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Uncertain Centralized/Decentralized Production-Distribution Planning Problem in Multi-Product Supply Chains: Fuzzy Mathematical Optimization Approaches

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ABSTRACT

Complex and uncertain issues in supply chain result in integrated decision making processes in supply chains. So decentralized (distributed) decision making (DDM) approach is considered as a crucial stage in supply chain planning. In this paper, an uncertain DDM through coordination mechanism is addressed for a multi-product supply chain planning problem. The main concern of this study is comparison of DDM approach with centralized decision making (CDM) approach while some parameters of decision making are assumed to be uncertain. The uncertain DDM problem is modeled through fuzzy mathematical programming in which products' demands are assumed to be uncertain and modeled using fuzzy sets. Moreover, a CDM approach is customized and developed in presence of fuzzy parameters. Both approaches are solved using three fuzzy mathematical optimization methods. Hence, the contribution of this paper can be summarized as follows: 1) proposing a DDM approach for a multi-product supply chain planning problem; 2) Introducing a coordination mechanism in the proposed DDM approach in order to utilize the benefits of a CDM approach while using DDM approach; 3) Modeling the aforementioned problem through fuzzy mathematical programming; 4) Comparing the performance of proposed DDM and a customized uncertain CDM approach on multi-product supply chain planning; 5) Applying three fuzzy mathematical optimization methods in order to address and compare the performance of both DDM and CDM approaches. The results of these fuzzy optimization methods are compared. Computational results illustrate that the proposed DDM approach closely approximates the optimal solutions generated by the CDM approach while the manufacturer's and retailers' decisions are optimized through a coordination mechanism making lasting relationship.
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Keywords: Supply Chain Management, Distributed Decision Making (DDM), Coordination Mechanism in Supply Chain, Fuzzy Mathematical Programming, Fuzzy Optimization

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1. INTRODUCTION

Supply Chain Management is an important issue in today's competitive business world. Companies need to have firm relationships and interactions with their suppliers for a successful Supply chain system (Meredith, 2007). The force of information technology can help supply chain members to establish partnerships for better supply chain system efficiency. However, the complete information sharing between the manufacturer and the third party logistic provider is not possible due to information privacy and thus the existing research results cannot be directly used for real supply chain planning problem (Stadtler, 2009). Distributed Decision Making (DDM) is a discipline of decision theory in which decision making power is distributed among several decision making units (Schneeweiss, 2003). These decisions are interrelated because one decision affects the outcome of another. It has been quite acknowledged that, whereas centralized approaches are theoretically better in pursuing global system performance, they have several drawbacks concerning operational costs, reliability, inventory costs and so forth (Ertogral, 2000). This is the reason why several researchers offer to use decentralized approaches for distributed production planning.

In real life supply chain planning problems several uncertainties occur. For instance the demand of each level of supply chain usually is mixed with a notable amount of uncertainty. In such situation CDM and DDM approaches may have different performances. Generally, CDM approach makes the process of decision making simple, although the operational cost of the supply chain dramatically will increase due to the course that all decision should be processed and made in a central unit. Moreover, high integration including both data, material, goods, and finance, should be implemented while using CDM approaches. The reliability of whole supply chain is involved in the reliability of central decision making unit. Whenever the central unit fails all units of supply chain will fail as they cannot make an independent decision. In such situation a DDM approach which empowers all levels of chain in order to make the related decisions while the workload of central unit is reduced may be a suitable alternative. The inventory, supply, purchase, production, transportation, and distribution are made by the associated units in supply chain while the central unit just co-ordinate, analyze, and ease these decisions. If one level of supply chain fails the performance of the chain will decrease although some parts may take the associated decisions.

Hence, the purpose of this study is to analyze and compare the performance of two classes of supply chain optimization approaches, i.e., CDM and DD Mapproaches, in presence of uncertainty using different types of fuzzy optimization methods. On the other hand, the main question of this study is: what are the profit and the optimal policy of centralized optimization in comparison with decentralize optimization in presence of uncertainty? Centralize and decentralized models proposed by Hegeman *et al*. (2014) are developed in presence of uncertainty modeled through fuzzy sets and are investigated using three fuzzy mathematical programming optimization approaches.

The paper is arranged as follows. Section 2 presents a brief literature review about centralized/decentralized models. Uncertain mathematical programming approaches in the supply chain as also reviewed in Section 2. Section 3 presents the proposed modeling in both deterministic and uncertain situations. The coordination mechanism is also presented in Section 3.The fuzzy solution approaches are developed in Section 4. The numerical results are presented and discussed in Section 5.The conclusion remarks and future research directions are presented in Section 6.

The contribution of this paper can be summarized as follows: 1) proposing a DDM through coordination mechanism for a multi-product supply chain planning problem; 2) Modeling the aforementioned problem through fuzzy mathematical programming; 3) Comparing the performance of proposed DDM and a customized uncertain CDM approach on multi-product supply chain planning; 4) Applying three fuzzy mathematical optimization methods in order to address and handle both DDM and CDM approaches.

2. LITERATURE OF PAST WORKS

Decentralized SCM coordination mechanisms usually follow one of three approaches including, inventory control, quantity discounts, and contracting (Schneeweiss, 2004). Most of the research in this area is based on the classic work proposed by Clark and Scarf (1960) in which the optimal inventory policies in two-echelon systems was discussed. Cachon (2001) first introduced the cooperation concept between the different agents in the supply chain using game theory concepts. The first DDM system analyzed by Cao and Chen (2006) was a decentralized facility location problem. They changed a decentralized two level nonlinear programming model into an equivalent linear single level model. Uncertainty plays an important role in supply chain management context. Three basic approaches, including: (1) fuzzy programming, (2) stochastic programming and (3) robust programming are used to cope with uncertainty. Uncertainty is usually considered in the model parameters including: demands, transportation costs, handling costs and so on. In SCM, the companies are not considered as independent entities, but interacting entities which need to coordinate and integrate their process along the SC. Thus, the uncertainties related to external and internal processes constitute a challenge for the coordinated and integrated processes of SCM (Nishi, 2007).

Jung *et al*. (2008) developed a decentralized supply chain planning framework based on minimal-information sharing between the manufacturer and a third party logistics provider. Each one used its own model and kept private information. The coordination mechanism certified local solutions, converged towards a feasible solution, although the levels did not cooperate as a team. Each level strived for local optimization. However, opportunistic behavior was not demonstrated as the information they exchanged was truthful. While the different levels in the proposed model by Jung *et al*. (2008) had to wait for input from the other level before proceeding to search for their new local optimum. Pibernik and Sucky (2006) pointed out that centralized decision making achieved better results than decentralized decision making. Although, they argued that there were two major drawbacks with implementing the centralized option, i.e., the necessary alignment of individual decisions to SC-wide objectives and SC-wide information sharing. On the other hand, decentralized systems tend to be more robust to failure than centralized systems. Centralized decision making is usually favored when the industry faces a complex yet static problem. Therefore, the adoption of centralized or distributed decision making at a specific temporal level will strongly depend on the SC and the problem under study.

Uncertainty in SCM optimization problems is typically incorporated into mathematical programming models. In some cases, uncertainty exists not due to randomness but fuzziness where doubt arises about the correctness of statements, exactness of concepts and judgments having little to do with occurrence of events (Luhandjula, 2007). This type of uncertainty is handled using fuzzy set theory which was developed by Zadeh (1965). Vahdani *et al*. (2012) defined a novel approach for designing a reliable network of production in closed loop supply chain under uncertainty. For this purpose, a centralized mathematical programming formulation was developed which minimized the total transport costs of a logistics network. To solve the model, a new hybrid solution was introduced by combining robust optimization approach and fuzzy multi objective programming that implemented using GAMS software.

Khalili-Damghani and Shahrokh (2014) proposed a multi-period multi-objective multi-product aggregate production planning problem. Three objective functions, including minimizing total cost, maximizing customer services level, and maximizing the quality of end product, were considered, simultaneously. Several constraints were also considered by Khalili-Damghani and Shahrokh (2014). The proposed problem was solved using Fuzzy Goal Programming (FGP) approach (Khalili-Damghani and Shahrokh, 2014). Min (2015) proposed a supply chain consisting of a manufacturer under emissions regulation and a permit supplier. Min (2015) developed a joint production quantity and investment strategy in order to reduce permit production cost decisions for centralized and decentralized supply chains. Min (2015) found analytically that the proposed cost-sharing contract with reasonable parameters could coordinate the supply chain whereas the wholesale price contract was not desirable to achieve the system-wide profit.

Zanjani *et al*. (2010) addressed a multi-echelon production planning problem based on non-homogeneous quality of materials. To solve the model robust optimization approach was applied. The implementation results of the proposed centralized model for a sawmill factory illustrated the importance of using robust optimization in generating more robust production plans in the uncertain environments compared with stochastic programming. Lin and Wang (2011) studied an integrated configuration Supply Chain network design problem under demand and supply uncertainty. They emphasized the strategic locating and capacity setting costs. Finally an L-shaped decomposition in the master problem was proposed for solving the model. Peidro *et al*. (2009) introduced a unique centralized/decentralized planning model considering production and distribution planning activities for a multi-echelon, multi-product and multi-period SC network. The model was formulated as a fuzzy mixed-integer linear programming (FMILP) in which, objective function was to minimize the total cost including production cost, the costs corresponding to idleness, inventory holding cost, and transport cost. Peidro *et al*. (2009) defined an approach to transform the proposed FMILP into an equivalent auxiliary crisp MILP model.

There are several fuzzy techniques applied on SC optimization models. The relevant research works in this area have been reviewed through the last decade and summarized in Table1. As can be seen in Table 1, the previous researches considered demand-side uncertainty (Hegeman *et al*., 2014; Arikan, 2013; Vahdani *et al*., 2012; Lu *et al*., 2012; Pishvae *et al*., 2011; Peidro *et al*., 2009; Xu *et al*., 2008; Selma *et al*., 2007; Shu *et al*., 2005). Moreover, it can be deduced that a large number of centralized mathematical programming models have been developed to simultaneously optimize the integration of the entire SC.

These researches addressed a wide variety of SC configurations ranging from the single-stage SC to the multi-stage SC. However, in the vast majority of these works, the conditions and justification for using the centralized approach were not explicitly described. Although there was an increasing number of contributions that combined mathematical programming approaches with the most realistic decentralized decision making, the SCs considered in most of these works were relatively simple in comparison with the real world SCs.

As the decisions in levels of supply chain are made under uncertain situations in real life problems, so the performance of two main approaches, i.e., CDM and DDM, in presence of uncertainty is interesting and is the main concern of this study. Due to our best knowledge there is no prior research work in this area. The main theme of this study is the comparison of performance of CDM and DDM approaches in presence of uncertainty. In this paper, the CDM approach proposed by Hegeman *et al*. (2014) is developed for an uncertain environment in which some parameters of decision making are mixed with vagueness and parameterized through fuzzy sets. Moreover, a DDM approach for a two level supply chain, including manufacturer's model and retailers' models, is proposed in uncertain environment. Then a coordination mechanism is proposed to joint the segments of DCM approach (i.e., manufacturer's model and retailers' model). On the other hand, both CDM and DDM approaches are

Table 1. Literature of uncertain centralized/decentralized approaches in SC

Research	Model			Objective function							Uncertainty of model			Solution approach
	Centralized	Decentralized	Other	Service level	Income	Cost: Inventory	Cost: Shortage	Cost: Transport	Cost: Ordering	Cost: Locating	Parameters	Supply	Demand	
Hegeman *et al*. (2014)	*	*				*	*	*					*	Exact using Cplex
Arikan (2013)	*					*			*		*	*		Exact using GAMS
Vahdani *et al*. (2012)	*							*		*	*			Exact using GAMS
Lu *et al*. (2012)	*	*			*	*							*	Exact using Lingo
Lin and Wang (2011)	*		*			*			*			*	*	Heuristic algorithm
Wang *et al*. (2011)	*		*					*		*				Normalized constraint method
Sawik (2011)	*						*					*		Exact using CPLEX
Pishvae *et al*. (2011)	*						*	*		*	*		*	Exact using CPLEX
Zanjirani *et al*. (2010)	*			*			*				*			Exact using CPLEX
Bassell and Gardner (2010)	*				*			*		*				Exact using GAMS
Li *et al*. (2010)	*				*		*	*		*		*		Game theory
Stadtler (2009)	*	*			*			*		*	*			Exact using GAMS
Peidro *et al*. (2009)	*	*						*			*	*	*	Fuzzy approach
Xu *et al*. (2008)	*			*			*	*		*	*		*	Genetic algorithm
Jung *et al*. (2008)	*	*						*						MILP
Selma *et al*. (2007)	*						*	*		*			*	Branch and Bound
Cao and Chen (2006)	*	*							*					Heuristic algorithm
Shu *et al*. (2005)	*							*	*	*			*	Column generation

extended into uncertain environment parameterized using fuzzy sets. Then, three fuzzy mathematical programming models are applied on the fuzzy models. The performance of fuzzy CDM and DDM approaches are compared using all three fuzzy mathematical programming using numerical examples. All the fuzzy mathematical programming approaches are coded and implemented using GAMS software.

3. MATHEMATICAL FORMULATION

As the model by Hegeman *et al*. (2014) is going to be extend here, so in this the centralized general production and distribution planning model proposed by Hegeman *et al*. (2014) is revisited.

3.1 Centralized model by Hegeman *et al.* (2014)

The centralized production and distribution planning problem considers a supply chain of manufacturing plants and retailers, with a planning horizon of multiple time periods. The manufacturing plants produce multiple items with a limited production capacity. The objective of the centralized planning problem is to maximize profits over the planning periods. The decision maker has all related data such as demands, inventories, production costs, and plans the production and distribution of final product items and subcomponent items. A mixed-integer model is employed to solve the centralized production and distribution planning problem. First, the used notations are presented as follows:

Indices:

i : Index of plants, $i \in (1, \cdots, I)$
j : Index of retailers, $j \in (1, \cdots, J)$
k : Index of items, $k \in (1, \cdots, K)$
t : Index of time periods, $t \in (1, \cdots, T)$

Parameters:

c_{ik} = processing cost of item k at plant i
s_{ik} = setup cost for item k at plant i
o_{ik} = processing time for item k at plant i
u_{ik} = setup time for item k at plant i
h^p_{ik} = inventory holding cost of item k at plant i period t

$$\gamma_{ik} = \begin{cases} 1 \text{ if plant } i \text{ can produce item } k \\ 0 \text{ if plant } i \text{ can NOT product item } k \end{cases}$$

$B_{ik'k}$ = required quantity of item k for the production of on item k' at plant i
L_i = production capacity of plant i
d_{ijk} = unit transportation cost of item k between plant i and retailer j
g = fixed cost per vehicle
B = fixed capacity per vehicle
E_{jkt} = demand for item k at retailer j in period t
F_{jkt} = total forecast demand for item k at retailer j in period t;
E_{jkt} = A part of F_{jkt}
p_{jk} = unit selling price of item k at retailer j
h^r_{jk} = inventory holding cost of item k at retailer j per period
w^r_j = capacity for units of inventory at retailer j
v_{jk} = stock out cost per unit of item k at retailer j
M = A large positive number

Decision Variables

x_{ikt} = quantity of item k produced in plant i in period t

$$y_{ikt} = \begin{cases} 1 \text{ if setup must be performed at plant } i \text{ for item } k \text{ in period } t \\ 0 \text{ otherwise} \end{cases}$$

a^p_{ikt} = level of inventory of item k at plant i in period t
c_{ikt} = quantity of item k consumed as subcomponent at plant i in period t
$qi_{ii'kt}$ = quantity of components k shipped from plant i to plant i' in period t
qj_{ijkt} = quantity of item k transported from plant i to retatiler j in period t
z_{ijt} = number of vehicles required for distribution from plant i to retailer j in period t
z_{jkt} = shortage volume of item k for retailer j in period t
si_{jkt} = outcome variable with available supply to be sent to retailer j in period t

The proposed centralized production distribution mathematical programming is as follows.

Model:

$$Max \quad Z = \sum_j \sum_k p_{jk} \sum_t (a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \tag{1}$$
$$-(\sum_i \sum_k \sum_t c_{ik} x_{ikt} + \sum_i \sum_k \sum_t s_{ik} y_{ikt} + \sum_i \sum_k \sum_t h^p_{ik} a^p_{ikt})$$
$$-(\sum_j \sum_k \sum_t h^r_{jk} a^r_{jkt} + \sum_j \sum_k \sum_t v_{jk} (F_{jkt}$$
$$-(a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt})))$$
$$-(\sum_i \sum_j \sum_t g \times z_{ijt} + \sum_i \sum_j \sum_k \sum_t d_{ijk}\, qj_{ijkt})$$

$$\sum_k (x_{ikt} o_{ik} + y_{ikt} u_{ik}) \le L_i, \quad \forall i, \forall t \tag{2}$$
$$x_{ikt} \le M y_{ikt}, \quad \forall i, \forall t, \forall k \tag{3}$$
$$x_{ikt} \le M \gamma_{ik}, \quad \forall i, \forall t, \forall k \tag{4}$$
$$c_{ikt} = \sum_{k'} B_{ik'k} x_{ik't}, \quad \forall i, \forall t, \forall k \tag{5}$$
$$c_{ikt} = \sum_{i'} qi_{i'ikt}, \quad \forall i, \forall t, \forall k \tag{6}$$
$$a^p_{ikt} = a^p_{ik(t-1)} + x_{ikt} - \sum_j qj_{ijkt} - \sum_{i'} qj_{ii'kt}, \quad \forall i, \forall t, \forall k \tag{7}$$
$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \ge E_{jkt}, \quad \forall j, \forall t, \forall k \tag{8}$$
$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \le F_{jkt}, \quad \forall j, \forall t, \forall k \tag{9}$$
$$\sum_k a^r_{jkt} \le w^r_j, \quad \forall j, \forall t \tag{10}$$
$$\sum_k qj_{ijkt} \le B z_{ijt}, \quad \forall i, \forall j, \forall t \tag{11}$$
$$a^p_{iko} = a^r_{jko} = 0, \quad \forall i, \forall j, \forall t \tag{12}$$
$$z_{ijt} \in z^+, x_{ikt} \ge 0, a^p_{ikt} \ge 0, y_{ikt} \in \{0,1\}, C_{ikt} \ge 0, \quad qi_{ii'kt} \ge 0, qj_{ijkt} \ge 0, a^r_{jkt} \ge 0, \forall i, \forall j, \forall t, \forall k \tag{13}$$

The objective function (1) expresses the overall net profit during all periods of planning. The objective function (1) is alculated by subtracting total costs from total revenue. Revenue is the total turnover of all retailers, computed by multiplying the selling price with sales $(a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{ikt})$. The costs include production-, inventory holding-, stock-out- and distribution costs.

Constraint (2) represents the production capacity limit at plant i in time period t. Constraint (3) ensures that the production is accomplished if setup is done in period t. Constraint (4) ensures that production of items is only allowed at a plant if that plant is capable of producing that item. Constraint (5) determines the amount of an item that is consumed for the production of the upper level items, by summing the products of the production quantities of the higher level items with the amount of lower level items consumed for their production. Constraint (7) assures the inventory balance at a plant, with both shipments to retailers and to other plants taken under consideration. Constraint (8) ensures that the 'core demand' is satisfied, whilst constraint (9) ensures that the sale is not more than the 'forecasted demand.' Constraint (10) applies the storage capacity for inventory held by retailers. The number of vehicles required for transportation of items to retailers is considered in constraint (11). Constraint (12) defines the initial inventory levels at both plants and retailers. The final constraint (13) enforces restrictions of non negativity, the integer and also the binary nature of decision variables.

The model calculates optimal production quantities x_{ikt} for all items at the various plants for all time periods and optimal amounts of qj_{ijkt} to be shipped to the retailers. It will balance setup with inventory holding costs and delivery costs with stock-out costs. It can so occur that not all forecasted demand is satisfied, although the inventory storage capacity at retailers exists to minimize the incidence of demand not being satisfied.

3.2 Decentralized (Distributed) Deterministic Model

The centralized deterministic model is decomposed into two separate models. These separate models, each pertain to a different decision maker, one that controls the manufacturing plants and distribution of items, and one that controls the retailers. A coordination mechanism is developed to link the two models and form the distributed deterministic model. The distributed decision making process is also presented to enhance clarity.

3.2.1 Manufacturer's Model

The first decision maker has control over the production of items in the plants, and their distribution to the retailers. It is assumed that distribution of items is a component of this decision maker's model because it is typically the manufacturer's responsibility to deliver a product to its customer. It is notable that the wholesale price of the manufacturer is ignored in this model. The notations used in manufacturer's model are as follows.

Indices:

$i = planta,\ \ i \in (1, \cdots, 3)$
$j = retailers,\ j \in (1, 2)$
$k = items,\ k \in (1, \cdots, 8)$
$t = time\ \ periods,\ t \in (1, \cdots, 5)$

Parameters:

All parameters are the same as in the centralized model, except for the following parameters that will be added in the manufacturer's model.

vi_{jk} = unit supply shortage penalty cost of retailer *j* for item *k*
SJ_{jkt} = requested supply quantity for item *k* by retailer *j* in period *t* (receive from *j*)

Considering the above notations, the manufacturer's model is proposed as follows.

Model:

$$Min\ \ Z_1 = \sum_i\sum_k\sum_t c_{ik}x_{ikt} + \sum_i\sum_k\sum_t s_{ik}y_{ikt} + (\sum_i\sum_k\sum_t h_{ik}^p a_{ikt}^p) + \sum_i\sum_j\sum_t gz_{ijt} + \sum_i\sum_j\sum_k\sum_t d_{ijk}qj_{ijkt}) + \sum_j\sum_k\sum_t vi_{jk}z_{jkt} \quad (14)$$

$$\sum_k (x_{ikt}o_{ik} + y_{ikt}\,u_{ik}) \le L_i, \qquad \forall i, \forall t \qquad (15)$$

$$x_{ikt} \le My_{ikt}, \qquad \forall i, \forall t, \forall k \qquad (16)$$

$$x_{ikt} \le M\gamma_{ik}, \qquad \forall i, \forall t, \forall k \qquad (17)$$

$$c_{ikt} = \sum_{k'} B_{ik'k}x_{ik't}, \qquad \forall i, \forall t, \forall k \qquad (18)$$

$$c_{ikt} = \sum_{i'} qi_{i'ikt}, \qquad \forall i, \forall t, \forall k \qquad (19)$$

$$a_{ikt}^p = a_{ik(t-1)}^p + x_{ikt} - \sum_j qj_{ijkt} - \sum_{i'} qj_{ii'kt}, \qquad \forall i, \forall t, \forall k \qquad (20)$$

$$\sum_i qj_{ijkt} + z_{jkt} = Sj_{jkt}, \qquad \forall j, \forall t, \forall k \qquad (21)$$

$$\sum_k qj_{ijkt} \le B \times z_{jkt}, \qquad \forall i, \forall j, \forall k \qquad (22)$$

$$\sum_i qj_{ijkt} = Si_{jkt}, \qquad \forall i, \forall j, \forall k \qquad (23)$$

$$a_{iko}^p = 0, \qquad \forall i, \forall k \qquad (24)$$

$$z_{jkt} \in \{0\} \cup z^+, \qquad \forall j, \forall t, \forall k \qquad (25)$$

The manufacturer does not know the actual demand for final products. The manufacturer only knows the requested supply quantities for each item per period as submitted by the retailers. This quantity is represented by a new parameter SJ_{jkt}. The manufacturer should endeavor to fill the requested supply quantities to the best of his ability, because it contributes to Supply Chain (SC) profitability. To make the model strive for this, a penalty will be incurred for every unit of unfilled requested supply. For this reason, a shortage penalty cost v_{ijk} and a shortage quantity decision variable Z_{jkt} have been defined.

The manufacturer has no knowledge of actual demand or of retail prices. Maximizing profit is thus not a valid objective for this model. Instead, the manufacturer will try to minimize its costs, whereas meeting supply, because that ought to contribute to SC profitability. The objective function (14) now only includes production, setup and inventory holding costs for the plants, distribution costs and supply shortage penalty costs. Because having shortage negatively affects the objective function, the model will try to fill all demand. The penalty cost per unit of shortage should be high enough for the manufacturer to generally prefer production and distributing to incurring the penalty. Constraints (15-20) are the same as in the centralized model, but constraint (21) replaces the constraints that ensured filling demand. It makes sure that the amount of an item shipped from all the plants to a retailer plus any shortage equals the requested supply quantity by that retailer for that item. If the shipped amounts do not suffice, the shortage is positive and the penalty will be incurred. Constraint (22) governs the amount of vehicles required for transportation of items to retailers. Constraint (23) calculates the supply of an item k that is available for a retailer in a period t. The decision variable, SJ_{jkt}, is the connection between the manufacturer's model and the retailers' model. On the other hand, it will be provided to retailers' model after the manufacturer's model has been solved. The retailers then know the available supply quantities that they can use to satisfy the demand. It will become clear that SJ_{jkt} is an input variable in the retailers' model, just like SJ_{jkt} which was for the manufacturer's model.

3.2.2 Retailers' Model

The second decision maker has control over the retailers. This can be a modeling choice, as each retailer could also have its own model, in which case the index j of the retailers would be forsaken. The notations and explanation of the model are as follows.

Indices:

j = retailers, $j \in (1, 2)$
k = items, $k \in (1, 2)$
t = time periods, $t \in (1, \cdots, 5)$

Parameters and variables are the same as in the centralized model, but the following parameters and variables will be added in retailers' model.

Si_{jkt} = offered supply quantity of item k to retailer j in period t $\begin{cases} \text{first iteration it is infinite} \\ \text{then, recevied from plants} \end{cases}$

Decision variables:

q_{jkt} = quantity of item k requested from plants by retailer j in period t
a^r_{jkt} = level of inventory of item k at retailer j in period t

Considering the above notations, the retailers model is proposed as follows.

Model:

$$Max \quad Z_2 = \sum_j \sum_k p_{jk} \sum_t (a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt}) - \tag{26}$$

$$((\sum_j \sum_k \sum_t h^r_{jk} a^r_{jkt} + \sum_j \sum_k \sum_t v_{jk} (F_{jkt} - (a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt}))$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \geq E_{jkt} \quad \forall j, \forall t, \forall k \tag{27}$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \leq F_{jkt} \quad \forall j, \forall t, \forall k \tag{28}$$

$$q_{jkt} \leq si_{jkt} \quad \forall j, \forall t, \forall k \tag{29}$$

$$\sum_k a^r_{jkt} \leq w^r_j \quad \forall j, \forall t \tag{30}$$

$$q_{jkt} \leq sj_{jkt} \quad \forall j, \forall t, \forall k \tag{31}$$

$$a^r_{jk0} = 0 \quad \forall j, \forall k \tag{32}$$

$$q_{jkt}, a^r_{jkt} \in \{0\} \cup Z^+ \quad \forall j, \forall k, \forall t \tag{33}$$

First of all, the index i for the plants is no longer present, because it does not matter for the retailers where their supply comes from, as long as it comes. The parameter SJ_{jkt}. is the only new parameter, and it shows the available supply of an item for a retailer in period t, which is received from the manufacturer's model. Just for the first iteration of the retailers' model it is assumed to be infinite. This is because of the distributed search for the optimal solution begins at the retailers. Because it does not matter from which plant the supply comes, the decision variable qj_{ikt} is changed into q_{jkt}. The latter decision variable only represents the item quantities requested by a retailer from the manufacturer as a whole. The objective function (26) maximizes the profits through maximizing sales and minimizing inventory holding costs and stock-out costs. Constraints (27) and (28) still exist to make sure 'core demand' is satisfied and 'forecasted demand' not exceeded, respectively. The smallest modification in these constraints is that $\sum_i qj_{ijkt}$ is replaced by q_{jkt}. Constraint (29) enforces that the requested amount of items from the manufacturer is at most equal provided amount by manufacturer. Constraint (30) is copied from the centralized model. The constraint (31) calculates the input variable for the manufacturer's model, SJ_{jkt}. It is simply greater than or equal to q_{jkt}. Constraints (32)-(33) are supplied to define the value and type of decision variables, respectively.

3.3 Coordination Mechanism

Information sharing in the distributed model is minimal, with only requested quantities and available quantities shared between the two decision makers. The Coordination mechanism is proposed in eleven steps as follows:

Step 1. Generate initial sales plan from the distribution plan.
Step 2. Compute initial demand $SJ^{initial}_{jkt}$
Step 3. Calculate Retailer's profit
Step 4. Generate production and distribution plan from the manufacturer's model.
Step 5. If there is production shortage in manufacturer's model, go to step 6, otherwise terminate the optimization procedure.
Step 6. Calculate available supply quantities Si_{jkt}.
Step 7. Calculate Manufacturer's cost
Step 8. Calculate gap
Step 9. Generate sales plan from retailer's model.
Step 10. If the core demand can be met go to step 11, otherwise terminate the optimization procedure.
Step 11. Calculate request quantity SJ_{jkt} and go to step 3.

Figure 1 shows the coordination mechanism designed for cooperation of manufacturer and retailer.

This mechanism guaranty the coordination mechanism among manufacturers and retailers in supply chain.

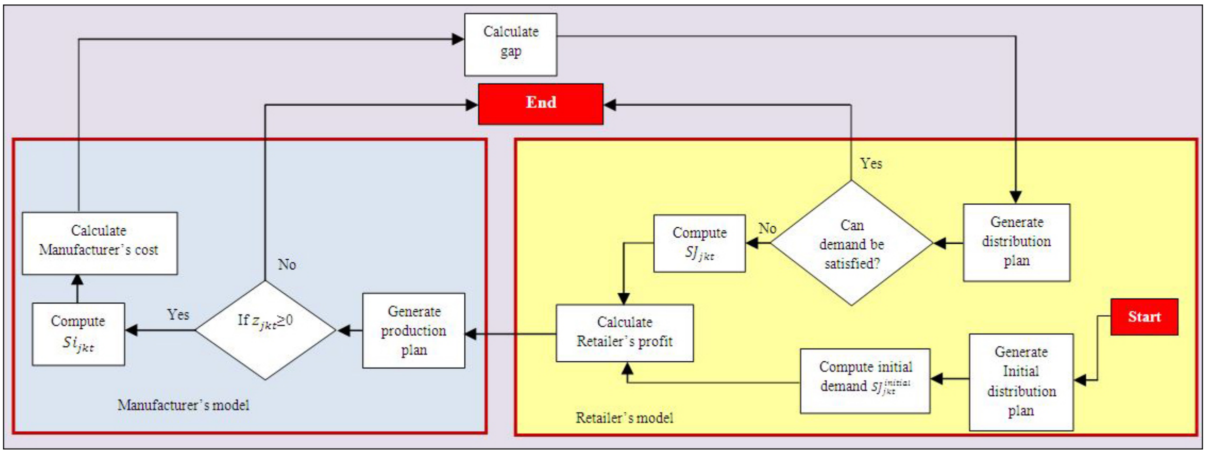


Figure 1. Coordination Mechanism.

3.4 Decentralized (Distributed) Model Under Uncertainty

In this section, the distributed deterministic model is adapted to account for uncertainty in demand. The retailers' model is the model that takes demand into account. In contrast, the manufacturer has no information on demand. The adaptation to account for uncertainty will be done exclusively on the retailers' model.

Two parameters were used to define the demand in the retailers' model i.e., 'core demand' and 'forecasted demand.' In possibility theory, these parameters are turned into diffuse coefficients. It is plausible that both parameters can turn out to be somewhat lower, or somewhat higher than initially thought. Consequently, a membership function that expresses that is needed. A triangular membership function is thus chosen to represent the uncertainty in demand parameters. It has a central value with a membership degree of one, and therefore the membership degree decreases the further the parameter moves away from the central value. Outside of two boundary values (one left and one right), the membership degree turns zero, meaning that it is not plausible that demand will take on values outside of a definite interval. Taking 'core demand as an example, the triangular fuzzy coefficient E is defined by atriple as (E_1, E_2, E_3). E_1 is the left boundary of the fuzzy set, E_2 the central value for which the membership degree equals to one, and E_3 is the right boundary of the set. Furthermore, it is reasonable to assume that there is less uncertainty for the 'core demand' in comparison with the 'forecasted demand' because, as it comes from a loyal customer base. A smaller range of values, therefore, belongs to the fuzzy set of core demand than of 'forecasted demand.'

3.4.1 Retailers' Model Formulation Under Uncertainty

The F_{jkt} parameter in the objective function will be replaced with the expected value, so that the model is also generally valid. Because the newly defined fuzzy demand parameters only appear on the right hand sides of the constraints, only the right hand sides of the constraints are affected. The new terms are factored out to preserve linearity. The model performance is evaluated under demand uncertainty by applying three selected fuzzy approaches.

Considering the above mentioned descriptions for uncertain demands the following parameters in CDM approach, and in the retailer's model of DDM approach are assumed to be mixed with uncertainty parameterized through fuzzy sets:

$\tilde{E}_{jkt}$ = fuzzy demand for item k at retailer j in period t
$\tilde{F}_{jkt}$ = total fuzzy forecast demand for item k at retailer j in period t

It is notable that these fuzzy parameters have been parameterized through fuzzy triangular numbers (TFNs). All other notations, parameters and indices are similar to those represented in crisp models.

4. SOLUTION PROCEDURES

In this section three fuzzy mathematical optimization approaches are briefly described and customized in order to solve both centralized and decentralized production distribution supply chain network design problem.

4.1 Jiménez Approach (Jiménez *et al.*, 2007)

This approach was first used to change the deterministic model into a fuzzy model. It was presented to combine fuzzy coefficients with trapezoidal membership functions into linear programming models. It is notable that the acceptable optimal solutions in degree α are not fuzzy numbers that makes it easier to take a decision in a simple way by solving a crisp parametric linear program.

For triangular functions, Jiménez *et al*. (2007) showed that the expected interval of fuzzy coefficient $\tilde{a}$ = (a_1, a_2, a_3), can be calculated using (34).

$$EI(\tilde{a}) = \left[E_1^{\tilde{a}}, E_2^{\tilde{a}}\right] = \left[\frac{1}{2}\times(a_1 + a_2),\ \frac{1}{2}\times(a_2 + a_3)\right] \quad (34)$$

And the expected value of fuzzy coefficient ã = (a_1, a_2, a_3) can then be calculated using (35)

$$EV(\tilde{a}) = \left(\frac{E_1^{\tilde{a}} + E_2^{\tilde{a}}}{2}\right) \quad (35)$$

Constraints change based on the type of inequality. The 'satisfy core demand' and 'not surpass forecasted demand' constraints affected in the retailers' model are in form of less than or equal, and greater than or equal constraints respectively which change as follows (36)-(37).

$$ax \ge b \to \left[(1-\alpha)E_2^{\alpha} + \alpha E_1^{\alpha}\right]x \ge \alpha E_2^{b} + (1-\alpha)E_1^{b} \quad (36)$$

$$ax \le b \to \left[(1-\alpha)E_1^{\alpha} + \alpha E_2^{\alpha}\right]x \le \alpha E_1^{b} + (1-\alpha)E_2^{b} \quad (37)$$

Where $\alpha \in [0, 1]$ is a parameter set by the decision maker. The F_{jkt} parameter in the objective function will be replaced with the expected value, so that the model is also generally valid. The new fuzzy retailers' model is thus re-formulated as:

Parameters:

α = degree of feasibility parameter set by decision maker

$\tau = (1-\alpha)$, complement of degree of feasibility parameter set by decision maker

$E_{\tilde{jkt}} = (E_1, E_2, E_3)_{jkt}$, demand for item k at retailer j in period t that must be filled

$F_{\tilde{jkt}} = (f_1, f_2, f_3)_{jkt}$, total forecast demand for item k at retailer j in period t that must be filled

Si_{jkt} = offered supply quantity of item k to ret: j in period t

$$\begin{cases}\text{first iteration it is infinite} \\ \text{then, recevied from plants}\end{cases}$$

Decision variable:

q_{jkt} = quantity of item k requested from plants by retailer j in period t

a^r_{jkt} = level of inventory of item k at retailer j in period t

Model:

$$Max \sum_j \sum_k p_{jk} \sum_t (a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt}) \quad (38)$$

$$-((\sum_j \sum_k \sum_t h^r_{jk} a^r_{jkt} + \sum_j \sum_k \sum_t v_{jk}(\frac{1}{4}F_1 + \frac{1}{4}F_2 + \frac{1}{4}F_2 + \frac{1}{4}F_3)$$

$$-(a^r_{jkt-1} + qj_{ijkt}))$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \ge \left(\frac{1}{2}\alpha E_2 + \frac{1}{2}\alpha E_3 + \frac{1}{2}\gamma E_1 + \frac{1}{2}\gamma E_2\right) \quad \forall j, \forall t, \forall k \quad (39)$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \le \left(\frac{1}{2}\alpha F_1 + \frac{1}{2}\alpha F_2 + \frac{1}{2}\gamma F_2 + \frac{1}{2}\gamma F_3\right) \quad \forall j, \forall t, \forall k \quad (40)$$

$$q_{jkt} \le si_{jkt} \quad \forall j, \forall t, \forall k \quad (41)$$

$$\sum_k a^r_{jkt} \le w^r_j \quad \forall j, \forall t \quad (42)$$

$$q_{jkt} \le sj_{jkt} \quad \forall j, \forall t, \forall k \quad (43)$$

$$a^r_{jk0} = 0 \quad \forall j, \forall k \quad (44)$$

$$q_{jkt}, a^r_{jkt} \in \{0\} \cup Z^+ \quad \forall j, \forall k, \forall t \quad (45)$$

4.2 Werners' Approach (Werners, 1987)

Werners (1987) introduced an interactive general system which supports a decision maker in solving programming models with fuzzy constraints and crisp goals. In the Werner's approach fuzzy constraints were converted to crisp constraints by linear membership functions. More details can be found in Werners (1987).

Solving the following LP problems a membership function for the objective function is built.

LP (b)

$Max\, C^t x$

Such that $A_i x \le b_i \quad i = 1, 2, \cdots, m \quad x \ge 0$ (46)

LP (b+p)

$Max\, C^t x$

Such that $A_i x \le b_i + p_i \quad i = 1, 2, \cdots, m \quad x \ge 0$ (47)

Whereas before p_i's the maximum tolerances from b_i Which are determined by decision makers for them constraints of "type 1 FLP." Let z_0 and z_1 be the optimal values of LP (b) and LP (b+p) respectively.

Continuously nondecreasing linear memberships function for objective function by using z_0 and z_1 are as follows:

$$\mu_0(c^T x) = \begin{cases} 1 & c^T x > z_1 \\ 1 - \dfrac{z_1 - c^T x}{z_1 - z_0} & z_0 \le c^T x \le z_1 \\ 0 & c^T x < z_0 \end{cases} \quad (48)$$

The membership functions of the constraints are the same as before, linearly decreasing over the tolerance interval p_i

$$\mu_0(c^T x) = \begin{cases} 1 & A_i x > b_i \\ 1 - \dfrac{A_i x - b_i}{p_i} & b_i \le A_i x \le b_i \\ 0 & A_i x < b_i + p_i \end{cases} \quad (49)$$

By using $\mu_i (i = 0, 1, 2, \cdots, m)$ and Bellman and Zadeh principle in decision making, the "type 1 FLP" is equivalent to the following crisp LP problem:

$$\begin{aligned}&Max\ \alpha\\&\mu_i(x)\ge\alpha,\quad i=1,2,\cdots,m\\&\alpha\in[0,1],\ \ x\ge 0\end{aligned}\tag{50}$$

Which on substitution for

$$\begin{aligned}&Max\ \alpha\\&C^t x>z_1-(1-\alpha)(z_1-z_0)\\&A_i x\le b_i+(1-\alpha)p_i,\quad i=1,2,\cdots,m\\&\alpha\in[0,1],\ \ x\ge 0\end{aligned}\tag{51}$$

The new fuzzy retailers' model is thus formulated as:

Parameters:

$E^{\sim}_{jkt}$ = Fuzzy demand for item k at retailer j in period t that must be filled

$F^{\sim}_{jkt}$ = Fuzzy total forecast demand for item k at retailer j in period t

Si_{jkt} = offered supply quantity of item k to ret: j in period t

$$\begin{cases}\text{first iteration it is infinite}\\\text{then, recevied from plants}\end{cases}$$

$\alpha\in[0,1]$ and $B=1-\alpha$

Decision variable;

q_{jkt} = quantity of item k requested from plants by retailer j in period t

a^r_{jkt} = level of inventory of item k at retailer j in period t

With $p_1, p_2=20$ the right margin and 141225 is upper bounded of objective function that allowed by the decision maker. The auxiliary parametric integer programming problem is:

Model:

$$Min\,B\tag{52}$$

$$a^r_{jkt-1}+\sum_i qj_{ijkt}-a^r_{jkt}\le F_{jkt}+20B\quad\forall j,\forall t,\forall k\tag{53}$$

$$a^r_{jkt-1}+\sum_i qj_{ijkt}-a^r_{jkt}\ge E_{jkt}-20B\quad\forall j,\forall t,\forall k\tag{54}$$

$$\sum_j\sum_k p_{jk}\sum_t(a^r_{jkt-1}+\sum_i qj_{ijkt}-a^r_{jkt})-((\sum_j\sum_k\sum_t h^r_{jk}a^r_{jkt}\tag{55}$$

$$+\sum_j\sum_k\sum_t v_{jk}(F_{jkt}-(a^r_{jkt-1}+\sum_i qj_{ijkt}-a^r_{jkt}))\ge 1255-10510\text{B}$$

$$B\in[0,1]\tag{56}$$

$$q_{jkt}\le si_{jkt}\quad\forall j,\forall t,\forall k\tag{57}$$

$$\sum_k a^r_{jkt}\le w^r_j\quad\forall j,\forall t\tag{58}$$

$$q_{jkt}\le sj_{jkt}\quad\forall j,\forall t,\forall k\tag{59}$$

$$a^r_{jk0}=0\quad\forall j,\forall k\tag{60}$$

$$q_{jkt},a^r_{jkt}\in\{0\}\cup Z^+\quad\forall j,\forall k,\forall t\tag{61}$$

4.3 Tan and Cao's Approach (Tan and Cao, 2005)

The classical LP problem is stated model (62).

$$\begin{aligned}&Max\,C^t x\\&A_i x\le b_i\ (i=1,\cdots,m)\\&x\ge 0\end{aligned}\tag{62}$$

Where A, b and c are crisp numbers. In most of the cases it is not possible to describe the constraints and the objective function in crisp terms and therefore usage of fuzzy linear programming offers the advantage that the decision maker can model the problem in accordance to the current state of information. In this approach fuzzy constraints are converted to crisp constraints by linear membership functions. By associating an objective function with an optimal value of parametric programming Tan and Cao (2005) defined a normal form of Fuzzy LP as model (63).

LP (α)

$Max\,C^t x$

Such that

$$A_i x\le b_i+(1-\alpha)p_i,\ \ i=1,2,\cdots,m,\ x\ge 0,\ \alpha\in[0,1]\tag{63}$$

Where α is a parameter on the interval [0, 1], and $p\ge 0$. x_α indicates the optimal solution to (LP_α), B_α and z_α the optimal basis vector and the optimal value of LP_α, respectively. The right hand side coefficient $b+(1-\alpha)p$ of the constraint condition in LP_α will vary with the changing of parameter α.

Let z_1 be an optimal value of LP_1 and z_0 be an optimal value of LP_0, $p_0=z_0-z_1>0$

Base on the above descriptions, Tan and Cao (2005) algorithm is summarized as follows:

Step 1. Solve linear programming problems (LP_1) And (LP_0).

Let the associated optimal solutions be x_0 and x_1, the optimal values be z_0 and z_1, and the optimal matrix of LP_0 be B_0

Step 2. Solve $\left[B_0^{-1}(b+(1-\alpha)p\right]_i=0$ (64)

Assume the solutions (65).

$$\alpha_1,\cdots,\alpha_{n-1},(0<\alpha_1<\cdots<\alpha_{n-1}<1)\tag{65}$$

$$\text{Let}\ \ \alpha_0=0,\ \ \alpha_n=1,\ \alpha=\alpha_1, k=1\tag{66}$$

Step 3. Solve (LP_α)

Let the optimal value be z_α. If $z_\alpha\le z_1+p_0\alpha$ go to Step 4, otherwise let $k=k+1, \alpha=\alpha_k$ go to step 3.

Step 4. Set the optimal alpha-level using (67)

$$\alpha^*=\frac{z_1\cdot\alpha_k-z_1\cdot\alpha_{k-1}-z_{\alpha_{k-1}}\cdot\alpha_k+z_{\alpha_k}\cdot\alpha_{k-1}}{z_{\alpha_k}-z_{\alpha_{k-1}}-\alpha_k\cdot p_0+\alpha_{k-1}\cdot p_0}\tag{67}$$

Step 5. Solve linear programming LP_{α^*}, and obtain optimal solution x_{α^*} and an optimal value z_{α^*}.

Let C be the constraint on domain X, where $c_\alpha = \{x | x \in X,\ C(x) \geq \alpha\}$. The fuzzy objective function can be defined as $z_\alpha = z_1 + p_0 \cdot \alpha$. So we can use the intersection of the fuzzy objective function $z_\alpha = z_1 + p_0 \cdot \alpha$ and $z_\alpha = C_{B\alpha} \cdot B_\alpha^{-1}(b + (1-\alpha)p)$ to find an optimal decision of LP.

Setting $p_1 = 10,\ p_2 = 20$ as the right margin allowed by decision maker. The associated auxiliary parametric integer programming problem is:

Model:

$$Max\, z_\alpha \tag{68}$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \geq 10(1-\alpha) \quad \forall j, \forall k, \forall t \tag{69}$$

$$a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \leq 20(1-\alpha) \quad \forall j, \forall t, \forall k \tag{70}$$

$$\alpha \in [0,1] \tag{71}$$

$$q_{jkt} \leq si_{jkt} \quad \forall j, \forall t, \forall k \tag{72}$$

$$\sum_k a^r_{jkt} \leq w^r_j \quad \forall j, \forall t \tag{73}$$

$$q_{jkt} \leq sj_{jkt} \quad \forall j, \forall t, \forall k \tag{74}$$

$$a^r_{jk0} = 0 \quad \forall j, \forall k \tag{75}$$

$$q_{jkt}, a^r_{jkt} \in \{0\} \cup Z^+ \quad \forall j, \forall k, \forall t \tag{76}$$

5. COMPUTATIONAL RESULTS

In this section the computational results for the both centralized and distributed approaches under uncertainty are discussed. Four different datasets were used to test the efficacy and applicability of proposed approaches. Three controllable parameters were chosen to be varied to create the different sets of benchmark instantces. First, demand was given two different behaviors. Both had the same total demand value, but in one instance the demand was stable over the periods, whereas in the other it was very erratic, that varying from near nothing to high peaks. Second, production capacity was varied. Low capacity meant that the production capacity constraints were very tight, and that it was never really possible to meet all demand.

Production/setup costs were the third parameter to be varied. Combinations of low unit production costs with high setup costs, and high unit production costs with low setup costs were made to change the decisions the manufacturer would make regarding batches. Low setup costs obviously encouraged smaller batches. Four combinations made and the data sets were generated using configuration presented in Table 2.

Input data of the problems are set, without loss of generality, through the preliminary studies. Core demand is generated from a uniform distribution on [50, 70]. Forecasted demand is determined by $1.5\times E_{jkt}$. Capacity of all vehicles is set to 100. Storage capacity at a retail outlet is set by $Max \sum_k E_{jkt}\ \ j = 1, 2, \cdots, J$.

Unit selling price is determined by multiplying C_{ik} by a uniform random number on [2.5, 3.5]. For the costs of set-up, holding at a plant and a retail outlet, and stock out, $s_{ik} = 300c_{ik}$, $h^p_{ik} = 0.2c_{ik}$, $h^r_{jk} = 0.1p_{jk}$ and $v_{ik} = 0.15p_{jk}$ are assigned, respectively. Unit transportation cost is set to the value between 20,000 and 40,000 in proportion to the Euclidean distance between the two locations. Coordinates of plants and retail outlets are randomly generated from a uniform distribution on [0, 10,000]. Fixed vehicle cost is set to 25,000. For sake of brevity the details of the created datasets are not presented. In order to make a better sense of data generated the full data of data set 1 are presented in Appendix A.

All data sets were solved using all three fuzzy mathematical optimization procedures. The results of both centralized method (CM) and decentralized method (DM) for data set 3 are presented in Table 3.

The explanation is that high setup costs may cause the manufacturer to not want to produce a batch of acertain item, if it has enough inventory to meet the agreed fill rate. Some of the forecasted demand can then not be met, resulting in lost sales and a sub optimal solution. CPU times were either very short, or extremely long. Three of the 12 runs of the manufacturer's model took 49, 54 and 58s respectively. This did not occur for the same dataset either, which seems to suggest that some combinations of data make the problem more difficult to solve optimally. The two most important observations come from the iterations column. In some cases, only one iteration is required because it is optimal for the manufacturer to deliver everything that is requested.

Table 2. Data used in test sets

Data Sets	Demand		Production Capacity		Production/setup cost		i	k	j	t
	Type	Value	Value	Distribution	Value	Distribution				
Set1	Fixed	60	Low	Uniform(500, 1,000)	Low	Uniform(500, 1,000)	3	8	2	5
Set2	Fixed	60	Low	Uniform(500, 1,000)	High	Uniform(10^5, 10^6)	3	8	2	5
Set3	Random	Uniform(50, 70)	High	Uniform(1,000, 5,000)	Low	Uniform(500, 1,000)	3	8	2	5
Set4	Random	Uniform(50, 70)	High	Uniform(1,000, 5,000)	High	Uniform(10^5, 10^6)	3	8	2	5

Table 3. Computational results (v_{ijk} = 60% of sales price)

Fuzzy Optimization Approach	Data sets	CDM*	DDM**	Gap	Gap%	Iterations	CPU time (s)
Jiménez (2007)	1	137325	126167	11158	8.13%	2	4s
	2	140177	138699	1478	1.05%	2	4s
	3	122982	109001	13981	11.37%	1	6s
	4	136242	136223	19	0.01%	3	2s
Tan and Cao (2005)	1	116345	109907	6438	5.53%	4	18s
	2	118111	118111	0	0.00%	2	54s
	3	120871	130212	-9341	-7.73%	1	58s
	4	144665	143770	895	0.62%	2	49s
Werners (1987)	1	151765	141255	10510	6.93%	2	2s
	2	139234	138629	605	0.43%	1	3s
	3	149872	144332	5540	3.70%	1	1s
	4	151765	151000	765	0.50%	2	5s

* Centralized Decision Method (CDM).
** Decentralized Decision Method (DDM).

Table 4. Sample Results of Coordination Mechanism for Data Set 1

	Iteration 1								Iteration 2							
Steps	$SJ_{jkt}^{initial}$	z_{jkt}	SI_{jkt}	SJ_{jkt}	E_{jkt}	Z_m	Z_r	Gap	$SJ_{jkt}^{initial}$	z_{jkt}	SI_{jkt}	SJ_{jkt}	E_{jkt}	Z_m	Z_r	Gap
Step 1	2578	-	infinite	-	-	-	-	-	-	-	-	-	-	-	-	-
Step 2	3822.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Step 3	-	-	-	-	-	-	37317	-	-	-	-	-	-	-	17380	-
Step 4	-	-	60.8	-	-	-	-	-	-	0	1004.4	-	-	-	-	-
Step 5	-	743.3	-	-	-	-	-	-	-	0	-	-	-	-	-	-
Step 6	-	-	84.7	-	-	-	-	-	-	-	-	-	-	-	-	-
Step 7	-	-	-	-	100	-	-	-	-	-	-	-	-	-	-	-
Step 8	-	-	-	-	-	137551	-	-	-	-	-	-	-	143547	-	-
Step 9	-	-	-	-	-	-	-	100234	-	-	-	-	-	-	-	126167
Step 10	-	-	-	-	100	-	-	-	-	-	-	-	-	-	-	-
Step 11	-	-	-	1102.2	-	-	-	-	-	-	-	-	-	-	-	-

This coincides with an Erratic demand behavior and high production capacity. Computational results illustrate that the proposed distributed model closely approximated the optimal solutions generated by the centralized model.

As mentioned, Table 3 shows the comparison of the proposed approaches (i.e., CDM, and DDM) through data set 3. Regardless of the demand pattern, the combination of High in production capacity and Low/High in Production/ setup cost generates poor quality solutions in comparison with other combinations. This implies that high capacity tightness of the manufacturer constrains the feasible region for production planning and makes retailer that has enough distribution capacity to be able to choose a locally optimized coordination plan that might be distant from the optimal coordination plan.

Finally, the Werner's approach in terms of the number of iterations, solution time, and objective function performs better than the other two methods on four data sets of this study.

In order to make a better sense of steps of proposed coordination mechanism between manufacturer and retailers in the supply chain, the results of Jiménez (2007) approach for data set1are presented in Table 4.

The content of Table 4 shows the convergence trend of the proposed algorithm during two iterations. As shown in Table 4, the coordination mechanism works suitably. The other data sets have followed the similar procedure and are not presented here for sake of brevity.

The Jiménez *et al*. (2007) approach used the expected interval and expected value of a fuzzy number to change the fuzzy mathematical programming into a crisp mathematical programming. The Werners (1987) method calculated the membership function of objective function and constraints of fuzzy mathematical programming. Then, using these membership functions and the max-min operator of bellman and Zadeh converted the fuzzy mathematical programming into the associated crisp ma-

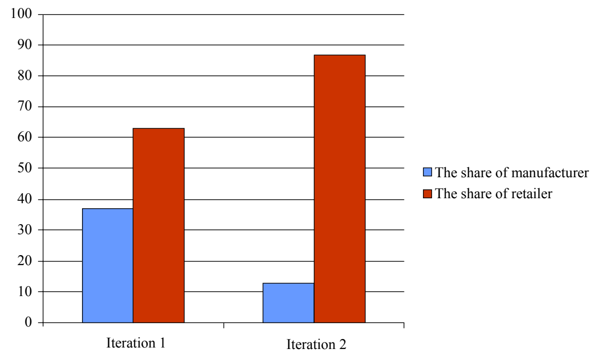


Figure 2. The Share of each channel.

thematical model. The Tan and Cao (2005) approach used the membership value of objective function, alpha-cut of fuzzy variables, and tolerance approach in fuzzy mathematical programming in order to convert the fuzzy mathematical programming into a crisp mathematical programming through an algorithmic approach.

Considering coordination mechanism retailer's profit is calculated in step 3 and Manufacturer's cost is calculated in step 7. The gap between retailer's profit and Manufacturer's cost is calculated in step 8. This gap is shown as DDM** in Table 3. In fact the difference between retailer's profit and Manufacturer's cost is called DDM** as total profit of decentralized decision making. We can calculate sharing profit according to sharing information by each channel. Figure 2 shows the share of each channel.

As the Table 4, Z_m is Manufacturer's cost value and Z_r is retailer's profit. In the first iteration retailer's profit is calculated 137,551 and Manufacturer's cost is calculated 37,317 and the gap is 100,234. In the first iteration 64% of the total profit of decentralized decision making is the share of manufacturer and 37% of it is the share of retailer. In the second iteration the profit increases and the cost decreases therefore the gap is improved. 87% of the total profit of decentralized decision making is the share of manufacturer and 13% of it is the share of retailer. According to Figure 2, Table 4 and coordination mechanisms it can be said that as long as the share of manufacturer increases and retailer share decreases the value of Z_{jkt} increases and therefore Shortage does not occur (step5). Also in Table3, for first data set 48% of the total gap is the share of decentralized and 52% of it is the share of centralized channel.

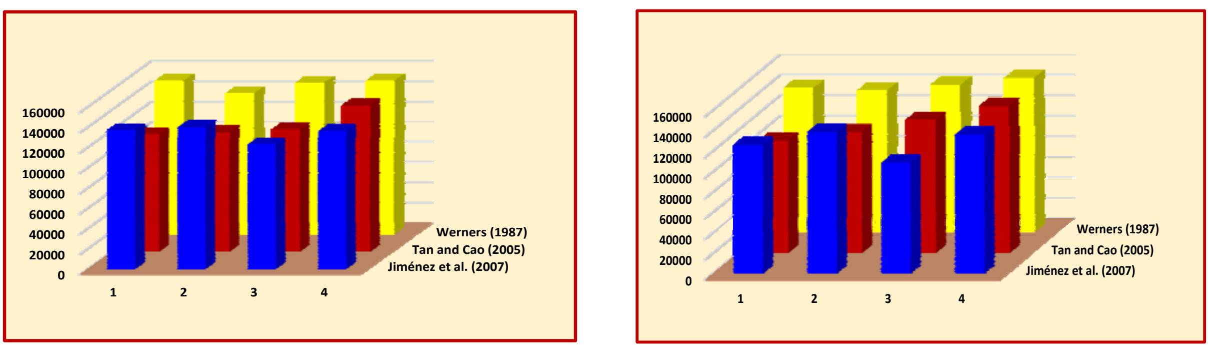
Figure 3 shows the Objective Function Values for both Centralized and Decentralized Methods for all 4 benchmark instances.

Three different methods were used to compare the solutions in centralized and decentralized situations. The biggest absolute gap in the objective value profit between the CM and the DM is 13,981, whilst the lowest gap is 0. In general, the DM looks to be performing reasonably well compared to the CM, with many distributed solutions being close to the optimal solution. The biggest absolute gaps occur when setup costs are high in relation to unit production costs. This corresponds to the odd datasets.

Although the results obtained from three fuzzy mathematical programming models are different, as the viewpoint of these fuzzy methods are different and variability in values of outcomes are seen, but the trend of outcome in all of the fuzzy mathematical approaches (i.e., Jiménez *et al*. (2007), Werners (1987), and Tan and Cao (2005)) support a single truth about performance of DDM approach in comparison with CDM approach. This truth is plotted in Figure 4.

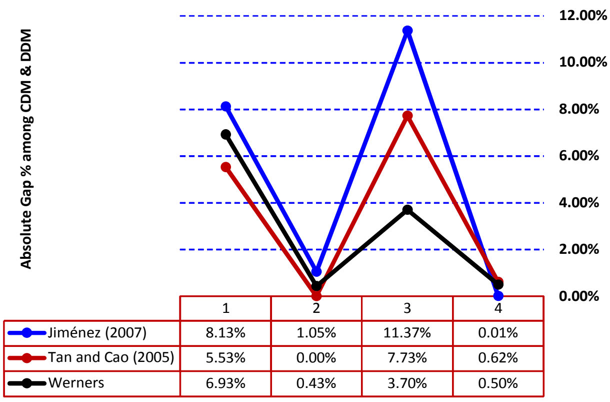
Figure 4 (a) shows absolute gap% among the CDM and DDM approaches in supply chain for each solution method and each data set. As shown in Figure 4(a) the smallest gap is seen in second data set while solved using Tan and Cao (2005) approach. The largest gap is seen in third data set while solved by Jiménez *et al.* (2007) approach.

The average gap for each solution method is represented in Figure 4(b). The average gap of all solution methods are similar and this shows that there is no significant difference between three fuzzy mathematical programming approach while handling CDM and DDM approaches. Moreover, the average gap of solution methods is near to 5% which is assumed as a suitable result. This means that the DDM and CDM approaches have very close results. This justify the application of proposed DDM approach.

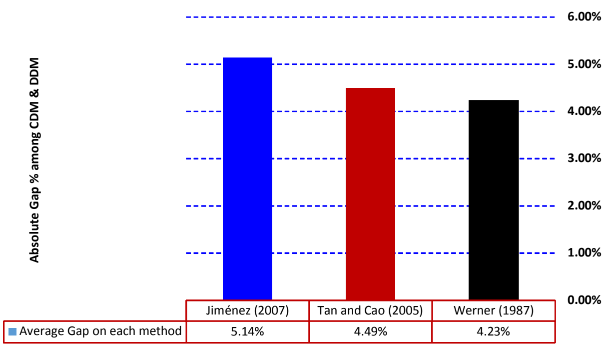


(a) Centralized method (b) Decentralized methods

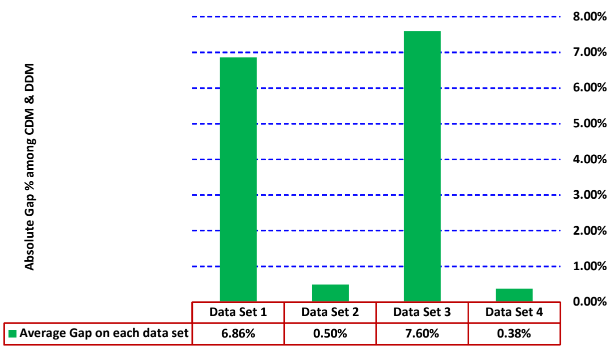
Figure 3. Objective function values for both centralized and decentralized methods.



(a) Gap analysis: all methods-all data sets



(b) Average gap: each method



(c) Average gap: each data set

Figure 4. Gap Analysis between objective function of DDM and CDM.

The average gap for each data is represented in Figure 4(c). The average gap of data set 2 and data set 4 are very small while average gap of data set 1 and data set 3 are close to 7%. The overall results illustrate that there is no significant difference between performance of CDM and DDM approaches while different data sets are used.

The main results concluded based on Figure 4 is that the DDM approach has a suitable and acceptable performance in comparison with CDM approach while the work load of the central unit in supply chain is interestingly reduced and the manufacturer and retailers can make the associated decisions independently through proposed coordination mechanism.

6. CONCLUSION REMARKS AND FUTURE RESEARCH DIRECTIONS

Optimization methods which enable collaborative decision making among the various levels of supply chain (i.e., suppliers, manufacturers, wholesalers, distributors, and retailers) are becoming increasingly more necessary in the present competitive environment. As a matter of fact, supply chain is the upstream fraction of the value chain activities. Also decision making approaches play significant role in this case. Hence, Decentralized/ Distributed Decision Making (DDM) approach has become a very critical issue towards efficient supply chain management. In DDM approach each level of supply chain may take the associated decisions independently while the management of the chain just facilities the co-ordination of the levels. In such situation, the workload of the central unit in supply chain is interestingly reduced while the levels have a certain amount of independency. Moreover, the reliability of supply chain will increased in DDM approach in comparison with CDM approach, as the central unit is not directly responsible for all decisions made, but it supervise and ease the decisions made. In contrast, the CDM approach all decisions are made in central unit and distributed among the levels, so failure of central unit means the failure of all or the main parts of the chain. Although it is notable that the integration and consistency of decision made in CDM approach cannot be neglected while the independency of DDM approach may cause some inconsistency in chain. The main question is that whether there is a possibility to propose an approach which inherits the advantages of both DDM and CDM approaches, i.e., the distributed work load and independent decision from DDM and consistency and integration form CDM? Moreover, as it is known, in real life supply chain problems several parameters of the chain may involve with a notable amount if uncertainty.

In this research a deterministic CDM approach was extended into uncertain situation. The uncertain CDM approach was modeled using fuzzy mathematical programming. Then, a DDM approach including manufacturer's model and retailers' model, were developed. In order to take the advantages of a CDM approach in the proposed DDM approach a coordination mechanism was attached. On the other hand, the DCM approach and the associated optimization models were equipped through a coordination mechanism developed among manufacturers and retailers. Both approaches, i.e., CDM and DDM, were developed to handle demand uncertainty by applying possibility theory parameterized through fuzzy sets. Finally, three fuzzy mathematical optimization procedures were proposed to solve the numerical example of both CDM and DDM approaches. The results were discussed and comparison was made on the results of centralized and decentralized models.

The main contribution of this research are as follows:

1) proposing a DDM approach for a multi-product sup-

ply chain planning problem while the production and distribution problems in supply chain are addressed independently in manufacturer and retailers;
2) Extension of both CDM and DDM approaches in presence of demand uncertainty using fuzzy sets;
3) Introducing a coordination mechanism in the proposed DDM approach in order to handle problems related to production and distribution in a two-echelon supply chain network through a decentralized approach and in order to utilize the benefits of a CDM approach while using DDM approach;
3) Modeling the aforementioned problem through fuzzy mathematical programming;
4) Comparing the performance of proposed DDM and a customized uncertain CDM approach on multi-product supply chain planning;
5) Applying three fuzzy mathematical optimization methods in order to address and compare the performance of both DDM and CDM approaches.

Moreover, one of the main contributions of the developed application is that it enables to show an insightful tool for decision makers dealing with uncertainty. One of the goals of this research was to illustrate how a proposed DDM for a supply chain planning problem under uncertainty can obtain solutions very close to those obtained by the centralized model. The results of these study showed that the DDM approach can present qualified solutions in comparison wit the solutions generated by CDM solutions while the managers utilize the benefits of a DDM approach for decision making in uncertain situations.

The main limitation and assumptions of these study are listed as below:
1) The coordination mechanism in this study was organized for a two echelon supply chain including production and distribution process, this may be extended for three or more echelons in future researches.
2) The uncertainty was considered in demand of the chain, while other parameters of the chains may have considerable amount of uncertainty. This can be conduct a future research.
3) They uncertainty was modeled through possibility set theory (i.e., fuzzy sets) parameterized through fuzzy numbers. Other types of uncertainty, including probabilistic theory or interval data can be investigated in future research works.
4) As a new concepts, i.e., equipment of DDM approach using a coordination mechanism under uncertainty in order to utilize advantages of both DDM and CDM approaches, were considered in this research, so a numerical example was investigated. In the future work, a real case study can be used in order to test the applicability of proposed approach.

It is expected that when the size of the numericalinstance grows, the CPU time of proposed solution approaches will be greater. The application of metaheuristics approaches and other soft computing techniques could be investigated by further research in order to consider large scale instances. Furthermore, other parameters, such as costs could be taken into consideration in uncertain environment.

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Appendix A: Simulated Data Set 1

Parameters regarding plants and items

	$k=1$						$k=2$					
	c_{ik}	s_{ik}	o_{ik}	u_{ik}	h_{ik}^p	γ_{ik}	c_{ik}	s_{ik}	o_{ik}	u_{ik}	h_{ik}^p	γ_{ik}
$i=1$	50	1500	101.4	24.2	10	1	70	2100	190.3	23.1	14	1
$i=2$	55	1650	160.5	6.8	11	1	75	2250	153.9	19.3	15	1
$i=3$	60	1800	130.0	14.8	12	1	80	2400	186.1	28.4	16	1

Parameters regarding retailers and items

	$k=1$			$k=2$		
	p_{jk}	h_{jk}^r	v_{jk}	p_{jk}	h_{jk}^r	v_{jk}
$j=1$	1765	176.5	264.75	1084	108.4	162.6
$j=2$	1654	165.4	248.1	1873	187.3	280.95

Parameters regarding demand of items in planning periods

	$t=1$		$t=2$		$t=3$		$t=4$		$t=5$	
	E_{jkt}	F_{jkt}	E_{jkt}	F_{jkt}	E_{jkt}	F_{jkt}	E_{jkt}	F_{jkt}	E_{jkt}	F_{jkt}
$j=1,\ k=1$	154.2	231.3	123.5	185.25	94.4	141.6	48.4	72.6	176.3	264.45
$j=1,\ k=2$	67.2	100.8	184.3	276.45	146.7	220.05	167.5	251.25	152.0	228
$j=2,\ k=1$	158.9	238.35	100.1	150.15	54.1	81.15	130.5	195.75	77.5	116.25
$j=2,\ k=2$	54.2	81.3	33.7	50.55	174.8	268.2	63.5	95.25	110.7	166.05

Parameter regarding unit transportation cost of items between plants and retailers

d_{ijk}	$k=1$	$k=2$
$i=1,\ j=1$	2376.3	3720.2
$i=2,\ j=1$	2784.6	2876.3
$i=3,\ j=1$	2567.7	2984.9
$i=1,\ j=2$	2387.3	3982.6
$i=2,\ j=2$	3093.4	3401
$i=3,\ j=2$	3937.5	3743.5

Note: *Inventory capacity at a retail outlet is set by* $w_j^r = Max_t \sum_t E_{jkt}$ $j=1, 2, \cdots, j$ *(i.e.,* $w_1^r = 328.3,\ w_2^r = 228.9$*) Core demand is generated from a uniform distribution on [0, 200]. Forecasted demand is determined by* $1.5 \times E_{jkt}$*. Capacity of all vehicles is set to 200. Fixed cost per vehicle is set to 2,500. For the costs of set-up, holding at a plant and a retail outlet, and stock out,* $h_{ik}^p = 0.2c_{ik}$*,* $h_{jk}^r = 0.1p_{jk}$ *and* $v_{jk} = 0.15p_{jk}$ *are assigned, respectively. Processing time for item k at plant i is generated from a uniform distribution on [100, 200]. Setup time for item k at plant i is generated from a uniform distribution on [0, 30]. Unit selling price of item k at retailer j is generated from a uniform distribution on [1,000, 2,000].*