A CLOSED-FORM SOLUTION FOR LOOKBACK OPTIONS USING MELLIN TRANSFORM APPROACH

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Abstract. Lookback options, in the terminology of finance, are a type of exotic option with path dependency whose the payoff depends on the optimal (maximum or minimum) underlying asset's price occurring over the life of the option. In this paper, we exploit Mellin transform techniques to find a closed-form solution for European lookback options in Black-Scholes model.

1. Introduction

Look-back options are one of the most popular path-dependent derivatives traded in markets all over the world. The payoffs of these options depend on the realized minimum or maximum asset price over the life of the option. Even though both analytic and numerical researches have done on the look-back options, we still have a lot of open problems to solve them. The significant contributions on the derivation of the closed solution of look-back option under Black-Scholes framework can be given as follows. Goldman et al. [6] and Conze and Viswanathan [2] studied exact formula for floating and fixed strike lookback options by utilizing the probabilistic approaches. Dai et al. [3] derived a closed-form solution for quanto lookback options. Also, He et al. [7] obtained joint density functions to apply them to numerical analysis or Monte-Carlo simulation for lookback option pricing.

This paper studies a technical work on the pricing of lookback options under Mellin transform method. The Mellin transform is an integral transform, which is regarded as the multiplicative version of the two-sided Laplace transform. Up to now, to find the analytic formula for the valuation of options, many researchers have used mainly probabilistic techniques. However, the pricing of a given option with probabilistic approaches requires the complexity of the calculation. To resolve the problem, we exploit the analytic approach using Mellin

transforms, which can be a very useful tool for the transformation of partial differential equations. In fact, Panini and Srivastav [9] derived the pricing formula of European and American vanilla options and basket options with the Mellin transform. Also, Panini and Srivastav [10] obtained the pricing of perpetual American options using Mellin transform techniques. Frontczak [5] applied the technique of the Mellin transforms to a partial integro-differential equation and found the pricing formula of options with jump diffusion models. Elshegmani and Ahmed [4] obtained analytical solution for an arithmetic Asian option using Mellin transforms. Also, Yoon and Kim [12] used double Mellin transforms to study European vulnerable options under stochastic (the Hull-White) interest rates as well as constant interest rate and Yoon [11] found a closed-form solution for European options in Black-Scholes model with stochastic interest rate using Mellin transform.

In this paper, we derive a closed formula for European lookback option (Floating strike lookback option) in Black-Scholes model using Mellin transform method. Before discussing this, we should consider the method of images mentioned in [1]. The method of images is closely connected with the reflection principle of the expectations solution. Based upon the PDE method of images, Buchen [1] derived the pricing formula of Barrier options more easily than the existing method. Using the method of images enables us to transform the P.D.E of the European lookback option with two conditions (boundary and final condition) into the P.D.E with the final condition of the extended range of underlying asset, and then we can solve the pricing formula of lookback option using Mellin transform approaches.

This article is organized as follows. Section 2 considers a floating strike lookback put option and obtains the partial differential equation for the option. Section 3 applies the method of images and the Mellin transform to obtain a closed-form analytic solution for the lookback option. The concluding remarks are mentioned in Section 4.

2. Floating strike lookback option

2.1. Model formulation

In this section, we consider the floating strike lookback option with the following underlying asset price model under a risk-neutral probability measure Q :

$$dS_t = rS_t dt + \sigma S_t dW_t, \tag{1}$$

where S_t is an underlying asset price and W_t is a 1-dimensional standard Brownian motion. By using the notation

$$\bar{S}_t = max_{u \leq t} S_u, \qquad\qquad H(S_t, \bar{S}_t) = \bar{S}_t - S_t, \tag{2}$$

we obtain the risk-neutral price of the floating strike lookback option, which is denoted by $v(t, s, y)$ at time $t \in [0, T]$:

$$v(t, s, y) = E^{Q,s,y}[e^{-r(T-t)}H(S_T, \bar{S}_T)|\mathcal{F}_t],$$

where $\mathcal{F}_t$ is a σ-algebra generated by the standard Brownian motion W up to time t, $S_t = s$ and $\bar{S}_t = y$. Then, from Feynman-Kac formula (cf, [8]), we obtain the following Black-Scholes-Merton P.D.E for $v(t, s, y)$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2} + rs\frac{\partial v}{\partial s} - rv = 0 \tag{3}$$

with the region $\{(t, s, y) : 0 \le t < T, 0 \le s \le y\}$. Also, the final and boundary conditions are given by

$$v(T, s, y) = H(s, y), \quad 0 \le s \le y,$$
$$\frac{\partial v}{\partial y}(t, y, y) = 0, \quad 0 \le t < T, \tag{4}$$

respectively.

2.2. The review of the P.D.E method of images : Up-and-Out Barrier option

Most of all, to find the closed solution of the above floating lookback option, we should consider the P.D.E method of images as seen in [1]. By the method of image solution, the P.D.E of up-and-out Barrier option $P(t, x)$ with underlying asset $X_t = x$

$$\mathcal{L}P = 0,$$
$$P(t, B) = 0, \quad 0 \le t < T, \tag{5}$$
$$P(T, x) = f(x), \quad 0 \le x < B$$

is transformed into

$$\mathcal{L}P = 0,$$
$$P(t, B) = 0, \quad 0 \le t < T,$$
$$P(T, x) = f(x)\mathbf{1}_{\{x<B\}} - \left(\frac{B}{x}\right)^{\frac{2r}{\sigma^2}-1} f(\frac{B^2}{x})\mathbf{1}_{\{x>B\}}, \quad 0 \le x < \infty, \tag{6}$$

where $\mathcal{L} = \frac{\partial}{\partial t} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2} + rx\frac{\partial}{\partial x} - r \cdot$ (Black-Scholes operator) and $f(x)$ is an expiry payout function.

2.3. The review of the Mellin transform approach

The method of the mellin transform enables us to derive an closed solution of the floating lookback option $v(t, s, y)$. For a locally Lebesgue integrable function $g(x)$, $x \in \mathbb{R}^+$, the Mellin transform $\mathcal{M}(g(x), w)$, $w \in \mathbb{C}$ is defined by

$$\mathcal{M}(g(x), w) := \hat{g}(w) = \int_0^\infty g(x)x^{w-1}dx,$$

and if $a < Re(w) < b$ and c such that $a < c < b$ exists, the inverse of the Mellin transform is expressed by

$$g(x) = \mathcal{M}^{-1}(\hat{g}(w)) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \hat{g}(w) x^{-w} dw.$$

3. The derivation of the floating strike lookback option price : Mellin transform approach

In this section, we are going to derive the closed formula of the lookback option using the method of images and the Mellin transform techniques mentioned above. Most of all, to solve the P.D.E of (3)-(4), we have to find the final condition $v(T, s, y)$ generated from the method of images. Then, we can obtain the closed solution the P.D.E of (3)-(4) by applying the Mellin transform.

If we differentiate both sides of the equation (3) with respect to y, we have

$$\frac{\partial v_y}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v_y}{\partial s^2} + rs \frac{\partial v_y}{\partial s} - r v_y = 0, \tag{7}$$

where $v_y = \frac{\partial v}{\partial y}$, and the terminal and boundary conditions are given by

$$\begin{aligned} v_y(t, s, y)|_{s=y} &= 0, \quad 0 \le t < T \\ v_y(T, s, y) &= \frac{\partial H}{\partial y}. \quad 0 \le s < y \end{aligned} \tag{8}$$

Now, let $u(t, s, y) = v_y(t, s, y)$. Then, equation (7)-(8) yield

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial s^2} + rs \frac{\partial u}{\partial s} - r u &= 0, \\ u(t, s, y)|_{s=y} &= 0, \quad 0 \le t < T \\ u(T, s, y) &= \frac{\partial H}{\partial y}, \quad 0 \le s < y. \end{aligned} \tag{9}$$

Here, it implies that equation (9) is the up-and-out Barrier option stated in (5) by regarding y as a barrier of s. Therefore, by the method of images mentioned in section 2.2, we have the following P.D.E of $u(t, s, y)$ with the terminal condition of the extended range of the underlying asset

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial s^2} + rs \frac{\partial u}{\partial s} - r u &= 0, \\ u(T, s, y) = \frac{\partial H}{\partial y}(s, y)\mathbf{1}_{\{s<y\}} - \frac{\partial H^*}{\partial y}(s, y)\mathbf{1}_{\{s>y\}}, \quad 0 \le s < \infty. \end{aligned} \tag{10}$$

Now, from the final condition $u(T, s, y)$ of (10), we have the relational expression

$$v(T, s, y) - v(T, s, s) = \int_s^y u(T, s, \xi)d\xi$$

$$= \int_s^y \frac{\partial H}{\partial \xi}(s, \xi)\mathbf{1}_{\{s<\xi\}}d\xi - \int_s^y \frac{\partial H^*}{\partial \xi}(s, \xi)\mathbf{1}_{\{s>\xi\}}d\xi \qquad (11)$$

$$= \mathbf{1}_{\{s<y\}}\left(H(s, y) - H(s, s)\right) + \int_y^s \frac{\partial H^*}{\partial \xi}(s, \xi)\mathbf{1}_{\{s>\xi\}}d\xi.$$

Hence,

$$v(T, s, y) = H(s, s)\mathbf{1}_{\{s>y\}} + H(s, y)\mathbf{1}_{\{s<y\}} + \mathbf{1}_{\{s>y\}}\int_y^s \frac{\partial H^*}{\partial \xi}(s, \xi)d\xi. \qquad (12)$$

Therefore, from (12), the equation (3)-(4) lead to

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 v}{\partial s^2} + rs\frac{\partial v}{\partial s} - rv = 0,$$

$$v(T, s, y) = H(s, s)\mathbf{1}_{\{s>y\}} + H(s, y)\mathbf{1}_{\{s<y\}} + \mathbf{1}_{\{s>y\}}\int_y^s \frac{\partial H^*}{\partial \xi}(s, \xi)d\xi \qquad (13)$$

in the region $\{0 \le s < \infty, \quad 0 \le t \le T\}$.

Now, to solve the P.D.E of the equation (13), if we use the substitutions $z = \frac{s}{y}$ and $q(t, z) = v(t, \frac{s}{y}, 1)$ (Reduction of Dimension) then, $v(t, s, y)$ satisfies

$$v(t, s, y) = yv\left(t, \frac{s}{y}, 1\right) = yq\left(t, \frac{s}{y}\right), \quad 0 \le t \le T, \ y > 0, \qquad (14)$$

and the equation (13) leads to the following equation

$$\frac{\partial q}{\partial t} + \frac{1}{2}\sigma^2 z^2 \frac{\partial^2 q}{\partial z^2} + rz\frac{\partial q}{\partial z} - rq = 0, \quad 0 \le t < T, \quad 0 < z < \infty$$

$$\theta(z) = q(T, z) = H(z, z)\mathbf{1}_{\{z>1\}} + H(z, 1)\mathbf{1}_{\{z<1\}} + \mathbf{1}_{\{z>1\}}\int_1^z \frac{\partial H^*}{\partial \xi}(z, \xi)d\xi. \qquad (15)$$

From $H(s, y) = y - s$ in (2), we have $H(z, z) = 0$, $H(z, 1) = 1 - z$ and $\frac{\partial H}{\partial \xi}(z, \xi) = 1$. Also, the method of image solutions mentioned in [1] yields

$$\frac{\partial H^*}{\partial \xi}(z, \xi) = \left(\frac{\xi}{z}\right)^{\frac{2r}{\sigma^2}-1} \frac{\partial H}{\partial \xi}(\frac{\xi^2}{z}, \xi) = \left(\frac{\xi}{z}\right)^{\frac{2r}{\sigma^2}-1}. \qquad (16)$$

Hence, the $\theta(z)$ in (15) satisfies

$$
\theta(z) = (1-z)\mathbf{1}_{\{z<1\}} + \mathbf{1}_{\{z>1\}} \int_1^z \left(\frac{\xi}{z}\right)^{\frac{2r}{\sigma^2}-1} d\xi,
$$

$$
= (1-z)\mathbf{1}_{\{z<1\}} + \mathbf{1}_{\{z>1\}} z^{2k_3} \int_1^z \xi^{k_1-1} d\xi, \quad (\text{where } k_3 = \frac{1-k_1}{2}, \ k_1 = \frac{2r}{\sigma^2})
$$

$$
= (1-z)\mathbf{1}_{\{z<1\}} + \mathbf{1}_{\{z>1\}} z^{2k_3} \left(\frac{1-z^{k_1}}{k_1}\right)
$$

$$
= (1-z)\mathbf{1}_{\{z<1\}} - \frac{1}{k_1}\mathbf{1}_{\{z>1\}} \left(z^{2k_3}-1\right) + \frac{1}{k_1}\mathbf{1}_{\{z>1\}}(z-1).
$$

(17)

Theorem 3.1. *Under the given final and boundary conditions in (4), the closed formula of the floating strike lookback put option option is given by*

(18)
$$
v(t,s,y) = \left(1+\frac{\sigma^2}{2r}\right) s \mathcal{N}\left(d_1\left(T-t,\frac{s}{y}\right)\right) + e^{-r(T-t)} y \mathcal{N}\left(-d_2\left(T-t,\frac{s}{y}\right)\right)
$$
$$
- \frac{\sigma^2}{2r} e^{-r(T-t)} \left(\frac{y}{s}\right)^{\frac{2r}{\sigma^2}} s \mathcal{N}\left(-d_2\left(T-t,\frac{y}{s}\right)\right) - s,
$$

where $\mathcal{N}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^w e^{-\frac{1}{2}\eta^2} d\eta$ and $d_{1,2}(T-t,s)$ are the functions defined by

$$
d_{1,2}(T-t,\varsigma) = \frac{1}{\sigma\sqrt{T-t}}\left(\ln \varsigma + \left(r\pm\frac{1}{2}\sigma^2\right)(T-t)\right),
$$

respectively.

Proof. First, to solve the P.D.E of (15), we use the following relation

$$
q(t,z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \hat{q}(t,\omega) z^{-\omega} d\omega,
\tag{19}
$$

where $\hat{q}(t,\omega)$ is the Mellin transform of $q(t,z)$.

Then, the P.D.E of (15) yields

$$
\frac{d\hat{q}}{dt} + \left(\frac{\sigma^2}{2}\left(\omega^2+\omega\right) - r\omega - r\right)\hat{q} = 0,
\tag{20}
$$

which has the general solution

$$
\hat{q}(t,\omega) = \hat{\theta}(\omega) e^{-\frac{1}{2}\sigma^2\left\{\omega^2+(1-k_1)\omega-k_1\right\}t},
\tag{21}
$$

where $\hat{\theta}(\omega)$ is the Mellin transform of $\theta(z) = q(T,z)$ and $k_1 = \frac{2r}{\sigma^2}$. Hence, the Mellin inverse of (19) gives

$$
q(t,z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \hat{\theta}(\omega) e^{-\frac{1}{2}\sigma^2\left\{\omega^2+(1-k_1)\omega-k_1\right\}(T-t)} z^{-\omega} d\omega.
\tag{22}
$$

In (22), if we define $\gamma(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{-\frac{1}{2}\sigma^2\left\{\omega^2+(1-k_1)\omega-k_1\right\}(T-t)}z^{-\omega}d\omega$, then it satisfies $\gamma(z) = e^{-k_2\left(\frac{k_1+1}{2}\right)^2}\frac{1}{2\pi i}\frac{z^{k_3}}{\sigma\sqrt{2\pi(T-t)}}e^{-\frac{1}{2}\left(\frac{\log z}{\sigma\sqrt{T-t}}\right)^2}$, where $k_2 = \frac{\sigma^2}{2}(T-t)$ and $k_3 = \frac{1-k_1}{2}$. (Refer to [9])

Lemma 3.2. *Relation to multiplicative convolution* *Let A and B be function from $\mathbb{R}_+^n$ into $\mathbb{C}$. Let $\hat{A}(w)$ be the Mellin transform of $A(x)$ and $\hat{B}(w)$ be the Mellin transform of $B(x)$ given by $\hat{\bullet}(w) = \int_0^\infty \bullet(x)x^{w-1}dx$.*
Then, the Mellin convolution of A and B is given by the inverse Mellin transform of $\hat{A}(w)\hat{B}(w)$ as follows

$$A(x) * B(x) = M_w^{-1}\left[\hat{A}(w)\hat{B}(w); x\right] = \int_0^\infty u^{-1}A\left(\frac{x}{u}\right)B(u)du,$$

where $A(x)*B(x)$ is the symbol of the Mellin convolution of A and B and M_w^{-1} is the symbol of the inverse Mellin transform.

In (17), if we define $(1-z)\mathbf{1}_{\{z<1\}}$, $-\frac{1}{k_1}\mathbf{1}_{\{z>1\}}\left(z^{2k_3}-1\right)$ and $\frac{1}{k_1}\mathbf{1}_{\{z>1\}}(z-1)$ as $h_1(z)$, $h_2(z)$ and $h_3(z)$, respectively, then $\theta(z) = h_1(z)+h_2(z)+h_3(z)$. Also, since $\hat{\theta}(\omega)$ is the Mellin transform of $\theta(z)$ and $e^{-\frac{1}{2}\sigma^2\left\{\omega^2+(1-k_1)\omega-k_1\right\}(T-t)}$ is the Mellin transform of $\gamma(z)$, $q(t,z)$ in (22) leads to the following formula by using the relation of the Mellin convolution mentioned in Lemma 1 :

$$
\begin{aligned}
q(t,z) &= \int_0^\infty \theta(p)\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= \int_0^\infty (h_1(p)+h_2(p)+h_3(p))\,\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= \int_0^\infty h_1(p)\gamma(\frac{z}{p})\frac{1}{p}dp + \int_0^\infty h_2(p)\gamma(\frac{z}{p})\frac{1}{p}dp - \int_0^\infty h_3(p)\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= q_1(t,z) + q_2(t,z) + q_3(t,z).
\end{aligned}
\tag{23}
$$

Hence, by transforming variables, they satisfy

$$
\begin{aligned}
q_1(t,z) &= \int_0^\infty h_1(p)\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= \frac{e^{-k_2\left(\frac{k_1+1}{2}\right)^2}}{\sigma\sqrt{2\pi(T-t)}}\int_0^1 \frac{z^{k_3}}{p^{k_3+1}}(1-p)e^{-\frac{1}{2}\left(\frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)^2}dp, \\
&= e^{-r(T-t)}\mathcal{N}(-d_2(T-t,z)) - z\mathcal{N}(-d_1(T-t,z)), \\
&\left(\text{by using } z^* = \frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)
\end{aligned}
\tag{24}
$$

where $d_{1,2}(T - t, z) = \frac{1}{\sigma\sqrt{T-t}} \left[\log z + \left(r \pm \frac{\sigma^2}{2} \right)(T - t) \right],$

$$
\begin{aligned}
q_2(t, z) &= \int_0^\infty h_2(p)\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= -\frac{1}{k_1}\frac{e^{-k_2\left(\frac{k_1+1}{2}\right)^2}}{\sigma\sqrt{2\pi(T-t)}} \int_1^\infty \frac{z^{k_3}}{p^{k_3+1}}(p^{2k_3} - 1)e^{-\frac{1}{2}\left(\frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)^2} dp, \\
&= -\frac{1}{k_1}\frac{e^{-k_2 k_1}}{\sigma\sqrt{2\pi(T-t)}}z^{2k_3} \int_1^\infty \left(\frac{z}{p}\right)^{-k_3}\frac{1}{p}e^{-\frac{\left(\log\frac{z}{p}\right)^2 + k_2^2(k_1-1)^2}{4k_2}} dp \\
&\quad + \frac{1}{k_1}\frac{e^{-k_2\left(\frac{k_1+1}{2}\right)^2}}{\sigma\sqrt{2\pi(T-t)}} \int_1^\infty \frac{z^{k_3}}{p^{k_3+1}}e^{-\frac{1}{2}\left(\frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)^2} dp, \\
&= -\frac{1}{k_1}\frac{e^{-r(T-t)}}{\sigma\sqrt{2\pi(T-t)}}z^{2k_3} \int_1^\infty \frac{1}{p}e^{-\frac{1}{2}\left(\frac{\log\left(\frac{z}{p}\right)+2k_2k_3}{\sqrt{2k_2}}\right)^2} dp \\
&\quad + \frac{1}{k_1}\frac{e^{-k_2\left(\frac{k_1+1}{2}\right)^2}}{\sigma\sqrt{2\pi(T-t)}} \int_1^\infty \frac{z^{k_3}}{p^{k_3+1}}e^{-\frac{1}{2}\left(\frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)^2} dp, \\
&= -\frac{1}{k_1}z^{2k_3}e^{-r(T-t)}\mathcal{N}\left(-d_2\left(T-t, \frac{1}{z}\right)\right) + \frac{1}{k_1}e^{-r(T-t)}\mathcal{N}(d_2(T-t, z)), \\
&\quad \left(\text{by using } \hat{z} = \frac{\log\left(\frac{z}{p}\right)+2k_2k_3}{\sqrt{2k_2}}, \text{ and } z^* = \frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}} \right)
\end{aligned}
\tag{25}
$$

and

$$
\begin{aligned}
q_3(t, z) &= \int_0^\infty h_3(p)\gamma(\frac{z}{p})\frac{1}{p}dp, \\
&= \frac{1}{k_1}\frac{e^{-k_2\left(\frac{k_1+1}{2}\right)^2}}{\sigma\sqrt{2\pi(T-t)}} \int_1^\infty \frac{z^{k_3}}{p^{k_3+1}}(p - 1)e^{-\frac{1}{2}\left(\frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}}\right)^2} dp, \\
&= \frac{1}{k_1}z\mathcal{N}(d_1(T-t, z)) - \frac{1}{k_1}e^{-r(T-t)}\mathcal{N}(d_2(T-t, z)). \\
&\quad \left(\text{by using } z^* = \frac{\log(\frac{z}{p})}{\sigma\sqrt{T-t}} \right)
\end{aligned}
\tag{26}
$$

Finally, by combining (23), (24), (25) and (26), it leads to the following equation

$$
\begin{aligned}
q(t,z) &= e^{-r(T-t)}\mathcal{N}(-d_2(T-t,z)) - z\left(1 - \mathcal{N}(d_1(T-t,z))\right) \\
&\quad - \frac{\sigma^2}{2r}z^{2k_3}e^{-r(T-t)}\mathcal{N}(-d_2(T-z,z^{-1})) + \frac{\sigma^2}{2r}e^{-r(T-t)}\mathcal{N}(d_2(T-t,z)) \\
&\quad + \frac{\sigma^2}{2r}z\mathcal{N}(d_1(T-t,z)) - \frac{\sigma^2}{2r}e^{-r(T-t)}\mathcal{N}(d_2(T-t,z)) \\
&= \left(1 + \frac{\sigma^2}{2r}\right)z\mathcal{N}(d_1(T-t,z)) + e^{-r(T-t)}\mathcal{N}(-d_2(T-t,z)) \\
&\quad - \frac{\sigma^2}{2r}e^{-r(T-t)}z^{1-\frac{2r}{\sigma^2}}\mathcal{N}(-d_2(T-t,z^{-1})) - z. \quad\quad (27)
\end{aligned}
$$

By using $z = \frac{s}{y}$ and (14), we obtain the closed formula of the floating strike lookback put option as follows :

$$
\begin{aligned}
v(t,s,y) &= \left(1 + \frac{\sigma^2}{2r}\right)s\mathcal{N}\left(d_1\left(T-t,\frac{s}{y}\right)\right) + e^{-r(T-t)}y\mathcal{N}\left(-d_2\left(T-t,\frac{s}{y}\right)\right) \\
&\quad - \frac{\sigma^2}{2r}e^{-r(T-t)}\left(\frac{y}{s}\right)^{\frac{2r}{\sigma^2}}s\mathcal{N}\left(-d_2\left(T-t,\frac{y}{s}\right)\right) - s.
\end{aligned}
$$

$$(28)$$

The proof is completed. $\qquad\square$

4. Conclusion

In this paper, we have demonstrated that a closed form solution for the European floating lookback option can be derived by taking advantage of the method of images and Mellin transform approaches. The Mellin transform methods help us resolve the complexity of the calculation in comparison to the probabilistic techniques, Fourier transforms and the method of change of variables in other types of options as well as the lookback options. Finally, studies of the Mellin transforms for many other derivatives are also currently underway.

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