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The Use of Traditional Algorithmic Versus Instruction with Multiple Representations: Impact on Pre-Algebra Students' Achievement with Fractions, Decimals, and Percent

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The purpose of this study was to investigate the impact of multiple representations on students' understanding of fractions, decimals, and percent. The instructional approach integrating multiple representations was compared to traditional algorithmic instruction, a form of direct instruction. To examine and compare the impact of multiple representations instruction with traditional algorithmic instruction, pre and post tests consisting of five similar items were administered with 87 middle school students. Students' scores in these two tests and their problem solving processes were analyzed quantitatively and qualitatively. The quantitative results indicated that students taught by traditional algorithmic instruction showed higher scores on the post-test than students in the multiple representations group. Furthermore, findings suggest that instruction using multiple representations does not guarantee a positive impact on students' understanding of mathematical concepts. Qualitative results suggest that the limited use of multiple representations during a class may have hindered students from applying their use in novel problem situations. Therefore, when using multiple representations, teachers should employ more diverse examples and practice with multiple representations to help students to use them without error.

Ⅰ. Introduction

Rational numbers are one of the most important mathematical concepts that middle school students study but many students struggle with (Donovan & Bransford, 2005). While these struggles can be attributed to many factors, classroom instruction is partly to be criticized. Traditionally, instruction on rational numbers in the U.S. is algorithmic, rule-based, and relies heavily on sets of procedures aimed at making students quick and accurate when solving problems (National Research Council [NRC], 2001). Often when rational numbers and operations are introduced, classroom time is devoted towards symbolic manipulations, instead of

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seeking the meaning of these numbers (Moss, 2002) and students are often taught to rely on memorized rules rather than to develop deep and practical understanding of rational numbers (Donovan & Bransford, 2005).

Traditional algorithmic instruction often begins with teachers stating an algorithm (e.g., "to divide by a fraction, invert and multiply"), teacher-led demonstrations of how the algorithm works by presenting several examples, and then student practice, independently or in groups, on similar exercises. While algorithmic approaches have been found to be efficient methods for teaching students how to solve problems (Newton & Sands, 2012), major issues arise when, as a result of these approaches, students begin to view mathematics as sets of rules and give up their own mathematical sense-making while carrying out the steps of an algorithm (Fosnot & Dolk, 2002). The National Research Council (NRC) finds that the "rules for manipulating symbols are being memorized but students are not connecting those rules to their conceptual understanding nor are they reasoning about the rules" (NRC, 2001, p. 234). The unintended consequence is that many students are not engaged in their learning of mathematics, forget important mathematical concepts from year to year, and are not fully prepared for higher-level mathematics (Rasmussen et al., 2011).

In efforts to promote deeper understanding of mathematical topics various methods have been used and recommended. One teaching approach that has shown to be effective is the use of multiple representations (MR), such as diagrams, graphical displays, and symbolic expressions, to help students make better sense of mathematics and develop deeper conceptual understanding (Fosnot & Dolk, 2002; Ng & Lee, 2009; van den Heuvel-Panhuizen, 2003). A considerable amount of evidence demonstrates that, if used properly, the use of MR can significantly enhance student learning in complex domains (Ainsworth, Bibby, & Wood, 2002; Schnotz & Bannert, 2003). However, research also suggests that simply providing learners with MR does not necessarily result in flexible knowledge acquisition (Ainsworth et al., 2002). Ng and Lee (2009) recommend that students must be given multiple experiences where they get: 1) the opportunity to reflect on representations, 2) the option to make modifications, and 3) the final choice in the selection of solution strategy. Van den Heuvel-Panhuizen (2003) asserts that, "It is not the models in themselves that make the growth in mathematical understanding possible, but the students' modeling activities" (p. 29). Therefore, providing diverse activities using MR can be employed for those students who have difficulties in understanding rational numbers.

II. Multiple Representations

Mathematical representations can play a critical role in teaching mathematics by allowing teachers to explain mathematical concepts to students in a meaningful way and interpret students' representations to evaluate their understanding. Previous research suggests that the implementation of MR in mathematics classroom is an effective instructional approach to improve student understanding on mathematical concepts such as fractions, decimals, and percent (NRC, 2001;

Lamon, 2001; National Council of Teachers of Mathematics [NCTM], 2000). Engaging students in mathematics through MR helps them better visualize, simplify, and make sense of abstract mathematical topics (NRC, 2001; NCTM, 2000). Lamon (2001) found that by using different representations of rational numbers students gained a deeper understanding of the concepts and were better able to transfer the gained knowledge and understanding from one model to another. Another study conducted by Niemi (1996) investigated the relationship between student fraction knowledge and the level of representational skill. More than 500 fifth grade students were asked to represent their conceptual knowledge on fractions in their own ways. The findings suggest that using MR flexibly is a key characteristic of skilled problem solvers and that students who have better representational skills are more likely to show better performance in solving problems.

Multiple representations instruction is not only a meaningful mean of providing effective instruction but also a formative assessment tool that can be used to gauge students' understanding of mathematics concepts. Previous studies have recognized that representations might be valuably used as a source of information about student mathematical knowledge (Duval, 1999; Greeno & Hall, 1997; Heritage & Niemi, 2006; NCTM, 2000). Through the use of MR students reveal whether, how, and to what extent they understand mathematics concepts. Teachers can monitor student progress, predict the extent of student understanding of concepts and observe the source of student misconceptions. Such information can help teachers evaluate whether students have achieved mastery goal and make decisions about curriculum and, if additional time should be afforded, to explore the mathematical concept further (Heritage & Niemi, 2006; NCTM, 2000).

Student learning and misconceptions about fractions, decimals, and percent are often impacted by the types of mathematical representations that their teachers use and the learning experiences that their teachers provide. It is not the MR used but rather the instructional decisions about the representations that will be used that play a major role in facilitating students' learning. Muzheve and Capraro (2012) showed that the representations used by teachers in mathematics classrooms influenced the types of representations that their students used when learning how to convert fractions, decimals, and percent. Moreover, these researchers found that the representations used by teachers in mathematics classroom were often the same or similar to those in classroom textbooks (Muzheve & Capraro, 2012). Unfortunately, as these representations are common in textbooks, they may become overused by teachers. Jigyel and Afamasaga-Fuata'I (2007) found that students were very familiar with geometric figures such as circles or rectangles, however they lacked in skills relating to numeric and symbolic representations of fractions (Jigyel & Afamasaga-Fuata'i, 2007). In the study examining the use of number lines as MR tool, Shaughnessy (2011), found that when students were asked to label points on the number line using fractions or decimals they were more likely to label appropriately when the interval on the number line was equally partitioned than when the interval was unequally partitioned. This misconception might be attributed to limited teacher usage, student exposure to number lines, and experiences where number lines were only partitioned equally.

Ⅲ. Theoretical Framework

Learning theorists have highlighted the importance of using MR and providing children with varied experiences where they manipulate representations to gain deeper understanding of abstract concepts. Piaget's theory of cognitive development emphasizes the necessity of providing young children with experiences where concrete objects are used to facilitate learning before using more abstract representations (Piaget, 1957). Similarly, Bruner (1966) suggests three levels of representational thought: enactive, iconic, and symbolic. According to Bruner (1966), the learning of a new concept occurs through a process of internalizing one's environment and proceeds through: (a) an "enactive" level where learning occurs from direct experiences and acting on concrete objects; (b) an "iconic" level where learning occurs from forming images of the concrete constructions and using visual models such as pictures and diagrams; and (c) a "symbolic" level where individuals use and manipulate abstract symbols and notations to represent concepts. Bruner (1966) refers to the work of Piaget, stating that "what is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought" (p. 38). Both Bruner (1966) and Piaget (1957) note the importance of helping students internalize representations into mental models and construct their own knowledge of concepts.

Unlike Piaget (1957) however, Bruner (1966) does not associate the modes of representational thought and learning to particular ages and stages of cognitive development. Instead Bruner (1966) asserts that individuals proceed from concrete to abstract modes of thought regardless of their age and that the learning of concepts comes from multiple experiences with manipulating concrete objects, pictorial representations, and abstract symbols. This suggests that based on their learning experiences children are able to think at an abstract level, to a certain degree, and that adults may need to be provided with experiences in which they are introduced to new concepts by manipulating concrete objects. Due to the diverse student experiences with mathematical concepts and knowledge levels, the use and manipulation of MR in mathematics classrooms is essential for learning. Furthermore, because students are first introduced to mathematical concepts by their teacher the types of representations that the teacher uses have a major impact on student learning of mathematical concepts.

Ⅳ. Purpose of Study

The purpose of this study was (1) to investigate how the use of MR in mathematics lessons helps students solve contextual word problems on fraction, decimal, and percent and (2) to examine whether students' errors on the problems were removed or retained after interventions. To answer the first research question, traditional algorithmic

[TA] group was compared to MR group. In addition, it is noted that the current study was not only to compare the number of students who got correct answers in MR and TA groups but also to analyze how each different treatment intervention influenced students' errors on fractions, decimals, and percent and how it contributed to the remediation of those errors.

Ⅴ. Methodology

The present study utilized a general typology of mixed methods research which was classified as an embedded mixed research design gathering only one data type (i.e., 5th grade students' written responses) (Cresswell & Plano, 2011; Teddlie & Tashakkori, 2009). Then, the qualitative data were transformed to quantitative data using the criteria: correct answer (1) and incorrect answer (0). The qualitative and quantitative data was analyzed and interpreted and the interpretation was drawn from the both analyses. The mixed methods research will be superior to a monomethod (qualitative or quantitative only) study because it complements the weaknesses of quantitative and qualitative approaches alone (Johnson & Onwuegbuzie, 2004). Specifically, student's error, which was the core topic of this study, was hard to be examined only with quantitative approach because the data interpretation was possible depending on the problem solving procedures that students described on the test.

1. Participants

This study was conducted in an urban middle school in the Midwestern region of the United States. Almost half, 44.17% of students in the middle school were classified as economically disadvantaged, and a small percentage, 3.39%, were English Language Learners (ELLs). Participants for this study included eighty-nine advanced skills 7th graders enrolled in four pre-algebra sections. Before the current study was implemented, these students had not been taught how to use TA and MR for transforming among fractions, decimals, and percents. Over half were females (52.8%), and all students were 11-13 years old. Students came from very diverse ethnic backgrounds including 14.6% African American, 12.4% were Asian American, 6.7% were Hispanic, 43.8% were White, 21.3% were Mixed Race, or 1.1% indicated "Other". The classroom teacher involved in the study had over six years of teaching experience at the urban middle school. This same teacher taught all participants in the study. Before the study was implemented, the teacher had participated in professional development on using multiple representations in middle grade mathematics classrooms. In both MR and TA groups, her instruction emphasized teaching for understanding and providing students with opportunities to solve real world problems using mathematics via multiple strategies.

2. Study Design

This study used a quasi-experimental design using two intact groups from four pre-algebra class sections with the same teacher. Group 1, MR group (N=45), experienced lessons that emphasized the use of MR and Group 2, TA group (N=42), received lessons where algorithms were emphasized.

a. Interventions: MR versus TA Lessons

Lessons within the two approaches differed in the type of mathematical representations and activities that students were asked to engage in. In MR lessons students were provided the opportunities of exploring multiple mathematical representations through the following three steps: 1) representing mathematical equations, 2) transforming among percent, fraction, and decimal, and 3) solving word problems using MR. Representations and models taught and used by students included chunking, number lines, double number lines, percent bars, ratio tables, and writing equations. For example, in the MR lesson the teacher provided instruction, examples, and guidance to students on how to use visual percent bars to find 20% of 20 (see figure V-1).

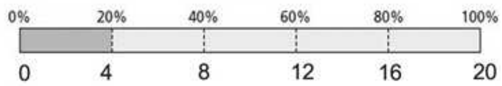


Figure V-1. Using Percent Bars to Find 20% of 20

b. Instruments

We gathered qualitative data using five mathematical items for each pre- and post-test. The items of the pre- and post-test assessed the participants' problem solving skills regarding the topic of fraction, decimal, and percent. The pre- and post-test consisted of the same types of questions to decrease Type I error, which might result from different type of test items. For example, first items of the pre- and post-test were to find a part of whole. The pre-test item was 'what is 25% of 32?' and the post-test item was 'what is 25% of 24?' Item 2, Item 3, Item 4, and Item 5 were contextual problems, whereas Item 1 was a non-contextual problem.

c. Procedures

Before the module on fractions, percent, and decimals, students completed a pre-knowledge test. During the 5-day module students experienced MR or TA lessons depending on the group that they were in. For each module day students were in class for about 50 minutes. Following the completion of the module, students filled out the post-knowledge test.

3. Data Analysis

To analyze the data, we employed an embedded mixed methods analysis (Cresswell & Plano, 2011; Teddlie & Tashakkori, 2009). According to Cresswell and Plano (2011), the embedded research design occurs "when the researcher collects and analyzes both quantitative and qualitative data within a traditional quantitative or qualitative" (p. 71). In the current study, a quantitative strand was added within a qualitative design. In the quantitative strand, two researchers carefully coded 176 pre- and post-assessment written responses as correct (1) and incorrect (0) answers. We performed a Wilcoxon signed ranks and Mann-Whitney U tests using Statistical Package for Social Science (SPSS) version 22.0 (2013) to determine if there were any changes pertaining to the correctness of students' answers during pre and post-assessments. Wilcoxon signed rank test and Mann-Whitney U-test are used for data sets having a dependent variable of ordinal or continuous level and an independent variable consisting of two categorical groups (Huck, 2008). The two tests do not assume normality of data, which gives more flexibility with the collected data (Huck, 2008). Wilcoxon signed rank test is for two categorically related groups and Mann-Whitney U-test is for two categorically independent groups.

In the qualitative strand, a researcher and graduate research assistant looked through students' worksheets including answers and solving process, and coded mathematical errors concerning fractions, decimals, and percent. To scrutinize the impact of MR on students' problem solving process, one class was randomly selected from each MR and TA groups. As students took pre- and post-tests, there were four possible cases: 1) Type A: correct answer in pre-test and correct answer in post-test, 2) Type B: correct answer in pre-test and incorrect answer in post-test, 3) Type C: incorrect answer in pre-test and correct answer in post-test, and 4) Type D: incorrect answer in pre-test and incorrect answer in post-test. The main interests of this research were Type B, Type C, and Type D. Specifically, for each item, students who gave incorrect answers in the pre- or post- test were selected as focus group for qualitative analysis. Therefore, the first step in the data analysis was to classify students into the four groups depending on their answers of each item. Through this step, only students in groups of Type B, Type C, and Type D were selected for the next step.

After determining the classification, each student's worksheets for pre- and post-tests were repeatedly examined and compared with each other in order to reveal differences and changes in the student's problem solving procedures. If students did not show their work in solving problems, these cases were excluded from the qualitative analysis.

Ⅵ. Results/Findings

1. Comparison between Pre and Post Tests

Wilcoxon signed ranks test was employed to compare student's achievement in pre- and post-tests. Two Wilcoxon signed ranks tests were run separately for each MR and TA group. The summary of results is reported in Table IV-1. For the MR group, the difference between positive ranks and negative ranks was statistically significant for Item 1 and Item 5. With Item 1, there was only one student who answered correctly in the pre-test and incorrectly in the post-test. Whereas, nine students who gave incorrect answers in the pre-test answered Item 1 correctly in the post-test. With Item 5, there were three students who answered correctly in the pre-test and incorrectly in the post-test. Whereas, 17 students who gave incorrect answers in the pre-test could answer correctly in the post-test on Item 5. The results of the Wilcoxon signed ranks tests for Items 2, 3, and 4 were not statistically significant. This means that the numbers of students who answered correctly in the pre-test and incorrectly in the post-test were not statistically significantly different with the numbers of students who

Table VI-1. Comparison between Pre- and Post Tests

MR

	Post1-Pre1	Post2-Pre2	Post3-Pre3	Post4-Pre4	Post5-Pre5
Negative Differences	1	9	8	7	3
Positive Differences	9	5	6	7	17
Ties	35	31	31	31	25
Total	45	45	45	45	45
Exact Sig.	0.021*	0.424	0.791	1.000	0.003*

TA

	Post1-Pre1	Post2-Pre2	Post3-Pre3	Post4-Pre4	Post5-Pre5
Negative Differences	0	9	5	0	0
Positive Differences	8	7	14	20	25
Ties	34	26	23	22	17
Total	42	42	42	42	42
Exact Sig.	0.008*	0.804	0.064	0.000*	0.000*

Note. MR=Multiple representation group; TA=Traditional algorithm group; Negative Differences= ‘Post<Pre’; Positive Differences= ‘Post>Pre’; Ties= ‘Post=Pre’; *p<0.05.

answered incorrectly in the pre-test and correctly in the post-test, respectively.

For the TA group, the difference between positive ranks and negative ranks was statistically significant for Items 1, 3, 4, and 5. As similar to the MR group, there were more students who answered correctly in the post-test, whereas incorrectly in the pre-test in the Item 1 and Item 5. However, as different with the MR group, more students who gave incorrect answers in the pre-test could answer correctly in the post-test, Items 3 and 4. With Items 1, 4, and 5, there was no student who answered correctly in the pre-test and incorrectly in the post-test. Eight, 20, and 25 students could get correct answers on Items 1, 4, and 5 (respectively) of the post-test, who could not answer correctly in the pre-test. With Item 3, five students answered incorrectly in the pre-test and correctly in the post-test, whereas 14 students could answer incorrect in the pre-test and correct in the post-test.

2. Comparison between MR and TA groups

To compare MR and TA students’ achievement, Mann-Whitney U-tests were employed for pre- and post-test separately. MR and TA groups did not show any differences with Items 1 and 2 in both pre-and post-tests. In the Item 3 of the pre-test, there was no difference between MR and TA. After the treatment, however, with Items 3 more TA students answered correctly than MR students.

In addition, with Item 4 and 5, the differences between TA and MR students in the pre-test were statistically significant with the mean rank of MR was higher than the mean rank of TA. After the treatment, TA group had higher mean rank than MR group in the Item 4 and the difference between TA and MR students had been removed in the Item 5 (p>0.05).

3. Impact of MR on Students' Errors

According to the findings of qualitative analysis, MR and TA approaches had different impacts on students' understanding on fraction, decimal, and percent. The frequencies and percentages of each type were reported in Table VI-2. Overall, the students in the MR group showed more cases of Type B and Type D, and fewer Type C than TA students, whereas the students in TA groups showed more cases of Type C than MR students. We excluded the description on the cases of Type A in the following.

a. Item 1

Most students in MR and TA were classified as Type C, except Type A. No student was identified as Type B and there was only one student in Type D. That is, most students, except the cases of Type A, gave incorrect answers in the pre-test, but correct answers in post-test. There were six and seven students categorized as Type C in MR and TA groups, respectively. They were all showed the same pattern, 'incorrect answer in pre-test and correct answer in post-test', however the ways to get correct answer in the post-test were different. The error concluding incorrect answer in the pre-test was same to students in both MR and TA

Table VI-2. Frequencies and Percentages of each case in MR and TA groups

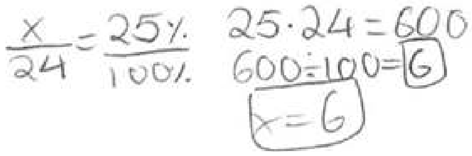
MR

Type of Case	Item 1	Item 2	Item 3	Item 4	Item 5
Type A	16 (72.7%)	12 (54.5%)	6 (27.3%)	6 (27.3%)	9 (40.9%)
Type B	0 (0%)	5 (22.7%)	4 (18.2%)	5 (22.7%)	3 (13.6%)
Type C	6 (27.3%)	1 (4.5%)	3 (13.6%)	4 (18.2%)	9 (40.9%)
Type D	0 (0%)	4 (18.2%)	9 (40.9%)	7 (31.8%)	1 (4.5%)
Total	22	22	22	22	22

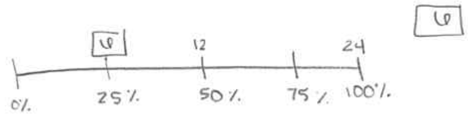
TA

Type of Case	Item 1	Item 2	Item 3	Item 4	Item 5
Type A	15 (65.2%)	13 (56.5%)	10 (43.5%)	10 (43.5%)	7 (30.4%)
Type B	0 (0%)	2 (8.7%)	1 (4.3%)	0 (0%)	0 (0%)
Type C	7 (30.4%)	5 (21.7%)	8 (34.8%)	8 (34.8%)	12 (52.2%)
Type D	1 (4.3%)	3 (13.0%)	4 (17.4%)	5 (21.7%)	4 (17.4%)
Total	23	23	23	23	23

groups. That is, the students showed the same error to compute whole number with percent number (e.g., 25%-32 or 32/25%) in the pre-test. In the post-test, MR students tried to make pictorial representations to solve the Item 1, whereas students in TA group were more likely to use cross multiplications (see Figure VI-1). Of six students in MR group five students represented a number line having percent numbers and the matching whole numbers in the post-test. The teacher had the strategy to write whole numbers and percent numbers on the upper and under sides of number lines for MR group. This might help MR students to differentiate percent numbers from whole numbers. The error to compute whole number with percent number (e.g., 25%-32 or 32/25%) in the pre-test was removed in the post-test. On the other hand, no students in TA group used pictorial representations such as number line for solving the problem. They made an equation representing fractional ratio.



a. Student in TA group



b. Student in MR group

Figure VI-1. Students' Problem Solving Response to Item 1

b. Item 2

The major difference between MR and TA groups regarding the pattern of types was that there were more cases of Type B in MR group and more cases of Type C in TA group than the counterpart group respectively. Five students in MR group were identified as Type B whereas two students in TA group were so. However, only one student in MR was categorized in Type C whereas five students in TA group were so. The reason why there were more Type B students in MR group was due to the students' misuse of pictorial representation and misunderstanding of the 'whole.' Of five Type B students in MR group three students tried to draw pictorial representations to solve the problem. In this process, the students depicted the problem, 'Three friends share two large pizzas' using two circles and tended to think of one circle as a whole and 100%, which showed the misconception on the concept of whole. Although the other two Type B students in MR group did not use pictorial representations in solving the problem but used algorithmic procedures for solving, they still had misconceptions of the whole and it caused an error in solving the problem. There were two Type B students in TA group for Item 2. One student showed a similar pictorial representation and misconception of whole and another student gave an incorrect answer simply because of a computational mistake.

Students showed the same misconception that percent and whole numbers could be computed together in the pre-test (e.g., students transformed 1/4 to 25% and added 0.3 to it.), however each student group had different strategies to fix their errors in the post-test. There were more students in TA group who got a correct answer on Item 2 of the post-test among those who answered incorrectly in the pre-test. Only one student was Type C in MR group whereas five students were Type C in TA group. The student who was Type C in MR group not only used a pictorial representation, but also approached the problem with the algorithmic method. This student represented the correct concept of whole, drew one circle as a whole, and transformed fraction and decimal numbers to percent numbers to compute the part that 'Susan eat'. All the five Type C students in TA group got correct answers using algorithmic procedures by transforming numbers into a single type: decimal or percent. Finally, 4 and 3 students were identified as Type D in MR and TA groups, respectively. There were significant differences between MR and TA groups in terms of changes in their problem solving process.

c. Item 3

The students in MR and TA groups showed quite different patterns of dispersion across Type B, C, and D in Item 3. Four, three and nine students were Type B, C, and D in MR group, whereas one, eight, and four students were Type B, C, and D in TA group. The main reason for this difference was because students in TA group who got incorrect answers in pre-test were more likely to answer correctly in post-test than those in MR group. That is, of the 12 students in MR group who answered incorrectly in the pre-test three students could get correct answer in post-test whereas nine students could not get correct answer in post-test as well. Two of the three Type C students in MR group used percent bars and could get correct answers. However, two of nine Type D students in MR group had problems with the percent number (15%) of Item 4, which was not a benchmark percent number. They could interpret the word problem and draw percent bar to figure out '15% of $45'. However, pictorial representations sometimes allowed students computing limited benchmark numbers such as 25%, 50%, and 75%. Another five Type D students in MR group could not connect the question 'a 15% increase in pay ($45)' to '15% of 45$' and tried to divide 15 by 45 without any units such as % or $. As other two Type D students in MR group got the increased total pay as the answer, therefore, they were recoded as wrong answers.

In TA group, there were most Type C students for Item 3. Only one student was Type B. This student tried to arithmetic approach without any representations and made a computational error. All eight Type C students in TA group used cross multiplication approach or directly get a multiplication equation (e.g., 450.15=). On the other hand, four students in TA group were Type D and one of them had the error having the whole as 115% instead of 100%. Except the student, the remaining three Type D students could not understand the problem itself and get a right equation or algorithmic approach.

d. Item 4

Students' patterns for Item 4 in terms of Type B, C, and D were almost same to the patterns for

Item 3. The difference between Item 3 and Item 4 was that Item 3 asked the percent number of a part when the whole and part numbers were given whereas Item 4 asked the part number when the percent number of a part and whole number were given. Therefore, basically if students understand the concept, ‘a part of a whole’ and ‘percent’ then they should have answered both Item 3 and Item 4 correctly. Five, four, and seven students in MR group were Type B, C, and D, respectively. All five Type B students in MR group could get correct answers using the computation (i.e., 15/40=0.375 (37.5%)) in the pre-test. However, they had wrong answers in the post-test because of various reasons. Two of them had the computation, 50/15=3.333⋯ without any other representations. This indicated that these students did not understand why 15 needed to be divided by 40 in the pre-test, because these pre- and post-test items had the same structure and only different numbers. Other two Type B students in MR group tried to use double number lines having whole numbers and percent numbers simultaneously, but could not get the final answer. This error was more likely to be caused from the issue regarding benchmark percent numbers mentioned before. Last, one Type B student in MR group did not show the problem solving process. Therefore, it was not possible to figure out the reason of an error.

Three of four Type C students in the MR group continued to use the simple division (15/50=0.3 (30%)) and the remaining one student employed a percent bar with Item 4. In addition, among the seven Type D students in MR group, five students tried to draw percent bars to solve the problem, however they could not figure out where the score 15 should be located and get the final answers. Other two remaining students approached the item just with simple arithmetic such as 50/15 or 1550 without any explanation of why they did it.

Of 22 students in TA group eight and five students were Type C and D. No Type B student was in TA group. All Type C students in TA group used the cross multiplication approach except one student who used the simple division (15/50=0.3 (30%)). The interesting thing to note among the Type D students in TA group was that two of them had the error type having wrong equation such as ‘part/whole = whole/part’. That is, using TA approach without understanding led students to these errors/misconceptions. The remaining Type D students in TA group did not show the problem solving process, so that it was impossible to analyze them.

e. Item 5

The number of Type D students in MR group was dramatically decreased whereas Type C students increased for Item 5. Three, nine, and one student in MR group were Type B, C, and D, respectively. All three Type B students in MR group tried to solve the problems using algorithmic equations. However, they had the correct equation $8,000/$40,000=0.2 (20%) for the pre-test and incorrect equation $24,000/$6,000=4. As mentioned similarly in the Item 4, two Item 5 of each pre- and post-test have same structure and students answer correctly in pre-test and incorrectly in post-test. This indicated that students did not understand the content and got correct answers almost by chance. All nine Type C students used simple division without any representations even though they were taught how to use MR in MR group. Finally, one Type D student explained the problem solving procedure as 0.240000.6000=0.144, which was the way this student tried in the pre-test as well.

On the other hand, the trend in TA group for Item 5 was similar to the trend for Item 4. Twelve and four students in TA group were Type C and D, respectively. Eight of 12 Type C students in TA group used the cross multiplication, 6,000/24,000=x/100, and other remaining four students used the simple division 6,000/24,000=0.25 (25%). The four Type D students in TA group had incorrect answers with the following reasons. One of them made the incorrect equation, 24,000/600=x/100, another had 24,000/6,000=4. The remaining two students were not analyzed because one of them made an arithmetic error and another student did not explain the problems solving procedure.

Ⅵ. Discussion

The current study investigated the impact of MR on students' answers and solving processes for the fractions, decimals, and percent problems. The mixed methods approach employed in this study contributes to contextualizing the findings of how and why the TA and MR interventions influence students' problem solving processes and answers differently. Four sections of a pre-algebra course were taught how to solve problems involving fractions, decimals, and percent using either TA or MR methods and student outcomes were compared. Results found that students who were taught by TA instruction showed more items correct than those who were taught by using MR. This result was contrast to the expectation rooted in the literature and previous studies concerning the instruction for fraction, decimal, and percent (Ng & Lee, 2009; NRC, 2001; Rasmussen et al., 2011).

The results of the quantitative analyses indicate that TA instruction is effective for helping students have correct answers. Even though previous research show that students were more familiar with visual figures rather than symbolic representations in learning fractions (Jigyel, & Afamasaga-Fuata'i, 2007), the participating students showed higher improvement in the post test items when they learned with TA instructional strategies. This improvement may be due to their familiarity with the application of algorithms such as cross multiplication as they had practiced several problems having similar patterns for solving. Students who were taught using an algorithmic approach just followed the instructed steps and got correct answers, which probably limited the errors in solving a same type of problem. As consistent with Newton and Sands' (2012) findings, it is easier for teachers and students to use algorithmic approaches in teaching and learning fractions, decimals, and percent.

According to the results of the qualitative analyses, traditional algorithmic approaches are not always beneficial for students learning about fractions, decimals, and percent. For example, students in TA group might answer questions correctly without deep understanding of mathematics content (Fosnot & Dolk, 2002; Newton & Sands, 2012; NRC, 2001; Rasmussen et al., 2011). In the present study, some students in the TA group showed the errors with setting up proportions by incorrectly inverting fractions when using cross multiplication. This type of error apparently indicates that those students do not understand the logic behind setting up proportions and cross multiplication to solve, but just repeat memorized steps and calculations resulting in incorrect answers. To increase the effectiveness of using algorithms in mathematics, it is critical that teachers explain the embedded logic behind these types of algorithms.

The findings of this study indicated that employing MR methods in the classroom do not yield significant impacts on students' understandings for fractions, decimals, and percent, although previous research suggest that utilizing MR in mathematics lessons help students understand mathematics concept more deeply (Ainsworth et al., 2002; Fosnot & Dolk, 2002; Ng & Lee, 2009; Schnotz & Bannert, 2003; van den Heuvel-Panhuizen, 2003). In this study, some students in MR group had difficulties answering certain types of questions while several other students showed improvement in modeling problem situations and solving questions using pictorial representations such as bar graphs, pie charts, or percent bars. Students might not have had sufficient time to get familiar and effectively utilize the newly taught strategies to solve problems. For example, students often failed to answer correctly when the given percent in the question was not a benchmark number (e.g., 25%, 50%). Consequently, the MR approach could have positive impact on students' understanding on mathematics concepts if students would have had sufficient opportunities to learn and practice with the use of mathematical representations to solve varied types of questions (Ainsworth et al., 2002; Ng & Lee, 2009).

Although MR can provide effective solutions to promote meaningful learning and support students' conceptual understanding of mathematical concepts, merely including MR without careful planning and class implementation does not lead to student learning. Teachers need to prepare a lesson with comprehensive consideration of how to carry out the instruction including diverse problem situations and cases using MR. Furthermore, during lessons and practice time, teachers should allow sufficient time for students to evaluate, reflect on, and/or revise their own representations (Ng & Lee, 2009). The MR approach with lack of student self-reflection on the representations may cause them to merely apply the representations without understanding their mathematical meaning (Ng & Lee, 2009). Further, teachers should monitor student progress, provide constructive feedback, and guide them when building representations (Muzheve & Capraro, 2001). The encouragement of self-reflection, variability in the questions combined with timely support and guidance could prevent students from having misconceptions and positively impact the transfer of learning to novel problem situations (van Merriënboer & Kirschner, 2012). Specifically, the findings of the present study provide Korean educators and policy makers with educational implications as traditional algorithmic approach has been commonly implemented in Korean classrooms. It is not appropriate to regard simply TA limits and MR encourages students' mathematical thinking skills. Instead, teachers should recognize the strong and weak points of each TA and MR and apply them into lessons for teaching fractions, decimals, and percents.

A significant limitation of this analysis is the student level and reliance on student tests. This study provided findings from the secondary analysis of students' answers and their problem solving process. Therefore, there is a possibility that this type of data may be limited as not all students explicitly explained and/or drew their solution process. Therefore, future studies should include qualitative data such as classroom observations, data directly collected from students and teachers capturing their thought processes, perceptions, and experiences. In addition, teacher's pedagogical beliefs and practices should be included in future studies as research indicates a significant relationship between their beliefs and classroom implementation of various strategies (Stipek, Givvin, Salmon, & MacGyvers, 2001). Future research may also consider implementation of MR approaches in various content area of mathematics (e.g., geometry and probability). In addition, students' errors should to be developed into taxonomy of students' misconceptions of fractions, decimals, and percent, which could help researchers in data analysis and reporting.

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전통적 알고리즘 교수법과 다양한 표상을 활용한 교수법의 비교 : 분수, 소수, 퍼센트 내용을 중심으로

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본 연구는 수학의 다양한 표상이 학습자의 분수, 소수 및 퍼센트에 대한 이해에 어떤 영향을 주는지 분석하는 것을 목적으로 하였다. 다양한 표상을 활용한 교수법을 전통적 알고리즘 교수법과 비교하고자 87명의 중학교 학생들을 대상으로 사전, 사후 검사를 실시하였다. 사전, 사후 검사는 각각 5개의 비슷한 문항으로 구성되었으며, 문항에 대한 학생들의 답안을 양적, 질적으로 분석하였다. 양적 분석 결과에 따르면, 전통적 알고리즘 교수법으로 지도 받은 학생들이 다양한 표상을 활용한 교수법에 의해 지도받은 학생들에 비하여 높은 점수를 나타내었다. 또한, 다양한 표상을 활용한 교수법이 학생들의 수학적 개념에 대한 이해를 보장해 주지는 못함이 드러났다. 질적 분석 결과에 따르면, 수학 교실에서 다양한 표상을 제한적으로 활용할 경우, 오히려 다양한 수학적 표상은 학생들이 문장제 문제를 푸는 과정에서 응용을 방해하는 것으로 나타났다. 본 연구 결과에 따르면, 교사는 수학 교실에서 다양한 표상을 활용함에 있어서 반드시 여러 가지 예시와 연습을 통해 학습자들이 다양한 표상을 제대로 이해하고, 연습할 수 있도록 도와야 할 것이다.

* 키워드 : 다양한 표상(multiple representation), 전통적 알고리즘 교수법(traditional algorithmic instruction), 수학적 오류(mathematical error), 분수(fraction), 소수(decimal), 퍼센트(percent)

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Appendix A

Pre-Test Items

1. What is 25% of 32? Show your work and box your answer.
2. Three candidates participated in a school election. Bianca received $\frac{1}{4}$ of the votes, Chelsea received 0.30 of the votes, and Francisco received the rest of the votes. What percent of the votes did Francisco receive? Show your work and box your answer.
3. Jena is eating at Red Robin. Her total bill comes to $28. If she decides to leave a tip that is 15% of the total bill, how much should she leave for the tip? Show your work and box your answer.
4. 40 seventh grade girls are trying out for the basketball team, but only 15 can make the team. What percentage of the girls will make the team? Show your work and box your answer.
5. An employee earned $40,000 in a year and had $8000 of her earnings withheld for federal income tax. What percent was withheld? Show your work and box your answer.

Post-Test Items

1. What is 25% of 24? Show your work and box your answer.
2. Three friends share two large pizzas. Frank eats $\frac{1}{2}$ of the slices, William eats 0.20 of the slices, and Susan eats the rest of the slices. What percent of the pizza slices did Susan eat? Show your work and box your answer.
3. Randy cuts his neighbor's lawn for extra money. He has been doing such a great job that his neighbor gave him a 15% increase in pay. If Randy's original pay was $45, how much was his increase? Show your work and box your answer.
4. Cindy is on the girl's varsity basketball team. In last week's game the team scored 50 points. If Cindy scored 15 of those points, what percent of the points were made by Cindy? Show your work and box your answer.
5. Susie pays $24,000 in a year on bills. Of these, her biggest yearly bill is rent at $6,000. What percent of he yearly bills is used for rent? Show your work and box your answer.