.

The entire trajectory from the earth to the moon is classified into three zones as follow: 1) the zone from launching to entrancing Earth orbit, 2) a transition zone between the orbits of Earth and the Moon, called the transfer lunar trajectory 3) the zone that entering the lunar orbit and the landing section. Among them, intensive control and communication is performed at the launching and landing parts, 1) and 3), which are unstable. Therefore, communication between the ground station and the explorer such as accurate trajectories should be performed. On the other hand, the transfer lunar trajectory is a relatively stable section. Therefore, the accurate trajectory data can be computed in the ground station and be transmitted to the explorer once, and the explorer can compute accurate trajectories using its on-board computer. Because the size of the accurate trajectory data is enormous, transmitting them to the explorer with no loss of information would be a burden. To minimize the problem, it would be better to reduce the data size while the accuracy of the trajectory is maintained. In this paper, such problems are addressed and methods to solve them are presented.

1.3 Proposed Models

The initial values for an orbit are determined at the TLI stage, and the orbit is precisely predicted using a high-

performance computer at the ground station. Then, the computed data are compressed into the form of control points of B-spline through the proposed procedure satisfying a given tolerance (position 3 km, velocity 3 m/s). The compressed data are sent to the explorer, which then reconstructs the orbit based on the compressed parameter. Accordingly, it not only enhances the orbital prediction accuracy, but also decreases the workload and transmission time required by the explorer to recognize its position.

The feasibility and performance of the method are demonstrated with direct transfer trajectory data for five days and WSB orbits for three months and compared with an existing method such as a method using polynomial regression [23].

The contributions of this study in terms of the lunar explorer operation are as follows:

1) A unique on-board orbit propagator for deep space is developed. The onboard orbit propagator for a deep space probe has not been addressed so far, and only a method for orbit propagation of a low-orbit satellite was discussed. Research on compression and transfer of the lunar explorer orbital data as a secondary navigation system has not been addressed so far. The proposed method is a first attempt to address this issue.

2) An improved compression algorithm for processing orbit data of the deep space explorer is proposed and compared with the existing methods [19, 20, 24, 25]. Compression ratio is maximized by driving the optimal degree and number of control points of a B-spline, generate orbit interpolating with reference to time axis, and using 2-norm tolerance.

3) An application to actual deep space probes is presented. A computation algorithm used at the ground station and on the lunar explorer and the data type for transferring presented.

This paper is structured as follows: Section 2 presents various properties of the orbit of the lunar explorer. Section 3 introduces deep space communications and their constraints. Section 4 describes the theory of interpolation using B-spline. Section 5 defines the formulation of the problem in this work and presents a procedure for improving the compression ratio based on the interpolation method described in Section 4. Section 6 describes the proposed methodology to be applied to the lunar explorer. Lastly, Sections 7 and 8 present the simulation results and conclusions, respectively.

2. Transfer Orbit of the Lunar Explorer

The orbital trajectory between the lunar and the earth orbits is called the transfer orbit. The fuel consumption of an explorer is influenced by the gravitational fields of the earth and the moon. There are two types of trajectories to the moon: the direct transfer trajectory and the WSB transition orbit. Using the direct transfer trajectory, an explorer can reach the moon in a short time, and therefore, relatively high precision is not required compared with the WSB transition orbit because the reduced time to reach the moon may lead to a less accumulation of errors along the trajectory, which can relax the requirement of high precision. On the other hand, the WSB transition orbit requires relatively longer time for an explorer to reach the moon, and high precision and less fuel are required at the trans-lunar injection burn since the Lagrange point between the earth and the sun are used [26, 27].

2.1 Direct Transfer Trajectory

The lunar explorer moves from the earth parking orbit to the transfer orbit using a chemical propulsion system, such as a kick motor. The direct transfer trajectory takes place over a 3–5 day transition period. Then, the explorer enters the lunar orbit as shown in Fig. 1. The direct transfer trajectory has the following characteristics: the period for entering the moon orbit is short, the mission is simple, and two opportunities for launching are available each day. This trajectory was used for Apollo, and was recently applied to lunar explorers such as Lunar Prospector [28] and Lunar Reconnaissance Orbiter LRO [29, 30].

Hiten spacecraft [31, 32] and the GRAIL spacecraft [33-35] entered lunar orbit using this trajectory.

2.2 WSB Transfer Trajectory

WSB transfer trajectory takes about 3–4 months to reach the lunar orbit from the earth parking orbit as shown in Fig. 2. Since the probe should reach Lagrange points as far from the earth as approximately 1.5 million km, the accuracy of the launch vehicle needs to be considerably high, and the communication equipment installed to the probe needs to
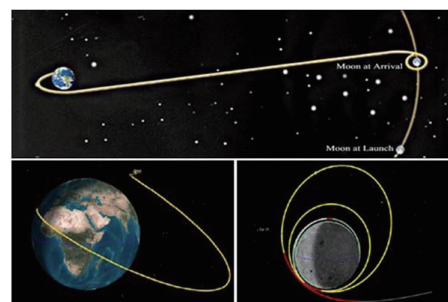

Fig. 1. Direct Transfer Trajectory(LRO) [29]

be excellent. However, the velocity increment to enter the lunar orbit is reduced up to 25%; therefore, the weight of explorer can be reduced.

3. Deep Space Communication and Constraints

Reducing the communication load of the lunar explorer is an important issue. Deep space communication is conducted in order to transmit tele-commands and receive telemetry and measured data for various purposes. Deep Space Network (DSN) has been operated with four antennae in three regions (Goldstone, Madrid, Canberra), which provide general service [36-38].

A directional parabolic antenna of immense size is used for deep space telecommunication. The visibility radio band increases as the communication distance increases; accordingly, a small number of antennae are often used due to the size of the antennae and their corresponding visibility bands. The communication time of the lunar explorer using deep space communication is severely limited, therefore, requiring that the probe orbital data be transmitted compactly. As the amount of communication data decreases, it becomes increasingly less influenced by the antenna frequency band; therefore, low-frequency antennae can be used.

4. Data Compression Based on Power and B-spline

The basic idea of orbital data compression is to identify a function that approximates the orbit while using a smaller amount of data, without losing critical information. In this section, a theoretical background of approximation is presented.

4.1 Data Approximation

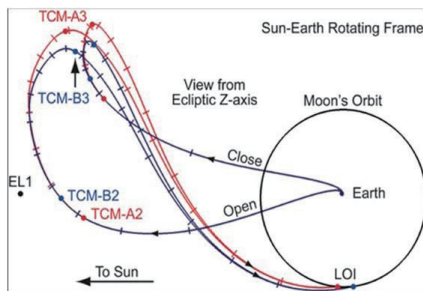Consider m points $\mathbf{p}_i=(t_i, x_i, y_i, z_i)(i = 1, ..., m)$, in 4D space,

Fig. 2. WSB Transfer Trajectory(GRAIL) [33]

which contain no loops or self-intersection. Assume that there is a function f as follows:

$$\mathbf{f}(s) = \sum_{j=0}^{n} \mathbf{a}_j \phi_{j,k}(s) \tag{1}$$

where $\phi_{j,k}$ is the j-th basis function of degree k, s is a parameter, n is the number of coefficients, and $\mathbf{a}_j$ is the j-th coefficient. Then the following equation should be satisfied at each point:

$$\mathbf{p}_i \cong \mathbf{b_i} = f(s_i) = \sum_{j=0}^{n} \mathbf{a}_j \phi_{j,k}(s_i), 1 \leq i \leq m \tag{2}$$

Here, s_i is the parametric value corresponding to $\mathbf{P}_i$ and the unknowns is $\mathbf{a}_j$. The numbers of the unknowns and equations are $n+1$ and m, respectively, and unique values of $\mathbf{a}_j$ are obtained only when $n+1 \leq m$. In matrix form, Equation (2) is given by $\mathbf{b}=\mathbf{\Phi} \times \mathbf{a}$, where $\mathbf{b}$ is an $m+1$ matrix of $\mathbf{b}_i$, $\mathbf{\Phi}$ is an $m \times (n+1)$ matrix of $\phi_{j,k}$, and $\mathbf{a}$ is an $(n+1) \times 1$ matrix of $\mathbf{a}_j$. This matrix equation is linear and can be solved using various numerical techniques such as singular value decomposition [19]. Once the values of $\mathbf{a}_j$ are determined, Equation (1) approximates (or interpolates when $m \times (n+1)$) the given data points. The approximation results are affected by the basis functions used. In this paper, two basis functions are considered: power and B-spline basis functions.

4.2 Power Basis Function

The power basis uses $\phi_{j,k}(s)=s^k$ with $k=j$, resulting in the power basis polynomial. The parameter s corresponds to time. When $m=n+1$, the polynomial interpolates the given data points, which is desirable to achieve high accuracy. However, from the standpoint of numerical stability and data reduction, it should be avoided since the degree of the polynomial could become very high since the number of the given data points is large, oscillation of the interpolated function could occur and no reduction of data is expected.

In order to handle these problems efficiently, the input data set is subdivided into n_μ segments, and the data points belonging to each segment are approximated using a polynomial regression method, separately. Equation (1) can be written as Equation (3), which is solved using singular value decomposition.

$$\begin{bmatrix} 1 & s_1 & s_1^2 & ... & s_1^n \\ 1 & s_2 & s_2^2 & ... & s_2^n \\ ... & ... & ... & ... & ... \\ 1 & s_m & s_m^2 & ... & s_m^n \end{bmatrix} \cdot \begin{bmatrix} a_0^x & a_0^y & a_0^z \\ a_1^x & a_1^y & a_1^z \\ ... & ... & ... \\ a_n^x & a_n^y & a_n^z \end{bmatrix} \approx \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ ... & ... & ... \\ x_m & y_m & z_m \end{bmatrix} \tag{3}$$

243

4.3 B-spline Basis Scheme

This approach uses B-spline as a basis function in Equation (1). The B-spline basis function is defined over a knot vector $\mathbf{T}=\{u_1, u_2, u_3, \ldots, u_{k+n+1}\}$ whose elements $u_l\,(l=1, \ldots k+n+1)$ are non-decreasing numbers [19]:

$$\phi_{i,0}(s) = \begin{cases} 1 & if \ u_i \le s < u_{i+1} \\ 0 & otherwise \end{cases}$$

$$\phi_{i,k}(s) = \frac{s-u_i}{u_{i+k}-u_i}\phi_{i,k-1}(s) + \frac{u_{i+k+1}-s}{u_{i+p+1}-u_{i+1}}\phi_{i+1,k-1}(s) \tag{4}$$

Using this definition, the B-spline curve in 4D space is given by:

$$\mathbf{c}(s) = \sum_{j=1}^{n} \mathbf{a}_j \phi_{j,k}(s) \tag{5}$$

Where $\mathbf{c}(s) \approx \mathbf{p}=(t(s), x(s), y(s), z(s))$ and $\mathbf{a}_j = (a_j^t, a_j^x, a_j^y, a_j^z)$. Additionally, $t(s)$ is the time for command, $x(s)$, $y(s)$, and $z(s)$ are positional and velocity information, and s is a parameter corresponding to $(t(s), x(s), y(s), z(s))$.

In order to use the B-spline scheme for fitting, the parameter values of s should be estimated. Namely, proper parametric values to each input point should be assigned, which is called the parametrization. Estimating appropriate parameter values is critical for good fitting. Various approaches for parameterization have been presented in the literature [25]. However, research on this issue is on-going since there is no single method that works for every case. In this work, the chord length parameterization method [25] is chosen. The chord length parametrization method was usually chosen for practical applications because it estimates parameters by considering the distribution of data points efficiently without involving any complicated computation. This method takes into account the distances between two consecutive points for parameter estimation. Given points $\mathbf{p}_i(i=1, 2, 3, \ldots, m)$, the parameter values s_i for each point are estimated as follows: for the i-th point, the parameter value is $s_i=s_{i+1}+\Delta_i/L$, $i=(1, 2, 3, \ldots, m)$, $\Delta_i=|\mathbf{p}_i-\mathbf{p}_{i-1}|$, $L=\sum\Delta_i$. Here, $s_1=0$ and $0\le s_i\le 1$.

Once parameter values are obtained, the coefficients in Equation (5), called the control points, can be computed by solving

$$\begin{pmatrix} t_i \\ x_i \\ y_i \\ z_i \end{pmatrix} = \sum_{j=0}^{n} \begin{pmatrix} a_j^t \\ a_j^x \\ a_j^y \\ a_j^z \end{pmatrix} \phi_{j,k}(s_i) \tag{6}$$

5. Proposed Procedure

5.1 Problem Description

Given a set of reference data, a B-spline curve is computed, which approximates the reference with errors within a predefined tolerance. For compression, the number of control points should be minimized while the maximum error of the compressed one to the reference data must be within the given tolerance. The proposed procedure is expressed as a function f. The control points and a generated curve are obtained with the degree, the number of control points, and reference orbital data. 2-norm constraints are used to reflect the geometric shape and to enhance the compression ratio. This problem is formulated as an optimization problem as follows:

- Objective: min(The size of parametic data)
- [Control points, Generated orbit data] = f($\overline{X}$, $\overline{\overline{I}}$)
- Decision variable $\overline{X}$ = (degree, the nubmer of control points)
- Input data $\overline{\overline{I}}$ = (Reference orbit data)

Reference orbit data = [t, Px_o^i, Py_o^i, Pz_o^i, Vx_o^i, Vy_o^i, Vz_o^i], Generated orbit data = [t, Px_r^i, Py_r^i, Pz_r^i, Vx_r^i, Vy_r^i, Vz_r^i]

- Tolerance

- $3 \text{ km} > \max\left(\sqrt{(Px_o^i - Px_r^i)^2 + (Py_o^i - Py_r^i)^2 + (Pz_o^i - Pz_r^i)^2}\right)$

- $3 \text{ m/s} > \max\left(\sqrt{(Vx_o^i - Vx_r^i)^2 + (Vy_o^i - Vy_r^i)^2 + (Vz_o^i - Vz_r^i)^2}\right)$

Here, X_o^i, Y_o^i, Z_o^i are the components of the ith position of the original data, X_r^i, Y_r^i, Z_r^i are the components of the ith data of the reproduced data, Vx_o^i, Vy_o^i, Vz_o^i are the components of the ith velocity vector of the original data, and Vx_r^i, Vy_r^i, Vz_r^i are the ith velocity vector of the reproduced data.

5.2 Procedure

The entire procedure is shown in Fig. 3. A method for orbit data compression is considered, which finds the optimal degree and control points that approximate the given reference data within a predefined tolerance. The control points are calculated by applying the B-spline scheme after parameterization of the original data using the chord length method as described in Section 4.3. A curve is generated based on the calculated control points. The reproduced data may generate distortions in the spatial axes, i.e. x, y, and z as well as in the time axis. The time axis of the generated curve completely matches with the original curve points, and the spatial data of the generated curve are interpolated according to the matched time with

the tolerance satisfied.

To find the optimal degree and the number of control points, the following B-spline characteristics are used: 1) as the degree of the curve increases, the number of control points of a curve for approximation is reduced, 2) for a degree exceeding a certain value, the number of control points is increased to approximate the data. Using these two properties, a procedure is proposed. The number of control points is increased for the initial degree (generally three) until the tolerance limit is satisfied. Then, the obtained number of control points for the initial degree is the largest. Next, the degree is increased by one, and a curve with a new set of control points is obtained. These steps are repeated, and the optimum degree corresponding to the minimum number of control points is obtained.

5.2.1 Data Parametrization

The original data are parametrized to apply the B-spline scheme using the chord length method. The parameterized data are given as input to the procedure.

5.2.2 Computation of Control Points

The control points are obtained using the B-spline scheme with respect to the position and velocity sharing the same time axis. One control point has the values of $[t, x, y, z]$.

5.2.3 Generation of the Curve

The orbit is reproduced using the obtained control points and degree. It is regenerated as the number of the original data. The reproduced data cause distortions in the x, y, z axes as well as in the time axis.

5.2.4 Linear Interpolation with respect to the Time Axis

Once data are reproduced through the control points of a B-spline curve, the time axes of the original and generated data are matched to apply the tolerance constraints. The differences between the original and the reproduced orbit are checked in the form of 2-norm with respect to the spatial data, i.e. $x, y,$ and z axes, excluding the time axis.

5.2.5 Tolerance Satisfaction

The geometric error is considered by adopting the 2-norm tolerance. This shows improved compression compared to the existing method. Previously, each control point is extracted on the two-dimensional plane, i.e. the time axis and individual x, y, z axis [14, 15], where the time information data are overlapped. In other words, assuming the number of control points as N_{Cp}, the existing methods need the $N_{Cp} \times 6$; on the other hand, the proposed method requires $N_{Cp} \times 4$. As the curvature of the curve increases, the required number of control points also increases. However, even curvature that is simple in the 3-D space may be complex once projected into the 2-D space, which may require more control points.

5.2.6 Stop Condition

The optimal degree and the number of control points within the tolerance range in B-spline are shown in Fig. 4. As the degree increases, the number of required control points exponentially decreases. Once the degree exceeds a certain point, the graph flattens and linearly increases because the number of points requires at least degree + 1. The degree and the number of control points corresponding to the position of the graph where the flattening shape begins can
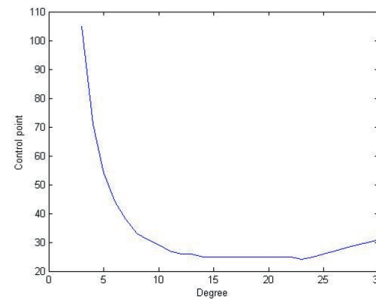
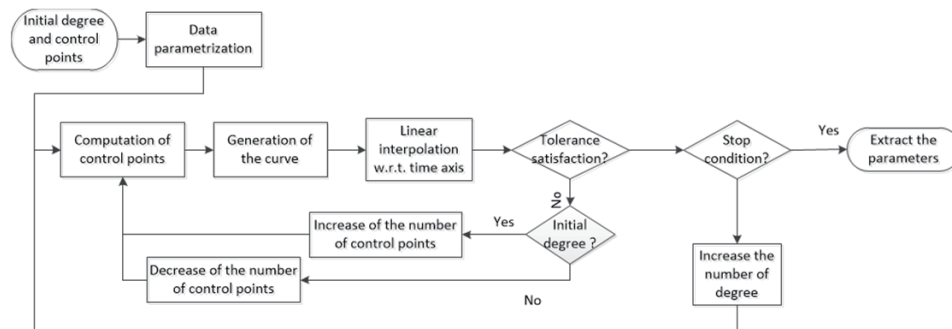Fig. 4. Example of control point according to the degree

Fig. 3. Proposed procedure

be regarded as the optimal values.

6. Application Within a Lunar Explorer

In order to use the proposed algorithm for a lunar explorer, the information transmitted to the explorer needs to be identified. Moreover, software in the explorer should be updated to handle the new algorithm.

6.1 Data Necessary for the B-spline Scheme

A command is given as a tuple of (t, x, y, z) to represent a position and a velocity. For a certain parameter s, a command is given by $\mathbf{c}_{pos}(s) = (c_{pt}(s), c_{px}(s), c_{py}(s), c_{pz}(s))$ and $\mathbf{c}_{vel}(s) = (c_{vt}(s), c_{px}(s), c_{py}(s), c_{pz}(s))$ for the position and velocity at s, respectively. Then the data that are necessary for the proposed method are given as follows:

- The numbers of the control points of the approximated B-spline curves with reference to the position and velocity: n_{pos}, n_{vel}
- The control points for position and velocity: $\mathbf{c}_{pos}(s) = (c_{pt}(s), c_{px}(s), c_{py}(s), c_{pz}(s))$ and $\mathbf{c}_{vel}(s) = (c_{vt}(s), c_{px}(s), c_{py}(s), c_{pz}(s))$
- The degree of the B-spline curve: k
- The knot vector: T
- The starting and ending times for the commands: t_s and t_e

Among them, the degree has a fixed value of three and the knot vector is assumed to have the following form:

$T=\{0, 0, 0, 0, j/r_t, 1, 1, 1, 1\}(j=1, ..., r_{t-1})$ with $r_t=n$-2. The knot vector then can be generated by the explorer using the same rule. Therefore, it does not have to be transmitted. The time function $c_t(s)$ is obtained using t_s and t_e. Therefore, all the information that must be transmitted to the explorer for correct reconstruction includes n_{pos}, n_{vel}, $\mathbf{c}_{pos}(s)$, $\mathbf{c}_{vel}(s)$, t_s and t_e.

6.2 System change

The current explorer system should be updated to use the proposed method. Two additional functions must be implemented in the explorer: knot vector generation, B-spline basis function evaluation.

6.2.1 Knot Vector Generation

The knot vector can be generated by the explorer. This generation process is straightforward after the number of control points is obtained from the ground station.

6.2.2 Basis Function Evaluation

The proposed method uses the B-spline basis function,

which requires a new routine for computation. Therefore, the ground station as well as the satellite should have the same routine for B-spline evaluation. The function can be implemented using recursion or using the De Boor Cox algorithm [39].

7. Algorithm Application and Results

In this section, the proposed algorithm and methodology are applied to the direct transfer and WSB trajectory calculated using Satellite Took Kit (STK). The polynomial regression method is applied to the orbit data for comparison.

A polynomial of degree five is considered in this paper for polynomial based data compression, which is determined by analyzing the relation between the polynomial degree and the compressed data. Trajectory data are taken for analysis of data compression as shown in Fig. 5, which shows that there is no obvious relation between the degree and the compression ratio. In fact, the polynomial of degree five shows the best compression rate. As the degree of a polynomial increases, the stability in fitting becomes worse. Therefore, a polynomial of lower degree is preferred in most cases.

The fifth order polynomial is expressed in the form of $f(t)=a_0+a_1t+a_2t^2+a_3t^3+a_4t^4+a_5t^5$. The compressed data consist of the time data that indicate the segment boundary and polynomial coefficients, The polynomial methods are independently applied to the spatial axes x, y, and z with time axis t. Exceeding a given tolerance range, it is further divided into smaller segments.

7.1 Simulation Condition

Orbit data are extracted through STK/Astrogator. The simulation conditions used in this test are shown in Table 1. The conditions can be set in STK.
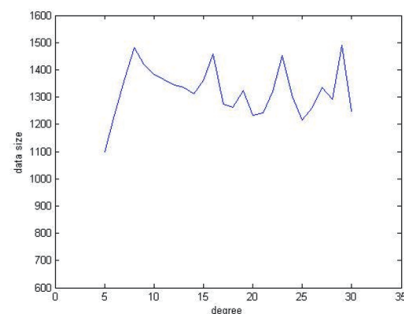
The data to be compressed are the time and positions

Fig. 5. Correlation between polynomial degree and compressed data volume

(*Px*, *Py*, *Pz*) and the velocity (*Vx*, *Vy*, *Vz*) as shown in Table 2, which are defined using the J2000 coordinate system of the earth. The data are extracted at one-minute intervals, the direct transfer trajectory has 6,480 rows with a short period of transition; WSB trajectory has 164,822 rows with a very long period of transition.

The original data are compressed in the form of control points of the B-spline. The compressed control points are transferred to the lunar explorer through tele-commands. The control points are independently calculated for position and velocity.

7.2 Results of Applying Direct Transition Trajectory

The control points are obtained by using the proposed method for position or velocity. The tolerances for position and velocity are assumed to be 3 km and 3 m/s, respectively. The number of control points according to the degree is the
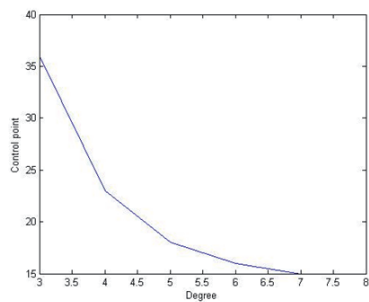
shown in Fig. 6.

The points where the curve does not increase are extracted. The data required for transmission include the degree, the number of control points, the control point values, and the initiation and end times [18]. Each of the control points has four-dimensional information, i.e., time and *x*, *y*, *z*. The degree, the number of control points, and the optimized compression ratio are shown in Table 3. The original orbit data, the calculated control points, and the reproduced orbit data are shown in Fig. 7.

The errors between the reference and the generated orbit are represented in Fig. 8.
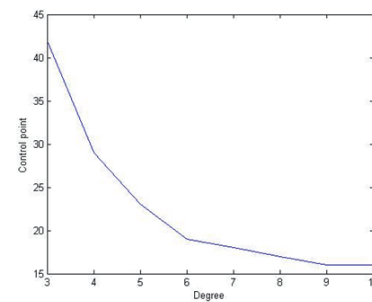
The results of applying the polynomial regression are shown in Table 4. The regression with the 5th order polynomial is applied for each of the spatial axes *x*, *y*, and *z*. The regression with the 5th order polynomial is currently considered in practice. Polynomials of higher degrees could be considered for more compression. However, the regression using polynomials of higher degrees could become unstable. Therefore, the degree should be maintained to a reasonable

Table 1. Simulation conditions of orbit data

Numerical condition	Runge–kutta 78th
Earth gravitational field	JGM 16 × 16
Moon gravitational field	LP165 8 × 8
Third body perturbation	Earth, Sun, Moon
Coordinate frame	J2000
Reference data interval	60 seconds

Table 3. Application of the proposed algorithm

	Position	Velocity
Degree	7	9
The number of control points	15	17
The number of data	64	72
The number of original data	45360	
Data compression rate	99.70%	

Table 2. Orbital Data for Compression

Epoch	Position (km)			Velocity (km/s)		
(Julian Date)	*Px*	*Py*	*Pz*	*Vx*	*Vy*	*Vz*
2458850.287	41801.664	42182.506	58602.419	1.5694	1.88991	1.2346
2458850.288	41895.777	42295.849	58676.422	1.56769	1.88817	1.23219
…	…	…	…	…	…	…

(a) Position

(b) Velocity

Fig. 6. Variation of the number of control points as a function of degree

247

range. If the position and velocity exceed 3 km and 3 m/s, the segment is subdivided into smaller ones, and the same process is performed until the tolerance is satisfied.

7.3 Results of Applying WSB Transition Trajectory

The WSB trajectory is enormous compared to the direct transfer trajectory. With the curvature increasing, the number of the control points grows due to the characteristics of the B-spline. The three-dimensional curvature of the location and velocity according to the time axis are shown in Fig. 9. The graph shows that the curvature increases sharply

at a certain point, which indicates the earth-sun Lagrange point located 1 million km away from the earth. Arriving the Lagrange point, the explorer changes its route towards the moon, which means inflection in terms of orbital geometry. The number of control points exponentially increases at the Lagrange point. Thus, the data is processed by first dividing it at the Lagrange point into two segments. The control points are obtained for the classified position and velocity. The tolerances of position and velocity are 3 km and 3 m/s, respectively.

The number of control point variations according to the degree for each segment, and the simulation results are
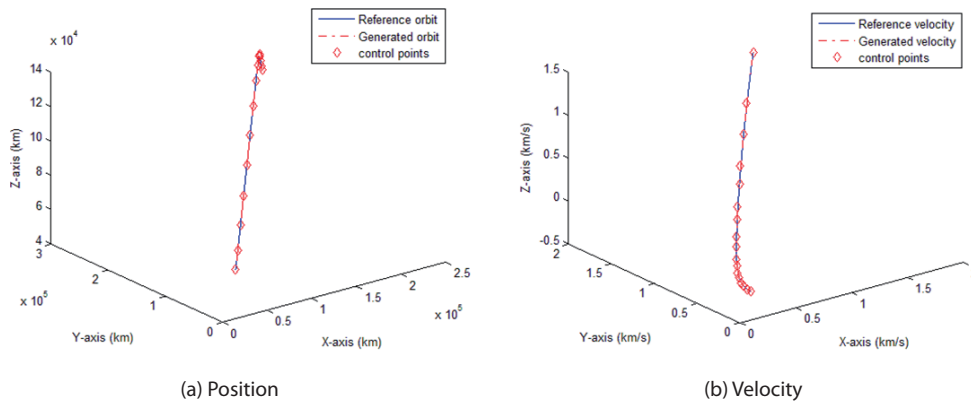

(a) Position   (b) Velocity

Fig. 7. Optimized control points and trajectories of direct transfer
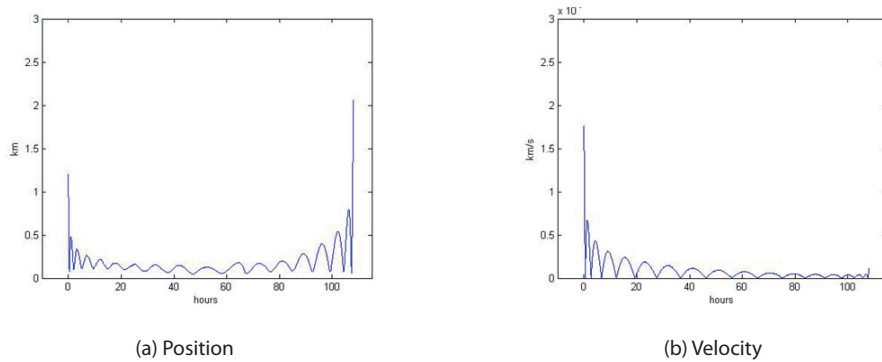

(a) Position   (b) Velocity

Fig. 8. Error histories between the reference and generated orbits of direct transfer trajectory

Table 4. Polynomial regression application

	Position		Velocity	
Axis	The number of segments	The number of data	The number of segments	The number of data
x	132	925	7	50
y	157	1100	7	50
z	129	904	9	64
Total	418	2929	23	164
Data compression rate				93.18%

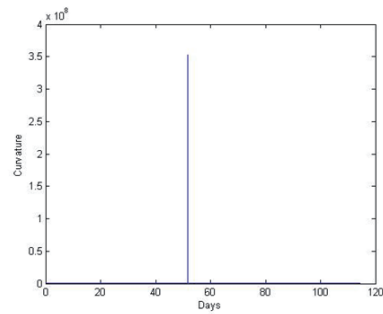shown in Fig. 10 and Table 5, respectively.

The original orbit, the calculated control points, and the reproduced orbit are depicted in Fig. 11.

Error histories of trajectory orbit are depicted in Fig. 12.

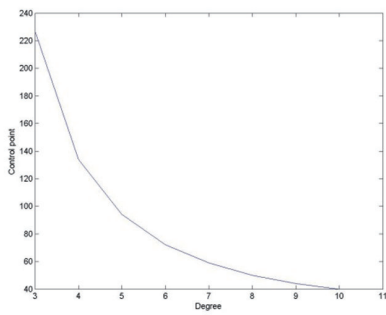The results of the polynomial regression are represented in Table 6.
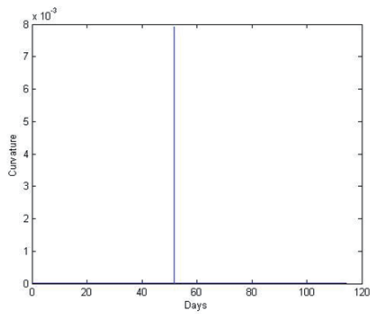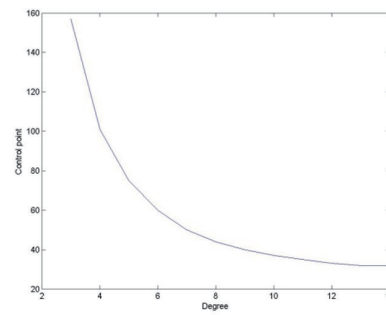

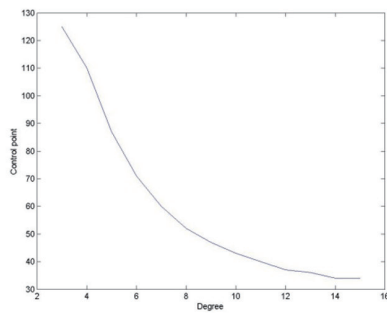(a) Curvature of position

(b) Curvature of velocity

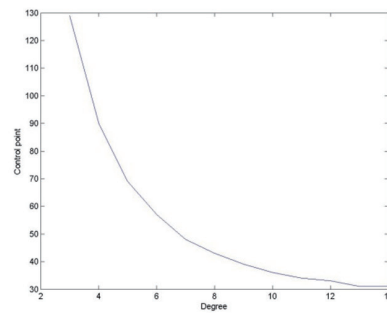Fig. 9. Curvature according to position and velocity

(a) First segment of position

(b) Second segment of position

(c) First segment of velocity

(d) Second segment of velocity

Fig. 10. Variation of the number of control points according to degree

Table 5. Application of the proposed algorithm

	Position		Velocity	
Segment	1	2	1	2
Degree	10	13	14	13
The number of control points	39	31	34	30
The number of data	160	128	140	124
The number of the original data		1153754		
Data compression rate		99.95%		

7.4 Comparison of Data Compression Rates

The comparison of the proposed method and the polynomial regression are shown in Table 7. For direct transfer data the compression ratios of 93.18 % and 99.7 % are obtained for the polynomial regression and the proposed algorithm, respectively. For the WSB data, 93.18 % and 99.7 % reduction are achieved for the polynomial regression and the proposed algorithm, respectively.

Using the size of the data compressed by the proposed algorithm as a numerator and the size of the data compressed by the polynomial regression as a denominator, the ratios are 4.4% (direct transition data) and 9.73% (WSB data), respectively. Transmitting the date using polynomial regression took 100 seconds, it takes 4.4 and 9.73 seconds

Table 6. Application of the polynomial regression method

Axis	Position		Velocity	
	The number of segments	The number of data	The number of segments	The number of data
x	390	2731	22	155
y	245	1716	15	106
z	125	876	13	92
Total	760	5323	50	353
Data compression rate				99.51%

(a) Position (b) Velocity
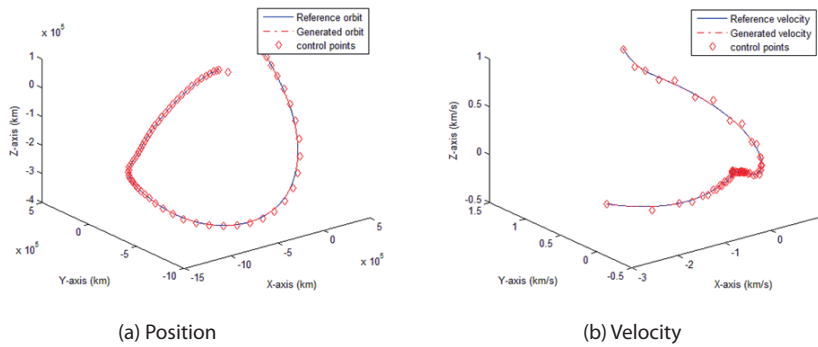
Fig. 11. Optimized control points and trajectories of WSB transfers
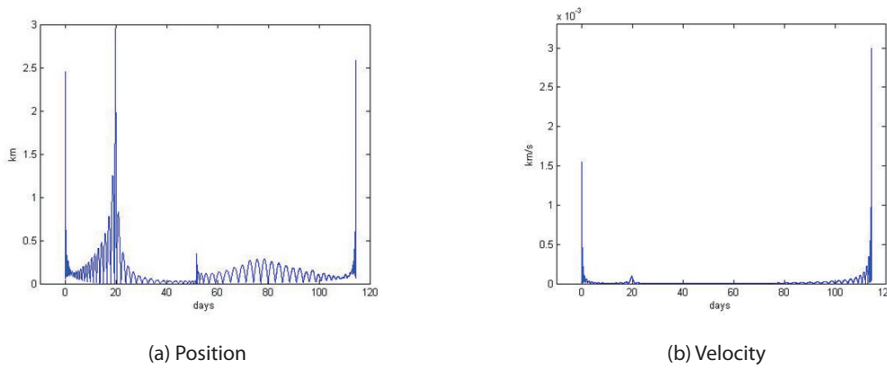

(a) Position (b) Velocity

Fig. 12. Error histories between the reference and generated orbits of WSB trajectory

Table 7. Comparison of compression rates

	Direct transfer trajectory		WSB trajectory	
	Polynomial regression	Proposed method	Polynomial regression	Proposed method
The size of compressed data (byte)	6186	272	11352	1104
Compression rate	93.18%	99.70%	99.51%	99.95%
The size of original data (byte)	90720		2307508	
$\dfrac{\text{Proposed method}}{\text{Polynomial regression}} \times 100\%$	4.40%		9.73%	

for direct transfer and WSB trajectory, respectively, using the proposed methods. Therefore, the proposed algorithm shows excellent compression efficiency for the orbital data transmission.

Figure 8 and Fig. 12 satisfy the given tolerance. The initial and final errors of the compression results are relatively large compared to those of the other sections. This is caused by the B-spline approximation scheme with free-end conditions at the initial and final positions. Here, the free-end condition is the one that no position constraint or any higher derivative constraint is enforced. In general having such a free end condition may require less control points in the approximation while the tolerance condition is satisfied. Such errors at both ends can be reduced by introducing position constraints, or by using a less tolerance, either of which may result in more control points. However, because the main purpose of the proposed method is to compress the orbit data as compactly as possible within a user specified tolerance, we use the free end conditions at a loss of accuracy at the initial and final positions.

8. Conclusions

In this study, an on-board orbit propagator and orbital data compression using a B-spline are proposed for a lunar explorer. A model for the on-board orbit propagator of a lunar explorer is designed to decrease the workload of the ground station and the explorer. A huge amount of data is compressed with stability and high efficiency using B-spline. Furthermore, the compression ratio is maximized using various novelty methods. Applying WSB and direct transfer trajectory calculated by STK, a compression ratio of over 99% can be obtained, thus demonstrating the excellence of the proposed algorithm compared to polynomial regression. The changes of the system and the transfer data to be applied are specifically described. The proposed method can be used to reduce the communication data of a lunar explorer, and its application can ultimately be extended to deep space probes, such as a Mars explorer.

References

[1] Yang, W., Kim, S. and Choi, J., "Conceptual Design of a Launch Vehicle for Lunar Exploration by Combining Naro-1 and KSLV-II", *The Korean Sociaty for Aeronautical and Space science*, Vol. 42, 2014, pp. 654-660.

[2] Son, T., Na, K., Kim, J., Lim, J. and Kim, K., "Design of a Structural Model for Korean Lunar Explorer", *Journal of the Korean Society for Aeronautical & Space Sciences*, Vol. 41, 2013, pp. 366-372.

[3] Ju, G., Roh, W., Choe, N. and Lee, S. R., "A Feasibility Study on Korean Lunar Exploration Mission", *The Korean Society For Aeronautical And Space Sciences*, 2008, pp. 384-387.

[4] Hoots, F. R. and France, R. G., "Hybrid Ephemeris Compression Model", *Celestial Mechanics and Dynamical Astronomy*, Vol. 66, 1997, pp. 51-60.

[5] Salama, A. H., "On-board Ephemeris Representation for Topex/Poseidon", in *Astrodynamics 1989*, 1990, pp. 674-679.

[6] Segerman, A. M. and Coffey, S. L., "Ephemeris Compression Using Multiple Fourier Series", *American Astronautical Society*, Vol. 97, 1997, pp. 689.

[7] Jung, O., No, T., Kim, H. and Kim, E., "A Numerical Approach for Station Keeping of Geostationary Satellite Using Hybrid Propagator and Optimization Technique", *International Journal of Aeronautical and Space Sciences*, Vol. 8, 2007, pp. 122-128.

[8] Lee, D. J., No, T. S., Choi, S. W., Lee, S. R., Kim, H. J. and Alfriend, K. T., "Precise Ephemeris Reconstruction Using the Clohessy-Wiltshire Frame and Multiple Sequential Compressions", *Journal of Guidance, Control, and Dynamics*, Vol. 26, 2003, pp. 781-785.

[9] Choi, E. J., Yoon, J. C., Lee, B. S., Park, S. Y. and Choi, K. H., "Onboard Orbit Determination Using GPS Observations Based on the Unscented Kalman Filter", *Advances in Space Research*, Vol. 46, 2010, pp. 1440-1450.

[10] Aghav, S. and Gangal, S., "Use of Spaceborne GPS Navigation Sensor for Autonomous LEO Orbit Determination", in *Physics and Technology of Sensors (ISPTS), 2012 1st International Symposium on*, 2012, pp. 70-73.

[11] Gangal, S. A. S. A., "Development of On-Board Orbit Determination System for Low Earth Orbit (LEO) Satellite Using Global Navigation Satellite System (GNSS) Receiver", *presented at the Geospatial World Forum*, Hyderabad, India, 2011.

[12] Zhou, N., "Onboard Orbit Determination Using GPS Measurements for Low Earth Orbit Satellites", Queensland University of Technology, 2004.

[13] Spall, J. C., *Introduction to Stochastic Search and Optimization* 1ed.: Wiley-Interscience, 2003.

[14] Boikov, V., Khutorovsky, Z. N. and Alfriend, K. T., "Singular Value Decomposition and Least Squares Orbit Determination", in *AAS/AIAA Astrodynamics Conference, Mackinac Island, Michigan*, 2007.

[15] Khutorovsky, Z. N., Samotokhin, A. S. and Alfriend, K. T., "Guaranteed Approach for Orbit Determination with Limited Error Measurements", *Journal of Guidance, Control, and Dynamics*, Vol. 36, 2013, pp. 1186-1193.

[16] Marshall, P. M., "Least Squares Solutions in Statistical Orbit Determination Using Singular Value Decomposition", Monterey, California: Naval Postgraduate School, 1999.

[17] Ahn, H., Chung, D., Ko, K., Wang, S., Jung, O. and Choi, S., "Satellite Antenna Control: Design and Performance Validation Under Given TPF", in *Control Automation and Systems (ICCAS), 2010 International Conference on*, 2010, pp. 1527-1532.

[18] Choi, S., Jung, O., Kang, C., Kim, Y. and Chung, D., "An Algorithm to Eliminate TPF Discontinuity for LEO Satellite", in *Annu. Conf. Korean Soc. Aeronautical Space Sci.(KSAS), Pyungchang, Korea*, 2009.

[19] Ko, K., Ahn, H., Wang, S., Choi, S., Chung, O. and Chung, D., "A Study on Approximation of TPF Profile Using B-spline", in *Society of CAD/CAM Engineers*, 2011, pp. 421-427.

[20] Lee, J., Ahn, H., Wang, S., Choi, S., Chung, O., Chung, D. and Ko, K., "A Study on Efficient TPF Approximation for Low-orbit Satellites", in *The Korean Sociaty for Aeronautical and Space science*, 2011, pp. 1221-1224.

[21] Ditmar, L. X.-L. P., "Smoothing a Satellite Orbit on the Basis of B-spline and Regularization", *Chinese Journal of Geophysics*, Vol. 49, 2006, pp. 86-94.

[22] Ye-zhi, S., Yong, H., Xiao-gong, H., Pei-jia, L. and Jian-feng, C., "Spacecraft Orbit Determination with The B-spline Approximation Method", *Chinese Astronomy and Astrophysics*, Vol. 38, 2014, pp. 172-185.

[23] Lee, D., Gwon, J., Choi, S. and Lyu, D., "Onboard Orbit Propagator for Spacecraft", in *The Korean Sociaty for Aeronautical and Space science*, 2014, pp. 737-740.

[24] Piegl, L. and Tiller, W., *The Nurbs Book. Monograph in Visual Communication*, 1997.

[25] Hoschek, J., Lasser, D. and Schumaker, L. L., *Fundamentals of Computer Aided Geometric Design* vol. 1: AK Peters, Ltd., 1996.

[26] Adimurthy, V., Ramanan, R., Tandon, S. and Ravikumar, C., "Launch Strategy for Indian Lunar Mission and Precision Injection to the Moon Using Genetic Algorithm", *Journal of Earth System Science*, Vol. 114, 2005, pp. 711-716.

[27] Song, E., Park, C., Cho, S. and Roh, W., "Mission Design for a Lunar Orbiter Launched by KSLV-Ⅱ", *Aerospace Engineering and Technology*, Vol. 8, 2009, pp. 108-116, 7.

[28] Lozier, D., Galal, K., Folta, D. and Beckman, M., "Lunar Prospector Mission Design and Trajectory Support", *Advances in the Astronautical Sciences*, Vol. 100, 1998, pp. 297-312.

[29] Beckman, M., "Mission Design for the Lunar Reconnaissance Orbiter", in *Proceedings of the 29 th AAS G&C Conference Abstract AAS-07-057*, 2006.

[30] Keller, J. W. and Petro, N. E., "The Lunar Reconnaissance Orbiter-Six Years of Science and Exploration at the Moon", in *AAS/Division for Planetary Sciences Meeting Abstracts*, 2015.

[31] Uesugi, K., "Results of the MUSES-A "HITEN" mission", *Advances in Space Research*, Vol. 18, 1996, pp. 69-72.

[32] Lei, H. and Xu, B., "Transfers Between Libration Point Orbits of Sun–Earth and Earth–Moon Systems by Using Invariant Manifolds", *Journal of Engineering Mathematics*, 2015, pp. 1-24.

[33] Roncoli, R. B. and Fujii, K. K., "Mission Design Overview for the Gravity Recovery and Interior Laboratory (GRAIL) Mission", *Paper AIAA*, Vol. 8383, 2010.

[34] Zuber, M. T., Smith, D. E., Lehman, D. H., Hoffman, T. L., Asmar, S. W. and Watkins, M. M., "Gravity Recovery and Interior Laboratory (GRAIL): Mapping the Lunar Interior from Crust to Core", *Space Science Reviews*, Vol. 178, 2013, pp. 3-24.

[35] Antreasian, P. G., Bhat, R. S., Broschart, S. B., Chung, M. K., Criddle, K. E., Goodson, T. D., Hatch, S. J., Jefferson, D. C., Lau, El. L., Mohan, S., Parker, J. S., Roncoli, R. B., Ryne, M. S., Sweetser, T. H., You, T. H. and Young, B. T., "Navigation of the Twin GRAIL Spacecraft into Science Formation at the Moon", in *23 rd International Symposium on Space Flight Dynamics, Pasadena Convention Center October*, 2012.

[36] Clement, B. J. and Johnston, M. D., "The Deep Space Network Scheduling Problem", in *Proceedings of the National Conference on Artificial Intelligence*, 2005, pp. 1514.

[37] Clement, B. J. and Johnston, M. D., "Design of a Deep Space Network Scheduling Application", in *Proceedings of the International Workshop on Planning and Scheduling for Space*, 2006.

[38] Tai, W. S., Bhanji, A. M., Luers, E. B. and Shen, Y., "Deep Space Network Services Catalog", ed: Document, 2015.

[39] Patrikalakis, N. M. and Maekawa, T., *Shape Interrogation for Computer Aided Design and Manufacturing*: Springer Science & Business Media, 2009.