Optimal Power Allocation and Relay Selection for Cognitive Relay Networks using Non-orthogonal Cooperative Protocol

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Abstract

In this paper, we investigate joint power allocation and relay selection (PARS) schemes in non-orthogonal cooperative protocol (NOCP) based cognitive relay networks. Generally, NOCP outperforms the orthogonal cooperative protocol (OCP), since it can provide more transmit diversity. However, most existing PARS schemes in cognitive relay networks focus on OCP, which are not suitable for NOCP. In the context of NOCP, we first derive the joint constraints of transmit power limit for secondary user (SU) and interference constraint for primary user (PU). Then we formulate optimization problems under the aforementioned constraints to maximize the capacity of SU in amplify-and-forward (AF) and decode-and-forward (DF) modes, respectively. Correspondingly, we derive the closed form solutions with respect to different parameters. Numerical results are provided to verify the performance improvement of the proposed PARS schemes.

Keywords: Cognitive radio, relay networks, non-orthogonal cooperative protocol, power allocation, relay selection, channel capacity

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1. Introduction

Since most spectrum is used in a bursty fashion, cognitive radio (CR) proposed by Mitola [1], aims to improve the spectrum utilization efficiency by allowing secondary users (SUs) to share primary user's (PU's) spectrum as long as SU's transmit power does not exceed the interference tolerance of PU [2]. To use the spectrum efficiently while alleviate the interference to PU, cooperative relaying technique is now considered as a potential means for SU [3].

Various cooperative relaying schemes have been proposed to achieve the benefits such as improvement in link quality and reliability, and increase in coverage [4-8]. Among these schemes, amplify-and-forward (AF) and decode-and-forward (DF) schemes are the two widely used relaying protocols in cooperative networks [4]. Traditionally, orthogonal cooperative protocol (OCP) is mainly adopted in two-hop cooperative networks, where source and relay transmit signals in two adjacent non-overlapping timeslots alternately. Meanwhile, non-orthogonal cooperative protocol (NOCP) has been proposed in [9], where both source and relay transmit signals simultaneously in the second timeslot. It is shown in [9] that NOCP always outperforms OCP, as more transmit diversity gain can be achieved. To fully utilize the efficiency, we consider NOCP based CR relay networks, where a set of cognitive relays assist the secondary transmission while sharing the spectrum with PU.

In CR relay networks, it is essential to improve SU's performance through power allocation and relay selection (PARS). Under traditional OCP, various PARS schemes have been studied to meet different requirements [10-16]. To maximize the system capacity, a joint PARS scheme was investigated in [10] under limited interference constraint for PU. In CR AF relay network with multiple SUs, joint relay assignment and power allocation was further proposed in [11] to maximize the sum capacity of SUs. In addition to the interference constraint, the transmit power for SU is also constrained by transmit power limit in practice. Under the joint consideration of the interference constraint for PU and transmit power limit for SU, a simplified power allocation (PA) scheme was proposed in [12] for CR multi-node relay networks. In [13], the authors studied the PA scheme for CR networks with two SUs in both direct and relay-aided transmission scenarios. Further, to make a tradeoff between the achievable rate and the network lifetime, a distributed PARS scheme was studied in [14] for CR cooperative networks. The authors in [15] proposed the joint PARS schemes under guaranteed primary outage constraint in cognitive DF relay networks. For cognitive two-way communication, an optimal PARS scheme was proposed in [16] where a pair of cognitive transceivers communicate with each other assisted by a set of two-way relays.

To the best of our knowledge, few works have referred to the PARS schemes for NOCP based CR relay networks. Recently, the works in [17] and [18] studied the PA schemes in CR relay networks, where multiple non-orthogonal AF relays transmit over the same frequency band simultaneously. The authors in [19] further considered PA schemes in CR non-orthogonal two-way relay networks. The "non-orthogonal" in the mentioned studies [17-19] represents that multiple relays transmit over the same (non-orthogonal) frequency band. Moreover, the direct link between source and destination is not considered, i.e., source and relays should transmit in different non-overlapping (orthogonal) timeslots. Consequently, the mentioned studies are non-orthogonal in frequency domain, while orthogonal in the time domain. They still belong to the case of OCP protocol in essence. The studies in [20][21] considered NOCP protocol with AF and DF modes respectively. However, they focused on traditional cooperative communications systems, and the context of CR is not involved.

In NOCP based CR relay networks, secondary destination will get signals simultaneously from ST and SR in the second timeslot, while secondary destination will only get data from SR in OCP based CR relay networks. Consequently, the NOCP cooperative protocol shows great superiority as compared to the OCP protocol, for it can provide more transmit diversity. However, in the context of interference limited cognitive scenarios, the interference constraint for PU will undoubtedly be affected. Specifically, the interference for PU is caused by ST and SR simultaneously in the second timeslot. With the alterative interference constraint for PU, the PARS schemes will be different from that in OCP based CR relay networks.

In this paper, we investigate the NOCP based CR relay networks and propose the PARS schemes to maximize the SU's capacity for the AF and DF relay protocols, respectively. The main contributions of this paper are summarized as follows.

(1) We propose and formulate two optimization problems for PARS schemes in both the AF and DF NOCP based CR relay networks, subject to the joint constraint of transmit power limit for SU and alterative interference constraint for PU.

(2) The optimal solution for the PARS scheme in AF NOCP based CR relay networks is proved to be at the edge of the constrained conditions. Subsequently, an optimal solution is provided.

(3) In DF NOCP based CR relay networks, the corresponding optimal solution is achieved according to the channel quality of the direct link and relay-assisted link. SU can adjust its powers with different channel condition adaptively.

(4) The closed form solutions for the two optimization problems are derived according to different parameters such as channel conditions, interference constraint for PU and transmit power limit for SU.

The rest of this paper is organized as follows. In Section 2, we introduce the NOCP based CR relay network, and discuss power constraints for both ST and SR. In Section 3 and Section 4, we study PARS schemes to maximize the SU's capacity in AF and DF protocols, respectively. In Section 5, we present numerical results to illustrate the performance of the proposed PARS schemes. Then we conclude the paper in Section 6. The main notations used in this paper are summarized in **Table 1.**

Table 1. Notations

$P_{ST_i,1}$ and $P_{ST_i,2}$	Transmit power of ST in the first and second timeslot with respect to the *i*-th relay
P_{SR_i}	Transmit power of the *i*-th relay
u_{SR_i} and u_{SD}	Whitening noise caused by PT at the *i*-th relay and SD
n_{SR_i} and n_{SD}	AWGN noise at the *i*-th relay and SD
h_{i-j}	Channel coeffieient between transmitter *i* and receiver *j*
G_i	Normalized gain of the *i*-th relay in AF mode
I	The peak interference that PU can tolerate
$P_{\max}$	The maximum transmit power of ST and SR
$R(i)_{AF}$	The capacity with respect to the *i*-th relay in AF mode
$R(i)_{DF}$	The capacity with respect to the *i*-th relay in DF mode

2. System and Channel Model

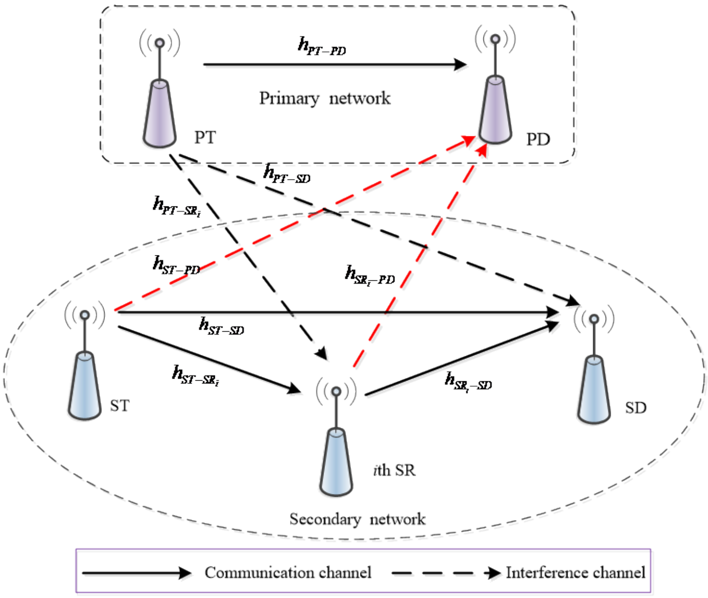

Fig. 1. System Model of NOCP based cognitive relay networks.

As is shown in **Fig. 1**, we consider a cognitive relay network involving a primary system and a secondary system. In the primary system, a primary transmitter (PT) sends data to a primary destination (PD). The secondary system is a cooperative relay system, which consists of a secondary transmitter (ST), a secondary destination (SD) and N secondary relays (SRs). In the system, the secondary transmission involves two timeslots. In the first timeslot, ST broadcasts messages to SRs and SD. While in the second timeslot, both ST and the selected best relay (BR) communicate to SD. Due to the simultanteous transmission of ST and BR in the second timeslot, this protocol is called NOCP protocol as in [9].

In order to guarantee the QoS requirement of the primary transmission, the transmit powers of ST and BR should be constrained to reduce the interference to PD. Assume that SU intends to reuse PU's frequency band to transmit its signal x_S ($E\left(|x_S|^2\right)=1$) to SD. The channel coefficeint between any transmitter $i \in \{ST,BR,PT\}$ and any receiver $j \in \{SD,SR,PD\}$ is denoted as h_{i-j}, which is invariant during the two successive timeslots. All the links are independently and identically distributed (i.i.d.) zero-mean Rayleigh flat fading and can be obtained by pilot aided channel estimation or CSI feedback [22,24]. The thermal noise at receiver n_j is modeled as additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2. The noise-whitening filter [25] is applied at SR and SD, so the interference at SU from PU can be modeled as zero mean AWGN with variance $\sigma_{SR_i}^2$ and σ_{SD}^2, respectively. Thus, the received signals at the *i*-th relay SR_i and SD during the first timeslot can be denoted as

$$y_{SR_i} = \sqrt{P_{ST,1}} h_{ST-SR_i} x_{S,1} + u_{SR_i} + n_{SR_i}, \tag{1}$$

$$y_{SD,1} = \sqrt{P_{ST,1}} h_{ST-SD} x_{S,1} + u_{SD} + n_{SD}, \tag{2}$$

where y_{SR_i} and $y_{SD,1}$ are the received signals at SR_i and SD, $P_{ST,1}$ represents the transmit power of ST, $x_{S,1}$ is the transmitted signal of SU, u_{SR_i} and u_{SD} represent the whitening noise at SR_i and SD caused by PT.

In the second timeslot, the received signals will be forwarded to SD with the help of SRs. In practice, two commonly used protocols for the relay retransmission are: AF and DF.

Case 1. AF mode: If the *i*-th relay is selected to forward the original signal, the received signal is multiplied by the gain of the *i*-th relay with G_i, and then it is retransmitted to SD. Thus, in the second timeslot, the received signal at SD can be expressed as

$$y_{SD,2}^{AF} = G_i \sqrt{P_{SR_i}} h_{SR_i-SD} y_{SR_i} + \sqrt{P_{ST_i,2}} h_{ST-SD} x_{S,2} + u_{SD} + n_{SD}, \tag{3}$$

where $y_{SD,2}^{AF}$ is the received signal of SD, $P_{ST_i,2}$ represents the transmit power of ST corresponding to the *i*-th relay and $x_{S,2}$ is a new signal transmitted by SU in the second timeslot [9][21]. It is clear from (3) that the choice of the relay gain G_i determines the equivalent SINR of SD. One choice for the gain was given in [4] as

$$G_i^2 = 1 \Big/ \left(P_{ST,1} \left| h_{ST-SR_i} \right|^2 + \sigma_{SR_i}^2 + \sigma_n^2 \right) \tag{4}$$

and (3) can now be reformulated as

$$y_{SD,2}^{AF} = \sqrt{P_{ST_i,2}} h_{ST-SD} x_{S,2} + G_i \sqrt{P_{SR_i} P_{ST,1}} h_{ST-SR_i} h_{SR_i-SD} x_{S,1} + \tilde{n} \tag{5}$$

with

$$\tilde{n} = \sqrt{P_{SR_i}} G_i h_{SR_i-SD} \left(u_{SR_i} + n_{SR_i} \right) + u_{SD} + n_{SD} . \tag{6}$$

Let $\sigma_i^2 = \sigma_{SR_i}^2 + \sigma_n^2$, $\upsilon^2 = \sigma_{SD}^2 + \sigma_n^2$, and $\omega_i = \sqrt{P_{SR_i} G_i^2 \left| h_{SR_i-SD} \right|^2 \sigma_i^2 + \upsilon^2}$, the receiver normalizes $y_{SD,1}$ and $y_{SD,2}^{AF}$ by factors υ and ω_i. This normalization does not alter the SINR but simplifies the ensuing presentation. Combining (2) and (3), the effective input-output relation can be summarized as [9]

$$\mathbf{y}_{AFi} = \mathbf{H}_{AFi} \mathbf{x} + \mathbf{n}, \tag{7}$$

where $\mathbf{y}_{AFi} = \left[y_{SD,1} / \upsilon ,\ y_{SD,2}^{AF} / \omega_i \right]^T$ is the received signal vector, $\mathbf{H}_{AFi}$ is the effective 2×2 channel matrix given by

$$\mathbf{H}_{AFi} = \begin{bmatrix} \sqrt{P_{ST,1}} h_{ST-SD} / \upsilon & 0 \\ G_i \sqrt{P_{ST,1} P_{SR_i}} h_{ST-SR_i} h_{SR_i-SD} / \omega_i & \sqrt{P_{ST_i,2}} h_{ST-SD} / \omega_i \end{bmatrix}, \tag{8}$$

$\mathbf{x} = \left[x_{S,1}, x_{S,2} \right]^T$ is the transmitted signal vector, and $\mathbf{n}$ is the circularly symmetric complex Gaussian noise vector with $E[\mathbf{n}] = \mathbf{0}$ and $E\left[\mathbf{n}\mathbf{n}^H\right] = \mathbf{I}_2$.

Case 2. DF mode: For the case of DF mode, the received signal at SD in the second timeslot can be expressed as

$$y_{SD,2}^{DF} = \sqrt{P_{SR_i}} h_{SR_i-SD} x_{S,1} + \sqrt{P_{ST_i,2}} h_{ST-SD} x_{S,2} + u_{SD} + n_{SD}. \tag{9}$$

The input-output relation for DF mode can now be summarized as

$$\mathbf{y}_{DFi}=\mathbf{H}_{DFi}\mathbf{x}+\mathbf{n}, \tag{10}$$

where $\mathbf{y}_{DFi}=\left[y_{SD,1}/\upsilon,\ y_{SD,2}^{DF}/\upsilon\right]^T$ is the received signal vector, $\mathbf{H}_{DFi}$ is the effective 2×2 channel matrix given by [9]

$$\mathbf{H}_{DFi}=\begin{bmatrix}\sqrt{P_{ST,1}}h_{ST-SD}/\upsilon & 0\\ \sqrt{P_{SR_i}}h_{SR_i-SD}/\upsilon & \sqrt{P_{ST_i,2}}h_{ST-SD}/\upsilon\end{bmatrix}. \tag{11}$$

Interference Constraint: In the context of interference limited cognitive scenarios, the transmit power of SU should be approximately controlled to protect PU. Here we take both the interference constraint I for PU and transmit power limit $P_{\max}$ for SU into consideration. With the considered NOCP protocol, ST transmits with power $P_{ST,1}$ in the first timeslot and the interference caused to PD is $P_{ST,1}\left|h_{ST-PD}\right|^2$. While in the second timeslot, ST and SR_i transmit simultaneously with respective powers $P_{ST_i,2}$ and P_{SR_i}, the resulted interference to PD is $P_{ST_i,2}\left|h_{ST-PD}\right|^2+P_{SR_i}\left|h_{SR_i-PD}\right|^2$. Then the transmit powers of ST and SR_i should satisfy

$$P_{ST,1}\left|h_{ST-PD}\right|^2\le I, \tag{12a}$$

$$P_{ST_i,2}\left|h_{ST-PD}\right|^2+P_{SR_i}\left|h_{SR_i-PD}\right|^2\le I, \tag{12b}$$

$$0\le P_{ST,1},P_{ST_i,2},P_{SR_i}\le P_{\max}. \tag{12c}$$

Equal Power Allocation Scheme: According to the constraints (12a) and (12c), the maximum feasible transmit power for ST in the first timeslot is $P_{ST,1}^{Best}=\min\left(I/\left|h_{ST-PD}\right|^2,P_{\max}\right)$. With respect to the constraints (12b), a simple but not optimal way to satisfy the interference constraint without cooperation between ST and SR would be to set the predetermined interference to half of the threshold, i.e., $P_{ST_i,2}\left|h_{ST-PD}\right|^2\le I/2$ and $P_{SR_i}\left|h_{SR_i-PD}\right|^2\le I/2$. Considering (12c), the feasible powers of $P_{ST_i,2}$ and P_{SR_i} are $P_{ST_i,2}=\min\left(P_{\max},\frac{I}{2}\Big/\left|h_{ST-PD}\right|^2\right)$ and $P_{SR_i}=\min\left(P_{\max},\frac{I}{2}\Big/\left|h_{SR_i-PD}\right|^2\right)$. We denote this scheme as the equal power allocation (EPA) on average scheme [26]. The EPA scheme allocates powers individually without the coordination between ST and SR, therefore, it cannot reach an optimal performance.

3. Optimal Power Allocation and Relay Selection Scheme for CR Systems with AF NOCP Protocol

In this section, based on the analysis of the system capacity, the PARS algorithm for CR relay networks with AF NOCP protocol is proposed.

3.1 Problem Statement

We employ an ergodic block-fading channel model and an i.i.d. Gaussian codebook. The capacity of the AF NOCP CR system is

$$R(i)_{AF} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_2 + \mathbf{H}_{AFi}\mathbf{H}_{AFi}^H\right). \tag{13}$$

Substitute (8) into (13) and further (13) can be simplified as

$$R(i)_{AF} = \frac{1}{2}\log_2\left(1+\left(\frac{P_{ST,1}}{\upsilon^2}+\frac{P_{ST_i,2}}{\omega_i^2}\right)\left|h_{ST-SD}\right|^2+\frac{P_{ST,1}P_{ST_i,2}}{\upsilon^2\omega_i^2}\left|h_{ST-SD}\right|^4+\frac{G_i^2P_{ST,1}P_{SR_i}}{\omega_i^2}\left|h_{ST-SR_i}\right|^2\left|h_{SR_i-SD}\right|^2\right). \tag{14}$$

Let

$$f_i\left(P_{ST,1}, P_{ST_i,2}, P_{SR_i}\right)=\frac{G_i^2P_{ST,1}P_{SR_i}}{\omega_i^2}\left|h_{ST-SR_i}\right|^2\left|h_{SR_i-SD}\right|^2+\left(\frac{P_{ST,1}}{\upsilon^2}+\frac{P_{ST_i,2}}{\omega_i^2}\right)\left|h_{ST-SD}\right|^2+\frac{P_{ST,1}P_{ST_i,2}}{\upsilon^2\omega_i^2}\left|h_{ST-SD}\right|^4 \tag{15}$$

The PARS scheme can be formulated with (12a), (12b), (12c), and (15) as

$$\left(P_{ST,1}^{Best}, P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right)=\arg\max_{P_{ST,1},P_{ST_i,2},P_{SR_i}} f_i\left(P_{ST,1}, P_{ST_i,2}, P_{SR_i}\right) \tag{16a}$$

$$BR=\arg\max_i f_i\left(P_{ST,1}^{Best}, P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) \tag{16b}$$

$$\text{s.t.} \qquad P_{ST,1}\left|h_{ST-PD}\right|^2 \le I, \tag{17a}$$

$$P_{ST_i,2}\left|h_{ST-PD}\right|^2+P_{SR_i}\left|h_{SR_i-PD}\right|^2 \le I, \tag{17b}$$

$$0 \le P_{ST,1}, P_{ST_i,2}, P_{SR_i} \le P_{\max}. \tag{17c}$$

3.2 PARS Scheme Realization

As can be seen, (16a) is a linear increasing function with $P_{ST_i,1}$. Take (17a) and (17c) into account, the best power of $P_{ST,1}$ should be $P_{ST,1}^{Best}=\min\left(I/\left|h_{ST-PD}\right|^2, P_{\max}\right)$. For simplicity, let $g_i(P_{ST_i,2}, P_{SR_i})=f_i(P_{ST,1}^{Best}, P_{ST_i,2}, P_{SR_i})-P_{ST,1}^{Best}\left|h_{ST-SD}\right|^2/\upsilon^2$, $G_i^{Best}=1\Big/\sqrt{P_{ST,1}^{Best}\left|h_{ST-SR_i}\right|^2+\sigma_{SR_i}^2+\sigma_n^2}$, $A_{SR_i}=\left(G_i^{Best}\right)^2P_{ST,1}^{Best}\left|h_{ST-SR_i}\right|^2\left|h_{SR_i-SD}\right|^2$, $B_{SR_i}=\left(G_i^{Best}\right)^2\left|h_{SR_i-SD}\right|^2\sigma_i^2$, and $C_{SR_i}=\left|h_{ST-SD}\right|^2\left(1+P_{ST,1}^{Best}\left|h_{ST-SD}\right|^2/\upsilon^2\right)$, in this case, $g_i\left(P_{ST_i,2}, P_{SR_i}\right)=\dfrac{A_{SR_i}P_{SR_i}+C_{SR_i}P_{ST_i,2}}{B_{SR_i}P_{SR_i}+\upsilon^2}$. The PARS problem can be rewritten as

$$\left(P_{ST,2}^{Best}, P_{SR_i}^{Best}\right)=\arg\max_{P_{ST_i,2},P_{SR_i}} g_i\left(P_{ST_i,2}, P_{SR_i}\right) \tag{18a}$$

$$BR=\arg\max_i g_i\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) \tag{18b}$$

$$\text{s.t.} \quad P_{ST_i,2}\left|h_{ST-PD}\right|^2+P_{SR_i}\left|h_{SR_i-PD}\right|^2 \le I, \tag{19a}$$

$$0 \le P_{ST_i,2}, P_{SR_i} \le P_{\max}. \tag{19b}$$

Define the feasible region of the power allocation as

$$\begin{aligned}\Omega &= \left\{ \left(P_{ST_i,2}, P_{SR_i}\right) \middle| P_{ST_i,2}\left|h_{ST-PD}\right|^2 + P_{SR_i}\left|h_{SR_i-PD}\right|^2 \le I, 0 \le P_{ST_i,2}, P_{SR_i} \le P_{\max} \right\} \\ &= \left\{ \left(P_{ST_i,2}, P_{SR_i}\right) \middle| 0 \le P_{ST_i,2} \le \min\left(\frac{I - \left|h_{SR_i-PD}\right|^2 P_{SR_i}}{\left|h_{ST-PD}\right|^2}, P_{\max} \right), 0 \le P_{SR_i} \le \min\left(\frac{I}{\left|h_{SR_i-PD}\right|^2}, P_{\max} \right) \right\}.\end{aligned} \tag{20}$$

The power allocation problem of (18a), (19a) and (19b) can now be simplified as

$$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg \max_{\left(P_{ST_i,2}, P_{SR_i}\right) \in \Omega} g_i\left(P_{ST_i,2}, P_{SR_i}\right). \tag{21}$$

Assume the edge of Ω is Ω_{Sub}, which is defined as

$$\Omega_{Sub} = \left\{ \left(P_{ST_i,2}, P_{SR_i}\right) \middle| P_{ST_i,2} = \min\left(\frac{I - \left|h_{SR_i-PD}\right|^2 P_{SR_i}}{\left|h_{ST-PD}\right|^2}, P_{\max} \right), 0 \le P_{SR_i} \le \min\left(\frac{I}{\left|h_{SR_i-PD}\right|^2}, P_{\max} \right) \right\}. \tag{22}$$

In the ensuing analysis, we use the following lemma, which will reduce the complexity of the power allocation algorithms, the proof of the lemma is shown in Appendix.

Lemma: The optimal power consumption from ST and SR is on the edge of Ω, in this case, the power allocation algorithm can be simplified as

$$\max_{(P_{ST_i,2}, P_{SR_i}) \in \Omega} g_i(P_{ST_i,2}, P_{SR_i}) = \max_{(P_{ST_i,2}, P_{SR_i}) \in \Omega_{Sub}} g_i(P_{ST_i\,2}, P_{SR_i}). \tag{23}$$

Based on this lemma, with different channel conditions, there will be five different cases for PA as shown in **Fig. 2**, where the circles identify the potential location of the optimal PA values of $P_{ST_i,2}$ and P_{SR_i}. The optimal power allocation will be one of the black circles of the five different cases as summarized in **Table 2**.

Once the optimal power allocation is performed, ST can select the relay that maximizes the capacity according to (16b) and then inform the selected relay to assist the transmission of the secondary link. For clarity, the proposed power allocation and relay selection scheme for AF NOCP CR systems is summarized in Algorithm 1.

Algorithm 1: Power allocation and relay selection in AF NOCP CR systems

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Step 1: Initialization
    • $i \leftarrow 0$, $R \leftarrow 0$, $I \leftarrow \varnothing$, where $I$ denotes the index of the selected relay.
    • The transmit power of ST in the first timeslot is calculated as $P_{ST,1}^{Best} = \min\left[ I / \left|h_{ST-PD}\right|^2, P_{\max} \right]$.
Step 2: $i \leftarrow i+1$
    • Calculate $I \big/ \left|h_{ST-PD}\right|^2$, $I \big/ \left|h_{SR_i-PD}\right|^2$ and $I \Big/ \left( \left|h_{ST-PD}\right|^2 + \left|h_{SR_i-PD}\right|^2 \right)$, decide which case belong to as shown in Fig. 2.
    • Calculate the optimal power allocation for the i-th relay as shown in Table 2.
    • Calculate the channel capacity $R_i$ with respect to the i-th relay using (14).
    • Update $I$ and $R$ :if $R_i > R$, then $R \leftarrow R_i$ and $I \leftarrow i$.
Step 3: Repeat Step 2 until $i = N$.
    • Finally, the index of selected relay and achievable maximum channel capacity can be found in $I$ and $R$, respectively.
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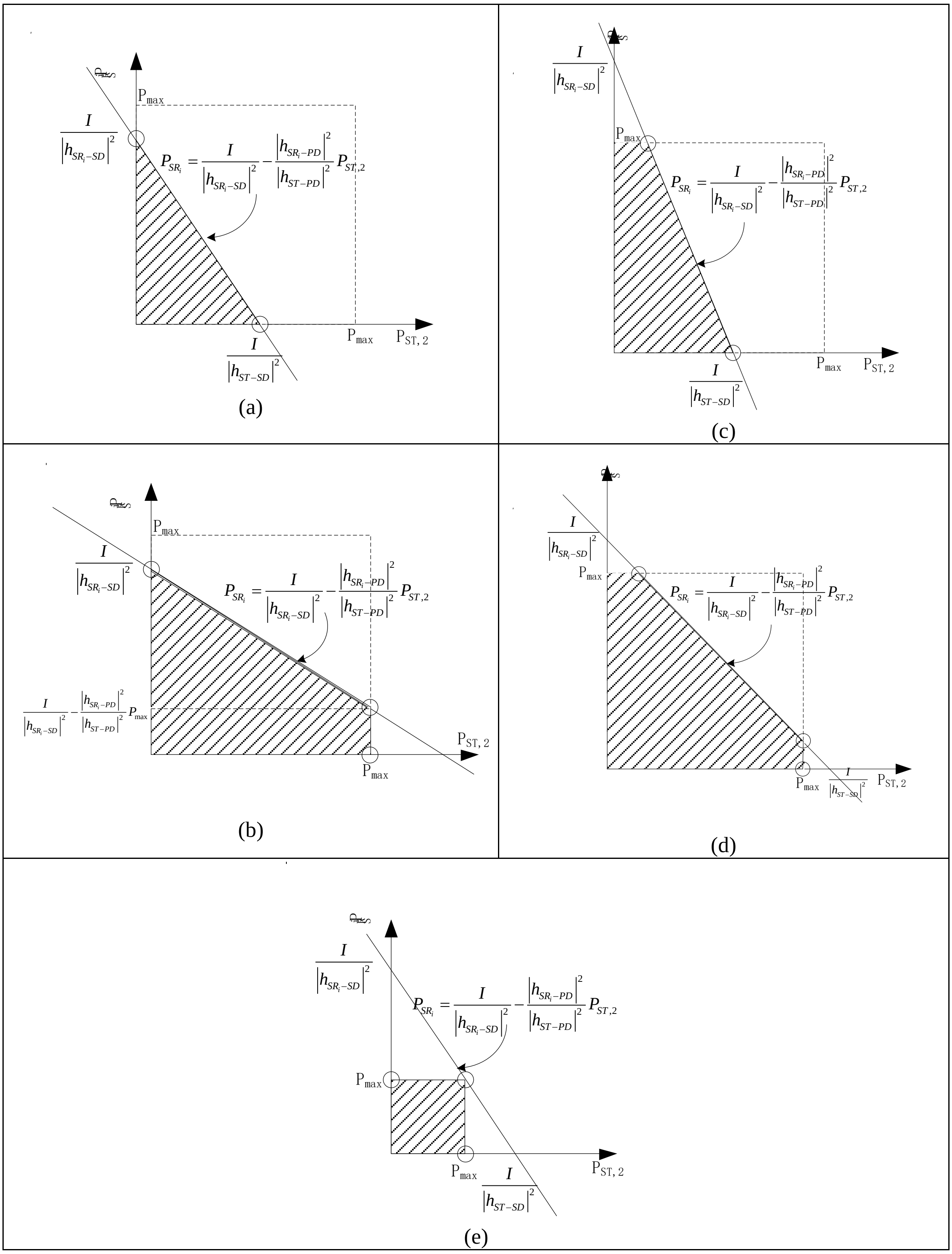

Fig. 2. Optimal power allocation values of $P_{ST_i,2}$ and P_{SR_i} for AF protocol with different channel conditions.

Table 2. Summarization of optimal power allocation of $P_{ST_i,2}$ and P_{SR_i} for AF NOCP CR system with different channel conditions

(a)	$0 < I \big/ \lvert h_{SR_i-PD} \rvert^2 \le P_{\max}$		$0 < I \big/ \lvert h_{ST-PD} \rvert^2 \le P_{\max}$
	$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max\left(g_i(\,0, I \big/ \lvert h_{SR_i-PD} \rvert^2), g_i(\,I \big/ \lvert h_{ST-PD} \rvert^2, 0)\right)$		
(b)	$0 < I \big/ \lvert h_{SR_i-PD} \rvert^2 \le P_{\max}$		$P_{\max} < I \big/ \lvert h_{ST-PD} \rvert^2$
	$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max\left(g_i\left(0, I \big/ \lvert h_{SR_i-PD} \rvert^2\right), g_i\left(P_{\max}, \left(I - \lvert h_{ST-PD} \rvert^2 P_{\max}\right) \big/ \lvert h_{SR_i-PD} \rvert^2\right), g_i\left(P_{\max}, 0\right)\right)$		
(c)	$P_{\max} < I \big/ \lvert h_{SR_i-PD} \rvert^2$		$0 < I \big/ \lvert h_{ST-PD} \rvert^2 \le P_{\max}$
	$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max\left(g_i\left(\left(I - \lvert h_{ST-PD} \rvert^2 P_{\max}\right) \big/ \lvert h_{SR_i-PD} \rvert^2, P_{\max}\right), g_i(\,I \big/ \lvert h_{ST-PD} \rvert^2, 0)\right)$		
(d)	$P_{\max} < I \big/ \lvert h_{SR_i-PD} \rvert^2$	$P_{\max} < I \big/ \lvert h_{ST-PD} \rvert^2$	$I \big/ \left(\lvert h_{SR_i-PD} \rvert^2 + \lvert h_{ST-PD} \rvert^2\right) \le P_{\max}$
	$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max\left(g_i(\frac{I - \lvert h_{SR_i-PD} \rvert^2 P_{\max}}{\lvert h_{ST-PD} \rvert^2}, P_{\max}),\ g_i(\,P_{\max}, \frac{I - \lvert h_{ST-PD} \rvert^2 P_{\max}}{\lvert h_{SR_i-PD} \rvert^2}), g_i(\,P_{\max}, 0)\right)$		
(e)	$P_{\max} < I \big/ \left(\lvert h_{SR_i-PD} \rvert^2 + \lvert h_{ST-PD} \rvert^2\right)$		
	$\left(P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max\left(g_i(P_{\max}, P_{\max}),\ \ g_i(\,P_{\max}, 0)\right)$		

4. Optimal Power Allocation and Relay Selection Scheme for CR systems with DF NOCP Protocol

In this section, we study the PARS algorithm for the CR system with DF NOCP protocol and derive the optimal solution to maximize the system capacity.

4.1 Problem Statement

Define

$$R(i)_{Total} = \frac{1}{2}\log_2 \det\left(\mathbf{I}_2 + \mathbf{H}_{DFi}\mathbf{H}_{DFi}^H\right), \tag{24}$$

$$R(i)_{Relay} = \frac{1}{2}\log_2\left(1 + P_{ST,1}\left|h_{ST-SR_i}\right|^2 \big/ \sigma_i^2\right), \tag{25}$$

$$R(i)_{Direct} = \frac{1}{2}\log_2\left(1 + P_{ST_i,2}\left|h_{ST-SD}\right|^2 \big/ \upsilon^2\right). \tag{26}$$

Combine (11) with (24), we have

$$R(i)_{Total} = \frac{1}{2}\log_2\left(P_{SR_i}\left|h_{SR_i-SD}\right|^2 \big/ \upsilon^2 + \prod_{m=1}^{2}(1 + P_{ST_i,m}\left|h_{ST-SD}\right|^2 \big/ \upsilon^2)\right). \tag{27}$$

The capacity for the DF NOCP CR system [9] is

$$R(i)_{DF} = \begin{cases} R(i)_{Total} & \text{for} \quad R(i)_{Relay} > R(i)_{Total} - R(i)_{Direct} \\ R(i)_{Relay} + R(i)_{Direct} & \text{for} \quad R(i)_{Relay} \le R(i)_{Total} - R(i)_{Direct}. \end{cases} \tag{28}$$

Therefore, the problem of PARS scheme to maximize the capacity can be formulated as

$$\left(P_{ST,1}^{Best}, P_{ST_i,2}^{Best}, P_{SR_i}^{Best}\right) = \arg\max_{P_{ST,1}, P_{ST_i,2}, P_{SR_i}} R(i)_{DF} \tag{29a}$$

$$BR = \arg\max_{i} R(i)_{DF} \tag{29b}$$

$$\text{s.t.} \qquad P_{ST,1}\left|h_{ST-PD}\right|^2 \le I, \tag{30a}$$

$$P_{ST_i,2}\left|h_{ST-PD}\right|^2 + P_{SR_i}\left|h_{SR_i-PD}\right|^2 \le I \ , \tag{30b}$$

$$0 < P_{ST,1}, P_{ST_i,2}, P_{SR_i} \le P_{\max}. \tag{30c}$$

4.2 PARS Scheme Realization

As can be seen, (29a) is an increasing function with $P_{ST,1}$, $P_{ST_i,2}$, and P_{SR_i}. Take (30a) and (30c) into account, the best transmit power of $P_{ST,1}$ should be $P_{ST,1}^{Best} = \min\left(I\big/\left|h_{ST-PD}\right|^2, P_{\max}\right)$. To optimize the system performance, without considering the constraints in (28) and we first aim at power allocation to maximize $R(i)_{Total}$, later the constraints of (28) will be considered and the power allocation will be adjusted to maximize $R(i)_{DF}$.

Define $D_{ST} = 1 + P_{ST,1}^{Best}\left|h_{ST-SD}\right|^2\big/\upsilon^2$, we have

$$f_i\left(P_{ST_i,2}, P_{SR_i}\right) = P_{SR_i}\left|h_{SR_i-SD}\right|^2\big/\upsilon^2 + D_{ST}\left(1 + P_{ST_i,2}\left|h_{ST-SD}\right|^2\big/\upsilon^2\right). \tag{31}$$

Let the optimal power allocation for maximizing $R(i)_{Total}$ be $\hat{P}_{ST_i,2}$ and $\hat{P}_{SR_i}$, then the power allocation problem can be formulated as

$$(\hat{P}_{ST_i,2}, \hat{P}_{SR_i}) = \arg\max_{P_{ST_i,2}, P_{SR_i}} f_i(P_{ST_i,2}, P_{SR_i}) \tag{32}$$

$$\text{s.t.} \qquad \text{(30b) and (30c)}$$

Define $g_i(P_{ST_i,2}, P_{SR_i}) = f_i(P_{ST_i,2}, P_{SR_i}) - \left(1 + P_{ST,1}^{Best}\left|h_{ST-SD}\right|^2\big/\upsilon^2\right)$, the power allocation problem can be further converted to

$$(\hat{P}_{ST_i,2}, \hat{P}_{SR_i}) = \arg\max_{P_{ST_i,2}, P_{SR_i}} g_i(P_{ST_i,2}, P_{SR_i}) \tag{33}$$

$$\text{s.t.} \qquad \text{(30b) and (30c)}$$

Without considering the power limits (30c), to obtain the optimal solution, the constraint (30b) should be satisfied with equality, which can be easily proved by the contradiction. The optimal power allocation for this case are denoted as $\tilde{P}_{ST_i,2}$ and $\tilde{P}_{SR_i}$, which should satisfy

$$\tilde{P}_{SR_i} = \left(I - \tilde{P}_{ST_i,2}\left|h_{ST-PD}\right|^2\right)\Big/\left|h_{SR_i-PD}\right|^2. \tag{34}$$

Thus, by substituting (34) into (33), $g_i\left(\tilde{P}_{ST_i,2}, \tilde{P}_{SR_i}\right)$ can be further expressed as

$$g_i(\tilde{P}_{ST_i,2}, \tilde{P}_{SR_i}) = \frac{I}{\upsilon^2}\frac{\left|h_{SR_i-SD}\right|^2}{\left|h_{SR_i-PD}\right|^2} - \frac{\tilde{P}_{ST_i,2}}{\upsilon^2}\left(\frac{\left|h_{SR_i-SD}\right|^2\left|h_{ST-PD}\right|^2}{\left|h_{SR_i-PD}\right|^2} - D_{ST}\left|h_{ST-SD}\right|^2\right). \tag{35}$$

Define $\Im = \frac{\left|h_{SR_i-SD}\right|^2 \left|h_{ST-PD}\right|^2}{\left|h_{SR_i-PD}\right|^2} - \left(1+\frac{P_{ST,1}^{Best}}{\upsilon^2}\left|h_{ST-SD}\right|^2\right)\left|h_{ST-SD}\right|^2$, for the case $\Im \geq 0$, $\tilde{P}_{ST_i,2}$ should be set as zero and all the power will be allocated to SR_i, and when $\Im < 0$, $\tilde{P}_{ST_i,2}$ will be maximized while $\tilde{P}_{SR_i}$ is set as zero. The power allocation scheme is summarized as

$$(\tilde{P}_{ST_i,2}, \tilde{P}_{SR_i}) = \begin{cases} \left(0, I\big/\left|h_{SR_i-PD}\right|^2\right) & \text{for} \quad \Im \geq 0, \\ \left(I\big/\left|h_{ST-PD}\right|^2, 0\right) & \text{for} \quad \Im < 0. \end{cases} \tag{36}$$

When the peak power constraints in (30c) are considered, the optimal power allocation to maximize $R(i)_{Total}$ should be further adjusted. For the case $\Im \geq 0$ and $I\big/\left|h_{SR_i-PD}\right|^2 < P_{\max}$, the power allocation is the same as (36). However, when $\Im \geq 0$ and $I\big/\left|h_{SR_i-PD}\right|^2 \geq P_{\max}$, the feasible way is to decrease the transmit power at SR_i and let it be $P_{\max}$. Taking into consideration that $g_i\left(P_{ST_i,2}, P_{SR_i}\right)$ is an increasing function of $P_{ST_i,2}$, and according to the constraints of (30b) and (30c), the optimal power allocation of $P_{ST_i,2}$ should be adjusted as $\hat{P}_{ST_i,2} = \min\left(\left(I - P_{\max}\left|h_{SR_i-PD}\right|^2\right)\big/\left|h_{ST-PD}\right|^2, P_{\max}\right)$. When $\Im < 0$, from a reasoning similar analysis with the case $\Im \geq 0$, the optimal power allocation for ST and SR_i are adjusted. Integrated from the above cases, the optimal solutions maximizing $R(i)_{Total}$ can be obtained as

$$(\hat{P}_{ST_i,2}, \hat{P}_{SR_i}) = \begin{cases} \left(0, I\big/\left|h_{SR_i-PD}\right|^2\right) \quad \text{for} \quad \Im \geq 0 \text{ and } I\big/\left|h_{SR_i-PD}\right|^2 < P_{\max} \\ \left(\min(\left(I - P_{\max}\left|h_{SR_i-PD}\right|^2\right)\big/\left|h_{ST-PD}\right|^2, P_{\max}), P_{\max}\right) \quad \text{for} \quad \Im \geq 0 \text{ and } I\big/\left|h_{SR_i-PD}\right|^2 \geq P_{\max} \\ \left(I\big/\left|h_{ST-PD}\right|^2, 0\right) \quad \text{for} \quad \Im < 0 \text{ and } I\big/\left|h_{ST-PD}\right|^2 < P_{\max} \\ \left(P_{\max}, \min(\left(I - P_{\max}\left|h_{ST-PD}\right|^2\right)\big/\left|h_{SR_i-PD}\right|^2, P_{\max})\right) \quad \text{for} \quad \Im < 0 \text{ and } I\big/\left|h_{ST-PD}\right|^2 \geq P_{\max} \end{cases} \tag{37}$$

Based on the above analysis, the optimal power allocation that can maximize $R(i)_{Total}$ is solved. However, only when the constraint $R(i)_{Relay} > R(i)_{Total} - R(i)_{Direct}$ is satisfied, $R(i)_{DF} = R(i)_{Total}$ and (37) is the optimal power allocation for DF NOCP CR systems. Once this constraint is not satisfied, the PA scheme needs to be revised to maximize $R(i)_{DF}$.

For the case $R(i)_{Relay} \leq R(i)_{Total} - R(i)_{Direct}$, we have $R(i)_{DF} = R(i)_{Relay} + R(i)_{Direct}$. Since $P_{ST,1}^{Best}$ has already been achieved as $P_{ST,1}^{Best} = \min\left(I\big/\left|h_{ST-PD}\right|^2, P_{\max}\right)$, which directly decides the value of $R(i)_{Relay}$, the power allocation should be adjusted to increase $P_{ST_i,2}$, further increase the value of $R(i)_{Direct}$ and also the value of $R(i)_{DF}$. In all, three cases are analyzed as follows.

Case 1: when $\Im \geq 0$ and $I \Big/ \left|h_{SR_i-PD}\right|^2 < P_{\max}$, the best power allocation that maximizes $R(i)_{Total}$ is $\left(\hat{P}_{ST_i,2}, \hat{P}_{SR_i} \right) = \left(0, I \Big/ \left|h_{SR_i-PD}\right|^2\right)$. In this case, $R(i)_{Relay} > R(i)_{Total} - R(i)_{Direct}$ equals $P_{ST,1}^{Best}\left(\frac{\left|h_{ST-SR_i}\right|^2}{\sigma_i^2} - \frac{\left|h_{ST-SD}\right|^2}{\upsilon^2}\right) > \frac{I\left|h_{SR_i-SD}\right|^2}{\upsilon^2\left|h_{SR_i-PD}\right|^2}$, and the power allocation scheme maximizing $R(i)_{Total}$ will be the best to maximize the capacity $R(i)_{DF}$.

However, if $R(i)_{Relay} \leq R(i)_{Total} - R(i)_{Direct}$ that is $P_{ST,1}^{Best}\left(\frac{\left|h_{ST-SR_i}\right|^2}{\sigma_i^2} - \frac{\left|h_{ST-SD}\right|^2}{\upsilon^2}\right) \leq \frac{I\left|h_{SR_i-SD}\right|^2}{\upsilon^2\left|h_{SR_i-PD}\right|^2}$, the optimal power allocation needs to be adjusted. We first neglect the power constraint of (30b), and it is easy to prove by contradiction that there will be an equilibrium point satisfying

$$\begin{cases} R(i)_{DF} = R(i)_{Relay} + R(i)_{Direct} \\ P_{ST_i,2}^{*}\left|h_{ST-PD}\right|^2 + P_{SR_i}^{*}\left|h_{SR_i-PD}\right|^2 = I. \end{cases} \tag{38}$$

Therefore, the equilibrium point $(P_{ST_i,2}^{*}, P_{SR_i}^{*})$ can be calculated as

$$\begin{cases} P_{ST_i,2}^{*} = \dfrac{I\left|h_{SR_i-SD}\right|^2 + P_{ST,1}^{Best}\left|h_{SR_i-PD}\right|^2\left(\left|h_{ST-SD}\right|^2 - \frac{v^2}{\sigma_i^2}\left|h_{ST-SR_i}\right|^2\right)}{\left|h_{ST-PD}\right|^2\left|h_{SR_i-SD}\right|^2 - \frac{P_{ST,1}^{Best}}{v^2}\left|h_{ST-SD}\right|^2\left|h_{SR_i-PD}\right|^2\left(\left|h_{ST-SD}\right|^2 - \frac{v^2}{\sigma_i^2}\left|h_{ST-SR_i}\right|^2\right)} \\ P_{SR_i}^{*} = I \Big/ \left|h_{SR_i-PD}\right|^2 - P_{ST_i,2}^{*}\left|h_{ST-PD}\right|^2 \Big/ \left|h_{SR_i-PD}\right|^2. \end{cases} \tag{39}$$

Take into consideration the maximum power limit constraints of (30c), the best power allocation with $P_{ST,1}^{Best}\left(\left|h_{ST-SR_i}\right|^2 \big/ \sigma_i^2 - \left|h_{ST-SD}\right|^2 \big/ \upsilon^2\right) \leq I\left|h_{SR_i-SD}\right|^2 \Big/ \left(\upsilon^2\left|h_{SR_i-PD}\right|^2\right)$ is given as

$$\left(P_{ST_i,2}^{\text{Best}}, P_{SR_i}^{Best} \right) = \left(\min(P_{ST_i,2}^{*}, I \big/ \left|h_{ST-PD}\right|^2, P_{\max}), \max(P_{SR_i}^{*}, 0)\right). \tag{40}$$

Case 2: when $\Im \geq 0$ and $I \Big/ \left|h_{SR_i-PD}\right|^2 \geq P_{\max}$: if $\left(I - P_{\max}\left|h_{SR_i-PD}\right|^2\right) \Big/ \left|h_{ST-PD}\right|^2 \geq P_{\max}$, the best power allocation is $\left(P_{ST_i,2}^{\text{Best}}, P_{SR_i}^{Best} \right) = \left(P_{\max}, P_{\max}\right)$ as $P_{ST_i,2}^{\text{Best}}$ is already the largest. If $\left(I - P_{\max}\left|h_{SR_i-PD}\right|^2\right) \Big/ \left|h_{ST-PD}\right|^2 < P_{\max}$, there exist two different possible situations.

- if $R(i)_{\text{Relay}} > R(i)_{Total} - R(i)_{Direct}$,

  i.e. $\frac{P_{\max}\left|h_{ST-PD}\right|^2\left|h_{SR_i-SD}\right|^2}{v^2\left|h_{ST-PD}\right|^2 + I\left|h_{ST-SD}\right|^2 - P_{\max}\left|h_{ST-SD}\right|^2\left|h_{SR_i-PD}\right|^2} < P_{ST,1}^{Best}\left(\frac{\left|h_{ST-SR_i}\right|^2}{\sigma_i^2} - \frac{\left|h_{ST-SD}\right|^2}{\upsilon^2}\right)$, the best power allocation should be $\left(P_{ST_i,2}^{\text{Best}}, P_{SR_i}^{Best} \right) = \left(\left(I - P_{\max}\left|h_{SR_i-PD}\right|^2\right) \Big/ \left|h_{ST-PD}\right|^2, P_{\max}\right)$.
- if $R(i)_{Relay} \leq R(i)_{Total} - R(i)_{Direct}$, similar to Case 1, the best power allocation is

$$\left(P_{ST_i,2}^{\text{Best}}, P_{SR_i}^{Best} \right) = \left(\min(P_{ST_i,2}^{*}, I \big/ \left|h_{ST-PD}\right|^2, P_{\max}), \min(\max\left(P_{SR_i}^{*}, 0\right), P_{\max})\right). \tag{41}$$

Case 3: when $\Im < 0$, no matter $I/|h_{ST-PD}|^2 < P_{\max}$ or $I/|h_{ST-PD}|^2 \geq P_{\max}$, $\hat{P}_{ST_i,2}$ will always be set to the largest transmit power allowed by the system, just as shown in (37). In this case, the best power allocation should be the same as (37) and no power allocation adjust is needed.

For the simplicity of analysis, let $P^{Best}_{ST,1}\left(\frac{|h_{ST-SR_i}|^2}{\sigma_i^2}-\frac{|h_{ST-SD}|^2}{\upsilon^2}\right)\frac{\upsilon^2|h_{SR_i-PD}|^2}{|h_{SR_i-SD}|^2}=\Re$ and

$$\frac{P^{Best}_{ST,1}\left(\frac{|h_{ST-SR_i}|^2}{\sigma_i^2}-\frac{|h_{ST-SD}|^2}{\upsilon^2}\right)\left(\nu^2|h_{ST-PD}|^2+I|h_{ST-SD}|^2-P_{\max}|h_{ST-SD}|^2|h_{SR_i-PD}|^2\right)}{|h_{ST-PD}|^2|h_{SR_i-SD}|^2}=\aleph$$

, integrated from the above analysis, the optimal solutions for the original problem (29a) with constraints (30a), (30b), and (30c) can be obtained as shown in **Table 3**. Once the optimal power allocation is performed, ST can select the relay that maximizes the capacity according to (29b) and then inform the selected relay to assist the transmission of the secondary link.

For clarity, the proposed power allocation and relay selection scheme for DF NOCP CR systems is summarized in Algorithm 2.

Table 3. Summarization of optimal power allocation of $P_{ST_i,2}$ and P_{SR_i} for DF NOCP CR system with different channel conditions

(a)	$\Im \geq 0$	$I < \min(P_{\max}|h_{SR_i-PD}|^2, \Re)$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(0, I/|h_{SR_i-PD}|^2\right)$	
(b)	$\Im \geq 0$	$\Re \leq I < P_{\max}|h_{SR_i-PD}|^2$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(\min(P^*_{ST_i,2}, I/|h_{ST-PD}|^2, P_{\max}), \max(P^*_{SR_i}, 0)\right)$	
(c)	$\Im \geq 0$	$\min\left(I/|h_{SR_i-PD}|^2, \left(I-P_{\max}|h_{SR_i-PD}|^2\right)/|h_{ST-PD}|^2\right) \geq P_{\max}$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(P_{\max}, P_{\max}\right)$	
(d)	$\Im \geq 0$	$\min\left(I/|h_{SR_i-PD}|^2, \aleph\right) \geq P_{\max} > \left(I-P_{\max}|h_{SR_i-PD}|^2\right)/|h_{ST-PD}|^2$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(\left(I-P_{\max}|h_{SR_i-PD}|^2\right)/|h_{ST-PD}|^2, P_{\max}\right)$	
(e)	$\Im \geq 0$	$I/|h_{SR_i-PD}|^2 > P_{\max} > \max\left(\left(I-P_{\max}|h_{SR_i-PD}|^2\right)/|h_{ST-PD}|^2, \aleph\right)$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(\min\left(P^*_{ST_i,2}, I/|h_{ST-PD}|^2, P_{\max}\right), \min\left(\max\left(P^*_{SR_i}, 0\right), P_{\max}\right)\right)$	
(f)	$\Im < 0$	$I < P_{\max}|h_{ST-PD}|^2$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(I/|h_{ST-PD}|^2, 0\right)$	
(g)	$\Im < 0$	$I \geq P_{\max}|h_{ST-PD}|^2$
	$\left(P^{Best}_{ST_i,2}, P^{Best}_{SR_i}\right)=\left(P_{\max}, \min\left(\left(I-P_{\max}|h_{ST-PD}|^2\right)/|h_{SR_i-PD}|^2, P_{\max}\right)\right)$	

Algorithm 2: Power allocation and relay selection in DF NOCP CR systems

```
Step 1: Initialization
    • $i \leftarrow 0$, $R \leftarrow 0$, $I \leftarrow \varnothing$, where $I$ denotes the index of the selected relay.
    • The transmit power of ST in the first timeslot is calculated as $P_{ST,1}^{Best} = \min\left[ I / \left| h_{ST-PD} \right|^2, P_{\max} \right]$.
Step 2: $i \leftarrow i+1$
    • Calculate $\left| h_{SR_i-PD} \right|^2$, $\aleph$, $\Re$ and $\Im$, decide which case belong to as shown in Table 3.
    • Calculate the optimal power allocation for the i-th relay as shown in Table 3.
    • Calculate the channel capacity $R(i)_{DF}$ with respect to the i-th relay using (28).
    • Update $I$ and $R$ :if $R(i)_{DF} > R$, then $R \leftarrow R(i)_{DF}$ and $I \leftarrow i$.
Step 3: Repeat Step 2 until $i = N$.
    • Finally, the index of selected relay and achievable maximum channel capacity can be found in
      $I$ and $R$, respectively.
```

5. Simulation Results

In this section, we evaluate the performance of the proposed PARS schemes through Monte-Carlo simulations for AF and DF NOCP CR systems, respectively, and compare them with the schemes using equal power allocation (EPA) with relay selection for NOCP CR systems, and the scheme using PA in [23] with relay selection for OCP CR systems. It is noted that the proposed schemes and the scheme in [23] can be performed in either a centralized or a distributed manner as in [27]. All of these schemes need to obtain the channel conditions of the interference link from SU to PU and the transmission link of SU by pilot aided channel estimation or CSI feedback. Consequently, they have almost the same implementation complexity in terms of acquiring channel conditions. However, once all the channel conditions are obtained, the proposed schemes have slightly more computational complexity than the scheme in [23], this is because that the proposed schemes need to trade off the powers between ST and SR in the second timeslot while the scheme in [23] doesn't. We will show that the proposed schemes can provide much superior performance than the scheme in [23], which is verified by simulation results later. We assume that the channel coefficients are i.i.d. and follow Rayleigh distribution. The following parameters are used throughout this section: $\sigma_n^2 = -20$ dBW and $\sigma_{SR_i}^2 = \sigma_{SD}^2 = -18$ dBW.

Fig. 3 describes SU capacity versus SU transmit power limit $P_{\max}$ with $N = 4$ relays and $I = -18$ dBW. As the figure shows, with the increase of $P_{\max}$, the proposed PARS schemes achieve better capacity performance as compared to the traditional schemes for the NOCP and OCP systems in both the DF and AF relaying modes. As $P_{\max}$ increases, the performance difference among the three different schemes becomes obvious. However, with the increase of $P_{\max}$, the performance improvement of the secondary system firstly increases, and after a certain level of $P_{\max}$, the performance improvement slows down. This phenomenon can be explained as follow: when $P_{\max}$ is in small region, the transmit power of SU will be dominantly decided by $P_{\max}$, and will become larger with the increase of $P_{\max}$ on condition that the interference constraint of PU is satisfied. However, with the continuous increase of $P_{\max}$, the interference constraint of PU becomes the dominant factor to determine SU transmit power, which will scarcely achieve to $P_{\max}$.

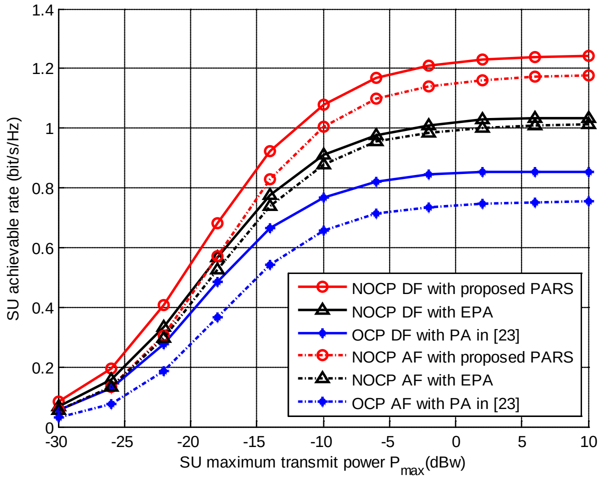

Fig. 3. SU capacity versus SU transmit power limit $P_{\max}$

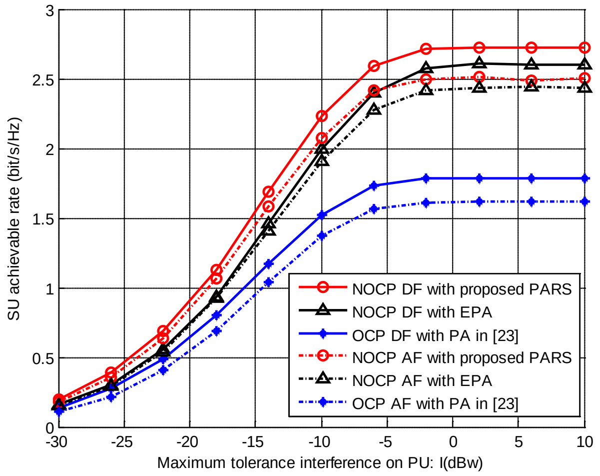

Fig. 4. SU capacity versus PU maximize tolerance interference I

Fig. 4 shows SU capacity versus the PU maximum interference tolerance level I with $N=4$ relays and $P_{\max}=-8$ dBW. It can be seen from **Fig. 4**, with the increase of I, the system performance is improved for the reason that more transmit power is allowed for SU. However, the difference among the three different schemes is neglectable when I is in low regions, especially for the NOCP protocol system. With the increase of I, the performance difference becomes obvious. The capacity ceiling is achieved when I is in high regions, which is due to the fact that when I is large, the SU transmit power is dominantly decided by the SU maximum power, which is a constant and do not vary with PU interference tolerance level.

In **Fig. 5**, we compare capacity obtained by SU system under different schemes with increasing number of relays and keeping interference threshold as $I=-18$ dBW, and the maximum transmit power $P_{\max}=-8$ dBW. As the result shows, the performance of the proposed schemes outperforms the others. It should be noted in **Fig. 5** that the capacity increases linearly with the increase of the relays number, as more spatial diversity is gained. Also it can be observed from the result that with relay selection, the performance of the system with DF protocol improved significantly as compared with the AF protocol system.

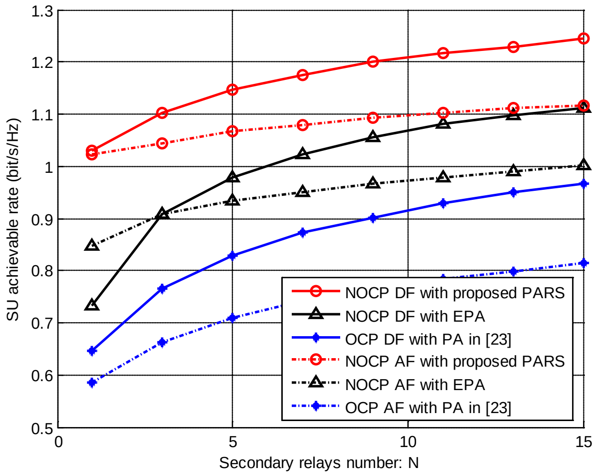

Fig. 5. SU capacity versus secondary relays number N

6. Conclusions

In this paper, the optimal power allocation and relay selection schemes are studied to improve the channel capacity of SU for non-orthogonal cooperative protocol based CR relay networks with both the AF and DF protocols respectively, subject to interference constraint for PU and transmit power limit for SU. In AF non-orthogonal cooperative protocol based CR relay networks, the optimal solution is proved to be at the edge of the constrained conditions and an intuitive solution is provided. In DF non-orthogonal cooperative protocol based CR relay networks, the corresponding optimal solution is achieved according to the channel quality of the direct link and relay-assisted link. Finally, the closed form expressions for the power allocation and relay selection schemes are derived with respect to different parameters. The performance of the proposed schemes was illustrated for different operating conditions and shown to outperform the other schemes.

Appendix

We give the proof of the Lemma in this appendix.

Proof: For $\forall\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega$,

$$\partial g_i\left(P_{ST_i,2}, P_{SR_i}\right)\Big/\partial P_{ST_i,2} = C\Big/\left(B_{SR_i}P_{SR_i}+\upsilon^2\right)>0 \tag{42}$$

always holds.

Define the feasible region of P_{SR_i} as $\Omega_{P_{SR_i}} = \left\{P_{SR_i}\,\middle|\,0\le P_{SR_i}\le \min\left(I\Big/\left|h_{SR_i-PD}\right|^2, P_{\max}\right)\right\}$, we have $P_{SR_i}\in\Omega_{P_{SR_i}}$, $g_i\left(P_{ST_i,2}, P_{SR_i}\right)$ increases progressively with $P_{ST_i,2}$. In this case, for $\forall\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega$, the equation

$$g_i(P_{ST_i,2}, P_{SR_i}) \le g_i\left(\min\left((I - \left|h_{SR_i-PD}\right|^2 P_{SR_i}) \Big/ \left|h_{ST-PD}\right|^2, P_{\max} \right), P_{SR_i} \right) \tag{43}$$

is always satisfied, and further we can get

$$\max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \le \max_{P_{SR_i}\in\Omega_{P_{SR_i}}} g_i\left(\min\left((I - \left|h_{SR_i-PD}\right|^2 P_{SR_i}) \Big/ \left|h_{ST-PD}\right|^2, P_{\max} \right), P_{SR_i} \right) \tag{44}$$

As it is easy to get

$$\max_{P_{SR_i}\in\Omega_{P_{SR_i}}} g_i\left(\min\left((I - \left|h_{SR_i-PD}\right|^2 P_{SR_i}) \Big/ \left|h_{ST-PD}\right|^2, P_{\max} \right), P_{SR_i} \right) = \max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega_{Sub}} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \tag{45}$$

Therefore

$$\max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \le \max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega_{Sub}} g_i\left(P_{ST_i,2}, P_{SR_i} \right). \tag{46}$$

Since $\Omega_{Sub} \subset \Omega$, the equation

$$\max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega_{Sub}} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \le \max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega_{Sub}} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \tag{47}$$

comes into existence forever. Take into consider both (46) and (47), we can get the conclusion

$$\max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega} g_i\left(P_{ST_i,2}, P_{SR_i} \right) = \max_{\left(P_{ST_i,2}, P_{SR_i}\right)\in\Omega_{Sub}} g_i\left(P_{ST_i,2}, P_{SR_i} \right) \tag{48}$$

directly. This completes the proof.

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