

Normalizing interval data and their use in AHP

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Entani and Tanaka (2007) presented a new approach for obtaining interval evaluations suitable for handling uncertain data. Above all, their approach is characterized by the normalization of interval data and thus the elimination of redundant bounds. Further, interval global weights in AHP are derived by using such normalized interval data. In this paper, we present a heuristic method for finding extreme points of interval data, which basically extends the method by Entani and Tanaka (2007), and also helps to obtain normalized interval data. In the second part of this paper, we show that the solutions to the linear program for interval global weights can be obtained by a simple inspection. In the meantime, the absolute dominance proposed by the authors is extended to pairwise dominance which makes it possible to identify at least more dominated alternatives under the same information.

Key Words : Normalization; Interval local and global weights, Interval priority weights, Extreme points, Pairwise dominance

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Received : June 7, 2016 Revised : June 10, 2016 Accepted : June 13, 2016
Publication Type : Concise paper Corresponding Author : Byeong Seok Ahn

1. Introduction

In decision-making under incomplete information, we often encounter interval expression about decision parameter(s) intended to alleviate the burdens of precisely specifying them due to reasons of time pressure, lack of data and domain knowledge, limited attention and information processing capabilities and so on. The interval weights in multi-criteria decision-making (MCDM) problems may be sometimes inconsistent in the sense that there are no feasible weights available,

or redundant in the sense that a range of interval weights can be tightened with no loss of information. The inconsistency of interval weights is easily checked by computing the sum of lower and upper bounds. Thus, if the sum of lower bounds exceeds one or if the sum of upper bounds is less than one, the interval weights turn out to be inconsistent because there are no feasible weights. The redundancy that has to be removed, however, is somewhat difficult to identify. With regard to this, we find that Campos, Huete, and Moral (1994) presented a number of basic operations

necessary to develop a calculus with probability intervals as an interesting tool to represent uncertain information. In the meantime, they proposed a recursive algorithm to develop the extreme probabilities by using an implicit tree search where each node is a partial probabilities and a child node represents a refinement of its parent node by increasing one component of probabilities. Wang and Elhag (2006) introduced normalization methods for interval weights which are classified into independent and dependent subject to the requirement of sum to unity constraint. They further extended their methods to fuzzy weights by using α -cuts and the extension principle. Some errors in their development were pointed out and corrected by Li, Wang, and Li (2009). Pavlačka (2014) studied the problem of normalization of a fuzzy vector of weights that expresses the joint probability distribution of initial weights.

Entani and Tanaka (2007) introduced a new efficient normalization method for interval weights while we find numerous normalization methods for crisp data. See more on this in Sugihara et al., (2004) and Tanaka et al., (2004).

This paper aims not to criticize their approach but to suggest a heuristic method to normalize interval data via extreme points, which also helps to achieve the normalization of interval data and thus the elimination of redundant bounds. To do so, we present two methods which are distinct from previous methods. In the first, we formulate a linear program (LP) for normalizing interval

weights based on Entani and Tanaka (2007). The optimal solution to the LP program simply reveals the normalized interval weights. Another approach is to find their extreme points by inspecting the end points (i.e., lower and upper bounds) of interval weights, based on the fact that, given n interval weights, every extreme point is composed of at least $(n - 1)$ end points.

Further, in the final stage of obtaining interval global weights in the framework of the analytic hierarchy process (AHP), we solve LPs by a simple inspection since they belong to a simple knapsack problem. The individual approach proposed by the authors is also extended to pairwise dominance which makes it possible to identify at least more dominated alternatives under the given information.

This paper is organized as follows. In Section 2, we present an LP to normalize interval weights on the basis of Entani and Tanaka (2007) and a heuristic method to find their extreme points. In Section 3, along with the absolute dominance, pairwise dominance applies when referenced priority weights and local weights are specified by interval weights. Concluding remarks follow in Section 4.

2. Normalizing interval weights and finding their extreme points

Entani and Tanaka (2007) presented an efficient normalization method to eliminate any redundancy

in interval weights with no loss of information.

Definition 1. An interval weight vector $W = (W_1, \dots, W_n)$ is said to be normalized if and only if

$$\sum_{i=1}^n \bar{w}_i - \max_j (\bar{w}_j - \underline{w}_j) \geq 1, \forall j$$

$$\sum_{i=1}^n \underline{w}_i + \max_j (\bar{w}_j - \underline{w}_j) \leq 1, \forall j$$

where $W_i = [\underline{w}_i, \bar{w}_i], \forall i$.

The inequalities in Definition 1 can be equivalently rewritten as follows:

$$\sum_{i \neq j} \bar{w}_i + \underline{w}_j \geq 1, \forall j$$

$$\sum_{i \neq j} \underline{w}_i + \bar{w}_j \leq 1, \forall j.$$

Given the interval weight vector W , a normalized interval weight vector $W' = (W'_1, \dots, W'_n)$ can be obtained by solving the following LP problem (P1):

(P1): An LP for normalizing interval weights

$$\text{Maximize } \sum_{i=1}^n (\bar{w}'_i - \underline{w}'_i)$$

s.t.

$$\sum_{i \neq j} \underline{w}'_i + \bar{w}'_j \leq 1, \forall j$$

$$\sum_{i \neq j} \bar{w}'_i + \underline{w}'_j \geq 1, \forall j$$

$$\bar{w}'_j \leq \bar{w}_j, \underline{w}'_j \geq \underline{w}_j, \forall j$$

$$\underline{w}'_j \geq \underline{w}'_j, \forall j$$

Suppose that a set of interval weights W is specified as

$$W = \{0.2 \leq W_1 \leq 0.4, 0.1 \leq W_2 \leq 0.3, 0.4 \leq W_3 \leq 0.7, 0.2 \leq W_4 \leq 0.3, \sum_{i=1}^4 W_i = 1\}.$$

The first condition for the feasibility of these weights is to satisfy $\sum_{i=1}^4 \underline{w}_i \leq 1$ and

$\sum_{i=1}^4 \bar{w}_i \geq 1$ and the weights-set W passes the feasibility test by yielding $\sum_{i=1}^4 \underline{w}_i = 0.9 < 1$ and $\sum_{i=1}^4 \bar{w}_i = 1.7 > 1$. Subsequently, we formulate an LP problem to obtain a tightened weights-set $W' = [\underline{w}'_i, \bar{w}'_i], i = 1, \dots, 4$ as follows:

$$\text{Maximize } \sum_{i=1}^4 (\bar{w}'_i - \underline{w}'_i)$$

s.t.

$$\bar{w}'_1 + 0.7 \leq 1, \bar{w}'_2 + 0.8 \leq 1, \bar{w}'_3 + 0.5 \leq 1, \bar{w}'_4 + 0.7 \leq 1$$

$$\underline{w}'_1 + 1.3 \geq 1, \underline{w}'_2 + 1.4 \geq 1, \underline{w}'_3 + 1 \geq 1, \underline{w}'_4 + 1.4 \geq 1$$

$$\bar{w}'_1 \leq 0.4, \bar{w}'_2 \leq 0.3, \bar{w}'_3 \leq 0.7, \bar{w}'_4 \leq 0.3$$

$$\underline{w}'_1 \geq 0.2, \underline{w}'_2 \geq 0.1, \underline{w}'_3 \geq 0.4, \underline{w}'_4 \geq 0.2$$

$$\bar{w}'_1 \geq \underline{w}'_1, \bar{w}'_2 \geq \underline{w}'_2, \bar{w}'_3 \geq \underline{w}'_3, \bar{w}'_4 \geq \underline{w}'_4$$

The optimal solution to the problem is to cut down the upper bounds of $W_i, i = 1, 2, 3$ from 0.4 to 0.3, from 0.3 to 0.2, and from 0.7 to 0.5 respectively, as can be seen in W' below:

$$W' = \{0.2 \leq W'_1 \leq 0.3, 0.1 \leq W'_2 \leq 0.2, 0.4 \leq W'_3 \leq 0.5, 0.2 \leq W'_4 \leq 0.3, \sum_{i=1}^4 W'_i = 1\}.$$

Therefore, we can use W' instead of W equivalently when solving any decision-making problems constrained by W .

Normalizing interval weights by eliminating redundant (upper or lower) bounds implies that such removed bounds never appear in the set of extreme points of interval weights. In other words, we can obtain normalized interval weights by finding extreme points of interval weights and then

selecting their coordinate-wise maximum and minimum values. Moreover, finding the extreme points of interval weights is not only helpful in removing, if any, their redundant bounds but also in solving decision-making problems with interval weights as shown in Section 3.

Now, we present how to find the extreme points of interval weights (see also Arbel (1989) for the case of ratio bounds) via LP (P2) below, which has any permutation in the objective function coefficients (Ahn and Park, 2014):

(P2): Enumeration method

Maximize $\sum_{i=1}^n \sigma(i) W_i$

s.t. $W_i \in W = \{w_i \leq W_i \leq \bar{w}_i, i = 1, \dots, n,$

$W_1 + \dots + W_n = 1\}$

where $\sigma(i)$ is the i th element of a permutation of $I = \{1, 2, \dots, n\}$.

Then, the optimal solution to (P2) corresponding to one extreme point is obtained by allotting w_i to all W_i and successively allotting the residual weight $(1 - \sum_{i=1}^n w_i)$ to W_i having the largest $\sigma(i)$ to its fullest extent, then to the second largest $\sigma(i)$, \dots until the residual weight is completely used. In this manner, we can find all the extreme points by permuting objective function coefficients. In the worst-case, one has to consider $n!$ different programs even though many extreme points are surely duplicated. In general, however, one only has to consider a smaller number of programs unless the sum of lower bounds is too

small to consider many different permutations. Later, we present how to find all extreme point via a heuristic approach instead of the enumeration method.

Let us find the extreme points of W :

$W = \{0.2 \leq W_1 \leq 0.4, 0.1 \leq W_2 \leq 0.3, 0.3 \leq W_3 \leq 0.5, 0.2 \leq W_4 \leq 0.3, \sum_{i=1}^4 W_i = 1\}$.

Considering the LP problem

$\max\{4W_1 + 3W_2 + 2W_3 + 1W_4 | W_i \in W, i = 1, \dots, 4\}$,

we first allot the lower bounds of the interval weights to each W_i , yielding $(W_1, W_2, W_3, W_4) = (0.2, 0.1, 0.3, 0.2)$ with the residual weight

0.2. Then, we allot 0.2 to W_1 because it has the largest coefficient and can also accommodate it.

The resulting extreme point is rendered to be $(0.4, 0.1, 0.3, 0.2)$. Since the residual weight is

completely allotted in W_1 , there is no need to consider other permutations starting from 4, which

saves us $3!$ problems to solve. When the coefficient 4 is placed in the second or the third position in the objective function, all the residual

weight 0.2 can be allotted to W_2 or W_3 , thus yielding the extreme points $(0.2, 0.3, 0.3, 0.2)$

and $(0.2, 0.1, 0.5, 0.2)$ respectively. Finally, we can allot only 0.1 to W_4 to its fullest with the

coefficient 4 in the fourth and accordingly, the residual weight 0.1 should be allotted to W_i

having the next largest coefficient 3, which yields three extreme points as follows:

$(0.3, 0.1, 0.3, 0.3), (0.2, 0.2, 0.3, 0.3), (0.2, 0.1, 0.4, 0.3)$.

In summary, we list all the extreme points in a matrix M :

$$M = (m_{ij}) = \begin{pmatrix} 0.2 & 0.2 & 0.3 & 0.2 & 0.2 & 0.4 \\ 0.1 & 0.2 & 0.1 & 0.3 & 0.1 & 0.1 \\ 0.4 & 0.3 & 0.3 & 0.3 & 0.5 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.2 & 0.2 & 0.2 \end{pmatrix} \quad (1)$$

Every extreme point in M shares some common feature in that it is composed of either $(n-1)$ (i.e., the first three columns) or n end points of W_i (i.e., the remaining three columns), which is restated in Theorem 1.

Theorem 1. If interval weights $W = (W_1, \dots, W_n)$, $W_i = [\underline{w}_i, \bar{w}_i] \forall i$ are consistent, their extreme points are determined by selecting at least $(n-1)$ end points (lower or upper bounds) of the interval weights.

Proof. If interval weight vector is consistent, it holds that $\sum_{i=1}^n \underline{w}_i \leq 1$ and $\sum_{i=1}^n \bar{w}_i \geq 1$. If $\sum_{i=1}^n \underline{w}_i = 1$, then $\bar{w}_i = \underline{w}_i$, $i = 1, \dots, n$, thus yielding only one extreme point. If $\sum_{i=1}^n \underline{w}_i < 1$, we successively assign remaining weight depending on the magnitude of any permuted coefficients if available according to (P2), starting with the lower bounds. If the remaining weight is completely used at the largest coefficient, then the allocation ends and the extreme points is constructed by using n end points. If there is still remaining weight since the upper bound corresponding to the coefficient is less than the

remaining weight, it will be allocated next largest coefficient, which yields extreme point composed of n end points if the upper bound of second largest coefficient is equal to the remaining weight and $(n-1)$ otherwise. In this manner, we can conclude any extreme points are constructed by using at least $(n-1)$ end points, noting that the specific permutation is not assumed (Ahn and Park, 2014).

Corollary 1. All extreme points of interval weights can be identified with a finite number of inspections of end points, namely $\sum_{i=0}^{n-1} \binom{n}{i}$ in the case of using n end points plus $\sum_{i=1}^{n-2} \binom{n}{i} \binom{n-i}{n-i-1}$ in the case of using $(n-1)$ end points.

Proof. The consistent and normalized interval weights are transformed to ones with zero lower bounds by making the change of variables $W'_i = W_i - \underline{w}_i$, $i = 1, \dots, n$, thus resulting in $0 \leq W'_i \leq \bar{w}_i - \underline{w}_i$, $i = 1, \dots, n$, $\sum_{i=1}^n W'_i = 1 - \sum_{i=1}^n \underline{w}_i \geq 0$. The number of inspections for n end points counts (1) all the upper bounds, (2) any one lower bound (i.e., zero) and all the other upper bounds, (3) any two lower bounds and all the other upper bounds and so on, which sums to $\sum_{i=0}^{n-1} \binom{n}{i}$. The number of inspections for $(n-1)$ end points counts (1) any one of n zero

bounds with any $(n-2)$ upper bounds among $(n-1)$ upper bounds plus one interior weight, (2) any two of n zero bounds with any $(n-3)$ upper bounds among $(n-2)$ upper bounds plus one interior weight, and so on, which sums to $\sum_{i=1}^{n-2} \binom{n}{i} \binom{n-i}{n-i-1}$. Combining these results proves the assertion of Corollary 1.

Consider a set of interval weights W again, which was already shown in the enumeration method:

$$W = \{0.2 \leq W_1 \leq 0.4, 0.1 \leq W_2 \leq 0.3, 0.3 \leq W_3 \leq 0.5, 0.2 \leq W_4 \leq 0.3, \sum_{i=1}^4 W_i = 1\}.$$

First, we convert W to W' of which the lower bounds are all set to zero by letting $W'_i = W_i - \underline{w}_i$, $i = 1, \dots, 4$:

$$W' = \{0 \leq W'_1 \leq 0.2, 0 \leq W'_2 \leq 0.2, 0 \leq W'_3 \leq 0.2, 0 \leq W'_4 \leq 0.1, \sum_{i=1}^4 W'_i = 0.2\}.$$

At this point, we can find all extreme points based on Theorem 1 and Corollary 1. But, for computational ease, we further make the change of variables to convert W'_i s to integer-valued ones, say $W''_i = 10W'_i$, $i = 1, \dots, 4$:

$$W'' = \{0 \leq W''_1 \leq 2, 0 \leq W''_2 \leq 2, 0 \leq W''_3 \leq 2, 0 \leq W''_4 \leq 1, \sum_{i=1}^4 W''_i = 2\}.$$

It is noteworthy that only integer-valued W''_i is allowed to be an element of extreme point. Therefore, for example, a vector $(0.5, 0.5, 0, 1)$

cannot be chosen an extreme point of W'' since it violates Theorem 1. According to Corollary 1, we can find two sets of extreme points, E_1 and E_2 comprised of four and three end points respectively,

$$E_1 = \{(2, 0, 0, 0), (0, 2, 0, 0), (0, 0, 2, 0)\}$$

$$E_2 = \{(1, 0, 0, 1), (0, 1, 0, 1), (0, 0, 1, 1)\}.$$

A sequence of operations to go back to W_i , specifically, dividing each element of extreme point by 10 and then adding lower bounds will result in the extreme points as shown in (1).

Consider another interval weights given as $W_1 = [0.12, 0.35]$, $W_2 = [0.25, 0.45]$, and $W_3 = [0.32, 0.41]$ with $W_1 + W_2 + W_3 = 1$. If

we perform a sequence of operations such that we subtract each lower bound from W_i and then multiply the resulting weights by 100,

it follows that $W''_1 = [0, 23]$, $W''_2 = [0, 20]$, and $W''_3 = [0, 9]$ with $W''_1 + W''_2 + W''_3 = 31$.

Then, reverse operations reveal the extreme points in terms of W''_i and W_i respectively:

$(23, 8, 0), (23, 0, 8), (11, 20, 0), (2, 20, 9), (22, 0, 9)$

in terms of W''_i and $(0.35, 0.33, 0.32), (0.35, 0.25, 0.40), (0.34, 0.25, 0.41), (0.23, 0.45, 0.32), (0.14, 0.45, 0.41)$ in terms of W_i . The normalized interval weights are thus shown by $W_1 = [0.14, 0.35]$, $W_2 = [0.25, 0.45]$, and $W_3 = [0.32, 0.41]$.

The component-wise minimum and maximum of extreme points give the normalized interval

weights, specifically, $W_i = [\min_j(m_{ij}), \max_j(m_{ij})]$, $i = 1, \dots, n$ where m_{ij} is the i th component of the j th extreme point. In the matrix M in (1), we find that a set of original interval weights is already normalized.

3. Identifying non-dominated alternatives

In the framework of the AHP, a decision-maker provides pairwise preference judgments between alternatives $A_i, i = 1, \dots, n$ with respect to each criterion and also pairwise preference judgments between criteria $C_k, k = 1, \dots, m$. According to Entani and Tanaka (2007), the local weight of alternative A_i under criterion C_k is denoted as $W_{ki} = [\underline{w}_{ki}, \bar{w}_{ki}]$, $\forall i$, and the referenced priority weight of criterion C_k is denoted as $P_k = [\underline{p}_k, \bar{p}_k]$, $\forall k$ with a single layer of hierarchy in the AHP. Then, a global weight of alternative A_i is in the form of interval weights since each input variable is specified by interval weights:

$$\text{Maximize (Minimize) } \sum_{k=1}^m P_k W_{ki}$$

- 1) Note that the constraints $\sum_{i=1}^n W_{ki} = 1, k = 1, \dots, m$ are unnecessary since each W_{ki} is assumed to be normalized.
- 2) Note that we would have to use w_{ki}^* instead of \bar{w}_{ki} for a maximization problem and w_{ki}^0 instead of \underline{w}_{ki} for a minimization problem if W_{ki} was not normalized:

$$w_{ki}^* = \max \{W_{kj} : \underline{w}_{kj} \leq W_{kj} \leq \bar{w}_{kj}, j = 1, \dots, i, \dots, n, \sum_{j=1}^n W_{kj} = 1\}$$

$$w_{ki}^0 = \min \{W_{kj} : \underline{w}_{kj} \leq W_{kj} \leq \bar{w}_{kj}, j = 1, \dots, i, \dots, n, \sum_{j=1}^n W_{kj} = 1\}.$$

s.t.

$$\sum_{k=1}^m P_k = 1$$

$$\underline{p}_k \leq P_k \leq \bar{p}_k, \forall k$$

$$\underline{w}_{ki} \leq W_{ki} \leq \bar{w}_{ki}, \forall k$$

where W_{ki} are normalized weights according to Definition 1¹⁾.

The programs proposed by Entani and Tanaka (2007) are further simplified by substituting \bar{w}_{ki} in a maximization and \underline{w}_{ki} in a minimization problem for W_{ki} , thus yielding equivalent LPs:

(Absolute dominance)

$$UB_i = \bar{w}_i = \text{Maximize } \sum_{k=1}^m P_k \bar{w}_{ki}$$

$$(LB_i = \underline{w}_i = \text{Minimize } \sum_{k=1}^m P_k \underline{w}_{ki})^2)$$

s.t.

$$\sum_{k=1}^m P_k = 1$$

$$\underline{p}_k \leq P_k \leq \bar{p}_k, \forall k$$

The LPs can be solved by inspection since they belong to a simple knapsack problem and detailed explanation is given later in pairwise dominance. According to pairwise dominance, alternative A_i at least dominates A_j if for any fixed set of feasible weights, the worst outcome in A_i at least exceeds

the best outcome in A_j .

Definition 2: Alternative A_i is at least preferred to A_j if and only if $\tau_{min}(A_i, A_j) \geq 0$ where

$$\tau_{min}(A_i, A_j) = \min\{\sum_{k=1}^m P_k(\underline{w}_{ki}(A_i) - \bar{w}_{kj}(A_j))\};$$

$$\sum_{k=1}^m P_k = 1, \underline{p}_k \leq P_k \leq \bar{p}_k, \forall k\}.$$

The set of dominated alternatives resulted from absolute dominance is a subset of the set of dominated alternatives from pairwise dominance due to the fact that $\tau_{min}(A_i, A_j) \geq LB_i - UB_j$ (Ahn, 2006). Thus, if alternative A_i absolutely dominates A_j , then A_i pairwise dominates A_j , but the reverse does not hold. Furthermore, the LP in Definition 2 also belongs to a class of knapsack problem and thus can be solved by the following procedure:

Step 1. Allot \underline{p}_k to all $(\underline{w}_{ki} - \bar{w}_{kj})$.

Step 2. Allot the residual weight $(1 - \sum_{k=1}^m \underline{p}_k)$ to the lowest $(\underline{w}_{ki} - \bar{w}_{kj})$ for some k if $(1 - \sum_{k=1}^m \underline{p}_k) \leq \bar{p}_k$; otherwise, allot $\bar{p}_k - (1 - \sum_{k=1}^m \underline{p}_k)$.

Successively allot the residual weight to the next lowest $(\underline{w}_{ri} - \bar{w}_{rj})$ for some $r \neq k$ and continue this step until all the residual weights are used up.

To exemplify this procedure, suppose that the referenced priority weights and local weights of two alternatives (A_1 and A_2) are given as in Table 1.

(Table 1) Referenced priority weights and local weights of two alternatives

Criteria	C_1	C_2	C_3
Referenced priority weights			
	[0.3, 0.4]	[0.2, 0.3]	[0.4, 0.5]
Local weights			
A_1	[0.25, 0.3]	[0.3, 0.4]	[0.45, 0.5]
A_2	[0.2, 0.3]	[0.4, 0.45]	[0.2, 0.3]

To establish pairwise dominance between A_1 and A_2 , we solve the following LP:

$$\tau_{min}(A_1, A_2) = \text{Minimize } P_1(0.25 - 0.3) + P_2(0.3 - 0.45) + P_3(0.45 - 0.3)$$

s.t.

$$0.3 \leq P_1 \leq 0.4, \quad 0.2 \leq P_2 \leq 0.3, \quad 0.4 \leq P_3 \leq 0.5,$$

$$P_1 + P_2 + P_3 = 1$$

First, we allot 0.3 to P_1 , 0.2 to P_2 , and 0.4 to P_3 . Next, we allot the residual weight of 0.1 to P_2 having the lowest coefficient, say $\min[-0.05, -0.15, 0.15] = -0.15$. The optimal weighting vector turns out to be $(P_1^*, P_2^*, P_3^*) = (0.3, 0.3, 0.4)$ with $\tau_{min}(A_1, A_2) = 0$, which means that A_1 at least dominates A_2 on the basis of pairwise dominance. The interval global priorities for alternatives A_1 and A_2 on the basis of absolute dominance will be $[\underline{w}_1(A_1), \bar{w}_1(A_1)] = [0.34, 0.42]$ and $[\underline{w}_2(A_2), \bar{w}_2(A_2)] = [0.24, 0.345]$ respectively. Even though the intervals show a strong tendency

of A_1 over A_2 (a very small overlapping between $\underline{w}_1(A_1)$ and $\overline{w}_2(A_2)$), it cannot be said that A_1 dominates A_2 based on absolute dominance.

4. Concluding remarks

In this paper, we presented an LP formulation to normalize interval weights, inspired by Entani and Tanaka (2007). We also showed that this goal can be achieved by finding their extreme points and presented how they can be found. Knowing the extreme points is not only helpful in removing redundant bounds which may be present in the interval weights but also in solving decision-making problems with interval weights. The absolute dominance proposed by the authors is also extended to pairwise dominance which makes it possible to identify at least more dominated alternatives under the same information.

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국문요약

구간데이터 정규화와 계층적 분석과정에서의 활용

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Entani and Tanaka(2007)는 불확실한 데이터를 처리하기에 적합한 구간 평가결과를 얻는 새로운 방법을 제시하였다. 무엇보다 그들의 방법은 구간 데이터를 정규화하여 redundant 데이터를 제거하는 데 특징이 있다. 더 나가 정규화된 구간데이터를 활용하여 계층분석과정(AHP)에서 최종 구간 우선순위벡터를 도출한다. 이 논문에서는 구간 데이터의 정규화 목적을 달성하기 위해 구간 데이터의 꼭지점을 구하는 쉽고 간편한 휴리스틱 방법을 제시한다. 한편 간단한 검사법을 활용하여 정규화된 데이터를 활용하여 최종 구간 우선순위벡터를 도출하는 방법을 제시하고자 한다. 아울러 Entani and Tanaka (2007)가 제시한 대안간 지배관계 규명 방법을 확장한 지배관계 규명 방법을 제시하고자 한다.

주제어 : 정규화, 최종 구간 우선순위 가중치, 꼭지점, 쌍비교 지배관계

논문접수일 : 2016년 6월 7일 논문수정일 : 2016년 6월 10일 게재확정일 : 2016년 6월 13일
원고유형 : 단편논문 교신저자 : 안병석

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